Computational Fluid Dynamics and Heat Transfer
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Lecture - 14
Introduction to Finite Element Method
(Elemental contributions and formation of Global Matrix)

Good morning everyone. Today we will continue with Finite Element Method.

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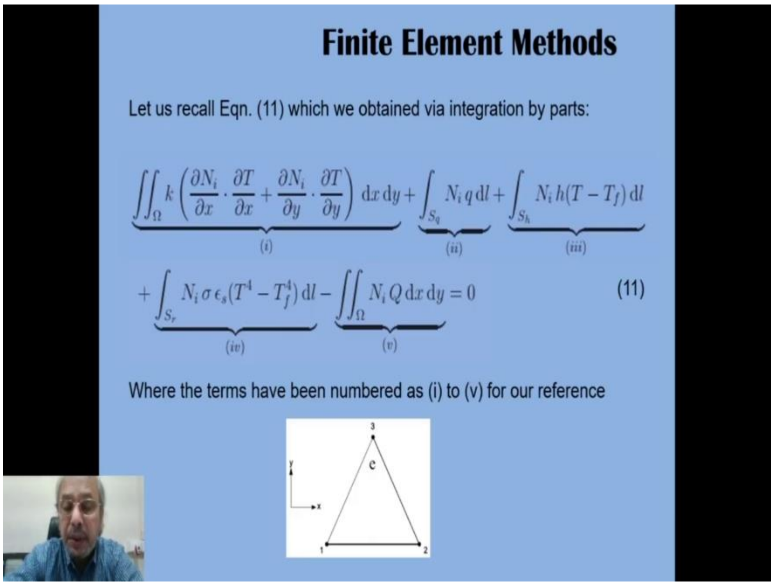

And if you recall, we obtained equation 11 via integration by parts and we aim at evaluating equation 11 on the element of interest. Basically, the element is a triangular element (3-noded triangular element) and we have already evaluated term one of equation 11 and term five. So, first term and fifth term we have already evaluated in the last lecture (in the last class). Today, we complete integration of other terms.

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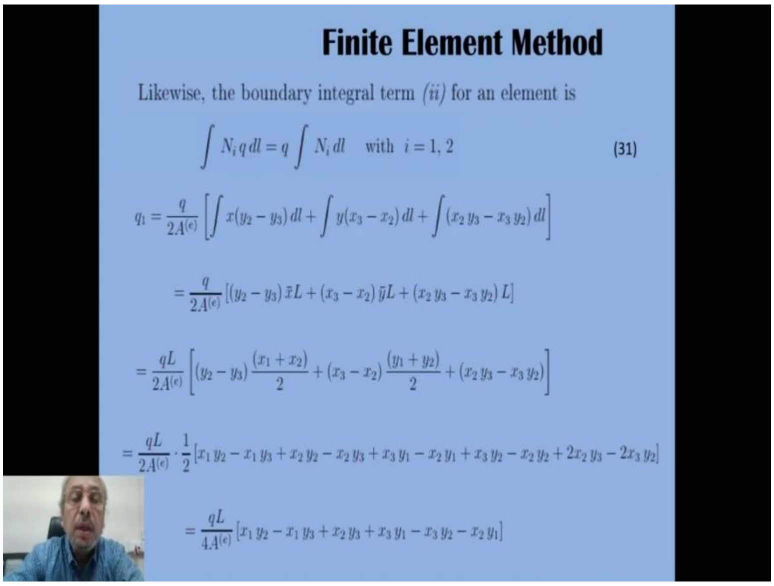

Now, next term we take up is basically term two and this is integration of heat flux on the confining surfaces that means, on the boundary of the domain. Now, if you look at the way we described the Greek topology. The way, the Greeks have been generated the triangular element are covering everywhere in the domain, but at the boundary, if you look at those triangular elements, it is obvious that line segments of the triangular element are falling on the boundary that means, boundary is covered by one side of the triangle.

So, all the triangles those are falling on the boundary, they are covering the boundary by one side of the individual triangles. So, basically, that means, that for the triangular element, which is falling on the boundary, we will have N_i in which is basically shape function at two points of each triangle that are following on the boundary.

So, when we perform the integral $\int N_i q dl$ we can write $q \int N_i dl$ where i is having a variation between 1 and 2. So, then we write N_1, the shape factor and as usual 2 into area ($2A^{(e)}$) is in the denominator that is determinant of the matrix, we are taking it outside the bracket, inside we are getting $\left[\int x(y_2 - y_3)dl + \int y(x_3 - x_2)dl + \int (x_2 y_3 - x_3 y_2)dl\right]$ and we are integrating over the line.

And in this line integral, we are following the method of Gaussian-quadrature where when the line integral is performed, the points may be identified, and weights may be assigned to the point. Here, we are we have taken $\bar{x}$ and $\bar{y}$ as those reference points which are basically a mean point between x_1 and x_2 and $\bar{y}$ is also y_1 plus y_2 by 2 $\left(\frac{y_1+y_2}{2}\right)$.

And then, we are performing this $\left(\frac{qL}{2A^{(e)}}\left[(y_2-y_3)\left(\frac{y_1+y_2}{2}\right)+(x_3-x_2)\left(\frac{y_1+y_2}{2}\right)+(x_2y_3-x_3y_2)\right]\right)$ integration and having performed the integration and carrying out the algebra, we get $\left(q_1=\frac{qL}{4A^{(e)}}[x_1y_2-x_1y_3+x_2y_3+x_3y_1-x_3y_2-x_2y_1]\right)$ where, L is a total length of the line element or the line segment. these terms $([x_1y_2-x_1y_3+x_2y_3+x_3y_1-x_3y_2-x_2y_1])$ can be reorganised as basically determinant of the matrix; that means, $2A^{(e)}$ $\left(2A^{(e)}=([x_1y_2-x_1y_3+x_2y_3+x_3y_1-x_3y_2-x_2y_1])\right)$.

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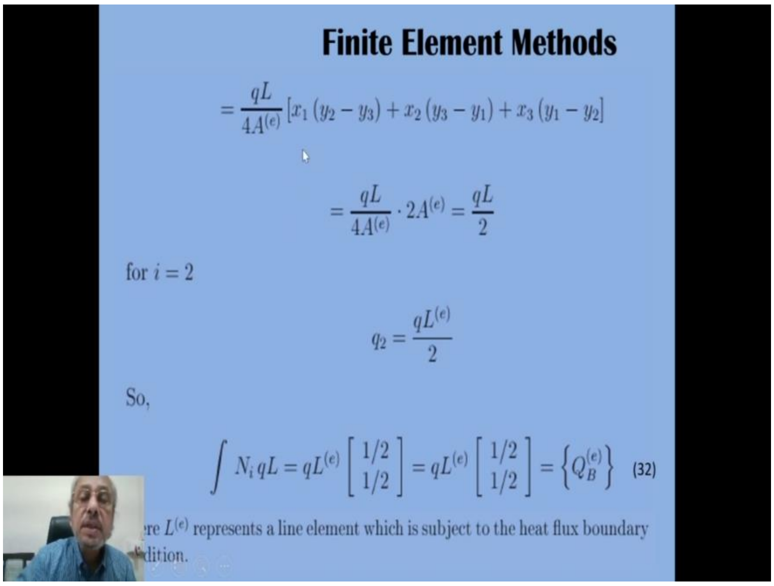

So, we have written that $q_1=\frac{qL}{4A^{(e)}}.2A^{(e)}$, so, $\frac{qL}{2}$ and next again, we I am going back by one slide, next again we substitute N_i, i equal to 2 so, N_2 and similarly, we integrate N_2 shape factor over the line segment and then again, we will find out $q_2=\frac{qL^{(e)}}{2}$ $\left(L^{(e)}=\text{ length of the line element}\right)$. So, this $(\int N_iqL)$ integration will give us $(\int N_iqL=qL^{(e)}\begin{bmatrix}\frac{1}{2}\\\frac{1}{2}\end{bmatrix}=\{Q_B^{(e)}\}$.

