

Computational Fluid Dynamics and Heat Transfer
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Lecture - 25
Turbulent Flow and Heat Transfer (preliminaries)

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Turbulent Flow and Heat Transfer (preliminaries)

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Good morning, everybody, today we will discuss about the preliminary aspects of Turbulent Flow and Heat Transfer. As we know many of the flows that we encounter are basically turbulent flows and it is very difficult to solve the governing equations for turbulent flows directly by using even computers because of the huge requirement of computing power huge requirement of the space.

So, the only feasible approach is through turbulence modelling. In this methodology the turbulent field is basically decomposed into time mean and fluctuating field time mean field and fluctuating field.

Equations are solved for the time mean values and closer models are written for the fluctuating components as such turbulent flow is very involved and it is to be understood first the physics of turbulence. So, today's lecture we will discuss some such preliminary aspects so to say foundation of turbulent flows.

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Turbulent Flow and Heat Transfer

Introduction

The following are the characteristics of Turbulent motion

- Irregularity

Complex variations of velocity, temperature, etc. with space and time (fluctuations) are the dominant properties of a turbulent flow. The irregular motion is generated due to random fluctuations. It is postulated that the fluctuations inherently come from disturbances (such as, roughness of the solid surface) and they may be either dampened out due to viscous damping or may grow by drawing energy from the free stream. At a Reynolds number less than the critical, the kinetic energy of flow is not enough to sustain the random fluctuations against the viscous damping and in such cases laminar flow continues to exist. At somewhat higher Reynolds number than the critical Reynolds number, the kinetic energy of flow supports the growth of fluctuations and transition to turbulence is induced.

Now, let me make some introductory comments on turbulent flows. The following are the characteristics features of turbulent motion one is irregularity complex variations of velocity temperature etcetera with space and time are the dominant properties of a turbulent flow.

The irregular motion is generated due to random fluctuations, it is postulated that the fluctuations in inherently come from disturbances such as, roughness of the solid surface they may be either damped out due to viscous damping or may grow by drawing energy from the free stream.

As a Reynolds number less than; at a Reynolds number less than critical the kinetic energy of flow is not sufficient to sustain the random fluctuations against the viscous damping. So, at a Reynolds number less than critical the kinetic energy of flow is not enough to sustain the random fluctuations against the viscous damping and in such cases laminar flow continues to exist.

At somewhat higher Reynolds number than the critical Reynolds number the kinetic energy of flow supports the growth of fluctuations and transition to turbulence is induced.

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- **Strong mixing**

High transfer of momentum, heat, mass by fluctuating turbulent motion - practically most important feature.

- **Turbulent motion always 3D**

For a parallel flow, it can be written that the axial velocity component is

$$u(y, t) = \bar{u}(y) + u'(\Gamma, t)$$

y is the normal direction, Γ is any space variable

Even if the bulk motion is parallel, the fluctuation u' being random varies in all directions. Now let us look at the continuity equation

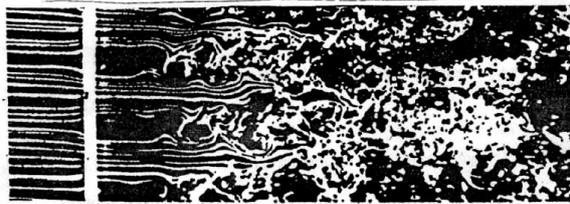


Figure 5.1: Turbulence is generated in the flow field.

Now, strong mixing is another feature heat transfer of or momentum transfer. So, these are enhanced due to the strong mixing. So, high transfer of momentum, heat, mass by fluctuating turbulent motion practically this is the most important feature.

Turbulent motion is always 3D, for a parallel flow usually in laminar parallel flow one component of velocity is nontrivial others other components are trivial. In a similar parallel flow situation if we consider that the velocity in the mainstream direction is a mean plus fluctuations and y is the normal direction to the flow uppercase gamma is any space variable fluctuations are expected to be in the space. Even if the bulk motion is parallel the fluctuation u prime being random varies in all directions.

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$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial u'}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Since $\frac{\partial u'}{\partial x} \neq 0$, the above equation depicts that y and z components of velocity exist even for the parallel flow if the flow is turbulent. We can write

$$u(y, t) = \bar{u}(y) + u'(\Gamma, t)$$

$$v = 0 + v'(y, t)$$

$$w = 0 + w'(y, t)$$

- **Turbulent motion carries vorticity - is composed of eddies interacting with each other**

Wide spectrum of eddy sizes and corresponding fluctuation frequencies are shown

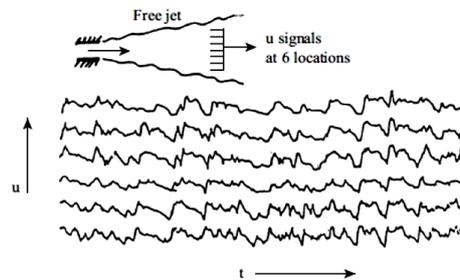
Now, let us look at the continuity equation. So, if we substitute u by $\bar{u} + u'$ we can write

in this way $\frac{\partial \bar{u}}{\partial x} + \frac{\partial u'}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$.

But $\frac{\partial u'}{\partial x}$ is not equal to 0, the above equation depicts that y and z components of velocity exist even for the parallel flow if the flow is turbulent; that means, u is \bar{u} plus the fluctuation in u even though there is no mean velocity in cross stream directions in y direction or z direction mean velocity we can write as 0, but there will be a fluctuating components in order to satisfy the continuity equation.

So that means, if there is fluctuation in any direction ideally the any component of motion it will induce fluctuations in all other directions or all other components of motion turbulent motion carries vorticity and vorticity is composed of eddies interacting with each other.

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- **Turbulent motion carries vorticity and consists of interacting eddies**
Wide spectrum of eddy sizes and corresponding fluctuation frequencies are shown. In a turbulent flow, energy is fed from large scales mainly to smaller scales by nonlinear processes.
- The term *homogeneous turbulence* implies that the velocity fluctuations in the system are random.
- The average turbulent characteristics are independent of the position of the fluid, i.e., invariant to axis translation.
- If the velocity fluctuations are independent of the axis of reference (invariant to axis rotation and reflection), the restriction leads to *isotropic turbulence*, which by definition is always homogeneous.

