Muffler Acoustics - Application to Automotive Exhaust Noise Control
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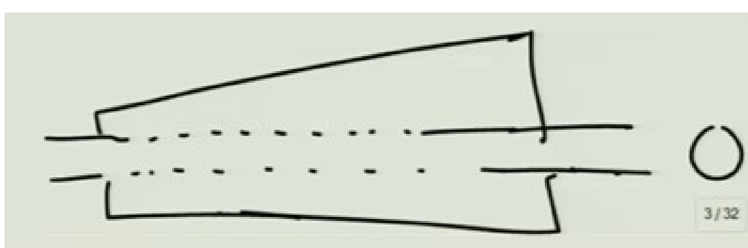
Lecture - 49 and 50
ITM analyses of Multi-pass perforated mufflers and Conical Concentric Tube Resonators (CCTRs)

Welcome back to this NPTEL course on Muffler Acoustics. So, we what we will do is that we will combine lectures 4 and 5 for this week of week 10 and you know this will be the final lecture on perforate muffler components. Because you know so far we have not really analyzed a three dimensional analysis of mufflers. So, next week we must start that and let us see how far we can go and maybe just get a glimpse of precipitate mufflers in the final week for this course.

But coming back to this lecture; what we will do is that we will focus on two methods;

- ITM Integrate Transfer Matrix
- CCTR Conical Concentric Tube Resonator

One the first one is integrated transfer matrix approach ok. Other one is the integrate transfer matrix approach and another one is the things we intend to squeeze out squeeze you know time to basically analyze you know as many configurations as possible. So, I thought it is a good idea to talk about the CCTR Conical Concentric Tube Resonator configuration.



You know I if you remember if you recall we, I promised to present some of the equations and analyze configurations muffler configurations like this one where you have this tapered pipe ok. We will take out time conical concentric tube resonator used a lot in 2 wheeler applications ok.

So, let us see how we go about it, but first main focus is to demonstrate to you an integrated transfer matrix approach using MATLAB thing on a very complicated a much more complicated system which has not only has perforated tubes, but also has baffles. So, I will present to you one such configuration and talk about what how does the transfer integrated transfer matrix looks like.

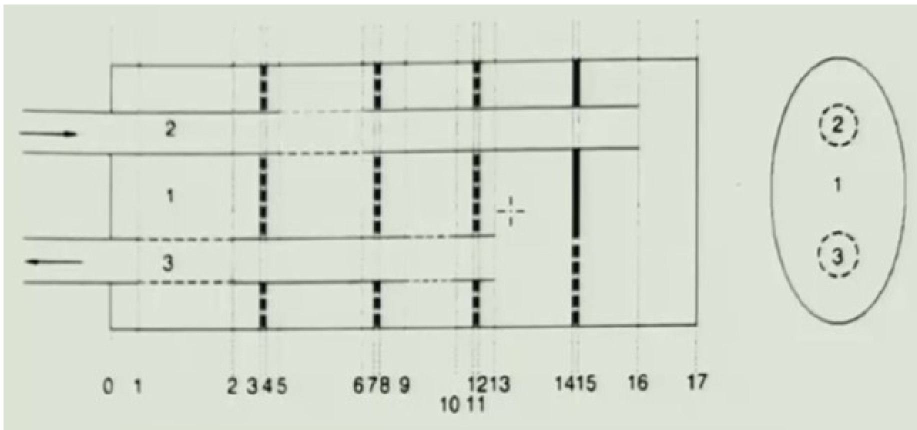


Figure 1: Schematic Diagram of Elnady *et al.* [1] muffler

So, you know for that to this end what we will do is that consider the configuration. So, this is the integrated transfer matrix approach what we will do for a flow reversal muffler configuration. But as you see it has vertical it has you know perforated pipes, but it also has baffles it also has vertical baffles.

So, how do we; how do we go about analyzing it? So, what integrated transfer matrix is nothing but it is just a very sophisticated form of network analysis whatever integrated ITM method can do, the same thing can very well be accomplished by network analysis what we presented in the beginning of this lecture.

You know analyze or derive some sort of a characterization matrix representation transfer matrix impedance matrix. So, whatever it is for a given muffler element and then apply junction loss and you know relate the unknown variables or some basically relate all the variables or express all the variables in terms of a few unknown variables and then take out certain a fraction or chunk of that matrix to get the overall transfer matrix or impedance matrix depending on the situation.

So, integrate transfer matrix does all these things in a bit perhaps in a bit more sophisticated manner I guess. What it does basically you know is that let us consider this muffler configuration, this was analyzed by Elnady from Egypt and other and his colleagues in Europe and this configuration promises to deliver a very good transmission loss performance especially in the low frequency region where other mufflers kind of do not do that well.

And the you know because of the perforates and the even though it is cross flow configuration because it is a perforated muffler component cross flow configuration is there back pressure is also not that high. So, it is like a win - win situation and I will draw I will plot out the transmission loss last. And you will see that in the presence of mean flow even a nominal mean flow of 0.05 will dramatically you know uplift the troughs that otherwise would occur.

So, basically all automotive mufflers have mean flow. So, automatically the performance of such complicated mufflers does goes up in the low frequency range in the presence of flow. So, coming back to this, this is the inlet what we are seeing this is the inlet and this is the outlet. What ITM does is that it divides now entire muffler into number of cross sections.