So, this is called $\{Q_B^{(e)}\}$. for the element. We have written also $L^{(e)}$ represents a line element which is subject to the specified heat flux condition. So, this is a contribution $\{Q_B^{(e)}\}$ per element is the contribution of the heat flux and the part of the boundary that the where constant heat flux is applied.

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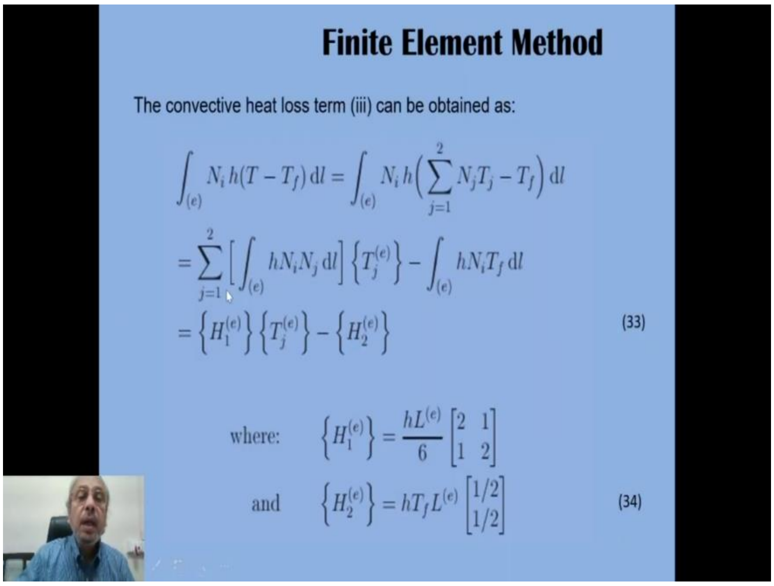

Now, we integrate the third term which is basically the convective heat loss. So, we have written here $\int N_i h(T - T_f)dl$ where N_i shape factor, h is a transfer coefficient and T_f is ambient temperature. Now, here again T is substituted by integral N_jT_j, where N_j is the shape factor and here also the triangular elements which are falling on the boundary, these are all line segments or lines, and these are having two points so, obviously, two shape factors and associated with the column vector for the temperature.

So, it is $(\sum_{j=1}^{2} N_jT_j - T_f)$. Now, we perform this $(\int N_i h(\sum_{j=1}^{2} N_jT_j - T_f)dl)$ integral in this $\left(\sum_{j=1}^{2}[\int_{(e)} h\, N_iN_j dl]\left\{T_j^{(e)}\right\} - \int hN_iT_f dl\right)$ manner and this $\left(\left\{T_j^{(e)}\right\}\right)$ is a column vector, here only two elements are there, and T_f the ambient temperature, this is also known we will come outside the integral.

Now, if we perform this you know two integrals, finally, we will get $\left\{H_1^{(e)}\right\}\left\{T_j^{(e)}\right\} - \{H_2^{(e)}\}$ and H_1 is what? H_1 is $\frac{hL^{(e)}}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and H_2 is $hT_f L^{(e)}\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$, again one array basically, a column matrix.

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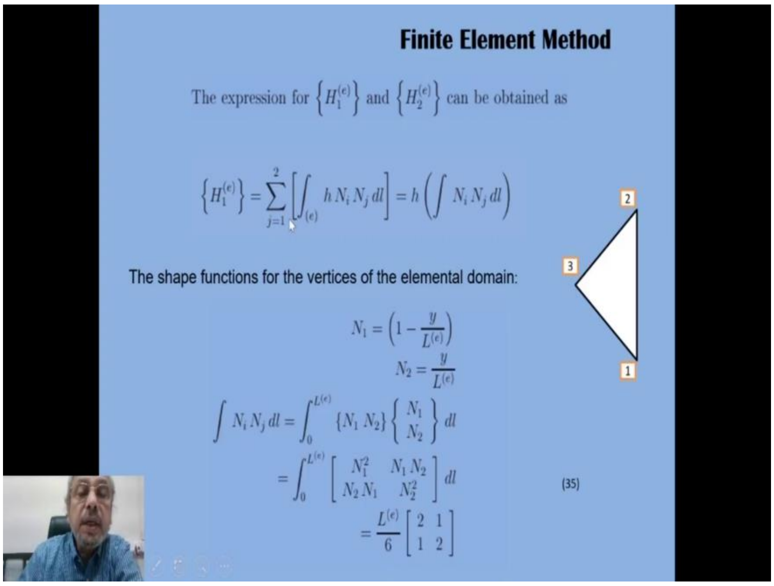

So, now evaluation of $\left\{H_1^{(e)}\right\}$ and $\left\{H_2^{(e)}\right\}$, we will discuss in the next slide. So, $\left\{H_1^{(e)}\right\}$ is as we said integral of $\sum_{j=1}^{2}[\int_{(e)} h\, N_i N_j dl]$ and we can write as $N_1 = \left(1 - \frac{y}{L^{(e)}}\right)$ and then, $\left(N_2 = \frac{y}{L^{(e)}}\right)$, very you know certain shape factors, then midpoint 1 and 2. If we say one is $\left(1 - \frac{y}{L^{(e)}}\right)$, two will be $\frac{y}{L^{(e)}}$ so that summation of the shape factors on anywhere on the segment has to be 1 that is a property of the shape factors.

So, we perform this $(\int N_i N_j dl)$ integral, you can see $\left(\int_0^{L^{(e)}} \{N_1\ N_2\}\begin{Bmatrix} N_1 \\ N_2 \end{Bmatrix} dl\right)$. Now, when we multiply, it becomes $\begin{bmatrix} N_1^2 & N_1 N_2 \\ N_2 N_1 & N_2^2 \end{bmatrix}$ and from here, after evaluating, we get $\frac{L^{(e)}}{6}$ and this shape factors finally, contributing to into $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ matrix. So, this is basically evaluation of $\left\{H_1^{(e)}\right\}$ and then, we will also find out $\left\{H_2^{(e)}\right\}$.

