Computational Fluid Dynamics and Heat Transfer

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Lecture - 26

Introduction to Mathematical Approaches to Turbulent Flows

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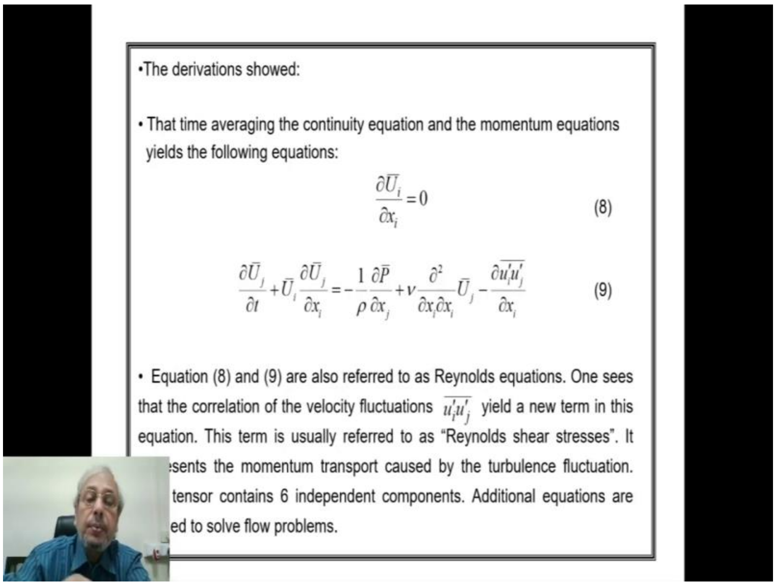

The derivations showed:

That time averaging the continuity equation and the momentum equations yields the following equations:

$$\frac{\partial \bar{U}_i}{\partial x_i} = 0 \qquad (8)$$

$$\frac{\partial \bar{U}_i}{\partial t} + \bar{U}_i \frac{\partial \bar{U}_j}{\partial x_i} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial x_j} + \nu \frac{\partial^2}{\partial x_i \partial x_j}\bar{U}_j - \frac{\partial \overline{u_i' u_j'}}{\partial x_i} \qquad (9)$$

Equation (8) and (9) are also referred to as Reynolds equations. One sees that the correlation of the velocity fluctuations $\overline{u_i' u_j'}$ yield a new term in this equation. This term is usually referred to as "Reynolds shear stresses". It presents the momentum transport caused by the turbulence fluctuation. The tensor contains 6 independent components. Additional equations are showed to solve flow problems.

Good morning everybody, today we will continue with Mathematical Approaches to Turbulent Flows. If you recall, what we did in the last class, we derived and established a paradigm, where the governing equations are time averaged, velocity based, continuity equation given by equation 8.

And again the 3 momentum equations, based on time average velocities, in 3 orthogonal directions and time averaged pressure. But, in this equation we get additional term which are basically, and these are you know terms actually we will get with each momentum equation, 3 terms from here. So, 3 into 3, 9 terms and these are basically called together they are called Reynolds stress tensor.

And the fluctuating components like u i prime u j prime over bar, that is the source of creation of the such terms. As I have already mentioned, for example, with x momentum equation it will be u 1 prime square bar. u 1 prime u 2 prime over bar, u 1 prime into u 3 prime over bar.

So, 3 quantities or you can say that, if you define them by u v w then u prime square bar u prime v prime over bar u prime w prime over bar. So like that you get with 1 momentum equation, with z momentum equation this contribution of the Reynolds stress. Now, that means, if you look into it, there are 6 additional unknowns in this tensor quantity I mean.

Whereas, the velocity is in 3 directions; time average velocity and time average pressure are basically to be solved from 4 equations. So, this adds 6 unknowns in this equation.

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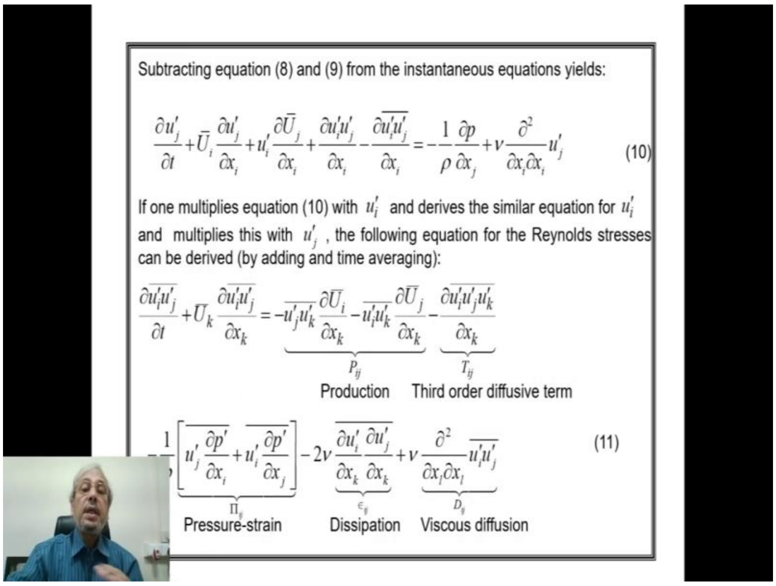

Subtracting equation (8) and (9) from the instantaneous equations yield:

$$\frac{\partial u_j'}{\partial t} + \bar{U}_i \frac{\partial u_j'}{\partial x_i} + u_i' \frac{\partial \bar{U}_j}{\partial x_i} + \frac{\partial u_i' u_j'}{\partial x_i} - \frac{\partial \overline{u_i' u_j'}}{\partial x_i} = -\frac{1}{\rho}\frac{\partial p}{\partial x_j} + \nu \frac{\partial^2}{\partial x_i \partial x_j} u_j' \qquad (10)$$

If one multiplies equation (10) with u_i' and derives the similar equation for u_i' and multiplies this with u_j', the following equation for the Reynolds stresses can be derived (by adding and time averaging):

$$\frac{\partial \overline{u_i' u_j'}}{\partial t} + \bar{U}_k \frac{\partial \overline{u_i' u_j'}}{\partial x_k} = \underbrace{-\overline{u_j' u_k'}\frac{\partial \bar{U}_i}{\partial x_k} - \overline{u_i' u_k'}\frac{\partial \bar{U}_j}{\partial x_k}}_{P_{ij}} - \underbrace{\frac{\partial \overline{u_i' u_j' u_k'}}{\partial x_k}}_{T_{ij}}$$

Production Third order diffusive term

$$-\underbrace{\frac{1}{\rho}\left[\overline{u_j' \frac{\partial p'}{\partial x_i}} + \overline{u_i' \frac{\partial p'}{\partial x_j}}\right]}_{\pi_{ij}} - \underbrace{2\nu \overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}}}_{\varepsilon_{ij}} + \underbrace{\nu \frac{\partial^2}{\partial x_i \partial x_j}\overline{u_i' u_j'}}_{D_{ij}} \qquad (11)$$

Pressure-strain Dissipation Viscous diffusion

Now, let us see how we can handle the Reynolds stress. See, it is probably not in congruous if we mention the creation of this Reynolds stress tensor mathematically, how we can reproduce that. See, which these are again go back, 8 and 9 these are time averaged equation. If we subtract time averaged momentum equation from instantaneous momentum equation.

Now, instantaneous equation, any velocity quantity for example, upper case U i is given by u i bar plus u i prime or upper case U j is instantaneous is U j upper case over bar mean quantity plus u j prime is fluctuating component. So, if we subtract the instantaneous the time mean quantity from the tau time mean momentum equations, from the momentum equations which are based on instantaneous velocities, we will get this equation in fluctuating component.

Which is del del t of u j prime plus U i bar into del del x i of u j prime plus u prime del del x i of U j bar plus del del x i of u i prime u j prime minus del del x i of u i prime into u j prime over bar, right hand side del del p of del p del x j plus nu del 2 del x i del x i into u j prime. So, if we multiply this equation, equation 10 with u i prime and you know just store it and then we derive a similar equation for u i prime.

This is equation for u j prime and we are multiplying it with u i prime, then we develop a equation in u i prime and multiply that equation with u j prime. And then; that means, we get equations in u i prime u j prime, actually after multiplication, then we add them and perform the time averaging. If we do that, we will get this these terms.

Now, in equation 10 there is one small typographical mistake, please read this term pressure term as del del x j of p prime, because instantaneous equation p is p bar plus p prime an average equation p is p bar. So, after subtracting this will be del del x j of p prime. Somehow, you know this has been a typographical mistake.

So, what I was referring to, that after performing these operations as I have explained, we get basically, governing equation for Reynolds stresses. I hope it is clear now. It is equation for u j prime we multiply this with u i prime. Similar way we derive an equation for u i prime and multiply that equation with u j prime, add these two equations and do the time averaging. So then, basically we will get this complete equation 11.

You can see this is del del t of u i prime u j prime over bar Reynolds stress. This is convective term of Reynolds stress convection of Reynolds stress, U k bar into del del x k of u i prime u j prime over bar.

Right hand side, this is production term, this is third order diffusive term, this is these two are together pressure strain term, this is dissipation term, and this is viscous diffusion term. This is equation 11.

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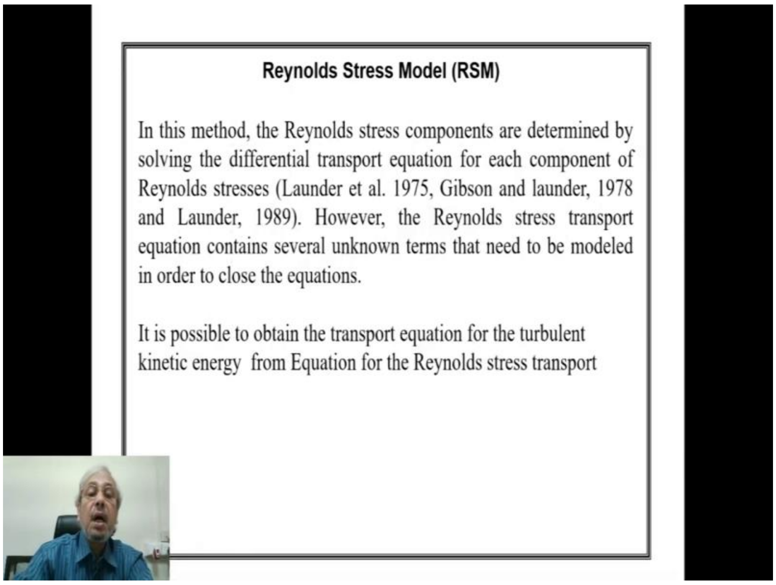

Reynolds Stress Model (RSM)

In this method, the Reynolds stress components are determined by solving the differential transport equation of each component of Reynolds stresses (Launder et al. 1975, Gibson and launder, 1978 and Launder 1989). However, the Reynolds stress transport equation contains several unknown terms that need to be modelled in order to close the equations.