So, 0 means you start from here and then you go up to the section 1 basically relates things from 0 to 1. So, how does it do? It divides this entire thing into 3 parts you know whether tubular elements is there here and here and the annular cavity. So, it basically writes you know things something like this.

$$\begin{bmatrix} S_{1,0} \\ S_{2,0} \\ S_{3,0} \end{bmatrix}_{6=1} = \underbrace{\begin{bmatrix} [P_{1,0\,1}]_{2\times2} & [O]_{2\times2} & [O]_{2\times2} \\ [O]_{2\times2} & [P_{2,0\,1}]_{2\times2} & [O]_{2\times2} \\ [O]_{2\times2} & [O]_{2\times2} & [P_{3,0\,1}]_{2\times2} \end{bmatrix}}_{A}{}_{6\times6} \begin{bmatrix} S_{1,1} \\ S_{2,1} \\ S_{3,1} \end{bmatrix}_{6=1} \quad (1)$$

$$\begin{bmatrix} S_{1,1} \\ S_{2,1} \\ S_{3,1} \end{bmatrix}_{6=1} \underbrace{\begin{bmatrix} [E_{1,3\,1\,2}(1:2,1:2)]_{2\times2} & [O]_{2\times2} & [E_{1,3\,1\,2}(1:2,3:4)]_{2\times2} \\ [O]_{2\times2} & [P_{2,0\,1}]_{2\times2} & [O]_{2\times2} \\ [E_{1,3\,1\,2}(3:4,1:2)]_{2\times2} & [O]_{2\times2} & [E_{1,3\,1\,2}(3:4,3:4)]_{2\times2} \end{bmatrix}}_{B}{}_{6\times6} \begin{bmatrix} S_{1,2} \\ S_{2,2} \\ S_{3,2} \end{bmatrix}_{6=1} \quad (2)$$

So, S 1 is just a state variable you know it comprises of P_1 v_1, S_2 0 comprises of P_2 v_2, S 3 comprises of P_3 v_3. And then the 0 means the null matrix and P matrix means is that the transfer matrix in this region ok. So, transfer matrix for the tubular element that comprises of cos j sin k_0 l j and though then the regular kind of thing and similarly here also it comprises the same thing and here also the same thing.

So, it is basically essentially a kind of a diagonal matrix where and all the non diagonal entries are null matrices these are your regular cos j sin k_0 l and that sort of thing for tubular elements. Now, come to the part between 1 to 2. Now this comprises a bit more complicated because 1 and 3 elements are conjoined or kind of coupled through a perforated thing, but 2 has this 2 just has a perforated thing.

So, 2 we do not have any problem here and the element I mean the relation between 1 and 3 is such that it is you take chunks of certain the overall 4 cross 4 matrix and dump in this first two rows and first two columns and remaining chunk can be dump can be sorry I am so sorry. So, the first two rows and two columns are somewhere in here and then it is remaining part is somewhere here and this is here and this is sort of here.

Here it requires a little bit of algebra I am just showing you the full picture one needs to work it out. So, basically the E matrix is really a 4 cross 4 matrix when you are taking the first two rows first two columns and then last two rows last two columns and then you know putting it and like this you know they will be the size will be 16 4 cross 4.

We are dumping it elsewhere and similarly now we come to section 2 to 3. So, 2 to 3 when you do that there is still no problem in the sense that you have tubular elements here, here, here. So, it is again the diagonal matrix as you see, but now comes an important part when you go from 3 to 4. Now, you have a vertical baffle.

Now, this what it does as its clear in the presence of mean flow this vertical baffle will also have its own effect in raising the trough the dissipative troughs ok and that is very very important. And because you know you have a nonzero react dissipative part and it will help in raising the troughs.

$$\begin{bmatrix} S_{1,2} \\ S_{2,2} \\ S_{3,2} \end{bmatrix}_{6\times1} = \underbrace{\begin{bmatrix} [P_{1,2\_3}]_{2\times2} & [O]_{2\times2} & [O]_{2\times2} \\ [O]_{2\times2} & [P_{2,2\_3}]_{2\times2} & [O]_{2\times2} \\ [O]_{2\times2} & [O]_{2\times2} & [P_{3,2\_3}]_{2\times2} \end{bmatrix}}_{C}{}_{6\times6} \begin{bmatrix} S_{1,1} \\ S_{2,1} \\ S_{3,1} \end{bmatrix}_{6\times1} \quad (3)$$

So, and this is related by a B matrix which is nothing but P_2 you know or rather P_4 minus P_3 the pressure just you know across this across the first section across the baffles is equal to $\rho_0 C_0$times the velocity. So, it leads us to this kind of a transfer matrix across 3 and 4 for the first annular region and for the tubular region it is just a conventional matrix and everything else is 0.

$$\begin{bmatrix} S_{1,0} \\ S_{2,0} \\ S_{3,0} \end{bmatrix}_{6=1} \underbrace{\begin{bmatrix} [E_{1,10\ 10}(1:2,1:2)]_{2\times2} & [O]_{2\times2} & [E_{1,10\ 10}(1:2,3:4)]_{2\times2} \\ [O]_{2\times2} & [P_{2,0\ 1}]_{2\times2} & [O]_{2\times2} \\ [E_{1,1\ 0\ 10}(3:4,1:2)]_{2\times2} & [O]_{2\times2} & [E_{1,1\ 0\ 10}(3:4,3:4)]_{2\times2} \end{bmatrix}}_{J}{}_{6\times6} \begin{bmatrix} S_{1,10} \\ S_{2,10} \\ S_{3,10} \end{bmatrix}_{6=1} \quad (10)$$

$$\begin{bmatrix} S_{1,10} \\ S_{2,10} \\ S_{3,10} \end{bmatrix}_{6=1} = \underbrace{\begin{bmatrix} [P_{1,10\ 11}]_{2\times2} & [O]_{2\times2} & [O]_{2\times2} \\ [O]_{2\times2} & [P_{2,10\ 11}]_{2\times2} & [O]_{2\times2} \\ [O]_{2\times2} & [O]_{2\times2} & [P_{3,10\ 11}]_{2\times2} \end{bmatrix}}_{K}{}_{6\times6} \begin{bmatrix} S_{1,11} \\ S_{2,11} \\ S_{3,11} \end{bmatrix}_{6=1} \quad (11)$$