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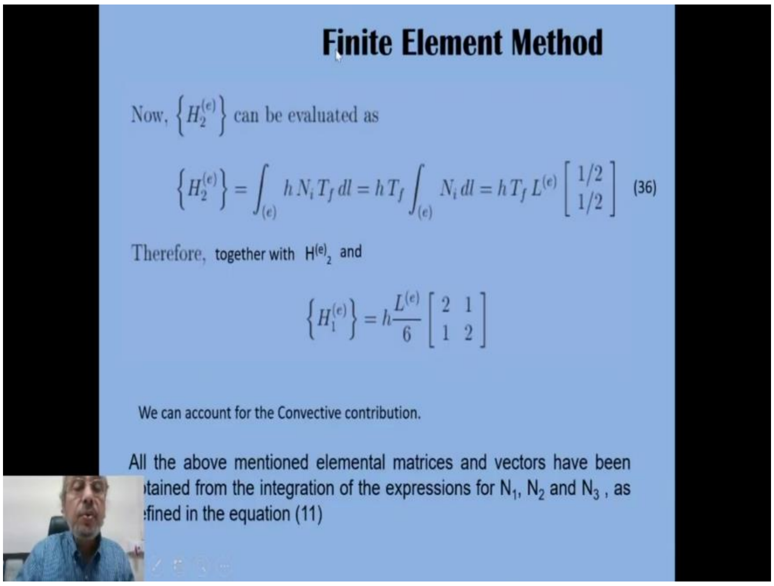

$\{H_2^{(e)}\}$ is $\int_{(e)} h\, N_i T_f dl$, h and T_f is known and constant rather here will come outside and then, it is $\int_{(e)} N_i\, dl$. So, $hT_f \int_{(e)} N_i\, dl = hT_f L^{(e)} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$. Basically, N_i evaluation when we integrate N_i, again this integration through Gaussian-quadrature philosophy has to be followed, just like what we did for the term two.

And we have seen there the final quantity becomes twice area of the triangle triangular element divided by four area of the triangular element $\frac{2A^{(e)}}{4A^{(e)}}$ that is half and half that forms a column matrix. Just not to confuse you, I will go back once to show this integration of N_i, how we did in the first slide (Refer Slide Time: 01:15).

So, this (Refer Slide Time: 01:15) is $\int N_i dl$. What I said finally we got whatever is a constant term, outside that is $\frac{2A^{(e)}}{4A^{(e)}}$ (Refer Slide Time: 05:22). So, basically, we get value of this integration as half. So, here also, we perform a similar integral $hT_f L^{(e)}$, we get outside the array and within the array, we get half and half.

So, therefore, together with $\{H_2^{(e)}\}$ and $\{H_1^{(e)}\}$, we can count, we can account for the convective contribution. So, $\{H_2^{(e)}\}$ and $\{H_1^{(e)}\}$ are the elemental contribution of convective energy transfer, convective heat transfer. All the above-mentioned elemental matrices and the vectors have been obtained from the integration of the expressions for N_1, N_2 and N_3 as defined in the equation 11.

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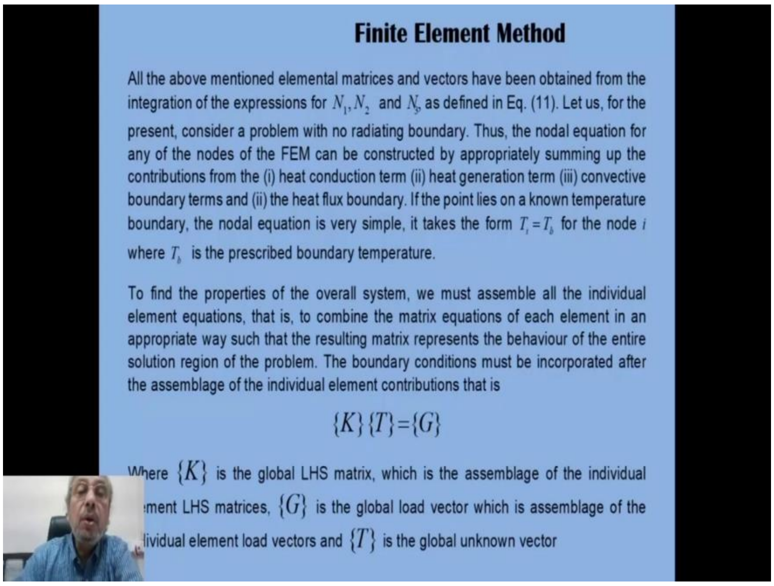

So, now we can say that all the above-mentioned elemental matrices and vectors have been obtained from the integration of the expressions for N_1, N_2 and N_3 as define in equation 11. Let us for the present; consider a problem with no radiating boundary. So, we have considered all the boundary conditions, but radiative boundary condition we have not evaluated, we will evaluate that little later.

So, let us imagine a domain, it does not have any boundary where radiative heat transfer is taking place. Thus, a nodal equation for any of the nodes of the FEM can be constructed by approximately summing up the contributions from the heat conduction term, heat generation term, convective boundary terms and the heat flux boundary.

If the points are such that few are lying on known I mean few are lying on your boundary where temperature is known, then obviously, for those points, we will prescribe known temperature. For example, T_i will be T_b for the node i where T_b is the prescribed boundary temperature. So, that is how the contributions will come for each term in equation 11 evaluated on elemental triangle I mean on individual elements triangles.

To find the properties of the overall system, we must assemble all the individual element equations that is, to combine the matrix equations for each element in an appropriate way.

Please note this very carefully. Such that the resulting matrix represents the behaviour of the entire solution region of the problem. The boundary conditions must be incorporated after the assemblage of the individual element contributions.

So, individual elements if there are 50 elements; from each 50 element, we will get one matrix multiplied by T_1, T_2, T_3 equal to some known vector. Now, we have to get all these contributions assembled in such a way that we can form a global matrix $\{K\}\{T\} = \{G\}$ where K is the global LHS matrix left-hand side matrix, which is the assemblage of the individual elemental LHS matrices.

So, that means, individual element we have seen, we get left-hand side matrix into as I was saying T_1, T_2, T_3 equal to the known vector on the right-hand side. So, if we assemble all the individual element left-hand side matrices, we get this global left-hand side matrix. $\{G\}$ is the global load vector. So, right-hand side known column vector is called load vector.

So, $\{G\}$ is a global load vector which is assemblage of the individual element load vectors and $\{T\}$ is a global unknown vector. As I mentioned earlier that each node we are identifying as $1, 2, 3$ I mean either 1 or 2 or 3 and based on an element, but all these nodes if we globally count maybe there are 100, 50 such nodes.

So, one node has an identification as a global node number also it as identification as a local node number, it maybe you know node one of 10^{th} element, node two of may be 31^{st} element and maybe node three of 21^{st} element. So, this relationship has also to be explained or fed into the program.