It is possible to obtain the transport equation for the turbulent kinetic energy from Equation for the Reynolds stress transport

Now, this equation 11 is Reynolds stress transport equation. This equation can also be modeled directly, this equation can also be model it is called Reynolds stress model. In this model, Reynolds stress components are determined by solving the differential transport equation for each component of Reynolds stresses.

And, this has been developed technique has been developed and very nicely explain in these three papers, one is Launder et al, probably it is Launder Reece and Rodi in 1990 1975, this is a journal of fluid mechanics paper, then Gibson and Launder and Launder 1989.

However, Reynolds stress transport equation contains several unknown terms that need to be modeled in order to close the equations. So, while discussing we have discussed about discussing about mathematical approaches. We have discussed about large dissimulation LDS, we have discussed about direct numeric simulation DNS, and we have also mentioned about Reynolds stress model.

But, all these techniques that we have mentioned so far, they are very involved. We need very large computational power and the detailed calculation procedure are to be discussed and taken up. But we are heading towards slightly less involved and easier approach for the time being. Now, it is possible to obtain the transport equation for turbulent kinetic energy from equation for the Reynolds stress transport, by doing a or bringing about if slight change. What is that?

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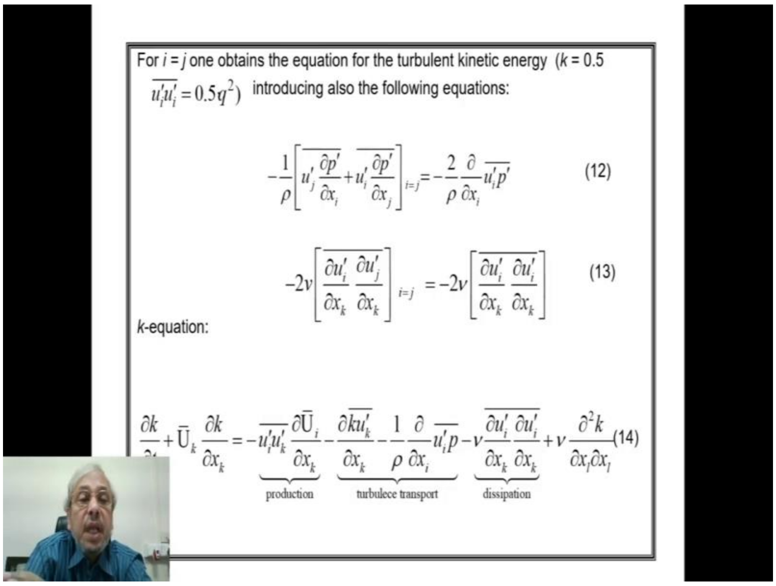

For $i = j$ one obtains the equation for the turbulent kinetic energy $\left(k = 0.5\overline{u_i'u_i'}q^2 = 0.5q^2\right)$ introducing also the following equations:

$$-\frac{1}{\rho}\left[\overline{u_j^{'}\frac{\partial p^{'}}{\partial x_i}}+\overline{u_i^{'}\frac{\partial p^{'}}{\partial x_j}}\right]_{i=j}=-\frac{2}{\rho}\frac{\partial}{\partial x_i}\overline{u_i^{'}p^{'}} \tag{12}$$

$$-2\nu\overline{\left[\frac{\partial u_i^{'}}{\partial x_k}\frac{\partial u_j^{'}}{\partial x_k}\right]}_{i=j}=-2\nu\overline{\left[\frac{\partial u_i^{'}}{\partial x_k}\frac{\partial u_i^{'}}{\partial x_k}\right]} \tag{13}$$

k-equation:

$$\underbrace{-\frac{1}{\rho}\left[\overline{u_j^{'}\frac{\partial p^{'}}{\partial x_i}}+\overline{u_i^{'}\frac{\partial p^{'}}{\partial x_j}}\right]}_{\pi_{ij}}-2\nu\underbrace{\overline{\frac{\partial u_i^{'}}{\partial x_k}\frac{\partial u_j^{'}}{\partial x_k}}}_{\varepsilon_{ij}}+\nu\underbrace{\frac{\partial^2}{\partial x_i \partial x_j}\overline{u_i^{'}u_j^{'}}}_{D_{ij}}$$

Pressure-strain Dissipation Viscous diffusion

$$\frac{\partial k}{\partial t}+\bar{U}_k\frac{\partial k}{\partial x_i}=\underbrace{-\overline{u_i^{'}u_k^{'}}\frac{\partial \bar{U}_i}{\partial x_k}}_{\text{production}}\underbrace{-\frac{\partial \overline{ku_k^{'}}}{\partial x_k}-\frac{1}{\rho}\frac{\partial}{\partial x_i}\overline{u_i^{'}p^{'}}}_{\text{turbulence transport}}\underbrace{-\nu\overline{\frac{\partial u_i^{'}}{\partial x_k}\frac{\partial u_i^{'}}{\partial x_k}}}_{\text{dissipation}}+\nu\frac{\partial^2 k}{\partial x_l \partial x_l} \tag{14}$$

For, if we substitute in governing equation for Reynolds stress transport, i equal to j one obtains the equation for turbulent kinetic energy, k is basically half u i prime u i prime over bar, this is kinetic energy half u square. So u, I mean fluctuating a component of velocity for turbulent kinetic energy. So, u i prime into u i prime over bar, you can call it 0.5 q square.

Now, if we do that the major changes, we get in the pressure strain correlation it becomes 2 bar minus 2 by rho del del x i of u i prime into p prime over bar and also the dissipation term which is in the previous equation. We have seen this is the dissipation term here, substituting i equal to j we get, minus 2 nu into del del x k of u i prime multiplied by del del x k of u i prime over bar. This is basically dissipation of turbulent kinetic energy.

So, from Reynolds stress transport equation we can derive governing equation for turbulent kinetic energy. We can call this quantity as k u i prime into u i prime over bar by 2. So, that is we can write del del t of k plus U k bar average velocity into del k del x k equal to the terms which we get on the right hand side are from Reynolds stress transport equation.

We if we analyze we will be able to say this term is production term and these two terms are turbulent transport term. Again, p prime will come here. So, equation 12 from here it comes, and this is dissipation because we are taking k which is u i prime into u i prime over bar by 2. So, these 2 is absorbed. So, nu this quantity which is dissipation plus nu this is diffusion of turbulent kinetic energy equation 14.

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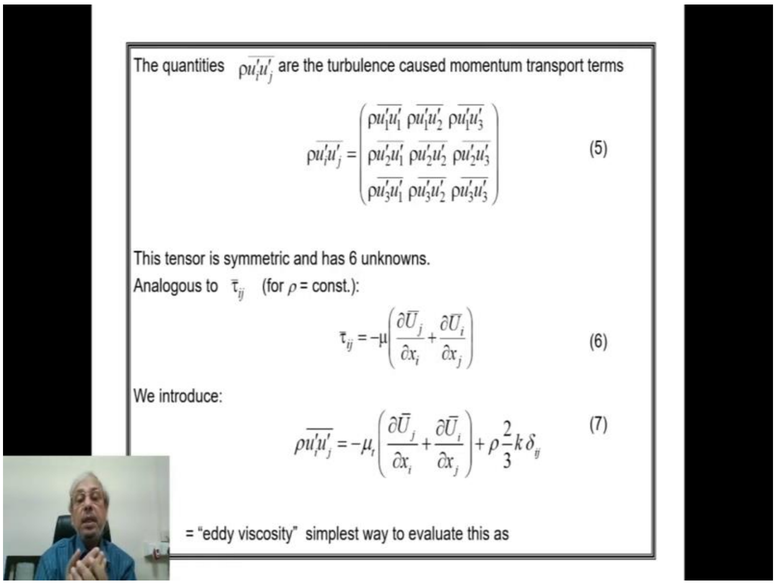

The quantities $\rho\overline{u_i^{'} u_j^{'}}$ are the turbulence caused momentum transport terms

$$\rho\overline{u_i^{'} u_j^{'}} = \begin{pmatrix} \rho\overline{u_1^{'} u_1^{'}} & \rho\overline{u_1^{'} u_2^{'}} & \rho\overline{u_1^{'} u_3^{'}} \\ \rho\overline{u_2^{'} u_1^{'}} & \rho\overline{u_2^{'} u_2^{'}} & \rho\overline{u_2^{'} u_3^{'}} \\ \rho\overline{u_3^{'} u_1^{'}} & \rho\overline{u_3^{'} u_2^{'}} & \rho\overline{u_3^{'} u_3^{'}} \end{pmatrix} \tag{5}$$

This tensor is symmetric and has 6 unknowns.
Analogous to $\bar{\tau}_{ij}$ (for $(\rho = \text{const})$:

$$\bar{\tau}_{ij} = -\mu\left(\frac{\partial \overline{U}_j}{\partial x_i} + \frac{\partial \overline{U}_i}{\partial x_j}\right) \tag{6}$$

$$\rho\overline{u_i'u_j'} = -\mu_t\left(\frac{\partial \overline{U}_j}{\partial x_i}+\frac{\partial \overline{U}_i}{\partial x_j}\right)+\rho\frac{2}{3}k\delta_{ij} \tag{7}$$

μ_t ="eddy viscosity" simplest way to evaluate this as

Now, we will you know see how we can utilize this k to close the time averaged governing equations. We will take eddy course 2 few earlier one or two earlier slides. If you recall that we have basically, rho u i prime u j prime over bar is a tensor quantity, if i equal to 1 j equal to 1, 2, 3 i equal to 2 j equal to 1, 2, 3 i equal to 3 j equal to 1, 2, 3, we get this tensor. This tensor is symmetric and having 6 unknown, because it is symmetric 1, 2, 3, 4, 5, 6, this 6 unknowns.

And, these are analogue us to basically the shear stress which arises out of molecular viscosity. So, we can introduce, we discussed it in the last class that this rho u i prime u j prime over bar, we can write mu t into shear stress del del x i of U j mean and del del x j of U i mean plus these terms which is which are comprising of kinetic energy and chronicle delta, and this mu t is the eddy viscosity. This is culmination of interaction between the fluctuating components.