We can look at this picture this is a free jet emanating from a nozzle and if we place sensors at different locations at the same x distance from the outlet of the nozzle we will be able to see the fluctuations at every locations and these fluctuations are such that they have no similarity if we look into signals at these 6 points, at any t they are not similar and they are also arbitrary at any instant for any particular signal.

So, turbulent motion carries vorticity and consists of interacting eddies. Wide spectrum of eddy sizes and corresponding fluctuation frequencies are shown here. In a turbulent flow energy is fed from large scale mainly to smaller scales by non-linear processes. The term homogeneous turbulence implies that velocity fluctuations in the system are random.

The average turbulent characteristics are independent of the position of the fluid that is invariant to axis translation. If the velocity fluctuations are independent of axis of reference that is invariant to axis rotation and reflection this condition leads to isotropic turbulence which by definition is always homogeneous.

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Classical Idealization of Turbulent Flows

Statistical calculation methods

- Details of turbulent fluctuations usually not of interest to engineers anyway
- Hence statistical approach is taken and turbulence is averaged out. Different trends of variation of the the mean and fluctuating components are shown in Figures 5.3 and 5.4.
- Statistical quantities:

$$u_i = \bar{u}_i + u'_i, P = \bar{P} + p', \bar{u} = \frac{1}{2t} \int_{t_0}^{t_0+t} u dt$$

Now, we will not take up various other issues about the turbulence physics we will take a route to the classical idealization of turbulent flows. So, that turbulent flow can be flow situations can be handled with good degree of accuracy also.

Now, usually for this we go for statistical calculation methods where details of turbulent fluctuations usually you know are not considered. Hence the statistical approach is taken and turbulence is averaged out. Different trends of variation of the mean and fluctuating components we will show in the in a figure in the next slide.

Where we can consider all the velocity components as the summation of mean and fluctuations pressure as mean pressure and fluctuating pressure and this mean is basically time mean component over a time interval.

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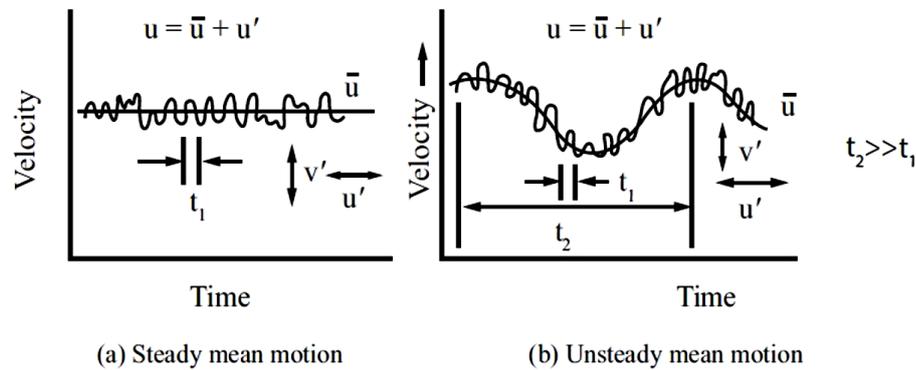


Figure 6.4 Steady and unsteady mean motions in a turbulent flow

$$\int_{t_0}^{t_0+\Delta t} u' dt = 0$$

However, the fluctuating components do not bring about the bulk displacement of a fluid element. The instantaneous displacement is $u' dt$ and if that is indeed not responsible for the bulk motion, we can conclude the above

So, here we have shown that a steady mean motion; that means, the instantaneous velocity can be decomposed into mean component and fluctuating component. Now, fluctuating components are such that if we perform the mean signal or the mean component then the mean is not varying with time. So, over a large time scale we can see mean is steady, but if we reduce the time scale then we can see the, we can observe or we can appreciate the fluctuations.

Similarly, this mean motion can also be unsteady mean motion. Here we can see the fluctuating components are there, but when we perform the average velocity, we will try to determine the average velocity and observe the variation of average velocity we will see the average velocity is also changing with time.

And you know this variation has a different time period which is much larger than the turbulent fluctuations or than the turbulence timescale. So, here the time period is very small for the fluctuating components. So, frequency is high, here the time period is large and variation of this frequency is much less. Now the fluctuating components do not bring about the bulk displacement of a fluid element.

The instantaneous displacement is $u' dt$ and if that is indeed not responsible for the bulk motion then we can write this integral t_0 to $t_0 + \Delta t$ the time window $u' dt$ this has to be 0. There is no bulk motion and these are all instantaneous displacement.

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Due to the interaction of fluctuating components, macroscopic momentum transport takes place. Therefore, interaction effect between two fluctuating components over long period is nonzero and this yields

$$\int_{-t}^t u' v' dt \neq 0$$

We take time average of these two integrals and write

$$\overline{u'} = \frac{1}{2t} \int_{-t}^t u' dt = 0$$

and

$$\overline{u'v'} = \frac{1}{2t} \int_{-t}^t u' v' dt \neq 0$$

Now, we can make a general statement with any two fluctuating parameters, say, with f' and g' as (f' and g' can be vectors or passive scalars)

$$\overline{f'} = \overline{g'} = 0 \quad \frac{\partial \overline{f'}}{\partial s} = \frac{\partial^2 \overline{f'}}{\partial s^2} = 0$$

and

$$\overline{f'g'} \neq 0 \quad \text{and} \quad \frac{\partial(\overline{f'g'})}{\partial s} \neq 0$$

Due to the interaction of fluctuating components macroscopic momentum transport takes place. Therefore, interaction effect between two fluctuating components over long period is non zero. So, between two fluctuating components like $u' v'$ this if we perform the integral over a time window dt is non zero.

We take time average of these two integrals; that means, this integral and this integral then u' prime bar that is average of u' that is 0 since this is this integral is 0. So, $\overline{u'}$ is 0 and $\overline{u'v'}$ bar since $u'v' dt$ this integral is non zero. So, $\overline{u'v'}$ this is non zero. So, $\overline{u'}$ is 0, but the quadratic quantities; that means, $\overline{u'v'}$, $\overline{u'w'}$, $\overline{u'u'}$ those are non zero.