$$\begin{bmatrix} S_{1,11} \\ S_{2,11} \\ S_{3,11} \end{bmatrix}_{6=1} = \underbrace{\begin{bmatrix} [B_{1,11\ 12}]_{2\times2} & [O]_{2\times2} & [O]_{2\times2} \\ [O]_{2\times2} & [P_{2,11\ 12}]_{2\times2} & [O]_{2\times2} \\ [O]_{2\times2} & [O]_{2\times2} & [P_{3,11\ 12}]_{2\times2} \end{bmatrix}}_{L}{}_{6\times6} \begin{bmatrix} S_{1,12} \\ S_{2,12} \\ S_{3,12} \end{bmatrix}_{6=1} \quad (12)$$

$$\begin{bmatrix} S_{1,12} \\ S_{2,12} \\ S_{3,12} \end{bmatrix}_{6=1} = \underbrace{\begin{bmatrix} [P_{1,12\ 12}]_{2\times2} & [O]_{2\times2} & [O]_{2\times2} \\ [O]_{2\times2} & [P_{2,12\ 13}]_{2\times2} & [O]_{2\times2} \\ [O]_{2\times2} & [O]_{2\times2} & [P_{3,12\ 13}]_{2\times2} \end{bmatrix}}_{M}{}_{6\times6} \begin{bmatrix} S_{1,13} \\ S_{2,13} \\ S_{3,13} \end{bmatrix}_{6=1} \quad (13)$$

So, following in this fashion we get this coupled thing for this one and we keep going in here and finally, we reach you know for the section here 14 to 15.

$$\begin{bmatrix} P_{1,0} \\ V_{1,0} \\ P_{2,0} \\ V_{2,0} \\ P_{1,0} \\ -V_{1,0} \end{bmatrix}_{6\times1} = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} & T_{15} & T_{16} \\ T_{21} & T_{22} & T_{23} & T_{24} & T_{25} & T_{26} \\ T_{31} & T_{32} & T_{33} & T_{34} & T_{35} & T_{36} \\ T_{41} & T_{42} & T_{43} & T_{44} & T_{45} & T_{46} \\ T_{51} & T_{52} & T_{53} & T_{54} & T_{55} & T_{56} \\ T_{61} & T_{62} & T_{63} & T_{64} & T_{65} & T_{66} \end{bmatrix} \begin{bmatrix} P_{1,13} \\ V_{1,13} \\ P_{2,13} \\ V_{2,13} \\ P_{1,13} \\ -V_{1,13} \end{bmatrix}_{6\times1} \quad (14)$$

$$\begin{bmatrix} P_{2,11} \\ V_{2,11} \end{bmatrix}_{2\times1} = \underbrace{\begin{bmatrix} P_{2,13\ 16} \end{bmatrix}}_{N}{}_{2\times2} \begin{bmatrix} P_{2,16} \\ V_{2,16} \end{bmatrix}_{2\times1} \quad (15)$$

So, 14 to 15 once you get there and 14 to 15 once you get there and then you get I am sorry 11 to 12, 12 to 13. So, what they have done is that, the authors have combined them you know basically you know kind of cascade it. So, you know what ITM does is basically it allows a very good I know basically extend the concepts of cascading the transfer matrix is all that we are doing for very simple system all this while, ok.

So, this is just a normal transfer matrix multiplication and then once you get p_1 v_1 p_2 v_2 p_3 and v_3 0 because direction has to be changed had 0th section to the 13th section until this part. After that something else will take over which I am going to talk about now.

So, we get this thing and then $p_{2,13}$ $v_{2,\ 13}$ and related to $p_{2,\ 16}$ and $v_{2,\ 16}$ that is somewhere here p_2 $p_{2,\ 13}$ and related to $p_{2,\ 16}$ and $v_{2,\ 16}$ by another sort of a transfer matrix that we see here you know it is a 2 cross 2 matrix. And another thing that I want to quickly mention here without going to too much details is that you know such elements are often seen in the commercial mufflers with the end chamber and with the extended inlet.

And then here it is much more complicated because what Elnady and others have done is that they have considered transverse plane wave propagation here. What I have also been using in some of my earlier papers and here there is a baffle. So, it is becomes much more challenging or complicated in a plane wave kind of a sense you know the waves go from here to here like the direction I am pointing.

And then when they come here they still are in the transverse same, but then in this part there is difference you know p 2 this minus this is there. And then so, and in this region the planar wave is there. So, you know all these things will become clear if you look at the lens of different constituent elements.

And finally, after all the algebra one can relate $p_{2,\,16}$ $v_{2,\,16}$ to p_3 and v3 p3 and $v_{3,\,13}$. So, what is p_3 and $v_{3,\,13}$? So, this is really your you know things here pressure and velocity here and $p_{2,\,16}$ and $v_{2,\,16}$ is somewhere here.

So, once we know the relation between you know these 2 guys, eliminating this and this and compassing all the other things that happen between and then you know once we have this sort of a thing we can relate $p_{2,\,13}$ $v_{2,\,13}$ to $p_{3,\,13}$ and $v_{3,\,13}$ and so, this matrix representation.

And finally, applying the boundary condition and the junction loss compatibility condition or the junction loss $p_{1,\,13}$ is equal to $p_{3,\,13}$ and $p_{3,\,13}$ is equal to $p_{3,\,13}$ dash and $v_{1,\,13}$ and $v_{3,\,13}$ is equal to minus v3 and 13 dash. And at the annular region $v_{1,\,0}$ is 0.