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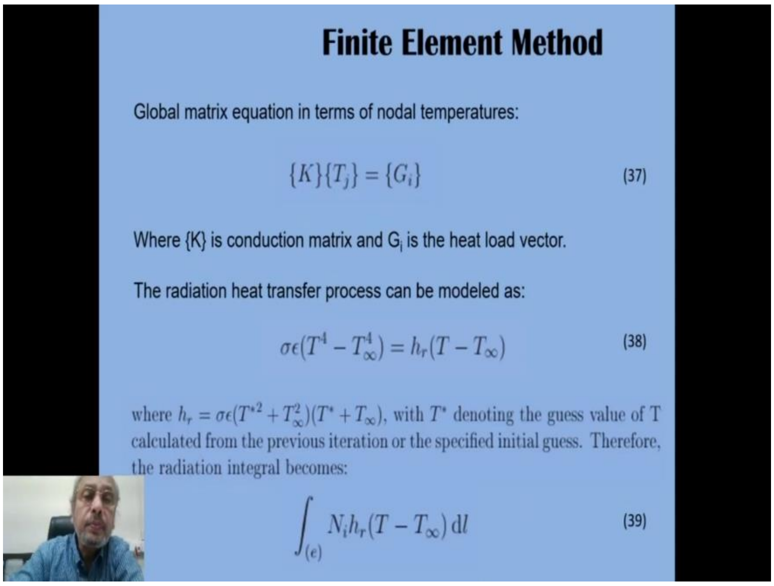

Now, global equation matrix in terms of nodal temperatures, we can just this is what I was saying $\{K\}\{T_j\}$ is may be 1 to 150, 1 to 1000 depending on number of global points in the domain equal to and also that will depend whether we discretize the domain by triangular elements or quadrilaterals that we will decide total number of points and $\{G_i\}$ is the heat load vector ($\{K\}\{T_j\} = \{G_i\}$). This we also explain in the last slide.

Now, the if we want to take care of radiation heat transfer, then obviously, we have to write down the equation which is a Stefan Boltzmann constant, $\sigma\epsilon(T^4 - T_\infty^4)$, T_∞ is ambient temperature, or you know this (T_∞) is equal to T_f here in this case to the power 4 equal to $h_r(T - T_\infty)$.

So, this ($\sigma\epsilon(T^4 - T_\infty^4)$) total radiative transport if we write down the way we write convective transport that means, some transport coefficient into the temperature difference, then it is possible to write this way h_r that is radiative heat transfer coefficient into $T - T_\infty$. As such there is no radiative heat transfer coefficient.

If the radiation is you know calculated this way, these are the properties emissivity is the surface property, sigma is the Stefan Boltzmann constant and then, the 4^{th} power of the temperature difference, but if we model this way ($\sigma\epsilon(T^2 + T_\infty^2)(T^* + T_\infty)$) , then h_r the radiative heat transfer coefficient will be defined obviously, we can find it out from here $h_r = \sigma\epsilon(T^2 + T_\infty^2)(T^* + T_\infty)$) where T^* is basically the guess value of temperature.

So, this T $(h_r(\boldsymbol{T} - T_\infty))$ if we take as T^*, then and this T $(\sigma\epsilon(\boldsymbol{T}^4 - T_\infty^4))$ also T^*, then this $(\sigma\epsilon(T^2 + T_\infty^2)(T^* + T_\infty))$ becomes a radiative heat transfer coefficient. So, T^* denoting the guess value of T calculated from the previous iteration on the specified initial guess or the specified initial guess and it will iteratively will correct till it reaches the final temperature. Therefore, the radiative total radiative heat transfer or the radiation integral that becomes integrated again over the line element or the line segment

$$\int_{(e)} h_r\, N_i(T - T_\infty)dl$$

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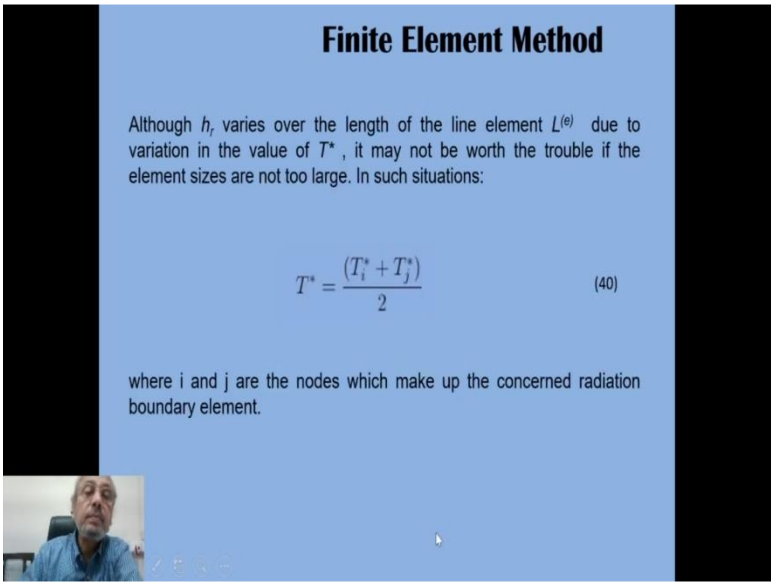

Although, h_r radiative heat transfer coefficient varies over the length of the line element due to variation in the value of T^*, it may not be worth the trouble if the element sizes are not too large. So, in such situation T^* can be taken again $\frac{T_i^* + T_j^*}{2}$ or T_1 plus T_2 that means, at the two points on the line segment divided by 2, or they are initial temperature by 2.

Or you know either initial guess or the way it is evolving T^* till it becomes T where i and j are the nodes which make the concerned radiation boundary element. The same thing if we say triangular element and the which are falling on the boundary, only their two points are lying on the boundary that means, one side is lying on the boundary, one segment is lying on the boundary and that will have two points that is the property of the boundary condition and property of the geometry.

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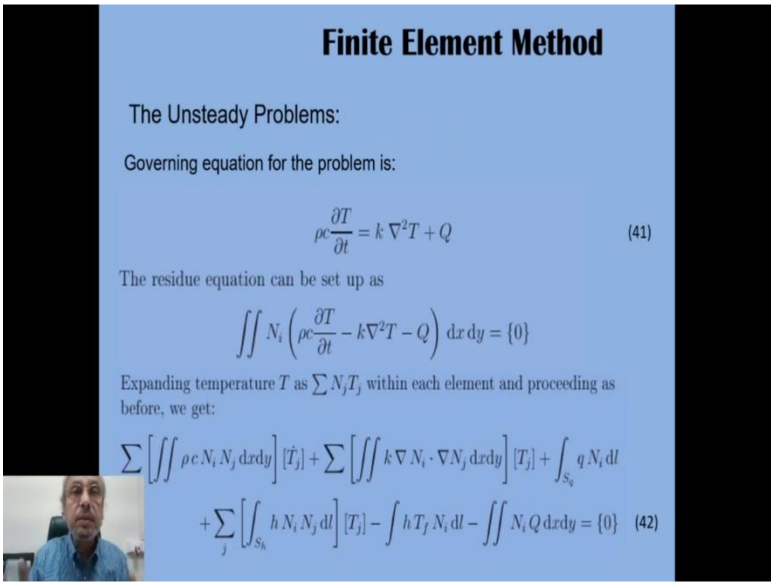

Now, see we can if we make the problem little more involved, we make it unsteady state problem, we can write down the governing equation as $\rho c\left(\frac{\partial T}{\partial t}\right) = k\nabla^2 T + Q$ and then again, we can integrate it over the entire domain with the help of shape functions. Basic idea is if shape function is correct, temperature is correct then eventually this ($\iint \left(\rho c\left(\frac{\partial T}{\partial t}\right) - k\nabla^2 T - Q\right) dx\, dy$) integral will produce 0. So, basically there will be no error, error will be minimised.