Now, we can make a general statement with any two fluctuating parameters say f' and g' and those f' and g' can be vectors or passive scalars. So, they can be fluctuations in

velocities or fluctuations in temperature. Now, $\overline{f'}$ equal to $\overline{g'}$ equal to 0, similarly $\frac{\partial \overline{f'}}{\partial s}$ equal to $\frac{\partial^2 \overline{f'}}{\partial s^2}$ that is also equal to 0. But $\overline{f'g'}$ is non zero and $\frac{\partial \overline{f'g'}}{\partial s}$ is also non zero.

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We shall state some rules of operating on mean time-averages herein. If f and g are two dependent variables and if s denotes any one of the independent variables x, y, z, t then

$$\frac{\partial \overline{f}}{\partial s} = \overline{\frac{\partial f}{\partial s}}; \quad \overline{\int f ds} = \int \overline{f} ds$$

$$u = \bar{u} + u', \quad v = \bar{v} + v', \quad w = \bar{w} + w', \quad p = \bar{p} + p'$$

Plug in continuity

$$\nabla \cdot \bar{u}_i = 0$$

and

$$\nabla \cdot u'_i = 0$$

So, we shall state some rules of operating on mean time averages if f and g are two dependent variables and s denotes; s denotes any one of the independent variables x, y, z or t we can write basically $\frac{\partial \overline{f}}{\partial s}$ equal to $\overline{\frac{\partial f}{\partial s}}$ or integral of $f ds$ over bar is integral of $\bar{f} ds$.

So, with this philosophy this is called Reynolds decomposition that is instantaneous quantities expressed in terms of average and fluctuation. So, $u = \bar{u} + u'$, $v = \bar{v} + v'$ and $w = \bar{w} + w'$ and $p = \bar{p} + p'$. If we plug in this into continuity equation and do-little algebraic operations we will get two results, one is divergence of \bar{u}_i equal to 0. That means $\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0$ and divergence of u'_i prime is 0; that means, $\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0$.

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We can write the x -momentum equation as

$$\rho \left(\frac{\partial u}{\partial t} + \frac{\partial(u^2)}{\partial x} + \frac{\partial(uv)}{\partial y} + \frac{\partial(uw)}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

The overbar operation on each term:

$$\overline{\frac{\partial(u+u')}{\partial t}} = \frac{\partial \bar{u}}{\partial t} + \frac{\partial \bar{u}'}{\partial t} = \frac{\partial \bar{u}}{\partial t}$$

$$\overline{\frac{\partial(\bar{u}+u')(\bar{u}+u')}{\partial x}} = \frac{\partial \bar{u}\bar{u}}{\partial x} + \frac{\partial(2\bar{u}u')}{\partial x} + \frac{\partial u'^2}{\partial x} = \frac{\partial(\bar{u}\bar{u})}{\partial x} + \frac{\partial u'^2}{\partial x}$$

$$\overline{\frac{\partial(\bar{u}+u')(\bar{v}+v')}{\partial y}} = \frac{\partial \bar{u}\bar{v}}{\partial y} + \frac{\partial(\bar{u}v')}{\partial y} + \frac{\partial(\bar{v}u')}{\partial y} + \frac{\partial u'v'}{\partial y} = \frac{\partial \bar{u}\bar{v}}{\partial y} + \frac{\partial u'v'}{\partial y}$$

$$\overline{\frac{\partial(\bar{u}+u')(\bar{w}+w')}{\partial z}} = \frac{\partial \bar{u}\bar{w}}{\partial z} + \frac{\partial(\bar{u}w')}{\partial z} + \frac{\partial(\bar{w}u')}{\partial z} + \frac{\partial u'w'}{\partial z} = \frac{\partial \bar{u}\bar{w}}{\partial z} + \frac{\partial u'w'}{\partial z}$$

$$\overline{\frac{\partial^2(u+u')}{\partial x^2}} = \frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}'}{\partial x^2} = \frac{\partial^2 \bar{u}}{\partial x^2}$$

$$\overline{\frac{\partial^2(u+u')}{\partial y^2}} = \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial^2 \bar{u}'}{\partial y^2} = \frac{\partial^2 \bar{u}}{\partial y^2}$$

$$\overline{\frac{\partial^2(u+u')}{\partial z^2}} = \frac{\partial^2 \bar{u}}{\partial z^2} + \frac{\partial^2 \bar{u}'}{\partial z^2} = \frac{\partial^2 \bar{u}}{\partial z^2}$$

Now, we can write x momentum equation in conservative form and apply this Reynolds decomposition and then average out the entire equation to see the time averaged equation for x momentum. So, the basically as I said that conservative form of equation

$$\rho \left(\frac{\partial u}{\partial t} + \frac{\partial(u^2)}{\partial x} + \frac{\partial(uv)}{\partial y} + \frac{\partial(uw)}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

You can easily see that if we find out the derivatives and then apply the continuity equation, we will get $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$ that is usually referred to as non-conservative form.

And we can just rewrite the same equation retaining everything intact in conservative form equal to $-\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$ and then as we decided we will apply Reynolds decomposition; that means, $\overline{\frac{\partial(\bar{u}+u')(\bar{u}+u')}{\partial t}}$ then we will get $\frac{\partial \bar{u}}{\partial t} + \frac{\partial \bar{u}'}{\partial t}$.

So, $\frac{\partial \bar{u}'}{\partial t}$ we have seen that is 0. So, this is $\frac{\partial \bar{u}}{\partial t}$. So, from this term after averaging out we get $\frac{\partial \bar{u}}{\partial t}$.

Similarly, let us look at the second term $\frac{\partial(\bar{u}+u')(\bar{u}+u')}{\partial x}$. So, we can write $\frac{\partial \bar{u}\bar{u}}{\partial x} + \frac{\partial(2\bar{u}u')}{\partial x} + \frac{\partial u'^2}{\partial x}$ and then as a result we will get $\frac{\partial \bar{u}\bar{u}}{\partial x} + \frac{\partial u'^2}{\partial x}$ bar u'^2 is non zero.