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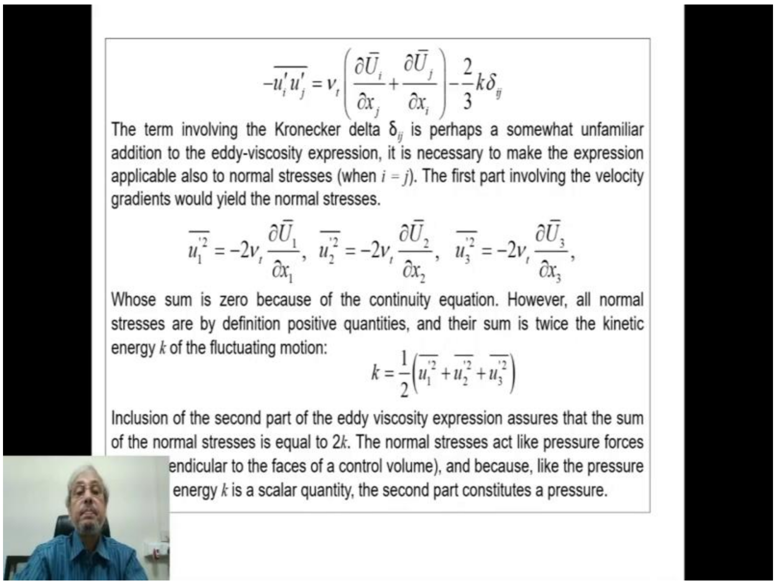

$$-\overline{u_i'u_j'} = \nu_t\left(\frac{\partial \overline{U}_i}{\partial x_j}+\frac{\partial \overline{U}_j}{\partial x_i}\right)-\frac{2}{3}k\delta_{ij}$$

The term involving the Kronecker delta δ_{ij} is perhaps a somewhat unfamiliar addition to the eddy-viscosity expression, it is necessary to make the expression applicable also to normal stresses (when $i = j$). the first part involving the velocity gradients would yield the normal stresses.

$$\overline{u_1'^2} = -2\nu_t \frac{\partial \overline{U}_1}{\partial x_1},\ \overline{u_2'^2} = -2\nu_t \frac{\partial \overline{U}_2}{\partial x_2},\ \overline{u_3'^2} = -2\nu_t \frac{\partial \overline{U}_3}{\partial x_3},$$

Whose sum is zero because of the continuity equation. However, all normal stresses are by definition positive quantities, and their sum is twice the kinetic energy k of the fluctuating motion:

$$k = \frac{1}{2}\left(\overline{u_1'^2} + \overline{u_2'^2} + \overline{u_3'^2}\right)$$

Inclusion of the second part of the eddy viscosity expression assures that the sum of the normal stresses is equal to 2*k*. the normal stresses act like pressure forces perpendicular to the faces of a control volume), and because, like the pressure, the kinetic energy *k* is a scalar quantity, the second part consumes a pressure.

And then I will also use these slide which I used in the last class. The same thing we have explained; u i prime u j prime over bar this is equal to nu t into the shear stress expression a nu t into the shear rate expression into the nu t it is basically shear minus two-third k del i j chronicle delta.

And purpose of adding this has been explained, so that the when we add normal stresses, we get a non zero value, which is nothing, but turbulent two times turbulent kinetic energy. Adding the normal stresses u i prime square bar plus u 1 prime square bar plus u 2 prime square bar plus u 3 prime square bar should give me 2 times k 2 times turbulent kinetic energy.

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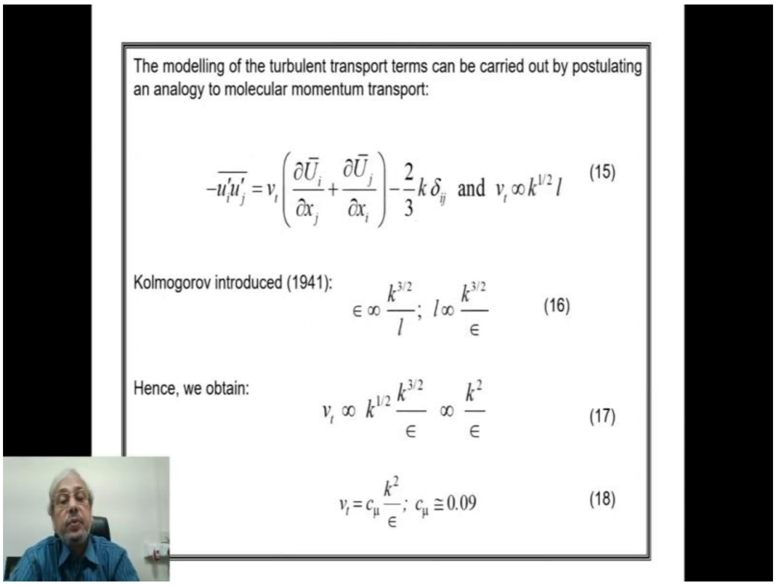

The modelling of the turbulent transport terms can be carried out by postulating an analogy to molecular momentum transport:

$$-\overline{u_i^{'}u_j^{'}} = \nu_t\left(\frac{\partial \overline{U}_i}{\partial x_j} + \frac{\partial \overline{U}_j}{\partial x_i}\right) - \frac{2}{3}k\delta_{ij} \quad \text{and} \quad \nu_t \infty k^{1/2} l \tag{15}$$

Kolmogorov introduced (1941):

$$\varepsilon \infty \frac{k^{1/2}}{l} \quad ; \; l \infty \frac{k^{1/2}}{\varepsilon} \tag{16}$$

Hence, we obtain:

$$\nu_t \infty k^{1/2}\frac{k^{3/2}}{\varepsilon} \infty \frac{k^2}{\varepsilon} \tag{17}$$

$$\nu_t \infty c_\mu \frac{k^2}{\varepsilon} \qquad c_\mu = 0.09 \tag{18}$$

So, this relationship we will try to make use of. So, we will accept this minus u i prime into u j prime over bar is nu t into the shear rate terms which is del del x j into U i mean del del x i into U j mean minus two-third k delta i j, and nu t is unknown. If nu t is known then we can really substitute this relationship in the Reynolds stress relationship of time average momentum equations.

But nu t is not known, and it was postulated that nu t is a function of k to the power half into l. many people try to now work on it you know, Bosnisk, then Kolmogorov, and it was suggested that this molecular viscosity I mean give some guidance to determine turbulent viscosity. How?

Molecular viscosity of the gas is are known that so function of molecular mean free path and the Brandon velocity which is root mean square velocity. So here, it is thought or Kolmogorov, finally suggested that turbulent viscosity is a function of turbulent velocity scale which is given by k to the power half that is you know substituting the RMS velocity in the molecular theory of gases.

And, then l turbulent length scale substituting basically the mean free path of collision that is a responsible for turbulent responsible for the molecular viscosity. And, we are substituting that mean free path when we are determining turbulent kinetic energy by turbulent lane scale.

And, Kolmogorov did another additional very important contribution, that this lane scale is link to dissipation right. So, lane scale varies at as k to the power 3 by 2 by epsilon. So, epsilon varies as k to the power 3 by 2 by l. So, turbulent lane scale varies as k to the power 3 by 2 divided by epsilon.

So, turbulent viscosity is a function of k to the power half, which is representative velocity scale, and l which is representative lane scale given by k to the power 3 by 2 divided by epsilon; that means, nu t varies as k to the power half into k to the power 3 by 2 divided by epsilon or nu t varies as k square by epsilon.

And then this can be I mean proportionality relationship can be converted into equality relationship, if we introduce a proportionality constant which is for such models given by c mu. And it was you know postulated then that was supported by accompanying experiments and it was found that c mu equal to 0.09.

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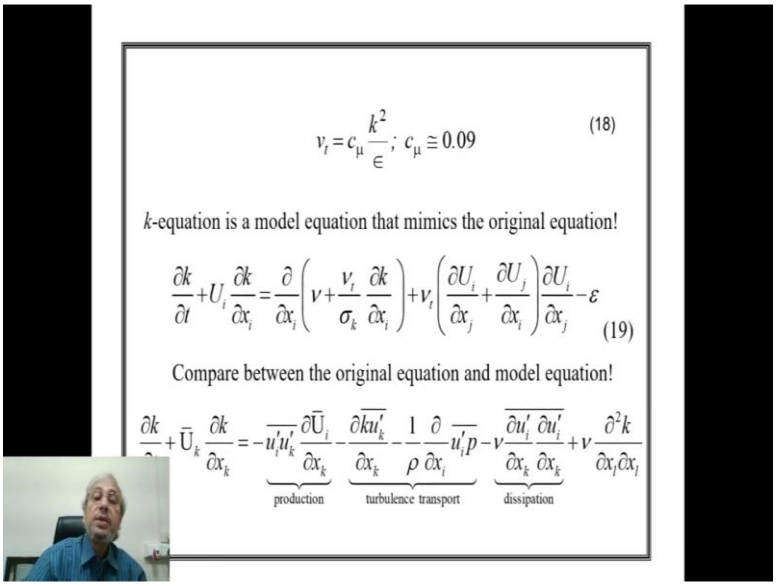

$$\nu_t \infty c_\mu \frac{k^2}{\varepsilon};\; c_\mu = 0.09 \tag{18}$$

k-equation is a model equation that mimics the original equation!

$$\frac{\partial k}{\partial t} + U_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\nu + \frac{\nu_t}{\sigma}\frac{\partial k}{\partial x_i}\right) + \nu_t \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j} - \varepsilon \tag{19}$$

Compare between the original equation and model equation!

$$\frac{\partial k}{\partial t} + \bar{U}_k \frac{\partial k}{\partial x_i} = \underbrace{-\overline{u_i^{'} u_k^{'}}\frac{\partial \bar{U}_i}{\partial x_k}}_{\text{production}} - \underbrace{\frac{\partial \overline{k u_k^{'}}}{\partial x_k} - \frac{1}{\rho}\frac{\partial}{\partial x_i}\overline{u_i^{'} p}}_{\text{turbulence transport}} - \underbrace{\nu \overline{\frac{\partial u_i^{'}}{\partial x_k}\frac{\partial u_i^{'}}{\partial x_k}}}_{\text{dissipation}} + \nu \frac{\partial^2 k}{\partial x_l \partial x_l}$$

So, nu t equal to c mu k square by epsilon, where c mu is given by 0.09. Together with that, another great contribution or simplification was done. If I am not wrong, Kolmogorov rota, and finally, the contribution came from Professor Brian Launder and Professor DB Spalding.

That, k was a model equation for k was set up. Where, this is the temporal variation of k, this is a convection of k, this is basically transport of k, and this is primarily diffusion transport where molecular viscosity and turbulent viscosity both are there, but we will see later this molecular velocity is much much smaller than turbulent viscosity.

So this can be neglected to, but right now we are retaining it. And this term is equivalent to production term minus dissipation. So, compare between the original equation and the model equation. This is the original equation just we have derived from Reynolds stress transport equation. So, left hand side exactly same, on the right hand side this was our production term.

And here, this production term is given by this term. This was our dissipation term. Dissipation term is coming here. And remaining terms, turbulent transport plus diffusion term due to molecular viscosity, they are clubbed together here. So this is basically, fitting in a model of turbulent kinetic energy transport through comparison with or comparing with the original turbulent kinetic energy transport equation.

While, production term is modeled by this term. Dissipation term is written, viscous diffusion and turbulent transport are clubbed together as this term. So, this k equation is model kinetic energy equation, and now you can see there is a model constant and we will discuss about determination of this model constants slightly later.