So, expanding temperature as $\sum N_j T_j$ within each element just the way we did in the earlier case steady state conduction case, we can see all the terms are again coming only additional term is unsteady arising out of unsteady term that is $\sum\left[\iint \left(\rho c N_i N_j\right) dx\, dy\right][\dot{T}_j]$. So, we are using here mass lumping concept and we are lumping this and multiplying it with basically this T_1, T_2, T_3, but its derivatives.

So, because these are, this ($[\dot{T}_j]$) quantities now time derivative. So, $\frac{\partial T_1}{\partial t}, \frac{\partial T_2}{\partial t}, \frac{\partial T_3}{\partial t}$ and ρc has been integrated over the domain using mass lumping concept.

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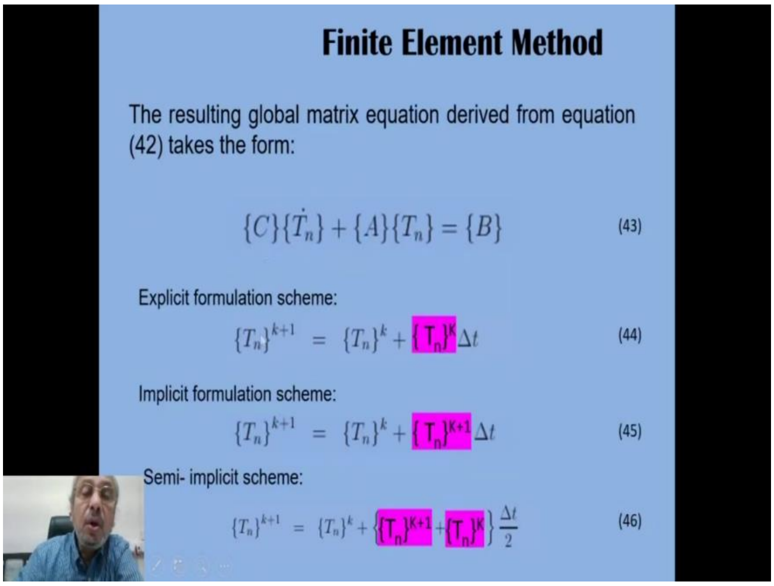

So, that then produces $\{C\}$ matrix basically, you know this $(\sum[\iint(\rho c N_i N_j) dx\, dy][\dot{T}_j])$ integration produces $\{C\}$ matrix into $\{\dot{T}_n\}$, this $\{\dot{T}_n\}$ is $\frac{dT_n}{dt}$ basically derivative quantities of the temperature, then $\{A\}\{T_n\}$ this is temperature array equal to $\{B\}$. So, this also can I mean from local, we can finally, form the global matrix and global vector and global heat flow of vector.

Now, to in order to solve this so, forming these $\{C\}$ matrix, $\{A\}$ matrix, $\{B\}$ matrix that is what is the major task from each element contribution, contribution from each element you know we get 3×3 matrix and then, this all these matrices are globally added up and form the global matrix and then again, this ($\{T_n\}$) is unknown temperature equal to the load vector, this time derivative of temperature ($\{\dot{T}_n\}$) also it is multiplied by the global matrix.

So, we have to now evaluate this (Eq. 43), we can go for explicit formulation. Since this is you know mass lumping concept has been used so, here we can write basically

$$\{T_n\}^{k+1} = \{T_n\}^k + \{T_n\}^K \Delta t.$$

Where, k is a time limit, all the terms shifted to the right-hand side.

Or the implicit formulation, implicit scheme we can write basically, I mean here, what we are doing? We are only writing this ($\{\dot{T}_n\}$) temperature derivative that means, $\{\dot{T}_n\}$ can be written as $\frac{\{T_n\}^{k+1}-\{T_n\}^k}{\Delta t}$ and how that $\{T_n\}^{k+1}$ can be evaluated in terms of $\{T_n\}^k$and Δt that basic scheme we have written here. It can be, this is (Eq. 44) I mean we already know; this is purely explicit, this (Eq. 45) is fully implicit. And if we write instead of writing this 44 or 45, if we write this ($\{T_n\}^{k+1}$ of 45) as average of two-time levels that means, $k+1$ and k by 2, then it becomes we have written here semi-implicit, or this is basically Crank-Nicolson implicit. So, it depends the way we want to progress.

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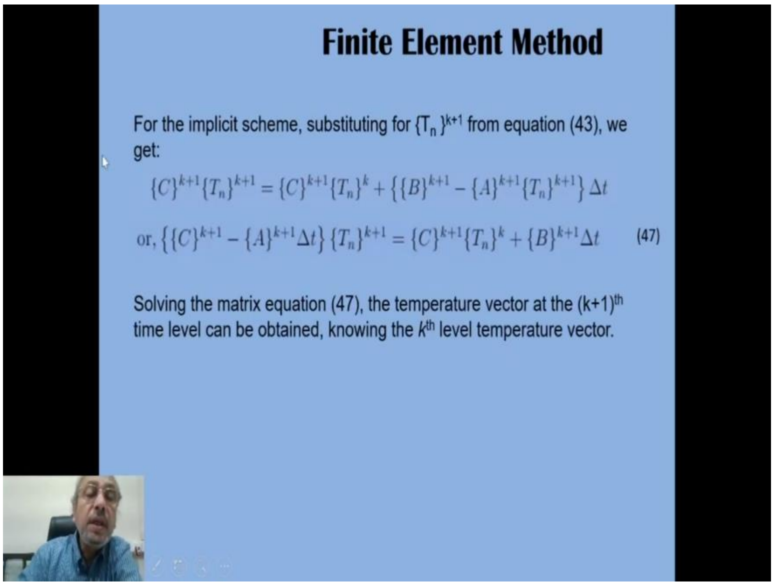

Say for example, if we want to do it through implicit formulation. Then in this ($\{C\}\{\dot{T_n}\} + \{A\}\{T_n\} = \{B\}$) in this relationship, we can substitute what we have written for implicit formulation that is $\{T_n\}^{k+1} = \{T_n\}^k + \{T_n\}^{k+1}\Delta t$. And if we do that, then you know we will get equation 47 finally, and equation 47, you can see can be solved for $k+1$ level temperatures, we can solve for $k+1$ level values from k^{th} level values.

So, solving matrix 47, the temperature vector and the $k+1$ time level can be obtained knowing the k^{th} level temperature vector. So, that is an overall solution strategy.