So, we again take up the I mean third term or second convective term $\frac{\partial(\bar{u}+u')(\bar{v}+v')}{\partial y}$. So, we can write $\frac{\partial \bar{u}\bar{v}}{\partial y} + \frac{\partial(\bar{u}v')}{\partial y} + \frac{\partial(\bar{v}u')}{\partial y} + \frac{\partial u'v'}{\partial y}$ entire terms operated by over bar.

And after applying over bar from first term we get $\frac{\partial \bar{u}\bar{v}}{\partial y}$ \bar{v}' is 0. From third term we will get del del y of v bar into u prime bar and u prime bar is 0. So, they will not contribute, but u prime v prime over bar which will produce $\frac{\partial u'v'}{\partial y}$ that is non zero that will be there. Then we take up the third term del del z of u bar plus u prime into w bar plus w prime over bar in the similar way finally, we will get del del z of u bar into w bar plus del del z of u bar u prime into w prime over bar.

$$\frac{\partial(\bar{u} + u')(\bar{w} + w')}{\partial z} = \frac{\partial \bar{u}\bar{w}}{\partial z} + \frac{\partial(\bar{u}w')}{\partial z} + \frac{\partial(\bar{w}u')}{\partial z} + \frac{\partial u'w'}{\partial z} = \frac{\partial \bar{u}\bar{w}}{\partial z} + \frac{\partial u'w'}{\partial z}$$

So, temporal term and the convective derivatives we have found out. Now this p is \bar{p} plus p' and if we apply over bar again \bar{p}' will be 0 we will get del p bar del x, we have not written it here separately, but substituting p equal to p bar plus p prime and applying over bar we will get $-\frac{\partial \bar{p}}{\partial x}$ and then the viscous terms $\frac{\partial^2(\bar{u}+u')}{\partial x^2} = \frac{\partial^2(\bar{u})}{\partial x^2} + \frac{\partial^2(u')}{\partial x^2} = \frac{\partial^2(\bar{u})}{\partial x^2}$.

Now, again this is the special variable, but being operated \bar{u}' which is 0. So, this will not finally contribute, we will get $\frac{\partial^2 \bar{u}}{\partial x^2}$ similarly from the second term we will get $\frac{\partial^2 \bar{u}}{\partial y^2}$ and from the third term we will get $\frac{\partial^2 u}{\partial z^2}$.

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Introduction of Reynolds decomposition into the Navier-Stokes equations and subsequent averaging and application of the laws of statistics leads to the appearance of turbulence correlations (turbulent or Reynolds stresses). For example, if we perform the aforesaid exercise on the x momentum equation, we obtain

$$\rho \left\{ \frac{\partial \bar{u}}{\partial t} + \frac{\partial(\bar{u}\bar{u})}{\partial x} + \frac{\partial(\bar{u}\bar{v})}{\partial y} + \frac{\partial(\bar{u}\bar{w})}{\partial z} \right\} = -\frac{\partial \bar{p}}{\partial x}$$

$$+ \mu \nabla^2 \bar{u} - \rho \left[\frac{\partial \overline{u'^2}}{\partial x} + \frac{\partial \overline{u'v'}}{\partial y} + \frac{\partial \overline{u'w'}}{\partial z} \right]$$

Introducing simplifications arising out of continuity equation we shall obtain

$$\rho \frac{\partial \bar{u}}{\partial t} + \rho \left[\bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} + \bar{w} \frac{\partial \bar{u}}{\partial z} \right] = -\frac{\partial \bar{p}}{\partial x}$$

$$+ \mu \nabla^2 \bar{u} - \rho \left[\frac{\partial \overline{u'^2}}{\partial x} + \frac{\partial \overline{u'v'}}{\partial y} + \frac{\partial \overline{u'w'}}{\partial z} \right]$$

So, having found out all these values if we plug in, we will be able to conclude that. So, we can say introduction of Reynolds decomposition into the Navier-Stokes equations and subsequent averaging and application of the laws of statistics leads to the appearance of turbulence correlations.

That is turbulent or Reynolds stresses for example: from the first equation now if I come back the with the contribution of individual terms and rewrite we will get

$$\rho \left\{ \frac{\partial \bar{u}}{\partial t} + \frac{\partial(\bar{u}\bar{u})}{\partial x} + \frac{\partial(\bar{u}\bar{v})}{\partial y} + \frac{\partial(\bar{u}\bar{w})}{\partial z} \right\} = -\frac{\partial \bar{p}}{\partial x}$$

$$+ \mu \nabla^2 \bar{u} - \rho \left[\frac{\partial \overline{u'^2}}{\partial x} + \frac{\partial \overline{u'v'}}{\partial y} + \frac{\partial \overline{u'w'}}{\partial z} \right]$$

However, these three terms which were produced while averaging the left-hand side $\frac{\partial \overline{u'^2}}{\partial x}$, $\frac{\partial \overline{u'v'}}{\partial y}$, and $\frac{\partial \overline{u'w'}}{\partial z}$ those will be there and they can be transferred to the right hand side and with the multiplier rho then we will get $\rho \left[\frac{\partial \overline{u'^2}}{\partial x} + \frac{\partial \overline{u'v'}}{\partial y} + \frac{\partial \overline{u'w'}}{\partial z} \right]$.

So, these three terms will act as additional stress terms with the viscous stress and they are called components of Reynolds stress which is basically culmination of the contribution of the fluctuating components due to Reynolds decomposition and subsequent averaging.

So, now again we can go back from this conservative form to non-conservative form and we can write

$$\rho \frac{\partial \bar{u}}{\partial t} + \rho \left[\bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} + \bar{w} \frac{\partial \bar{u}}{\partial z} \right] = - \frac{\partial \bar{p}}{\partial x}$$

$$+ \mu \nabla^2 \bar{u} - \rho \left[\frac{\partial \overline{u'^2}}{\partial x} + \frac{\partial \overline{u'v'}}{\partial y} + \frac{\partial \overline{u'w'}}{\partial z} \right]$$

just like the laminar flow terms only thing all the velocity components are replaced by their averaged time averaged quantities. So, these becomes time averaged x component of Navier-Stokes equations for turbulent flows.