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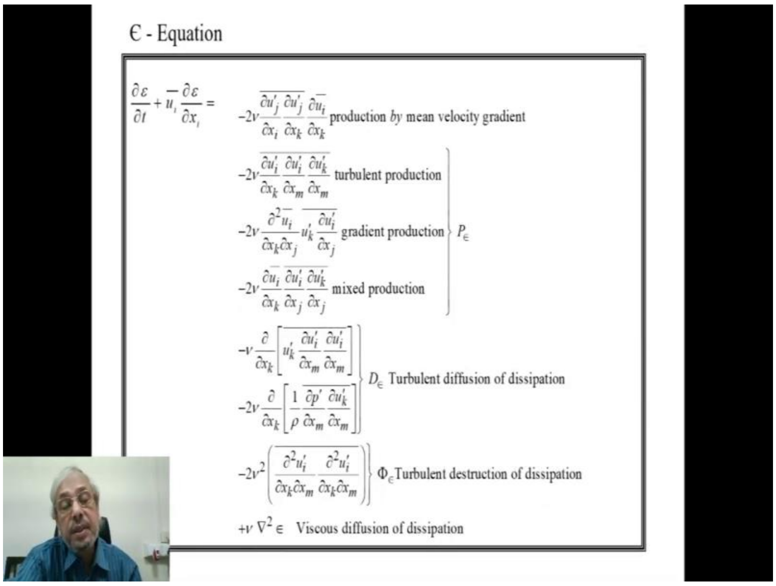

$$\frac{\partial \varepsilon}{\partial t} + \overline{u_i}\frac{\partial \varepsilon}{\partial x_i} = -2\nu \overline{\frac{\partial u_j^{'}}{\partial x_i}\frac{\partial u_j^{'}}{\partial x_k}}\frac{\partial \overline{u_i}}{\partial x_k} \text{ production by mean velocity gradient}$$

$$\left.\begin{aligned} &-2\nu \overline{\frac{\partial u_i^{'}}{\partial x_k}\frac{\partial u_i^{'}}{\partial x_m}\frac{\partial u_k^{'}}{\partial x_m}} \text{ turbulent production} \\ &-2\nu \frac{\partial^2 \overline{u_i}}{\partial x_k \partial x_j} \overline{u_k^{'} \frac{\partial u_i^{'}}{\partial x_j}} \overline{u_k^{'} \frac{\partial u_i^{'}}{\partial x_j}} \text{ gradient production} \\ &-2\nu \frac{\partial \overline{u_i}}{\partial x_k} \overline{\frac{\partial u_i^{'}}{\partial x_j}\frac{\partial u_k^{'}}{\partial x_j}} \text{ mixed production} \end{aligned}\right\} P_\varepsilon$$

$$\left.\begin{aligned} &-\nu \frac{\partial}{\partial x_k}\left[\overline{u_k \frac{\partial u_i^{'}}{\partial x_m}\frac{\partial u_i^{'}}{\partial x_m}}\right] \\ &-2\nu \frac{\partial}{\partial x_k}\left[\frac{1}{\rho}\overline{\frac{\partial p^{'}}{\partial x_m}\frac{\partial u_k^{'}}{\partial x_m}}\right] \end{aligned}\right\} D_\varepsilon \text{ Turbulent diffusion of dissipation}$$

$$\left.-2\nu^2 \left(\overline{\frac{\partial^2 u_i^{'}}{\partial x_k \partial x_m}\frac{\partial^2 u_i^{'}}{\partial x_k \partial x_m}}\right)\right\} \phi_\varepsilon \text{ Turbulent destruction of dissipation}$$

$$+\nu \nabla^2 \varepsilon \text{ Viscous diffusion of dissipation}$$

Now comes epsilon equation. You can see this is the full form of epsilon equation, which is really horrendous in terms of its complexity. You can see again, temporal derivative of epsilon dissipation of turbulent kinetic energy, convection of epsilon, convection of dissipation of turbulent kinetic energy.

And, right hand side we have production by mean velocity gradient, turbulent production of dissipation, gradient production of dissipation, mixed production of dissipation. This is another three production of dissipation. Turbulent diffusion of dissipation, these two terms. Turbulent distraction of dissipation, this term and viscous diffusion of dissipation.

Now, if you look into complexity of this equation, seemingly it is impossible to touch it with respect to the involvement the fluctuating components and mean components in a very complex manner.

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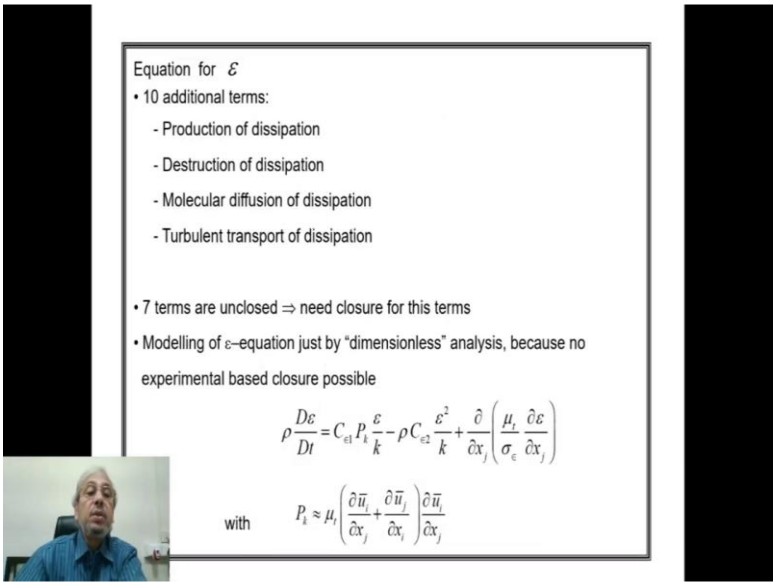

•10 additional terms:

-Production of dissipation

-Destruction of dissipation

-Molecular diffusion of dissipation

-Turbulent transport of dissipation

- 7 terms are unclosed ⇒ need closure for this terms
- Modelling of ε-equation just by "dimensionless" analysis, because no experimental based closure possible

$$\rho \frac{D\varepsilon}{Dt} = C_{\varepsilon 1} P_k \frac{\varepsilon}{k} - \rho C_{\varepsilon 2} \frac{\varepsilon^2}{k} + \frac{\partial}{\partial x_i}\left(\frac{\mu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x_j} \right)$$

$$\text{with } P_k \approx \mu_t \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \frac{\partial \bar{u}_i}{\partial x_j}$$

So, equation for epsilon, we have written 10 additional terms among them, production of dissipation, distraction of dissipation, molecule diffusion of dissipation, turbulent transport of dissipation, seemingly handling this term is impossible, as I said. But this group of scientists as I mentioning, thus started with Kolmogorov in Soviet Union.

Some work were done some work was done by Professor Rota in Germany and huge amount of work was done by Professor Spalding, Professor Launder, Professor Rodi, in

Imperial College. And then, this equation was developed. This is again a model equation. This is temporal plus convective transport of epsilon.

Here, production all the production terms have been modeled in a very novel way this, P k production of turbulent kinetic energy that has been taken. We have used this in turbulent kinetic energy production. It has been multiplied by epsilon divided by k and multiplied by a module constant to describe production of on all the production.

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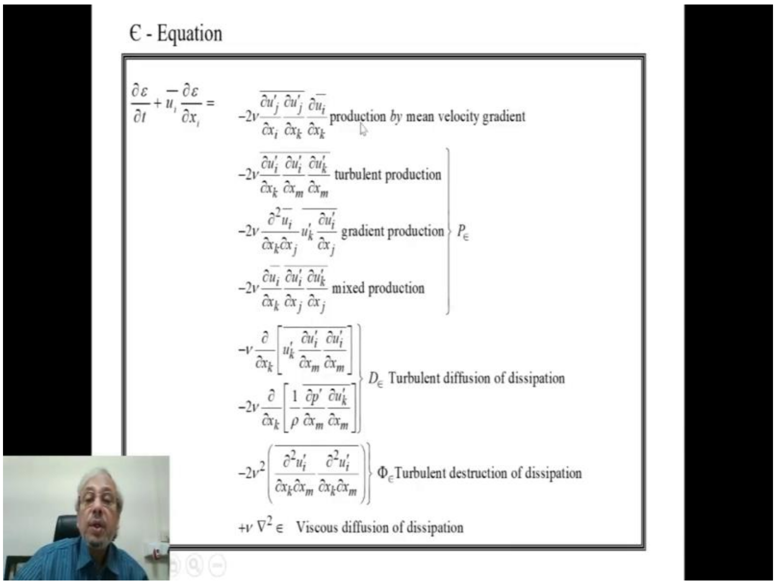

$$\frac{\partial \varepsilon}{\partial t} + \overline{u}_i \frac{\partial \varepsilon}{\partial x_i} = -2\nu \overline{\frac{\partial u_j^{'}}{\partial x_i} \frac{\partial u_j^{'}}{\partial x_k}} \frac{\partial \overline{u_i}}{\partial x_k} \text{production by mean velocity gradient}$$

$$\left.\begin{aligned} &-2\nu \overline{\frac{\partial u_i^{'}}{\partial x_k} \frac{\partial u_i^{'}}{\partial x_m} \frac{\partial u_k^{'}}{\partial x_m}} \text{turbulent production} \\ &-2\nu \frac{\partial^2 \overline{u_i}}{\partial x_k \partial x_j} \overline{u_k^{'} \frac{\partial u_i^{'}}{\partial x_j}} \overline{u_k^{'} \frac{\partial u_i^{'}}{\partial x_j}} \text{gradient production} \\ &-2\nu \frac{\partial \overline{u_i}}{\partial x_k} \overline{\frac{\partial u_i^{'}}{\partial x_j} \frac{\partial u_k^{'}}{\partial x_j}} \text{mixed production} \end{aligned}\right\} P_\varepsilon$$

$$\left.\begin{array}{l} -\nu \dfrac{\partial}{\partial x_k}\left[\overline{u_k \dfrac{\partial u_i'}{\partial x_m}\dfrac{\partial u_i'}{\partial x_m}}\right] \\ -2\nu \dfrac{\partial}{\partial x_k}\left[\dfrac{1}{\rho}\overline{\dfrac{\partial p'}{\partial x_m}\dfrac{\partial u_k'}{\partial x_m}}\right] \end{array}\right\} D_\varepsilon \text{ Turbulent diffusion of dissipation}$$

$$\left.-2\nu^2\left(\overline{\frac{\partial^2 u_i'}{\partial x_k \partial x_m}\frac{\partial^2 u_i'}{\partial x_k \partial x_m}}\right)\right\}\phi_\varepsilon \text{ Turbulent destruction of dissipation}$$

$$+\nu\nabla^2\varepsilon \text{ Viscous diffusion of dissipation}$$

So, you know production this if you look into it all these ways mixed production, gradient production, turbulent production of dissipation. And production by means of mean velocity gradient, it is I would say some kind of genius of them, that to get even an idea that these production terms can be model this way. So, this is kinetic energy production term, multiply it with epsilon divide it to by kinetic energy and model multiply with model constant.