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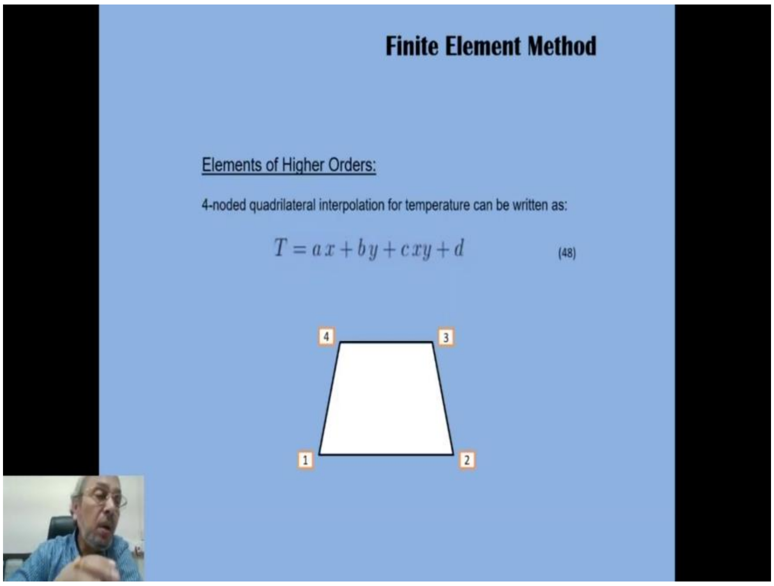

Now, instead of 3-noded element, we can have 4-noded quadrilaterals where T can be expressed as $ax + by + cxy + d$, arbitrary quadrilateral and again shape functions have to be determined which are you know at different points, shape functions will be N_i at this (at point 1) point will be having value 1, other points it will be having 0.

N_2 at this point it will be having value 1, other points it will be having value 0, similarly, N_3 and N_4 and first of all, we have to find it out the way we find out found out for triangular element and then, we have to perform the integration.

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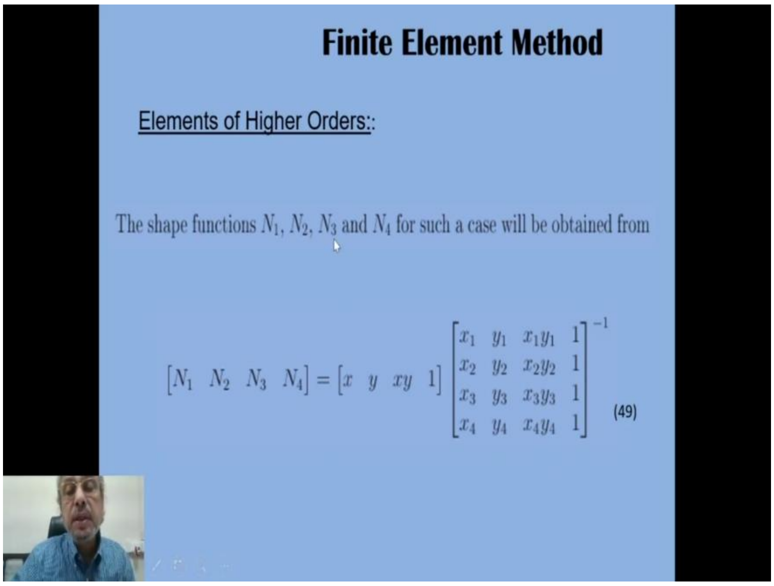

So, the shape functions N_1, N_2, N_3, N_4 for such a case will be obtained from basically, if you recall we obtain shape functions for triangular element as multiplication of $x, y, 1$ row matrix multiplied by inverse of $x_i\ y_i\ 1, x_j\ y_j\ 1, x_k\ y_k\ 1$ inverse of that matrix.

So, here also, we will following almost similar procedure, it will be $[x\ y\ xy\ 1]$ this is the row matrix multiplied by $\begin{bmatrix} x_1 & y_1 & x_1y_1 & 1 \\ x_2 & y_2 & x_2y_2 & 1 \\ x_3 & y_3 & x_3y_3 & 1 \\ x_4 & y_4 & x_4y_4 & 1 \end{bmatrix}^{-1}$ in a similar process, we can find out the shape functions.

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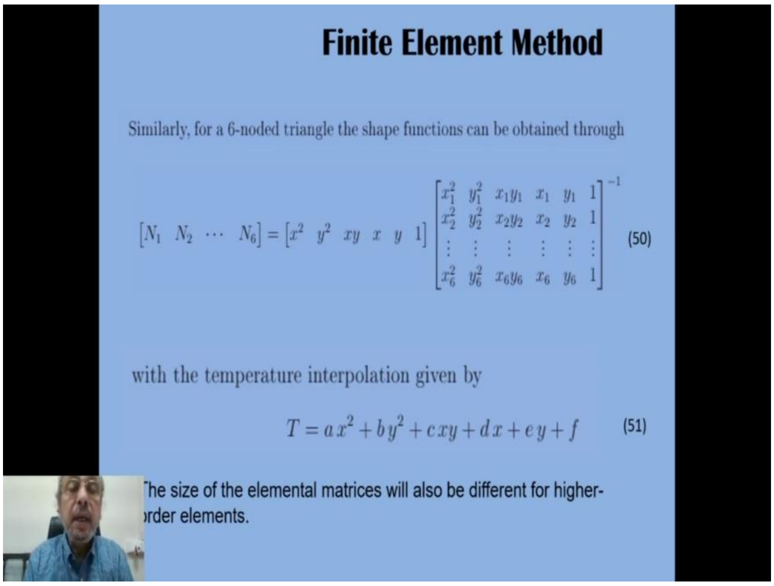

Similarly, if we take a 6-noded triangle the shape function just we add you know three more nodes on rectangular element and then the shape functions can be given by Eq.50 (Slide Time: 34:48). So, there the temperature interpolation is given by this ($T = ax^2 + cxy + dx + ey + f$). The size of the elemental matrices will also be different for higher-order elements.

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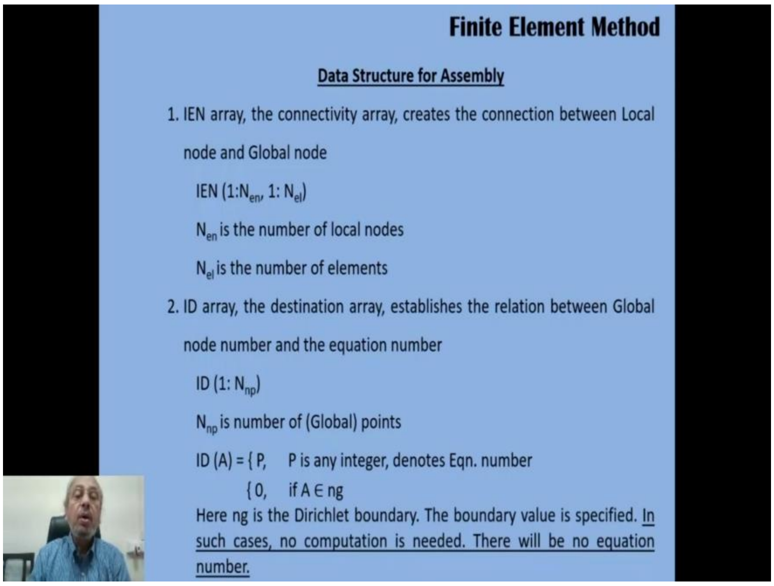

So, now, we have understood overall formulation strategy of finite element method and how it differs from finite difference and finite volume method. We mentioned finite volume is finite difference is basically you know difference quotient-based method whereas, finite volume is basically integral method, but the weighting function is one for each integral.