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similarly,

$$\begin{aligned} \rho \frac{\partial \bar{v}}{\partial t} + \rho \left[\bar{u} \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial \bar{v}}{\partial y} + \bar{w} \frac{\partial \bar{v}}{\partial z} \right] &= -\frac{\partial \bar{p}}{\partial y} + \mu \nabla^2 \bar{v} \\ &- \rho \left[\frac{\partial \overline{u'v'}}{\partial x} + \frac{\partial \overline{v'^2}}{\partial y} + \frac{\partial \overline{v'w'}}{\partial z} \right] \\ \rho \frac{\partial \bar{w}}{\partial t} + \rho \left[\bar{u} \frac{\partial \bar{w}}{\partial x} + \bar{v} \frac{\partial \bar{w}}{\partial y} + \bar{w} \frac{\partial \bar{w}}{\partial z} \right] &= -\frac{\partial \bar{p}}{\partial z} + \mu \nabla^2 \bar{w} \\ &- \rho \left[\frac{\partial \overline{u'w'}}{\partial x} + \frac{\partial \overline{v'w'}}{\partial y} + \frac{\partial \overline{w'^2}}{\partial z} \right] \end{aligned}$$

It is to be noted that the terms containing prime were not there in original NS equations.

$$\sigma_T = \begin{bmatrix} \sigma'_{xx} & \tau'_{xy} & \tau'_{xz} \\ \tau'_{xy} & \sigma'_{yy} & \tau'_{yz} \\ \tau'_{xz} & \tau'_{yz} & \sigma'_{zz} \end{bmatrix} = -\rho \begin{bmatrix} \overline{u'^2} & \overline{u'v'} & \overline{u'w'} \\ \overline{u'v'} & \overline{v'^2} & \overline{v'w'} \\ \overline{u'w'} & \overline{v'w'} & \overline{w'^2} \end{bmatrix}$$

σ_T is the Reynolds stress tensor and written in compact form as $-\rho \overline{u'_i u'_j}$

$$\sigma_{xx} = -\bar{p} + 2\mu \frac{\partial \bar{u}}{\partial x} - \rho \overline{u'^2}$$

$$\tau_{xy} = \mu \left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right) - \rho \overline{u'v'}$$

Similarly, if we apply exactly similar treatment on y momentum equation and z momentum equation then we will get time averaged y momentum equation which is

$$\rho \frac{\partial \bar{v}}{\partial t} + \rho \left[\bar{u} \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial \bar{v}}{\partial y} + \bar{w} \frac{\partial \bar{v}}{\partial z} \right] = -\frac{\partial \bar{p}}{\partial y} + \mu \nabla^2 \bar{v} - \rho \left[\frac{\partial \overline{u'v'}}{\partial x} + \frac{\partial \overline{v'^2}}{\partial y} + \frac{\partial \overline{v'w'}}{\partial z} \right]$$

$\nabla^2 \bar{v}$ that means, $\frac{\partial^2 \bar{v}}{\partial x^2} + \frac{\partial^2 \bar{v}}{\partial y^2} + \frac{\partial^2 \bar{v}}{\partial z^2}$. And then the fluctuating contribution of the fluctuating components. So, this is y-momentum equation for turbulent flows time averaged y momentum equation and this is time average z momentum equation.

$$\rho \frac{\partial \bar{w}}{\partial t} + \rho \left[\bar{u} \frac{\partial \bar{w}}{\partial x} + \bar{v} \frac{\partial \bar{w}}{\partial y} + \bar{w} \frac{\partial \bar{w}}{\partial z} \right] = -\frac{\partial \bar{p}}{\partial z} + \mu \nabla^2 \bar{w} - \rho \left[\frac{\partial \overline{u'w'}}{\partial x} + \frac{\partial \overline{v'w'}}{\partial y} + \frac{\partial \overline{w'^2}}{\partial z} \right]$$

So, these fluctuating components make a contribution in terms of additional stress and we can define them as Reynolds stress which are basically you can see σ'_{xx} , τ'_{xy} , τ'_{xz} again τ'_{xy} , σ'_{yy} , τ'_{yz} , τ'_{xz} and σ'_{zz} . So, these are basically

$$\sigma_T = \begin{bmatrix} \sigma'_{xx} & \tau'_{xy} & \tau'_{xz} \\ \tau'_{xy} & \sigma'_{yy} & \tau'_{yz} \\ \tau'_{xz} & \tau'_{yz} & \sigma'_{zz} \end{bmatrix} = -\rho \begin{bmatrix} \overline{u'^2} & \overline{u'v'} & \overline{u'w'} \\ \overline{u'v'} & \overline{v'^2} & \overline{v'w'} \\ \overline{u'w'} & \overline{v'w'} & \overline{w'^2} \end{bmatrix}$$

And if we add Reynolds; now this Reynolds stress tensor (σ_T) we can write them as $-\rho\overline{u'_i u'_j}$ in very succinct form and you know correct representation of the components and we can represent them by $-\rho\overline{u'_i u'_j}$.

Then together with viscous stresses the we can redefine basically the shear stress and the normal stress terms. So, total normal stress is, this was the normal stress for laminar flow plus this component which is $-\rho\overline{u'^2}$. Similarly, shear stress term this was you know of course, for laminar flow it was

$$\tau_{xy} = \mu \left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right) - \rho\overline{u'v'}$$

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- Averaged equations:

$$\frac{\partial \bar{u}_i}{\partial t} + u_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{P}}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\nu \frac{\partial \bar{u}_i}{\partial x_j} - \overline{u'_i u'_j} \right)$$

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0$$

We have more unknowns than number of available equations. The modified system of equations cannot be closed within itself unless empirical relations are supplied from experiments to correlate the fluctuating components with the mean motion. This is termed as the closure problem.