This is production of dissipation. And then, this is very simple. This is diffusion of dissipation you can see. And coefficient of viscosity turbulent viscosity comes and this is distraction of dissipation.

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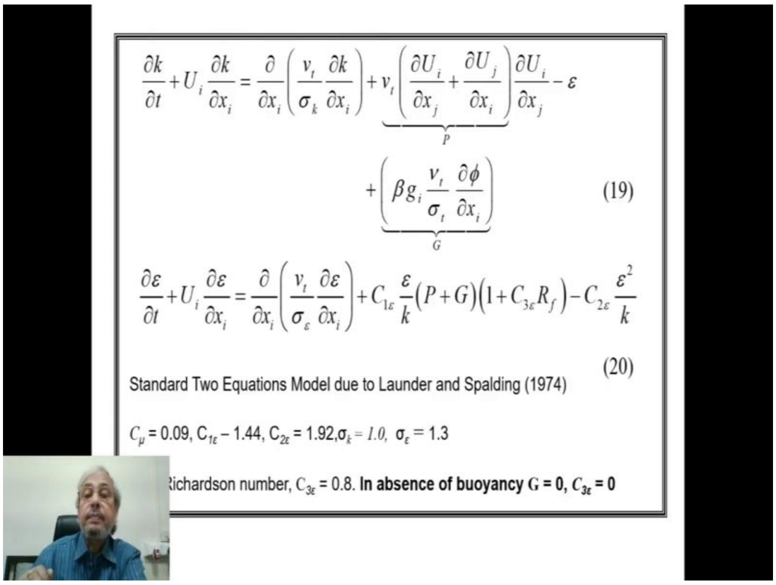

$$\frac{\partial k}{\partial t}+U_i\frac{\partial k}{\partial x_i}=\frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma}\frac{\partial k}{\partial x_i}\right)+\underbrace{\nu_t\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j}}_{P}-\varepsilon$$

$$+\underbrace{\left(\beta g_i\frac{\nu_t}{\sigma_t}\frac{\partial \varphi}{\partial x_i}\right)}_{G} \qquad (19)$$

$$\rho\frac{D\varepsilon}{Dt}+U_i\frac{D\varepsilon}{Dx_i}=\frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_j}\right)+C_{1\varepsilon}\frac{\varepsilon}{k}(P+G)(1+C_{3\varepsilon}R_f)-C_{2\varepsilon}\frac{\varepsilon^2}{k} \qquad (20)$$

Standard Two Equations Model due to launder due to Launder and Spalding (1974)

$C_\mu = 0.09, C_{1\varepsilon} = 1.44, C_{2\varepsilon} = 1.92, \sigma_\varepsilon = 1.3$

$C_{3\varepsilon} = 0.8$, in the absence of buoyancy $\mathrm{G} = 0, C_{3\varepsilon} = 0$

So, finally, this was the complete modeled k equation and epsilon equation. This is looking even more complex because I have added this term g which I have not introduced. This term C 3 epsilon and this term R f in this equation.

I had a purpose of introducing this, because if the flow field is thermal buoyancy dominated, thermal buoyancy is you know very very gearing component contributory component and if such flow fields are to be modeled then these terms are to be added.

There is, I am giving you a little background, in 80s and 90s in entire Europe, plenty of programs were taken up by different governments to clean the environment and major focus was the pollutions created by the chimney effluence, the you now hot gas thrown out by the chimney and also the hot industrial discharge into reverse and estuaries. Fishes were being destructed and lot of environmental imbalance started surfacing.

So, in order to remediate that, massive programs were taken. One program was to estimate such damages through modeling. And, what I know from my personal acquaintance with contemporary scientist, that Professor Wolfgang Rodi in University of (Refer Time: 38:14), he was handling a massive project.

And, as I said Professor Wolfgang Rodi was also earlier a member of Professor Spaldings research group and they modeled basically, and all these flows are reasonably high speed flow white high speed flows. So, modeling such chimney flows or discharge in rivers or estuaries these were done by Professor Rodi’s group.

And so, all these flows thermal buoyancy because, whether it is discharged in the river or discharge in the open ambients, basically these are hot gases or hot liquids, and thermal buoyancy effect was quite significant. So this phi is temperature, this is volume coefficient of expansion beta, and this is g generation term due to thermal buoyancy.

And, here also you can see, another model constant C 3 epsilon into R f, R f is a Richardson number, C 3 e is 0.8 and this is G, G is also sitting here. Now, if you exclude this seemingly unknown things, why seemingly this were unknown to us so far, otherwise you can consider this is the model we have discussed why.

We have one constant sigma k sigma, k is given by 1 and then the epsilon equation, if you exclude G, if we exclude this bracketed term which are due to thermal buoyancy effect, then basically temporal term, convective term this is, as I have already discussed diffusion of dissipation. This is production of anticipation. So, G is not there these, terms are not there. We discussed it earlier, P into epsilon by k into C 1 epsilon which is a modeling constant. It is given by 1.44.

Please ignore this minus sign, this is not minus this is equal to. Again this is a small typo. So C 1 epsilon is equal to 1.44, I am repeating C 1 epsilon is equal to 1.44 and C 2 epsilon is equal to 1.92 C t epsilon into epsilon square by k, this is distraction of dissipation term. And, this model was if we exclude the buoyancy part, original model was developed by Professor Launder and Professor Spalding.

The very famous paper published in 1974, but before that a book also came out from the same authors Launder and Spalding in 1972. So, in absence of buoyancy then G will be 0, C 3 epsilon will be 0.

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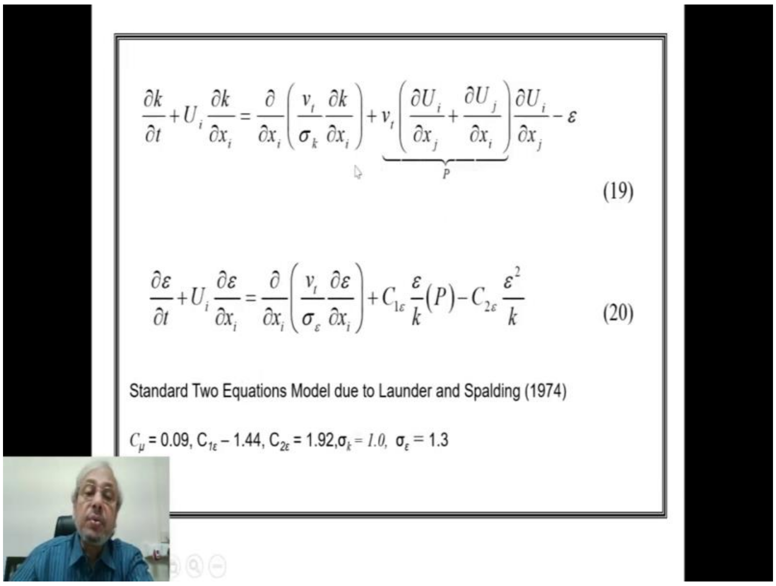

$$\frac{\partial k}{\partial t} + U_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{v_t}{\sigma}\frac{\partial k}{\partial x_i}\right) + \underbrace{v_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)}_{P}\frac{\partial U_i}{\partial x_j} - \varepsilon \qquad (19)$$

$$\rho\frac{D\varepsilon}{Dt} + U_i \frac{D\varepsilon}{Dx_i} = \frac{\partial}{\partial x_i}\left(\frac{v_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_j}\right) + C_{1\varepsilon}\frac{\varepsilon}{k}\left(P+G\right)\left(1+C_{3\varepsilon}R_f\right) - C_{2\varepsilon}\frac{\varepsilon^2}{k} \qquad (20)$$

Standard Two Equations Model due to launder due to Launder and Spalding (1974)

$C_\mu = 0.09, C_{1\varepsilon} = 1.44, C_{2\varepsilon} = 1.92, \sigma_\varepsilon = 1.3$

And then, probably you will be all of you will readily recognize, because this we have discussed. I just added the effect of thermal buoyancy, I could have avoided that, but just to emphasize its importance even in environmental flows I showed that slide.

So, basically this we have adequately discussed, temporal term, convective term, turbulent product kinetic energy production term, dissipation of kinetic energy, this is transport of kinetic energy through turbulent diffusion.

The diffusion coefficient is nu t and which is again turbulent viscosity eddy viscosity. So, we are determining k, but you know because k value will be needed for determining nu t, but we are using nu t also in this expression. You can say that the calculations are iterative. Once you know we will reach a situation when k and nu t will not change any more, k and nu t will not change anymore.

Similarly, del epsilon del t plus convective term equal to, as I said this is production of dissipation, this is distraction of dissipation, and this is basically diffusion of dissipation. And, C mu is a model constant which is needed for determining nu t, because nu t is C mu k square by epsilon. So, there C mu is needed, but here, in these two equations, the model constants are C 1 epsilon equal to 1.44.

I am repeating again, C 1 epsilon equal to 1.44, C 2 epsilon is 1.92, sigma k is 1 and sigma epsilon is 1.3. And these model constants were determined through series of experiments, experiments for shear flow, experiments for separated flow, like that series of experimental paradigms were created and the results were matched.

Although I cannot tell you typically a name of all experimentalists involved in it. As I said, Professor Launder, Professor Spalding US, under his leadership Professor Rodi, I mean these people were involved in the setting of theoretical model.

Similarly, counterpart experiments, what I know through my personal interaction or whatever you say personal link to Professor (Refer Time: 46:16) is again a big name, a very well known experimentalist in Germany. He is he was in University of Erlangen. He has retired now. He was one of the team members.

So, from him what I gather, that Professor JH Whitelaw ah, Professor Adrian Melling, Professor Doost and few others you know they participated in such you know experiments for. And I mean I am missing probably I mean I beg your pardon I it may be excused by other listeners too. I mean there are many other experimentalist. I also understand some experiments were done in Stanford University.

So, in nutshell what I should say that these where extensively validated with experimental data and then this model constants evolved.