$$\begin{bmatrix} P_{2,16} \\ V_{2,16} \end{bmatrix}_{2\times 1} = \begin{bmatrix} 1 & 0 \\ 1/Z & 1 \end{bmatrix} \begin{bmatrix} TM & for \\ variable & area\ duct \end{bmatrix} \left[B_{1,15\_14}\right]\left[P_{1,14\_1y}\right] \begin{bmatrix} P_{3,1y} \\ V_{3,1y} \end{bmatrix}_{2\times 1} \qquad (16)$$

$$\begin{bmatrix} P_{2,13} \\ V_{2,13} \end{bmatrix}_{2\times 4} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} P_{3,1y} \\ V_{3,1y} \end{bmatrix}_{2\times 4} \qquad (17)$$

$$P_{1,13} = P_{3,13} \qquad (18)$$

$$P_{3,1y} = P_{3,1y} \qquad (19)$$

$$V_{1,13} + V_{1,13} = -= V_{3,1y} \qquad (20)$$

$$V_{1,0} = 0 \qquad (21)$$

$$
\underbrace{\begin{bmatrix}
-1 & 0 & 0 & 0 & T_{11} & T_{12} & T_{13} & T_{14} & T_{15} & T_{16} & 0 & 0 \\
0 & -1 & 0 & 0 & T_{21} & T_{22} & T_{23} & T_{24} & T_{25} & T_{26} & 0 & 0 \\
0 & 0 & -1 & 0 & T_{31} & T_{32} & T_{33} & T_{34} & T_{35} & T_{36} & 0 & 0 \\
0 & 0 & 0 & -1 & T_{41} & T_{42} & T_{43} & T_{44} & T_{45} & T_{46} & 0 & 0 \\
0 & 0 & 0 & 0 & T_{51} & T_{52} & T_{53} & T_{54} & T_{55} & T_{56} & 0 & 0 \\
0 & 0 & 0 & 0 & T_{61} & T_{62} & T_{63} & T_{64} & T_{65} & T_{66} & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & Q_{11} & Q_{13} \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & Q_{21} & Q_{22} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}}_{R}{}_{12\times 12}
\begin{bmatrix} P_{1,0} \\ V_{1,0} \\ P_{2,0} \\ V_{2,0} \\ P_{1,13} \\ V_{1,13} \\ P_{2,13} \\ V_{2,13} \\ P_{3,13} \\ P_{3,13} \\ P_{3,1y} \\ P_{3,1y} \end{bmatrix}_{12\times 1}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ P_{3,0} \\ -V_{3,0} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{12\times 1}
\qquad (22)
$$

So, after apply after you apply all this condition and you lead to a big transfer matrix and then eliminating or inverting that transfer matrix and relating finally, you know what happens in p 2 v 2 to you know p 3 v 3 0, then that will give you the that will give you the final transfer matrix and compassing everything.

$$
\begin{bmatrix} P_{1,0} \\ V_{1,0} \\ P_{2,0} \\ V_{2,0} \\ P_{1,13} \\ V_{1,13} \\ P_{2,13} \\ V_{2,13} \\ P_{3,13} \\ P_{3,13} \\ P_{3,1y} \\ P_{3,1y} \end{bmatrix}_{12\times 1}
= inv(R)
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ P_{3,0} \\ -V_{3,0} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{12\times 1}
\qquad (23)
$$

$$
\begin{bmatrix} P_{2,0} \\ V_{2,0} \end{bmatrix}_{2\times 1}
= \underbrace{\begin{bmatrix} \tilde{R}_{3,5} & -\tilde{R}_{3,6} \\ \tilde{R}_{4,5} & -\tilde{R}_{4,6} \end{bmatrix}}_{TM}{}_{2\times 2}
\begin{bmatrix} P_{3,0} \\ V_{3,0} \end{bmatrix}_{2\times 1}
\qquad (24)
$$

```
function [ PORO ] = poro( nh, dh, d, Lp )
% To evaluate the porosity
%    nh = Number of holes
%    dh = diameter of holes
%    d = diameter of the pipe
%    Lp = length of the perforation

PORO = (nh*dh^2/(4*d*Lp));
end
```

```
function [ Z ] = Impedance_Transverse_Ellipse( x
%UNTITLED Summary of this function goes here
%   Detailed explanation goes here
%%
%%
%*********Transfer Matrix***********
y1 = x1/D1;
S1 = 2*D2*b*sqrt(y1 - y1^2);
Y1 = c/S1;
beta = k0*D1;
[ F1_y1, F2_y1, F1_d_y1, F2_d_y1 ] = Frobenius_E
Z = 1j*beta*Y1*F1_y1/F1_d_y1;
```

So, such a code has been written it has been shared by one of the colleagues which I like to demonstrate to you. So, it contains lots and lots of functions.

```
function [T] = twoduct_4x4matrix(d, poro, tw, d

% ------------------------------------------
% It evaluates and returns the 4x4 transfer mat
% It is calculated based on Bhushan Singh Gauta

% n = No. of interacting ducts == 2
% d = Matrix of diameters of outer shell and in
% lp = Length of perforated pipe in m
% poro = Porosity of the inner pipe in fraction
% tw = Perforated pipe thickness in m
% dh = Diameter of perforated hole in m
% M = Mach No. corresponding to convective effe
```

```
% T = Transfer matrix relates upstream acoustic
% to downstream
% ------------------------------------------------
n = 2;

%canonical differential matrix (Refer to Eqs.(3

q = sum(d.^2);                          % d(1)^2+d
Y = zeros(1,n);                         % Y = [0 0

Y(2) = c/(pi*0.25*(d(2))^2);            % Characte
Y(1) = c/(pi*0.25*(2*d(1)^2-q));        % Characte
```