Now, so averaged equations can be written in tensor notation as

$$\frac{\partial \bar{u}_i}{\partial t} + u_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{P}}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\nu \frac{\partial \bar{u}_i}{\partial x_j} - \overline{u'_i u'_j} \right)$$

Basically, this x momentum equation y momentum equation and z momentum equation for turbulent flow after time averaging, they can be written in this basically in this form which is basically a written using tensor notation.

If we expand $i = 1\ 2\ 3$, $j = 1\ 2\ 3$ we will get all the respective components for x y and z momentum equation and continuity equation is $\frac{\partial \bar{u}_i}{\partial x_i}$ here also $i = 1\ 2\ 3$ if we substitute we will get $\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0$. So, this is time averaged continuity equation.

Now, we have more unknowns than number of available equations. The modified system of equation cannot be closed within itself unless empirical relations are supplied from experiments to correlate the fluctuating components with the mean motion and this is called closure problem this may sound little complex to you.

But let me explain just consider laminar flow equations they are the x momentum equation y momentum equation z momentum equation and continuity equation. What are the unknowns? $u\ v\ w$ and p and equations you have x momentum y momentum z momentum and continuity 4 equations, 4 unknowns' system is perfectly well defined.

Now, here you have unknowns u average v average w average p average and additionally you have basically $\overline{u' u'}$ $\overline{u' v'}$ $\overline{u' w'}$ $\overline{v' v'}$ $\overline{v' w'}$ $\overline{w' w'}$. So, 1 2 3 4 5 6, 6 more unknowns these are symmetric terms. So, really not 9 more unknowns, it is 6 more unknowns.

So, 4 plus 6 10 unknowns and again equations are mean equations in x y and z direction and mean continuity equation so 4 equations. That is why it is being said that this problem cannot be closed now in order to close that you have to do something. So, that these fluctuating components can be expressed in terms of velocity gradients; that means, you know $\frac{\partial \bar{u}}{\partial x}$, $\frac{\partial \bar{u}}{\partial y}$, $\frac{\partial \bar{v}}{\partial x}$ and $\frac{\partial \bar{v}}{\partial y}$ which are basically stress components.

So, these fluctuating components have to be expressed in terms of time averaged velocity components to basically close the system.

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- Closure problem

The turbulent stresses need to be determined with the aid of a turbulence model.

- Here only models for all turbulent motions discussed, not subgrid-scale models for large-eddy simulations.

$$-\overline{u'_i u'_j} = \nu_t \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) - \frac{2}{3} k \delta_{ij}$$

The term ν_t is turbulent (eddy) viscosity. The term involving the Kronecker delta δ_{ij} in equation is perhaps a somewhat unfamiliar addition to the eddy-viscosity expression. It is necessary to make the expression applicable also to normal stresses (when $i = j$). The first part of involving the velocity gradients would yield the normal stresses.

$$\overline{u'^2} = -2\nu_t \frac{\partial \bar{u}}{\partial x}, \quad \overline{v'^2} = -2\nu_t \frac{\partial \bar{v}}{\partial y}, \quad \overline{w'^2} = -2\nu_t \frac{\partial \bar{w}}{\partial z}$$

And that is why turbulent this is the closure problem turbulent stresses need to be determined with the aid of a turbulence model. Here only models for turbulent motions are discussed. So, we have not yet introduced sub grid scale etcetera. So, this becomes irrelevant, this is for the turbulent motions and then we can say

$$-\overline{u'_i u'_j} = \nu_t \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) - \frac{2}{3} k \delta_{ij}$$

But we are writing additionally minus two-third k , this k is kinetic energy and δ_{ij} this δ_{ij} is Kronecker delta the term ν_t is the turbulent viscosity which is not known now, but it can be expressed in this way. So, if turbulent viscosity can be estimated, the term involving the Kronecker delta in this equation is so far unfamiliar. So, it is necessary to make explanation for this and k also the kinetic energy.

Now, if we as we said or proposed earlier fluctuating components can be expressed in terms of stress components and those stress components are defined as turbulent viscosity

multiplied by strain rate tensors and here all the velocity components are average velocity components. Now from this first part we can get u' when i and j are same; that means, $\overline{u'^2}$ is $-2\nu_t \frac{\partial \bar{u}}{\partial x}$

Similarly, there is no problem when we are writing $\overline{u'v'}$ or $\overline{u'w'}$, but again we are we will end up with a special situation if we write $i = j = 2$; that means, it is $\overline{v'^2}$ we will get. Again $-2\nu_t \frac{\partial \bar{v}}{\partial y}$ again when $i = j = 3$ for example, we will get $\overline{w'^2}$ and $-2\nu_t \frac{\partial \bar{w}}{\partial z}$.

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Whose sum is zero because of the continuity equation. However, all normal stresses are by definition positive quantities, and their sum is twice the kinetic energy k of the fluctuating motion:

$$k = \frac{1}{2} (\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$$

Inclusion of the second part of the eddy viscosity expression assures that the sum of the normal stresses is equal to $2k$. The normal stresses act like pressure forces (i.e. perpendicular to the faces of a control volume), and because like the pressure itself, the energy k is a scalar quantity, the second part of constitutes a pressure. Therefore, when equation is used to eliminate $\overline{u'_i u'_j}$ in the momentum equation and this second part can be absorbed by the pressure-gradient term so that in effect the static pressure is replaced as unknown quantity by the pressure $\bar{P} + \frac{2}{3}k$. Therefore the appearance of k in equation (5.21) does not necessitate the determination of k , it is the distribution of the eddy viscosity ν_t only that has to be determined. Here it may be mentioned that k can be linked to the intensity of turbulence which is given by

$$I = \sqrt{\frac{1}{3}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})} / U_\infty$$

Now, apparently there is no problem, but you know if we add up these three quantities, we will see sum is 0 because that will be minus twice ν_t into $\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0$ and that is continuity equation. So, sum is 0 because of the continuity equation; however, all normal stresses are by definition positive quantities and their sum turbulent normal stresses their sum is twice the kinetic energy k of the fluctuating motion.