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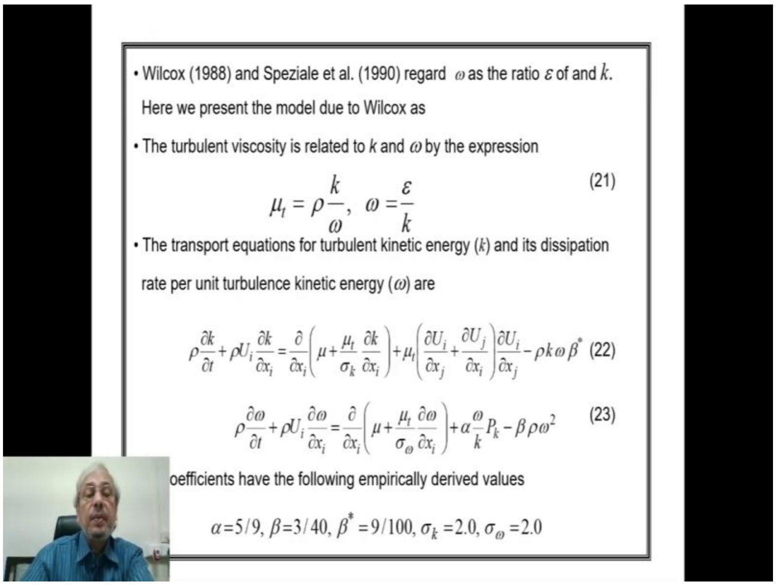

- Wilcox (1988) and Speziale et al. (1990) regard ω as the ratio ε of and k. Here we present the model due to Wilcox as
- The turbulent viscosity is related to k and ω by the expression

$$\mu_t = \rho \frac{k}{\omega}, \omega = \frac{\varepsilon}{k} \tag{21}$$

- The transport equations for turbulent kinetic energy (k) and its dissipation rate per unit turbulence kinetic energy (ω) are

$$\rho \frac{\partial k}{\partial t} + \rho U_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\mu + \frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_i}\right) + \mu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j} - \rho k \omega \beta^* \tag{22}$$

$$\rho \frac{\partial \omega}{\partial t} + \rho U_i \frac{\partial \omega}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\mu + \frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_i}\right) + \alpha \frac{\omega}{k} P_k - \beta \rho \omega^2 \tag{23}$$

$$\alpha = 0.09, \beta = 3/40, \beta^* = 9/100, \sigma_k = 2.0, \sigma_\omega = 2.0$$

Now, Wilcox and Speziale, they also much later you can see, but very significant 1988, subsequently 1990, they developed again yet another version of you know two equation model. I would again mention, since the turbulence is modeled by k modeled k equation and modeled epsilon equation, this paradigm is called two equations model of turbulence.

So, in 1988 and 1990, Professor Wilcox and Professor Speziale, they developed yet another two equation model and this time the turbulent viscosity is given by rho k by omega. This omega is not vorticity, this omega is different. This omega is dissipation rate per unit turbulent kinetic energy.

So, transport equations for turbulent kinetic energy k, this is slightly different than k equation of Launder and Spalding, and equation for dissipation rate per unit turbulent kinetic energy that is equation for omega. This is governing equation for omega. So omega is not vorticity, let me repeat again, it is dissipation rate per unit turbulence kinetic energy.

So, epsilon by k and nu mu t is given by rho into k by omega or nu t will be k by omega. And, this is a governing equation for omega. And here, you can see here also like Spalding Launders equation, sigma k they have used and they have use beta star, they have used beta. They have used alpha and they have used sigma omega.

So, alpha is 5 by 9, beta is 3 by 40, beta star is 9 by 100, sigma k is 2, sigma omega is 2. So, this was developed by them.

They have used it successfully, but more successfully or more I should say well known usage was done by Florian Menter. Menter developed a model which is called SST model of turbulence, shear stress transport model, where very close to the wall he used k omega and away from the wall this model was switched from k omega to k epsilon, that is called SST model. It is very popular and quite a few difficult flows it has performed very very nicely.

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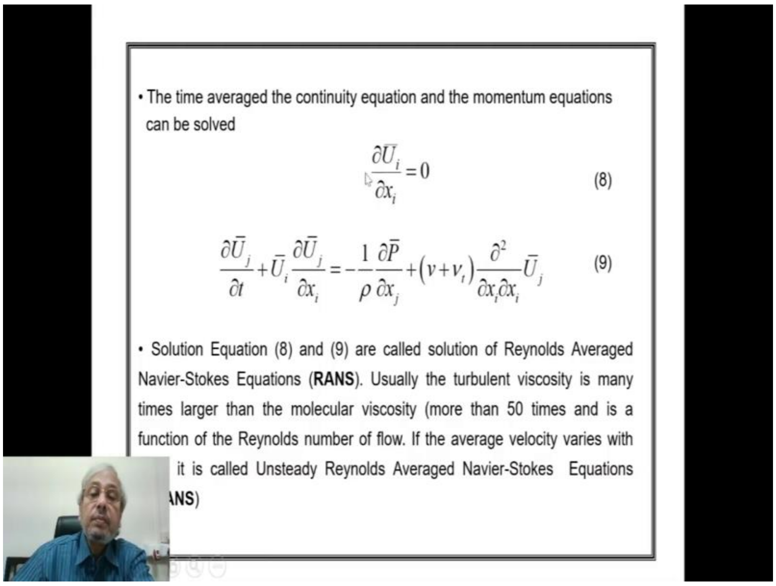

- The time averaged the continuity equation and the momentum equations can be solved

$$\frac{\partial \bar{U}_i}{\partial x_i} = 0 \qquad (8)$$

$$\frac{\partial \bar{U}_i}{\partial t} + \bar{U}_i \frac{\partial \bar{U}_j}{\partial x_i} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial x_j} + \nu \frac{\partial^2}{\partial x_i \partial x_j}\bar{U}_j - \frac{\partial \overline{u_i' u_j'}}{\partial x_i} \qquad (9)$$

- Solution Equation (8) and (9) are called solution of Reynolds Averaged Navier-Stokes Equations (RANS). Usually, the turbulent viscosity is many times larger than the molecular viscosity (more than 50 times and is a function of the Reynolds Averaged Navier-Stokes Equations (**RANS**)

So, let us come back to our discussion that in the time averaged continuity equation, time averaged momentum equation. Now, in this momentum equation nu we can use the molecular viscosity value and nu t will be created by C mu into k square by epsilon. And, k and epsilon are again field equations like momentum equations. So, every time step from the previously known values of u everywhere, this k and epsilon equations have to be solved.

Either these two or this is k omega, better to show k epsilon this is original. So, k and omega epsilon are to be solved, then that k and omega are to be used for creating nu t; that means, C mu into k square by epsilon. Then, we solve the time in momentum equations; that means, U i bar U j bar U k bar, in three orthogonal directions satisfy the continuity equation.

And, like the computational strategies we have outline so far in all our previous lectures, for computing momentum equation, continuity equation. They remain valid. Only the velocity components are time mean velocity components and this coefficient of viscosity is basically molecular viscosity plus turbulent viscosity. A turbulent viscosity will be calculated by again convection diffusion equation of k and epsilon.

And, I having done that, you know that value will be plugged in and the next time level values of velocities will be calculated. So, k and epsilon calculation will be trailing by you know one count. So, solution of equation 8 and 9 these are called solution of Reynolds average Navier-Stokes equations. Because this is these are solution of Reynolds averaged equations, so this is called Reynolds Average Navier-Stock equations. Reynolds averaged Navier-Stokes equation RANS.

Usually, the turbulent viscosity is many times larger than the molecular viscosity. More than 50 times, and it can be even more depending on the fluctuations in the flow. So, many a times this nu which is molecular viscosity can be neglected and is a function of Reynolds number of the flow.

So obviously, fluctuating components U i prime U j prime U k prime are function of Reynolds number and U i prime U j prime U k prime, they contribute to kinetic energy, they contribute to is dissipation rate. So; obviously, nu t is a function of Reynolds number. If the average velocity varies in time, now most of this RANS calculation we think that average velocity is a stationary random, it is not time varying it is dropped.

So that is why you know usual RANS calculation may not include this time marching, what I am discussed discussing is basically pseudo time marching; that means, we will be progressing, evolving as if we are evolving in time direction, but we will try to reach steady flow steady flow condition.

So, but what I would like to mention here, also a paradigm exists where time mean value is also a function of time. This may be you know low frequency oscillations, the time mean value. Time period is very large, much much larger than the turbulent flow time period. Fluctuating components time period is very small, frequency is very high several kilo Hertz, but the mean flow it may be also time varying, but time period is large. So that frequency is less, maybe few Hertz, but it can also happen.

So, basically we can say that if the average velocity varies with time it is called unsteady Reynolds Averaged Navier-Stokes equations or URANS. So, RANS and URANS cover this paradigm which we discuss today.

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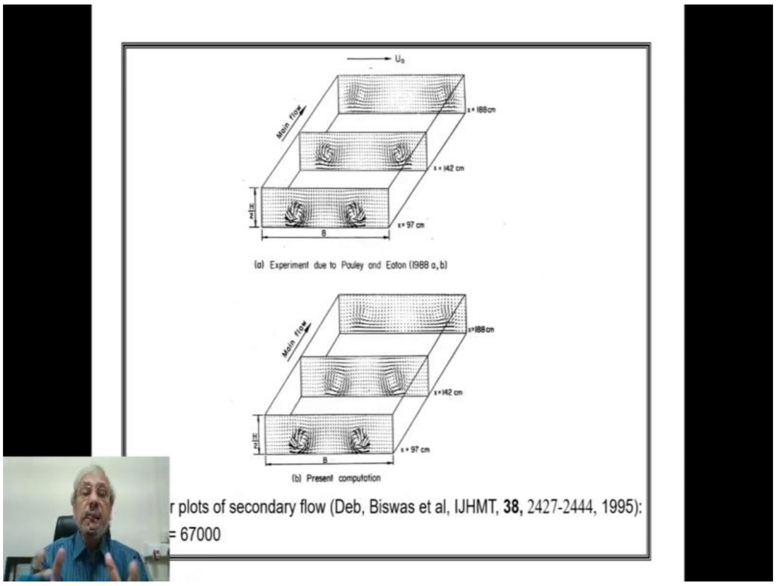

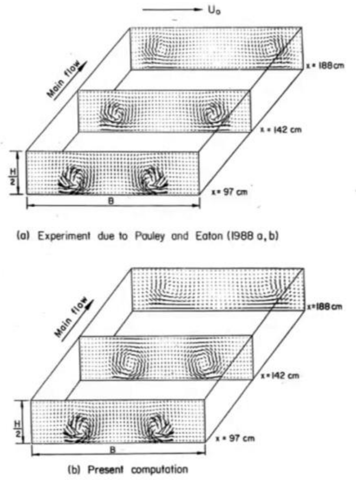

I will show you few very nice results, these are the flow fields behind two delta winglet peers sitting in a channel.

And this each delta winglet peer will create a longitudinal vertex. So, a peer of longitudinal vertices will be seen in the channel. So, if we cut say the flow at a cross section, will be able to see this counter rotating vertices. So, this was basically experiment of Professor Pauley and Professor John Eaton in Stanford University, very well known experiment. And these are basically computation of basically repeating the results computationally.