So, what I am going to do? I am just going to present only the bare important functions. So, and demonstrate to you one of the things in how mean how does it occur. Obviously, with flow one also has to do solve the network analysis. So, it is quite challenging to do analyze such a configuration which fortunately I am going to talk about I have the code.

```
clear all
close all
digits(64)
%%
%***************Input Data for calculation*****
di = 45e-3;      %Inlet pipe inner diameter
do = 57e-3;      %Outlet pipe inner diameter
t = 1.5e-3;      %Wall thickness of pipes and ba
dh = 5e-3;       %Diameter of holes in perforati
a = 97e-3;       %Minor axis of elliptical shell
b = 287e-3;      %Major axis of elliptical shell
D = sqrt(a*b);   %Equivalent diameter of ellipti
D_cd = sqrt(D^2-(di+2*t)^2-(do+2*t)^2);       %Eq
```

```
do = 57e-3;      %Outlet pipe inner diameter
t = 1.5e-3;      %Wall thickness of pipes and ba
dh = 5e-3;       %Diameter of holes in perforati
a = 97e-3;       %Minor axis of elliptical shell
b = 287e-3;      %Major axis of elliptical shell
D = sqrt(a*b);   %Equivalent diameter of ellipti
D_cd = sqrt(D^2-(di+2*t)^2-(do+2*t)^2);       %Eq
D_cs = sqrt(D^2-(di+2*t)^2);                  %Eq
D_ci = D_cs;                                  %Eq
D_co = sqrt(D^2-(do+2*t)^2);                  %Eq
NFP = 600;       %Number of frequency points in
Po = 101325;     %Mean pressure
Ma = 0.05;       %Mean flow Mach number in inlet
```

```
: inner diameter
ness of pipes and baffles
: holes in perforation
 of elliptical shell
 of elliptical shell
 diameter of elliptical shell
(do+2*t)^2);      %Equiv. Dia of cavity where inl
·                 %Equiv. Dia of cavity where onl
                  %Equiv. Dia of cavity excluding
·                 %Equiv. Dia of cavity excluding
frequency points in steps of 1Hz
ure
Mach number in inlet pipe
```

```
ty where inlet and outlet both pipes are presen
ty where only inlet pipe is present
ty excluding outlet pipe
ty excluding inlet pipe
```

So, this is the code and the inlet and outlet diameter pipe and this is a major and minor axis of the shell equal in diameter and then there number of things like equal and diameter of the cavity where inlet and outlet pipes are both present, there are number of such sections. So, these are some of the terminologies 600 is the number of frequency points in the range from 1 to 600 Hertz. So, we are dividing into 1 1 Hertz and everything else is same.

```
a = 97e-3;        %Minor axis of elliptical shell
b = 287e-3;       %Major axis of elliptical shell
D = sqrt(a*b);    %Equivalent diameter of ellipti
D_cd = sqrt(D^2-(di+2*t)^2-(do+2*t)^2);      %Eq
D_cs = sqrt(D^2-(di+2*t)^2);                 %Eq
D_ci = D_cs;                                 %Eq
D_co = sqrt(D^2-(do+2*t)^2);                 %Eq
NFP = 600;        %Number of frequency points in
Po = 101325;      %Mean pressure
Ma = 0;      %Mean flow Mach number in inlet pi
Temp = 273 + 15;            %Temperature of flui
c = sqrt(1.4*287*Temp);     %Speed of sound in m
Q0 = c*(0.25*pi*di^2)*Ma;   %Volume flow rate (c
```

So, let us do for a stationary medium by setting the inlet mark flow in the inlet pipe to be 0 and then this is a flow analysis. So, we also need to go to the solve network file solve network file which is I do not know where it is gone, but I will find out and figure out for you solve network well.

```
function [ Q, R, delta_p] = solve_flow_network
% For Elnady muffler configuration (DATE - 26/07
% To evaluate flow resistances, total resistance
% flow distribution and mach number calculation,
clc
clear all
% Density calculation
Po = 101325;
T = 15+273;
R = 287;
rho = Po/(R*T);      % Density
gamma  = 1.4;
c = sqrt(gamma*R*T);     % Velocity of sound
```

```
% Density calculation
Po = 101325;
T = 15+273;
R = 287;
rho = Po/(R*T);      % Density
gamma  = 1.4;
c = sqrt(gamma*R*T);     % Velocity of sound
d_0 = 45e-3;             % Diameter of inlet pipe
M = 0                % Mean flow Mach number in i
S_0 = 0.25*pi*d_0^2;     % Cross-sectional area o
Qo = c*S_0*M;            % Volume flow rate (cub.
Uo = M*c;
%% Calculation of flow resistances
```

```
Q5 = Qf(5); Q6 = Qf(6); Q7 = Qf(7);

Qf
for i = 1:NFP
    i
    f(i) = i;
    k0 = 2*pi*f(i)/c;
    %%
    %************************Section 0-1******
    l_0_1 = 39e-3;
    P1_0_1 = Pipe(0, 0.5*D_cd, l_0_1, c, k0);
    P2_0_1 = Pipe((Q0/Si)/c, 0.5*di, l_0_1, c,
    P3_0_1 = Pipe((Q0/So)/c, 0.5*do, l_0_1, c,
```