And finite element is again integral method to minimise the error and here, this is also a subdomain method, but this weighting function has to be determined and when we use a Galerkin weighted residual method, this weighting function is exactly equal to shape function.

So, how to really; now, you can see that or we can understand that in this entire formulation, the relationship between the nodes the local nodes that means, local nodes of the elements and the global node number, these are interrelated and these have to be sort of identified and store somewhere and this will help us to form the global matrix, global unknown vector and global load vector.

So, this bookkeeping is done through you know some data structure, I have given here a very elementary way, very elementary way of understanding and very you know easy example to follow. See if IEN is an array which is connectivity array, creates the connection between local node and global node.

So, IEN node will be written in this way $IEN(1\!:\!N_{en}, 1\!:\!N_{el})$ whereas, N_{en} is the number of local nodes and N_{el} is the number of elements. Just you know listen to this, we will take up an example, then it will be clear. Now, ID is an array that is called destination array, this establishes the relationship between global node number and the equation number.

So, IEN array is the local node number and the element number, relationship between them and second is an array which is basically you know will identified by the global points that means, and it will assign some value to the global points.

This will assign either value P or 0. P is any integer denotes the equation number from which equation it has come, or it connects to you know some specific equation and that connection, and 0 is if it is not connected to any equation and usually, these are Dirichlet boundary conditions. The boundary values are always specified in such cases, no computation is needed, there will be no equation number.

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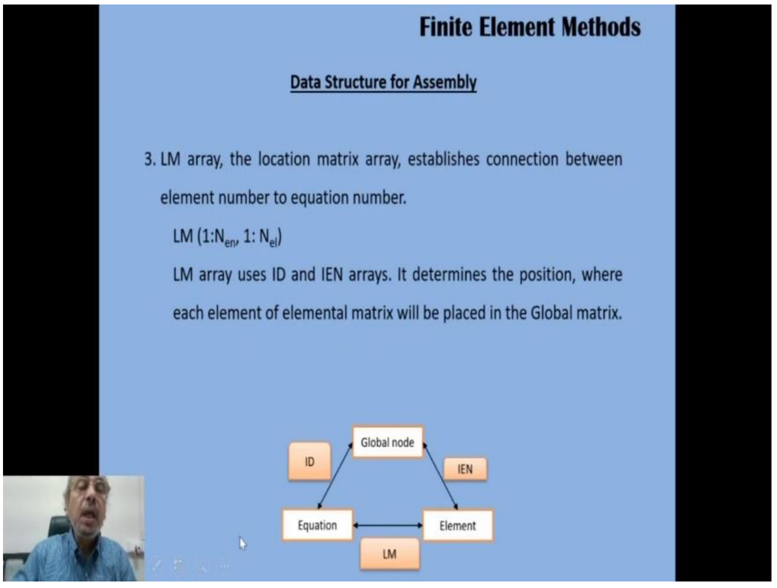

And another array is LM array; the location matrix array establishes connection between element number to equation number through the local node number. So, that is why here this is en which is basically local node number and el which is basically the element number. So, LM array uses ID and IEN arrays. It determines the position where each element of elemental matrix will be placed in the global matrix.

So, the relationship is established through these arrays. So, this is global node, this is equation, this is element, these two are connected by LM array, these two are connected by IEN array and these two are connected by ID array (refer Fig. in Slide Time: 40:40).

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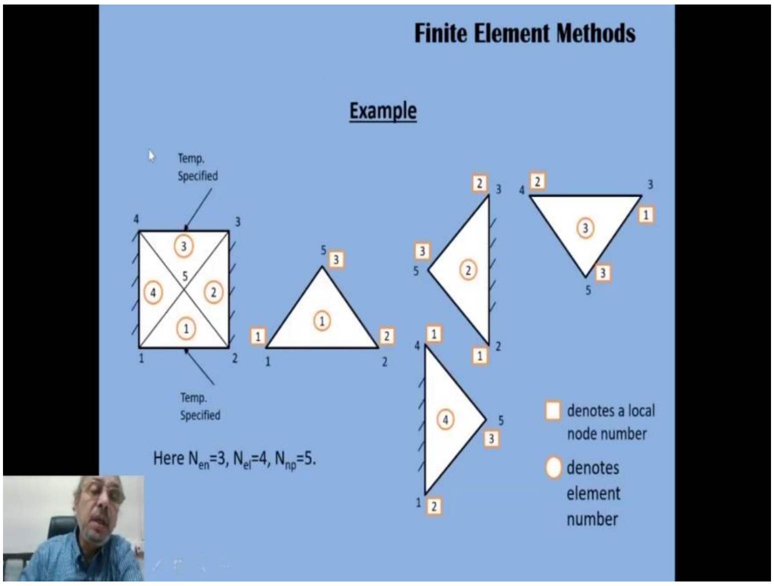


Now, we will take very easy example. See there is a square domain, and it has four triangles, and one side boundary is specified that is Dirichlet boundary condition, we know the temperature here. Another side, again temperature is specified. So, these two sides (top and bottom) are temperature specified. These two sides (left and right) may be insulated boundary.

Now, since the domain is divided by four triangular elements; element 1, element 2, element 3, element 4 and global nodes, first let us understand one this is obviously, globally 1st node, this is globally 2nd node, this is globally 3rd node, this is globally 4th node and this is globally 5th node (refer Fig. 1 of Slide Time: 41:53).

So, in the overall geometry, we have only these (refer Fig. 1 of Slide Time: 41:53) five global nodes 1, 2, 3, 4, 5. Now, if we separate out elements for example, element 1, element 1 we are separating out and we are and any element will have only three nodes 1, 2, 3 (refer Fig. 2 of Slide Time: 41:53) so, this square (rectangle in bottom right of Slide Time: 41:53) denotes local node number, and this circle (circle in bottom right of Slide Time: 41:53) denotes element number.

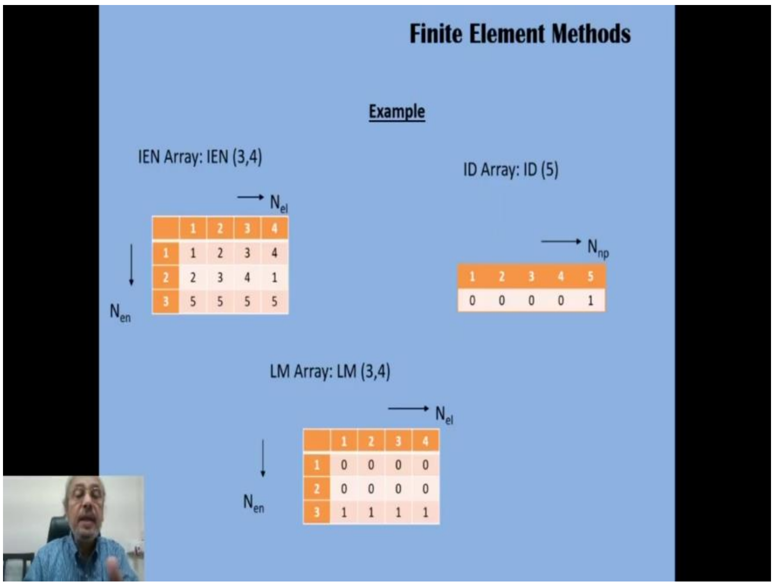
So, this (refer Fig. 2 of Slide Time: 41:53) is 1st element, we have taken out 1st element through the circle. Local node number 1, local node number 2, local node number 3 (refer Fig. 2 of Slide Time: 41:53) and this node, its global identification is 1, this node its global identification 2, but this node global identification is 5 (refer Fig. 2 of Slide Time: 41:53), then we are going at the 2nd element.