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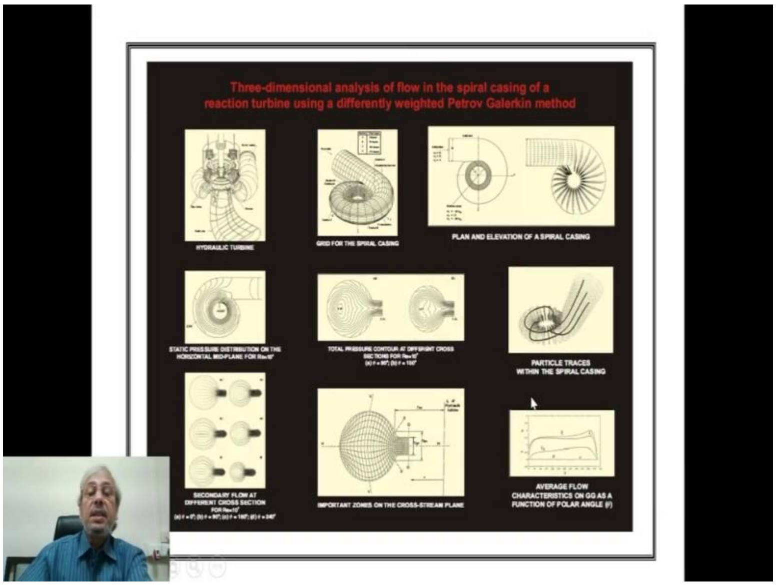

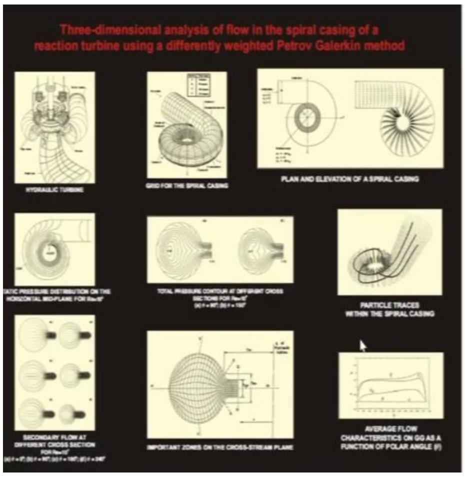

This is yet another complex flow. You can see you know, this is called spiral casing, spiral casing of a hydraulic turbine. So, very very important component, because you know if it is a reaction turbine, specially Francis turbine then you know kinetic energy will rotate the blades will rotate the turbine blades and also pressure energy will rotate the blades because of pressure difference.

So, and the these are many a times called 50 percent reaction turbine; that means, the work done is obtain 50 percent due to exchange of kinetic energy from the flowing stream to the blades, and 50 percent is obtained through the pressure expansion. So retaining pressure is very important and that is done by spiral casing. And this was a BHEL (Refer Time: 1:00:46) sponsored project.

BHEL (Refer Time: 1:00:50) Bhopal sponsored this project to us. It is very successfully delivered and calculation of flow field inside this spiral casing. You can see at different cross sections the flow fields and also pressure fields have been plotted.

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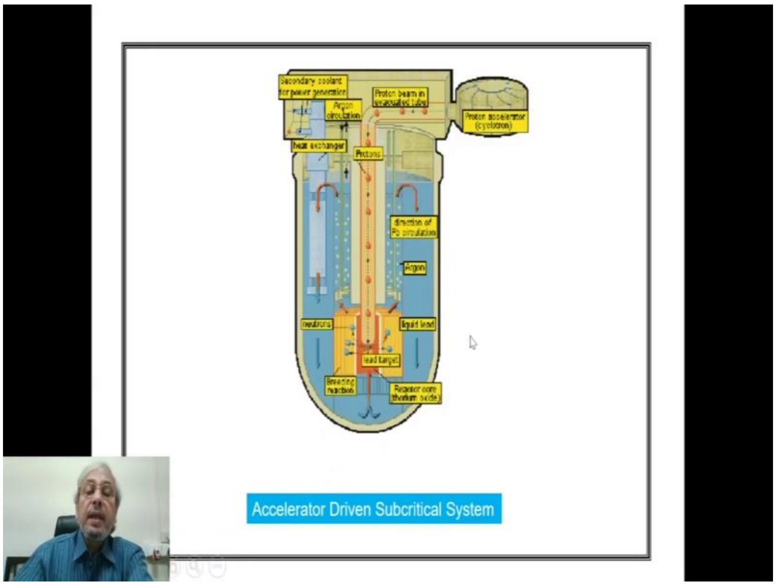

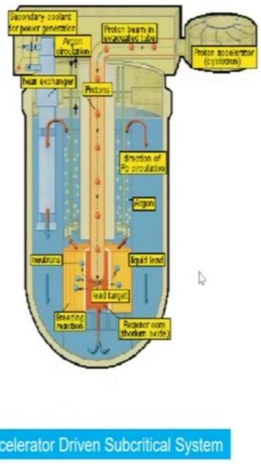

This is yet another very challenging problem. Basically, what you can see is accelerated driven subcritical system modeling of that it is basically led bismuth eutectic is a circulating fluid and it is being you know bombarded on a window by the proton beam and neutrons are generated here. Of course, neutronics is not our domain of activity.

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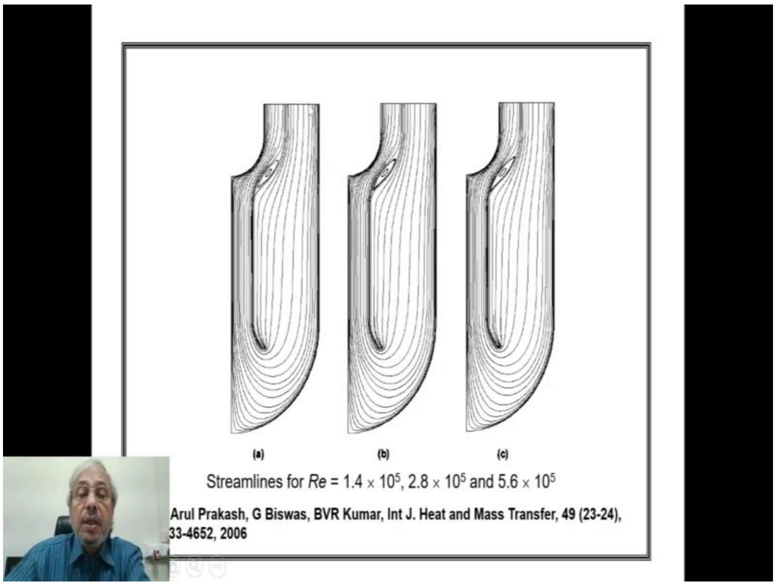

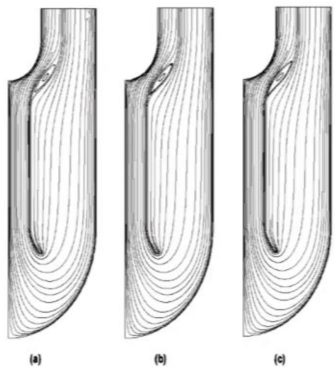

Streamlines for $Re = 1.4 \times 10^5$, 2.8×10^5 and 5.6×10^5

What we did is, because of repeatedly I mean repeated bombardment of proton beams, this window gets damaged. Huge amount of heat deposition takes place and the circulating led bismuth eutectic, the metal that basically removes the heat from this window surface. So, that heat removal calculation was done by again yet another group, Professor Arul Prakash and Professor BV Rathish Kumar. They were involved together with me in this calculation.

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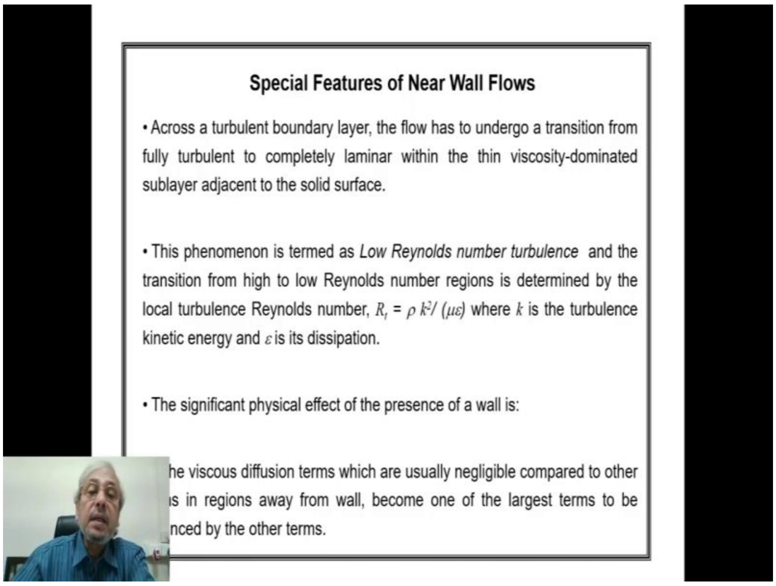

- Across a turbulent boundary layer, the flow has to undergo a transition from fully turbulent to completely laminar within the thin viscosity-dominated sublayer adjacent to the solid surface.
- This phenomenon is termed as Low Reynolds number turbulence and the transition from high to low Reynolds number regions is determined by the local turbulence Reynolds number, $R_t = \rho k^2/(\mu\varepsilon)$
- The significant physical effect of the presence of a wall is:

Now, I have not mentioned one thing. I have said that k and epsilon those are calculated in the flow field, but all of you know, in a turbulent flow very close to the wall, these fluctuating components die there is no turbulence. And the fluid layer flows to the wall is called viscous sub layer. Across a turbulent boundary layer the flow has to undergo a transition from fully turbulent to completely laminar within the thin viscosity dominated sub layer adjacent to the solids surface.

This phenomenon is termed as Low Reynolds number turbulence and the transition from high to low Reynolds number regions is determined by local Reynolds number. This local Reynolds number R t turbulent Reynolds number local turbulence Reynolds number determined by local turbulence Reynolds number R t given by rho k square by mu into epsilon. So, k is the turbulence kinetic energy and epsilon is the dissipation.

So, significant physical effect of the presence of wall is the viscous diffusion term which are viscous diffusion terms which are usually negligible compare to other terms in regions away from the wall, become one of the largest terms to be balanced by the other terms. So, this is also one issue close to the wall.