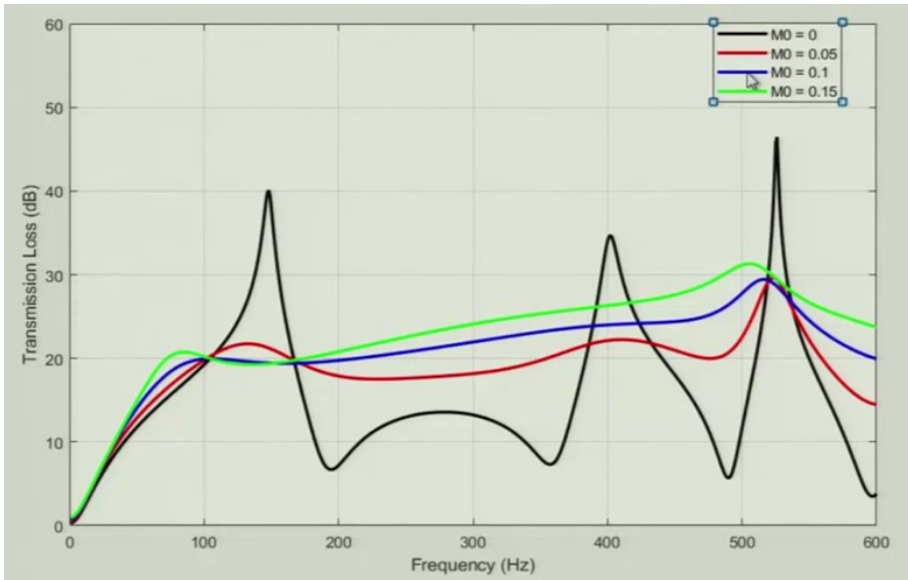
```
k0 = 2*pi*f(i)/c;
%%
%*************************Section 0-1******
l_0_1 = 39e-3;
P1_0_1 = Pipe(0, 0.5*D_cd, l_0_1, c, k0);
P2_0_1 = Pipe((Q0/Si)/c, 0.5*di, l_0_1, c,
P3_0_1 = Pipe((Q0/So)/c, 0.5*do, l_0_1, c,
A = [P1_0_1 zeros(2,2) zeros(2,2); zeros(2,
M_0_1 = [0 (Q0/Si)/c (Q0/So)/c];
%vpa(A,100);
%%
%*************************Section 1-2******
l_1_2 = 144e-3;
```

```
B(3:4,3:4) = P2_1_2;
M_1_2 = [abs((0.5*Q7/S_cd)/c) abs(((0.5*Q7+
%vpa(B,100);
%%
%*************************Section 2-3******
l_2_3 = 44e-3;
P1_2_3 = Pipe(abs((Q7/S_cd)/c), 0.5*D_cd, l
P2_2_3 = Pipe((Q0/Si)/c, 0.5*di, l_2_3, c,
P3_2_3 = Pipe(abs(((Q4+Q6)/So)/c), 0.5*do,
C = [P1_2_3 zeros(2,2) zeros(2,2); zeros(2,
M_2_3 = [abs((Q7/S_cd)/c) (Q0/Si)/c abs(((Q
%vpa(C,100);
%%
```

```
%vpa(O,100);
%%
%*******************Final Assembled Equatio
P = A*B*C*D*E*F*G*H*I*J*K*L*M; Q = N*O;
%vpa(P,100); %vpa(Q,100);
R = zeros(12,12);
R(1:4,1:4) = -eye(4); R(1:6,5:10) = P;
R(7:8,7:8) = -eye(2); R(7:8,11:12) = Q;
R(9,6) = 1; R(9,10) = 1; R(9,12) = 1;
R(10,5) = 1; R(10,9) = -1;
R(11,9) = 1; R(11,11) = -1;
R(12,2) = 1;
%vpa(R,100);
```

So, here also I am setting the mean flow to 0. So, there is no flow and then you know what it does is that NPF number of frequency steps and then it starts you know assimilating the transfer matrices across each section and then sort of multiplying them. You know so, this is these are the most important things A, you know exactly the same formulation what I had talked about B C and you know this code goes on and on.

So, I will be very concise here and kind of just present you the final thing P is equal to this and Q is equal to N Q and after all this algebra you know what you get is this thing and what you get is really this thing. And so, what I am going to do is run the code by this configuration.



So, these are the final results after you know varying a number of parameters in the code. So, what I did was that you know I varied the grazing flow Mach number in the inlet pipe starting from a stationary medium. So, M naught is equal to 0 let us you know I will you know the there are large number of such function files each doing some certain things like contraction it is calculating the losses due to contraction outlet or expansion.

And you know the Elnady's baffle expression transfer matrix for that and the number of such things the function does and this is based on my Frobenius series solution for the elliptical section and so on and so forth.

```
function [T] = Baffle_elnady(d, k0, c, poro, tw, d
xeta_baffle = perforate_impedance_baffle(k0, c, po

Y0 = c/(0.25*pi*d*d);
Z1 = Y0*xeta_baffle;

T = [1 Z1; 0 1];
end
```

```
function [ T ] = Elliptical_DS_TM( x1, x2, D1, D
%ELLIPTICAL_DS_TM function gives the Transfer Ma
%elliptical section.
%   Output T = Transfer Matrix or Four-pole para
%   Input x1 = Position of first section from to
%   section from bottom, D1 = Major axis, D2 = M
%   elliptical section, k0 = wave number.

%%
%*****Transfer Matrix of Elliptical section in u
[ T1 ] = Elliptical_SS_TM( x1, 0.5*D1, D1, D2, b
[ T2 ] = Elliptical_SS_TM( x2, 0.5*D1, D1, D2, b
D = T2(1,1)*T2(2,2) - T2(1,2)*T2(2,1);
```

You know so, and finally, all other results that we are seeing in this routine integrated transfer matrix thing. But now at the end of the day what we are interested in is the detailed analysis of the transmission loss. So, I will focus my attention on that itself without going too much into the you know the details of the code.

So, there are couple of things that have come out of this analysis is that. Firstly, mean flow significantly and dramatically influences the transmission loss characteristics it greatly enhances that. You know all this while; all this while you were talking about you know concentric tube resonators where the mean flow would just simply you know damp in the peaks the peaks would come down and the troughs would be slightly raised.

So, you would eventually conclude well flows doing good has some sort of an effect and we tended to ignore the convective effects of mean flow and retain the more important dissipative effects. Well in this paper we have also done the same in this analysis, but you know the profound effect of mean flow is that it completely changes the nature of the transmission loss behavior.

You know if you focus our attention on the Mach number 0 curve you see a peak and this is pretty normal you would expect such a peak and multiple such peaks because it really acts like a you know a quarter wave well a kind of you know a flow reversal kind of a configuration that it is and you would see a peak in the low frequency characteristic of the you know resonator thing.

But you know when, but when flow occurs hit the peaks definitely come down, but it has a dramatic leveling influence. So, when you have 0 mean flow, so, you can you know the transmission loss performance can be as low as you know 4 5 dB in 200 Hertz and continues to says not so good in the intermediate range from 200 Hertz to about 350 Hertz, 380 Hertz onwards until that.

But then even when there is a small amount of flow Mach number 0.05 you know it has such a good effect on the transmission loss attenuation that you know the straightaway the curve uplifts and you here you are getting almost like 20 dB or close 20 dB attenuation throughout the frequency band and the more the flow the better is the attenuation this is a there is a slope this is definitely an appreciable slope here and you know it really does the trick.

So, what I am; what I am trying to say is that mean flow in this case is very beneficial because of the dissipative effects of the flow characteristics, the curve really uplifts and that does a lot of good things to that attenuation performance and this we have not seen in all the configurations analyze so far.