2nd element again, it you know element circle denotes the element number, this is 2nd element (refer Fig. 3 of Slide Time: 41:53) and for the 2nd element, the local nodes are 1, 2, 3, but the global nodes, this is global node 2, this belongs to you know this side (right side) so, basically this 2 (refer top side of Fig. 3 in Slide Time: 41:53) is a local node, but its global identification is 3, global node. And last, you know 3 is the local node number, but its global node number is 5.

Now, take the 3rd element (refer Fig. 4 in Slide Time: 41:53). This is the element number again 1, 2, 3 these are the local node numbers, but with respect to global node numbers, this is 3, this is 4, this is 5 (refer Fig. 4 in Slide Time: 41:53). Finally, the 4th element, element 4, 1, 2, 3 are the local node numbers, but global node numbers are 1, 5, 4 (refer Fig. 5 in Slide Time: 41:53).

So, the N_{en} that is number of nodes local nodes 3, number of elements 4 and number of global node numbers are 5 and then, when you read you have to use this slide for the next slide without looking at this slide (Slide Time: 41:53), next slide cannot be made.

(Refer Slide Time: 45:44)



So, back and forth, you have to keep looking. If you form IEN array, IEN array this direction (horizontal right arrow) is basically number of elements and this direction (vertically downward arrow) is number of nodes. So, first two, 1st, 2nd, 3rd, 4th, this four elements 1st, 2nd, 3rd, 4th and for each element, we have node numbers 1, 2, 3 but at this position (2nd row 2nd column of table one in Slide Time: 45:44), we will write the global node number.

So, when 1 is the element number and 1 is a local node number of the 1st element, then what is the global node number of that? it is 1. So, this (refer Fig. 2 of Slide Time: 41:53) is the element number 1 and its local node number 1 so, what is its global node number? That is also 1. Then, element 2, its local node number 1, so, element 2, its local node number 1, what is its global node number? It is 2. So, global node number 2. Element 3, its local node number 1, let us look at it, element 3, local node number 1, but its global node number 3. So, the here, entry will be 3. Similarly, element 4, element 4 its local node number 1 is this node, but this node is basically globally number 4. So, globally number 4 [This paragraph explains the 2nd row entries of table 1 in Slide Time: 45:44].

Similarly, you know again the 1st element local node 2, what are the global node numbers? 2nd element, 2nd local node number what are the, what is the global node number? So, 3rd element again, 2nd local node number what is the global node number?

So, this way, we have to fill out this table. So, this is this array is called IEN array. Second array is ID; ID as I said earlier that it whether the nodes are connected with the equation or not that is what it writes. So, 1, 2, 3, 4, 5 here, 1 we can see it is falling on specified temperature boundary 3, 4 is also falling on specific temperature boundary.

So, no computation is needed, it is directly in known and for 5 computation is needed and we are writing it 0, 0, 0, for 1, 2, 3, 4 nodes and for 5th node, we are writing 1, 1 is the equation number. I have just given, it can be any you know the way we define equation numbers it is connected to equation and that equation, but 1 means it is connected to equation.

Similarly, now we will go for third array, LM array. As we defined here (refer Fig in Slide Time: 40:40), this is basically LM array and here, this is nodal node points (columns of table 3 in Slide Time: 45:44), this is (rows of table 3 in Slide Time: 45:44), the local node number of the element, but whether that node of that element needs equation or not. So, 1st element, 1st node, does not need equation. 2nd, similarly2nd element, 2nd element 1st node it also it is node 2, does not need equation falling on the boundary. 3rd element, again node 1, it is point 3 falling on the boundary does not need equation. So, that is how we are progressing.

Now, sorry this (table 3 in Slide Time: 45:44) is horizontal direction is N_{np}, N_{np} is I have done a mistake. N_{np} is the global node nodal node points, N_{np} is basically identification of, You read this again because without looking into the previous slide, I mentioned you earlier, I meant one may commit mistake like I did N_{np} is a global node number.

So, 1st global node number and it is very clear, then we do not need equation, 2nd we do not need equation, 3rd we do not need equation, 4th we do not need equation, point number 5 we need equation. So, point number 5, we need equation, and this is basically element node number whether we need equation for that particular node need equation or not.

Here, 1st element, 1st node we do not need equation, 2nd element 1st node so, all elements 1st node, we do not need equation. So, that means, this element 1st node, local 1st node is falling here, we do not need equation, 2nd element, local 1st node that means, point 2 we do not need equation. 3rd element 1 means a global 3 we known. 4th element 1 means global 4 known, we do not need equation.

Similarly, 1st element node 2, 2nd element node 2, 3rd element node 2, 4th element node 2 we have to see whether we need a equation, but 1st element node 3, 2nd element node 3, 3rd element node 3, 4th element node 3 all these nodes are basically 5th node [refer table 3 in Slide Time: 45:44]. 1st element node 3 is 5th, 2nd element node 3 is 5th, 3rd element node 3 is node 5, 4th element node 3 is node 5 and we need the equation for all this.

And this is how you know these arrays are formed. These arrays governed formation of as I have said formation of the global matrix. The connectivity array, this creates connection between local node and global node and these arrays are needed to form the global matrix, then global unknown vector and global known vector or the load vector.

(Refer Slide Time: 54:08)

So, as I said that this effort was to give an introduction to finite element method, we have not taken up any specific problem. If we take up a specific problem thing will be clearer. If we sort of get a few more lectures after covering the basic whatever is needed for this course, if we get time, we will fall back with I mean specific problem so that you know you can get hands on experience. But now, you know overall how a problem can be handled by finite element methods.

And this my class notes that I used, these are prepared from these two books very well-known books, one is Fundamentals of the Finite Element Method for Heat and Fluid Flow, Lewis Nithiarasu and Seetharamu and the second is Introduction to Finite Elements in Engineering, Chandrupatla and Belegundu, this is also a very well-known book. So, you can consult these two books, I will also circulate the notes.

And as I said the basic idea of formulating a problem based on finite difference, finite volume and finite element that was the you know initial target of our first few lectures or the first three modules you can say. 1^{st} module was based on how finite difference works and what is a best way to implement finite difference philosophy.

Then finite volume and then now, we have completed finite elements. From here onwards, we will go for you know full-fledged solvers for different flow and heat transfer situations. I will stop here today. Thank you very much. Thank you.