Now, kinetic energy k of the fluctuating motion can be defined as

$$k = \frac{1}{2} (\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$$

Now we can see why the second part was included. Inclusion of the second part of the eddy viscosity expression assures that the sum of three normal stresses is equal to twice k .

If k is this then sum of these three normal stresses has to be twice k and that is what is done by adding these with the general expression for Reynold stress. The normal stresses act like pressure forces that is perpendicular to the faces of a control volume and because like the pressure itself the kinetic energy k is a scalar quantity, the second part constitutes like a pressure.

Therefore, when equation this equation is used for eliminating $u_i' u_j'$ over bar in the momentum equation and the second part can be absorbed by the pressure gradient can be absorbed by the pressure gradient term so that effect of static pressure is replaced as unknown quantity and we can write that as $\bar{P} + \frac{2}{3}k$ and we can after having added that we can say this is the pressure now.

Therefore, the appearance of k in this particular equation does not necessitate determination of k it is the distribution of the eddy viscosity only that has to be determined here it may be also mentioned. So, basically the Reynolds stress terms we can express this way which will be basically these terms will contribute strain rate tensors after multiplying with ν_t they will be contributing to normal and tangential Reynolds stress components.

And this term will contribute to the normal Reynolds stress component, but will be absorbed in the pressure term; that means, modified equation for Reynolds average modified Reynolds average Navier-Stokes equations this will be absorbed in pressure. So, separately k need not be determined while solving the equation.

However, here it may be mentioned that k can be linked to intensity of turbulence and intensity of turbulence in a flow field has to be defined and this is usually recommended based on experimental observation and the this is basically intensity is root over one third; that means, average of $\overline{u'^2} + \overline{v'^2} + \overline{w'^2}$.

So, root over that divided by U_∞ that is the free stream velocity in short $I = \frac{\sqrt{\frac{u'^2 + v'^2 + w'^2}{3}}}{U_\infty}$.

So, this is basically definition of turbulent intensity (I) and this is basically definition of turbulent kinetic energy as such we can you know from comparing these two, we can see that k by U_∞ square is $1.5I^2$ that can be found out right.

So, I is the turbulent intensity definition is root over one-third or root over average of u prime square bar v prime square bar w prime square bar entire quantity under root divided by U_∞ , that is the turbulent intensity.

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REYNOLDS AVERAGED FORM OF ENERGY EQUATION

Let us consider a three-dimensional situation,

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right]$$

Using Reynolds decomposition of velocity and temperature

$$u = \bar{u} + u', \quad v = \bar{v} + v', \quad w = \bar{w} + w' \quad T = \bar{T} + T'$$

we get

$$\begin{aligned} \bar{u} \frac{\partial \bar{T}}{\partial x} + \bar{v} \frac{\partial \bar{T}}{\partial y} + \bar{w} \frac{\partial \bar{T}}{\partial z} &= \frac{\partial}{\partial x} \left(\alpha \frac{\partial \bar{T}}{\partial x} - \overline{u'T'} \right) \\ &+ \frac{\partial}{\partial y} \left(\alpha \frac{\partial \bar{T}}{\partial y} - \overline{v'T'} \right) + \frac{\partial}{\partial z} \left(\alpha \frac{\partial \bar{T}}{\partial z} - \overline{w'T'} \right) \end{aligned}$$

The terms $\overline{u'T'}$, $\overline{v'T'}$, and $\overline{w'T'}$ thus, cause additional heat flux in the x , y and z directions respectively, due to turbulent motion. The total heat flux in the three directions will therefore be given by

Now, we will look at Reynolds averaged form of energy equation. So, energy equation if we write as $\frac{\partial T}{\partial t}$, this t is time of temperature $+ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right]$.

And we applied Reynolds decomposition u is $\bar{u} + u'$, v is $\bar{v} + v'$, w is $\bar{w} + w'$ and T is

time average temperature plus fluctuating component of temperature this is instantaneous temperature.

And then just what we did for the momentum equation we substitute T by $\bar{T} + T'$ we write conservative form of convective terms then substitute u by $\bar{u} + u'$, T by $\bar{T} + T'$, v by $\bar{v} + v'$ and w by $\bar{w} + w'$ and apply over bar and operate following the laws of statistics.

So, and if we think that the mean temperature value is not a function of time; that means, steady state we can drop the contribution of this component and we can write

$$\begin{aligned} \bar{u} \frac{\partial \bar{T}}{\partial x} + \bar{v} \frac{\partial \bar{T}}{\partial y} + \bar{w} \frac{\partial \bar{T}}{\partial z} &= \frac{\partial}{\partial x} \left(\alpha \frac{\partial \bar{T}}{\partial x} - \overline{u'T'} \right) \\ + \frac{\partial}{\partial y} \left(\alpha \frac{\partial \bar{T}}{\partial y} - \overline{v'T'} \right) &+ \frac{\partial}{\partial z} \left(\alpha \frac{\partial \bar{T}}{\partial z} - \overline{w'T'} \right) \end{aligned}$$

$\overline{u'T'}$ $\overline{v'T'}$ $\overline{w'T'}$ over bar they cause additional heat flux in the x y and z direction due to turbulent mixing, due to the turbulent motion. The total heat flux in three directions.

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$$\begin{aligned} q_x'' &= -\rho c_p \left(\alpha \frac{\partial \bar{T}}{\partial x} - \overline{u'T'} \right); \quad q_y'' = -\rho c_p \left(\alpha \frac{\partial \bar{T}}{\partial y} - \overline{v'T'} \right); \\ q_z'' &= -\rho c_p \left(\alpha \frac{\partial \bar{T}}{\partial z} - \overline{w'T'} \right) \end{aligned}$$

As in the case of turbulent transport of momentum, it is convenient to define an eddy viscosity or turbulent viscosity, to study the turbulent transport of thermal energy, a turbulent thermal diffusivity α_t can be defined.