So, you know what exactly am I trying to say? So, one can all and other thing actually before we I conclude is that all this thing is happening in the very low frequency range look at the frequency 100 Hertz is giving you almost less than 100 Hertz is giving you almost 20 dB transmission loss in this range which is wonderful.

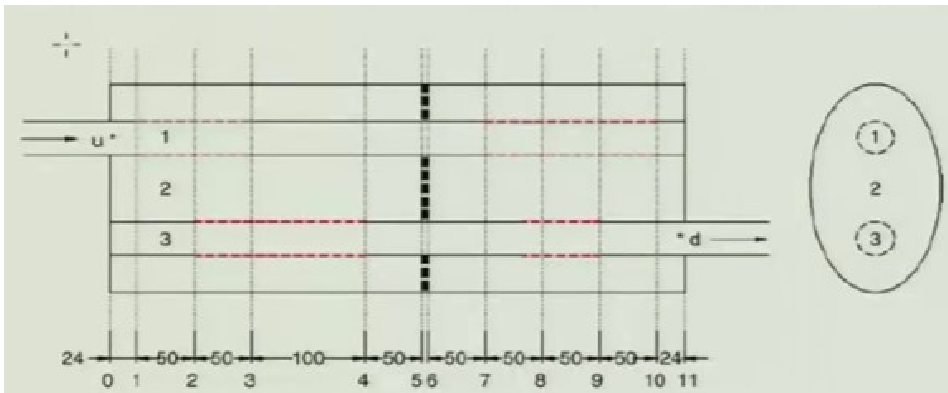
Because lot of engine noise is definitely at such low frequency 100 Hertz, 150 Hertz and then super harmonics like you know 300 Hertz, 450 Hertz and so on. So, if you can really get very good performance until about 600, 700 Hertz we are in business and all this is done by plane wave analysis.

So, plane wave is quite good, it is not that bad especially when you are analyzing a very complicated muffler the mode matching thing or you know the analytical mode matching, numerical mode matching or you know even simple 3D models would be very cumbersome.

So, you know instead of going for finite element analysis one can definitely do good in terms of plane wave analysis in the very low frequency range for automotive mufflers at least and you know then we are definitely in business. It is all about manipulating and setting up the cavities you know baffles and any complicated muffler can be analyzed by this powerful ITM method. It is just a matter of suitably combining the section into one and it can also be used to have considered dissipating material.

So, if you have dissipated materials then it will do even better in terms of attenuation and high frequencies. So, you know the idea is that for such configurations with vertical baffles and perforated pipes and those sort of things mean flow does a lot of it does a lot of good to the attenuation performance rather than just dampening the peaks and stuff like that.

So, what we will do is that we can just conclude that a large number of such configurations can be analyzed I will show you one such configuration if I can.

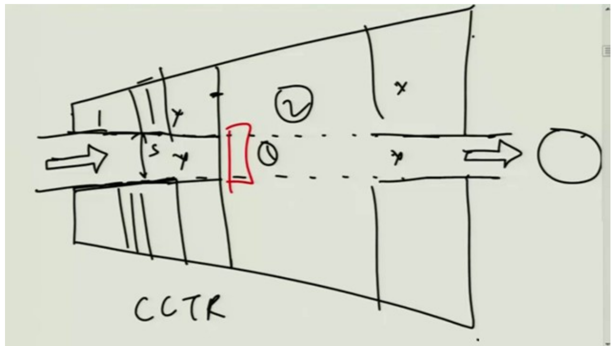


So, another such configuration is this one which is more like an expansion chamber with one baffle and this is even more simpler to analyze in the one that we the then the one we presented before. And here also with flow a lot of you know interesting things can happen it can straightaway lift the transmission loss curve.

You know, but this configuration also needs you know such you know to district to analyze the flow distribution in such complicated muffler or commercial mufflers and one also one also has to solve the flow how does the flow is distributed in different pipes. And for that we have to do the you know resistance analysis approach presented in Munjal's book and that requires separate analysis.

I am not going into that right now rather I just use the results that I presented use some of these formulations in the results that I presented so far. But it is possible in general to analyze any complicated muffler commercial mufflers using this ITM method along with the method of solving for the flow fields. So, once that is done you can obtain the transmission loss performance and you can one can stick up to low frequencies like up to 6, 700 Hertz and so on.

[FL] with this I will end the topic I will end this particular topic here and focus on this conical concentric tube resonator mufflers right now.



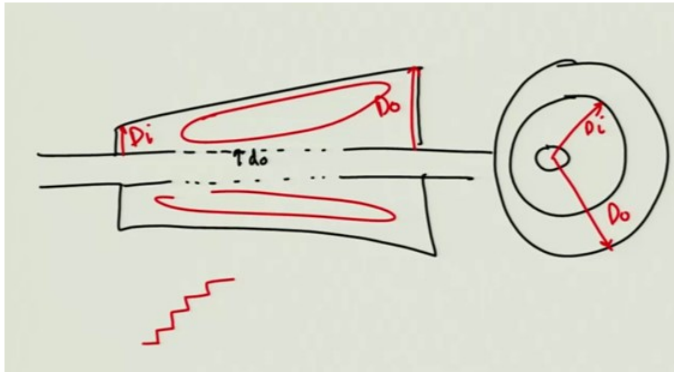
$$\frac{p_2}{\rho_0 C_0 V_1} = Z_1$$

$$\frac{p_2}{\rho_0 C_0 V_2} = Z_2 \; x = V$$

So, let us go to the let us briefly talk about this CCTR muffler configuration where you know when you know you have such a kind of a thing you know the only difference between this was you know suppose the taper was not there it was like this it would have been a CTR, but because we have a conical thing we have CCTR.

So, you know what we will do is that we will analyze we will develop the continuity equation in the region 1 and that in the region 2 and the annular region and let us say this is partially perforated only and you know develop relation between things here, here to this here and here. And derive transfer matrix and for a partially perforated tube conical CCTR configuration we will do the double tuning of such configuration exactly what we did for CTR.