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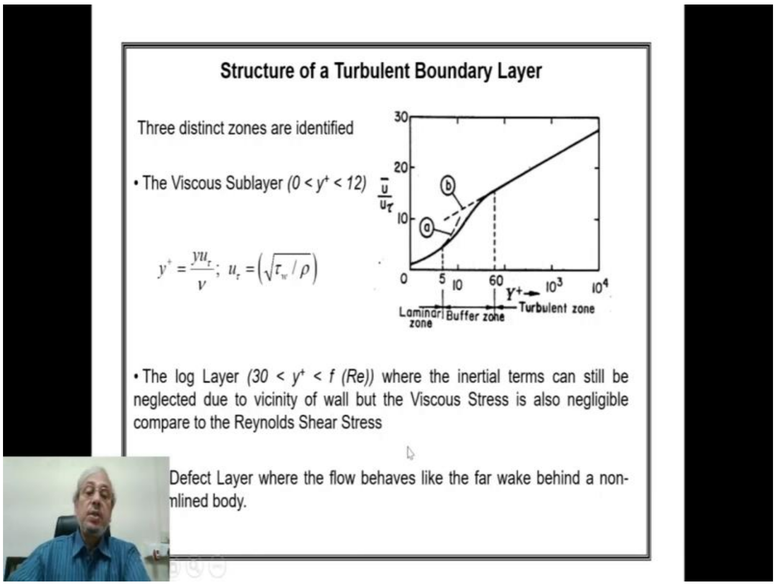
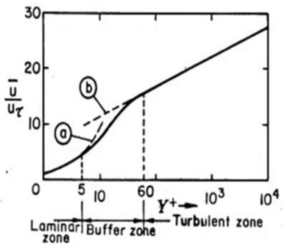


Three distinct zones are identified

- The viscous Sublayer $\left(0 < y^{+} < 12\right)$



$$y^{+} = \frac{yu_{\tau}}{v}; u_{\tau} = \left(\sqrt{\tau_{w}/\rho}\right)$$

The log layer $\left(30 < y^{+} < f\left(\mathrm{Re}\right)\right)$ where the initial terms can still be neglected due to vicinity of wall but the Viscous Stress is also negligible compare to the Reynolds Shear Stress

So, I will not go much detail here. What I will say that from the wall, if we progress in the normal direction then, up to a distance y plus equal to 12 we can call viscous sub layer. What is y plus? y plus is a nu scale, y plus is given by y u tau by nu. What is u tau? u tau is friction velocity and it is defined by root over tau w by rho root over shear stress by rho, will give u tau, u tau is friction velocity.

So, y into u tau by nu, it is becoming non dimensional. y is in meter u tau meter per second meter square per second nu meter square per second. So this is a non dimensional scale y plus, and this y plus is much much smaller than our usual scale we use in the channel height. You know you know usually, you know 1500 1600 y plus u needs may be you know found if we want to measure our channel height in usual laboratory experiments.

And, so you can imagine that y plus 12 is how small. So up to y plus we do not apply any boundary condition at the wall, because it is you know very difficult to apply boundary condition at the wall. Velocity profile gets in the turbulent zone log law profile, and log law profile log law is not valid at y equal to 0.

So, this part is kept and the boundary conditions are applied at y plus equal to 12 or may be slightly beyond 12. And, so the variation of velocity within the viscous sub layer is bridged by this concept, as if it is coming continuously in the y plus scale it as if it is varying linearly from 0 to the non zero value at y plus equal to 12, and y plus plus is equal to 12 it matches the value, that is there because of a logarithmic velocity profile. And again this is iterative calculation.

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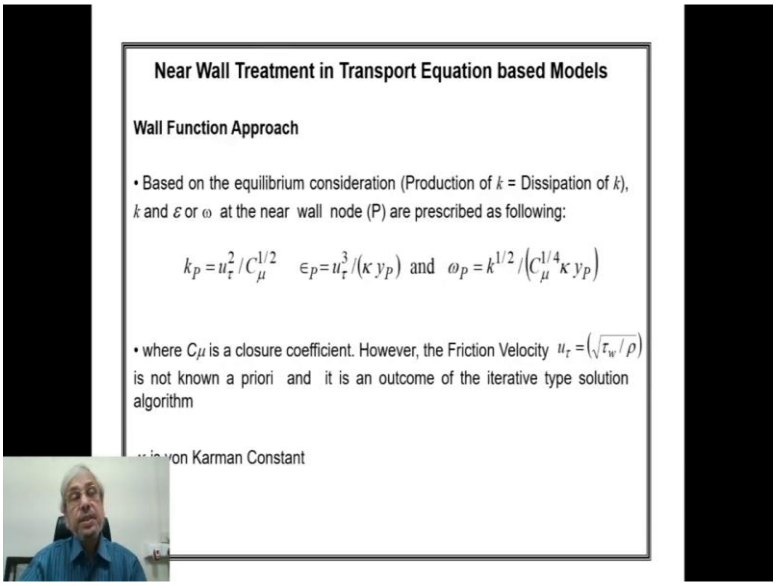

Near Wall Treatment in Transport Equation based Models

Wall Function Approach

- Based on the equilibrium consideration (Production of *k* =Dissipation of *k*), *k* and ε or ω at the near wall node (P) are prescribed as following:

$$k_p = u_\tau^2 \big/ C_\mu^{1/2} \qquad \varepsilon_p = u_\tau^3 \big/ (k y_P) \quad \text{and} \quad \omega_p = k^{1/2} \big/ \left(C_\mu^{1/4} \kappa y_P \right)$$

- Where C_μ is a closure coefficient. However, the Friction Velocity $u_\tau = \left(\sqrt{\tau_w / \rho} \right)$ is not known a priori and it is an outcome of the iterative type solution algorithm

Because, shear stress we calculate from the guess value, then from that shear stress root over tau by rho, we get friction velocity u tau from that we get y plus. So, this calculation is again done through iterations. Now, wall function approach that is why it is called wall function approach.

Based on the equilibrium consideration, production of k is equal to dissipation of k, k and epsilon or k and omega at the near wall node nearest node. So, in based on the computational scheme this should be located at y plus equal to 12 or may be the way you generate grid first grid point may be at y plus equal to 16, 18, 20 it is ok are prescribed in the following way.

k p the kinetic energy at that point from there onwards kinetic energy calculation is taken up by kinetic energy equation, and this k P is used as boundary condition. u tau square divided by C mu to the power half u tau friction velocity square by C mu to the power half. Epsilon P again epsilon equation is used in the whole field, but it does not come to the wall, it satisfy boundary condition is implemented through the near wall point boundary condition.

Epsilon P is u tau cube divided by kappa y P. This kappa is von Karman constant and omega P is k to the power half divided by C mu to the power one-fourth kappa into y P. Where, C mu is a closure coefficient 0.09, we have discussed it earlier. Friction velocity u tau equal to root over tau w by rho is not known a priori and it is an outcome of the iterative type of solution algorithm. And kappa is from Karman constant, its value is 0.4, 0.39, 0.4 around that.

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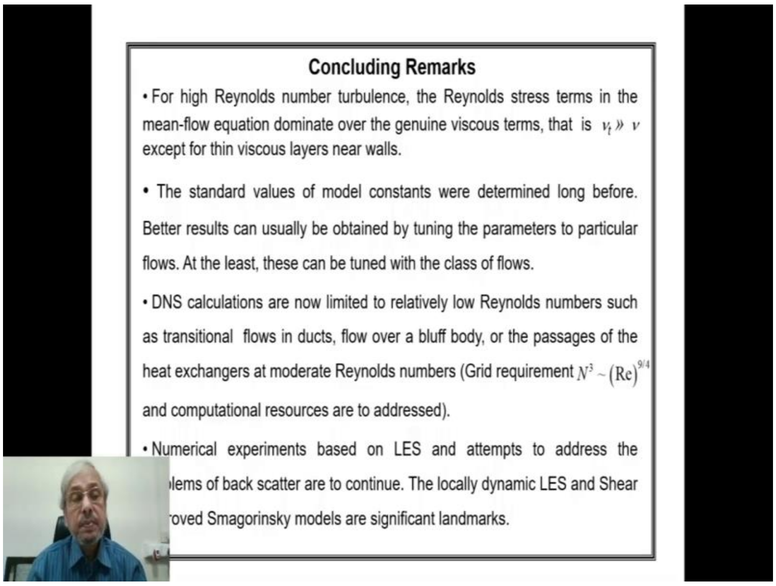

Concluding Remarks

- For high Reynolds number turbulence, the Reynolds stress terms in the mean-flow equation dominate over the genuine viscous terms, that is $\nu_t \gg \nu$ except for thin viscous layers near walls.

- The standard values of model constant were determined long before. Better results can usually be obtained by tuning the parameters to particular flows. At the least, these can be tuned with the class of flows.
- DNS calculations are now limited to relatively low Reynolds numbers such as transitional flows in ducts, flow over a bluff body, or the passages of the heat exchangers at moderate Reynolds numbers (Grid requirement $N^3 \sim (Re)^{9/4}$ and computational resources are to addressed).
- Numerical experiments based on LES and attempts to address the problems of back scatter are to continue. The locally dynamic LES and shear proved Smagorinsky models are significant landmarks.

So, concluding remarks, for high Reynolds number turbulence, Reynolds stress terms in the mean flow equation dominant over the genuine viscous terms, that is nu t is much much greater than nu, except for thin viscous layers near the walls. Near the wall of course, as I said viscous sub layer their viscosity plays a role and that is why there is no fluctuating component there.

The I mean not the on that is why I am saying fluctuating components are not there and viscosity molecular viscosity effect of that prevails, but outside viscous sub layer the turbulent viscosity is its a function of space of course, but it is the way k varies epsilon varies this will also vary, but it is much much bigger than nu molecular.

The standard values of model constants were determined long before. I mentioned about it about the experiments. Better results this were done for some class of experiments and values were obtained. Better results can usually be obtained by tuning the parameters two particular flows. At the least, these can be tuned with the class of flows. So they can be revisited for the further improvement.

DNS calculations are now limited to relatively low Reynolds numbers such as transitional flows in ducts, flow over bluff body, and flow over a bluff body, or the passage of the heat exchangers at moderate Reynolds number. And why Reynolds number is kept moderate? Because, the grid requirement, N is a grid in one direction. So, N cube is the grid requirement. It is proportional to Reynolds number to the power 9 by 4.

And obviously, for computing such flows, where you need 10 to the power 12, 10 to the power 14 type of grids in a domain, ordinary computers or even very good mainframe computers maybe inadequate. You really need high performance competing machine which are parallel machines called I mean supercomputers.

Numerical experiments based on LES and attempts to address the problems of back scatter. See, LES I did not discuss in detail. One of the problems in LES is back scatter, for which you can get basically Smagorinsky constant negative in some place places, which is you know undesirable. The locally dynamic LES and shear improved Smagorinsky models are significant landmarks.

So these have already developed, locally, dynamic LES and shear improved Smagorinsky model it is called SISM. These are significant advancement in LES and LES is coming up very fast, maybe in another 5 years it can be a usual tool to handle turbulence. It will be so, things will improve a lot, but you know till DNS and LES these two methods are elevated to that level.

And we have lot more powerful computers, this RANS and URANS will remain very reliable tool for, as I mentioned it earlier, engineering flows. Like you know if somebody wants to derived statistics related to turbulence physics, then without DNS or LES it is difficult.

But, you know if we want to calculate basically pressure drop, heat transfer, mass transfer, all such you know engineering parameters, then RANS or URANS are quite good to give, both the methods gave satisfactory results. Thank you very much for your interest, I hope it will enhance lot more after this lecture.

Thank you very much again.