The total heat flux in x, y and z directions can therefore be given as

$$\frac{q_x''}{\rho c_p} = -(\alpha + \alpha_t) \frac{\partial \bar{T}}{\partial x}, \quad \frac{q_y''}{\rho c_p} = -(\alpha + \alpha_t) \frac{\partial \bar{T}}{\partial y}, \quad \frac{q_z''}{\rho c_p} = -(\alpha + \alpha_t) \frac{\partial \bar{T}}{\partial z}$$

Now, we can write

$$q_x'' = -\rho c_p \left(\alpha \frac{\partial \bar{T}}{\partial x} - \overline{u'T'} \right); \quad q_y'' = -\rho c_p \left(\alpha \frac{\partial \bar{T}}{\partial y} - \overline{v'T'} \right);$$

$$q_z'' = -\rho c_p \left(\alpha \frac{\partial \bar{T}}{\partial z} - \overline{w'T'} \right)$$

So, this $-\overline{w'T'}$ multiplied with $-\rho c_p$ that is the additional contribution of the heat flux in z direction, similarly $-\overline{v'T'}$ multiplied with $-\rho c_p$ when it is multiplied that is the additional contribution in the y direction heat flux and $-\overline{u'T'}$ multiplied with $-\rho c_p$ over bar that is additional contribution in the x direction heat flux.

As in the case of turbulent transport of momentum it is convenient to define eddy viscosity or turbulent viscosity, we have already done that to study the turbulent transport of thermal energy a term is introduced here which is turbulent thermal diffusivity α_t . Physical diffusivity is alpha which is the property and this is not a property of the fluid, but property of the flow due to mixing is called alpha t turbulent thermal diffusivity.

The total heat flux in x, y and z directions now, therefore, can be redefined as $\frac{q_x''}{\rho c_p} = -\left((\alpha + \alpha_t) \frac{\partial \bar{T}}{\partial x} \right)$ equal to minus if this is expressed in terms of eddy diffusivity and the gradient then we can write $\frac{q_x''}{\rho c_p} = -\left((\alpha + \alpha_t) \frac{\partial \bar{T}}{\partial x} \right)$, $\frac{q_y''}{\rho c_p} = -\left((\alpha + \alpha_t) \frac{\partial \bar{T}}{\partial y} \right)$ and $\frac{q_z''}{\rho c_p} = -\left((\alpha + \alpha_t) \frac{\partial \bar{T}}{\partial z} \right)$.

So, these are now expressed in terms of temperature gradients and the entire temperature gradient and now we have little discussion about α_t .

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Like eddy viscosity, α_t is not a fluid property but depends on the state of turbulence. In fact the Reynolds analogy between heat and momentum transport suggests

$$\alpha_t = \frac{\nu_t}{\sigma_t}$$

The denominator σ_t is called turbulent Prandtl number. Experiments have shown that σ_t varies very little across the flow. Many models make use of σ_t as a constant. For the flow of air a value of 0.9 may be chosen.

Finally the transport equation may be written as

$$\begin{aligned} \bar{u} \frac{\partial \bar{T}}{\partial x} + \bar{v} \frac{\partial \bar{T}}{\partial y} + \bar{w} \frac{\partial \bar{T}}{\partial z} &= \frac{\partial}{\partial x} \left[(\alpha + \alpha_t) \frac{\partial \bar{T}}{\partial x} \right] \\ &+ \frac{\partial}{\partial y} \left[(\alpha + \alpha_t) \frac{\partial \bar{T}}{\partial y} \right] + \frac{\partial}{\partial z} \left[(\alpha + \alpha_t) \frac{\partial \bar{T}}{\partial z} \right] \end{aligned}$$

Like eddy viscosity, α_t is not a fluid property but depends on the state of turbulence. In fact, the Reynolds analogy between heat and momentum transport $\alpha_t = \nu_t$, but we obtain for closing the momentum equation $\frac{\nu_t}{\sigma_t}$, the denominator σ_t is called turbulent Prandtl number.

So, when we determine or find out ν_t to close the momentum equation that same ν_t if it is divided by σ_t which is Prandtl number we will get α_t which is turbulent or turbulent diffusivity or eddy diffusivity. Experiments have shown σ_t that is turbulent Prandtl number varies very little across the flow many models make use of σ_t that is turbulent Prandtl number as a constant. For the flow of air, a value of 0.9 may be chosen.

So, our task will be to find out correctly ν_t we will use that for closing momentum equation then we will divide the same value by σ_t , σ_t is turbulent Prandtl number and it is constant for a flow. For example, for here we can take 0.9 and then we will have α_t we will find out α_t , once α_t is known we will be able to close this equation which is basically time averaged energy equation

$$\begin{aligned} \bar{u} \frac{\partial \bar{T}}{\partial x} + \bar{v} \frac{\partial \bar{T}}{\partial y} + \bar{w} \frac{\partial \bar{T}}{\partial z} &= \frac{\partial}{\partial x} \left[(\alpha + \alpha_t) \frac{\partial \bar{T}}{\partial x} \right] \\ &+ \frac{\partial}{\partial y} \left[(\alpha + \alpha_t) \frac{\partial \bar{T}}{\partial y} \right] + \frac{\partial}{\partial z} \left[(\alpha + \alpha_t) \frac{\partial \bar{T}}{\partial z} \right] \end{aligned}$$

So, this α is physical property thermo physical property thermal diffusivity plus α_t which is basically due to turbulent mixing.

So, this is time averaged energy equation which has to be solved together with time averaged momentum equations; time average momentum equations will give us velocity components we will plug in those velocity components and then we can solve the energy equation for determining temperature.

But while we closing the momentum equation as I said determining ν_t is a challenge is a you know sort of involved task there are simple models, there are you know complex models in order to reach a more accurate simulation you must have heard about $k - \varepsilon$ model, you must have heard about large eddy simulation technique you have must have heard about DNS Direct Numerical Simulation technique.

So, we will if, we if time permits we will take up briefly those discussions, but you know ν_t has to be determined or estimated then momentum equation can equations can be closed and after ν_t having known ν_t if we know turbulent Prandtl number for a particular fluid like for air it is 0.9 then we can find out α_t which is thermal diffusivity turbulent thermal diffusivity, turbulent diffusivity and together with thermal diffusivity we can apply that to energy equation and solve for temperature.

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Thank you very much, thank you for your interest.