So, in this conical concentric tube resonator configuration CCTR you know you have its pretty much very similar to your CTR concentric tube resonator in which the outer volume is a uniform pipe, but in this case the difference is that the outer volume is a conical pipe.



So, you see you know I do not have space here, but what I could really do is basically just draw a small version here you know something like this and this kind of a thing and you have you know this sort of a thing. Now, we have this kind of a situation where we have the diameter D something like this and this is like this D_i and D_0.

So, we have this and this kind of a thing is there and the diameter here inside is say small d_0 which is uniform. So, you know this the internal pipe I am sorry the inner pipe or the airway can also be gradually varying in you know across its length, but we will consider a simple case and there is one of the reasons why we use a CCTR which you see a lot of such configurations in two wheelers.

Remember the photographs that we had seen probably week 6 of this lecture where we first introduced variable area ducts. Basically what happens is that it is a usual practice to exploit or employ the you know the attenuation resistance dissipation offered by the perforated components to achieve a lot of acoustic attenuation.

So, CTR is used concentric tube resonators are used, but usually they are they perform very well at relatively higher frequencies a low frequencies you typically have you know large volume. So, what this CCTR does is that you will have typically a relatively much larger volume here you know rather than having rather than making it something like this you know you often have space more towards you know the exhaust whether you know near the tail pipe.

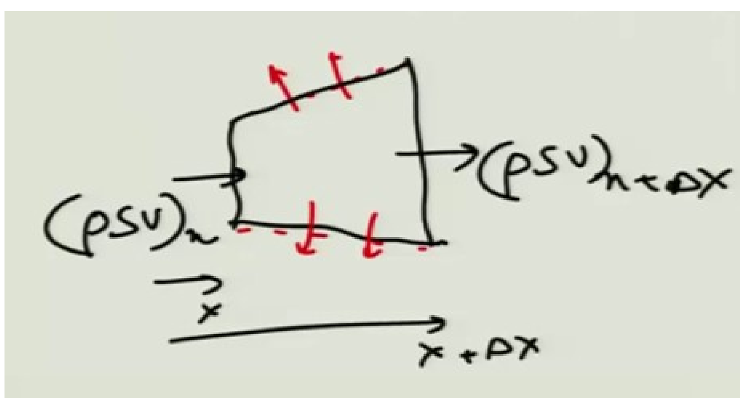
So, there you can have a relatively larger volume and this kind of a thing is there. So, it is the idea is to combine the low frequency attenuation that wherever a large volume is possible along with the attenuation or the resistance offered by the perforated components ok. And then so, that is how we combine the unique feature of perforated thing and as well as the large volume thing to achieve attenuation.

And also by the way you know gradually varying area can duct can also be thought of as you know notionally it can be thought of a step segment in the form of step segmentation approach where you kind of mimic the profile something like this kind of a thing. You know the idea is that this gradually varying area duct will always tend to reflect the acoustic power back.

So, in that way we, having a lot of advantages by using a CCTR configuration. So, let us get to the maths behind this. So, what I will do is sort of rub this guy and I need some space here. So, I will start from the control volume approach, but one thing we should keep here in mind is that I am not going to consider the effects of convective effects of mean flow I am going to sort of ignore it altogether just to keep the equation simple.

Of course, if you include mean flow effects you know there are more number of terms that will be added. So, and you will have a small amount of variation as we have been discussing throughout the course.

So, let us start with the you know an elementary control volume here in the region 1. So, what happens is [FL] is that you know let us say you have this control volume 1 you know something like this and this is S, ok. Let us say the internal the air airways also having gradual variation.



So, what we do basically is that you know let us say you have the mass flux that is entering is $(\rho SU)_x$ $(\rho SU)_{x+\Delta x}$ ok and this is x this is x + Δx ok.

So, now what is the and what is the total mass that is entering into the system? And of course, note that due to the perforated pipe here you know all across the surface some mass flux goes inside outside and there is a gradual you know exchange.

$$(\rho S_V)_x - (\rho SU)_{x+\Delta x} - S_C \rho U^* = \frac{\partial(\rho S \Delta x)}{\partial t}$$

Because density times cross section area into the distance. So, this is like the net mass that is temporal change of mass we have been. So, far I mean at this stage of this course; this term should be very familiar with you when you equate the mass influx and out flux across a control volume.

So, the standard we will apply the standard procedure. Note that S here can also be varying in general. What it means it is that this S is the cross sectional area you can see this is uniform pipe, but for generality what we will do is that we will keep S sort of varying and then we will assume that you know that S is remaining constant and we will drop off we will strike out certain terms.

So, for generality sake we will follow like this. So, then this becomes you can write this guy here and divide throughout S multiplied by

$$=> \quad \frac{\partial \rho}{\partial t} = \frac{(\rho SU)_x - (\rho SU)_{x+\Delta x}}{S\Delta x} - \frac{S_C \rho U^*}{S\Delta x}$$

So, we get this kind of thing. Now, what happens is that we should immediately sort of start to linearize this differential equation. But you know before we actually even attempt do that you know one can straightaway do the following.

So, what we do is basically,

You can just take the limit as this limit tends to 0 you know and you will see also here what will happen soon. And in the denominator you have S and here

$$=> \quad \frac{\partial \rho}{\partial t} = -\frac{1}{S}\frac{\partial(\rho SU)}{\partial x} - \frac{S_C \rho U^*}{S\Delta x}$$

So, here you will get $S_C \rho U^*$ as a radial velocity that goes along the perforate across the perforate I am sorry as S into Δx.

can also you can think of this as a function of actual coordinate z into delta x or actually I should write this is x ok. So, you get this and

$$=> \qquad \frac{\partial \rho}{\partial t} = +\frac{1}{S}\frac{\partial(\rho S U)}{\partial x} + \frac{\pi d \Delta x}{S \Delta x}\rho U^* = 0$$

So, this guy you know sort of goes away you know delta x delta x gets cancelled and what you really get,

$$=> \qquad \frac{\partial \rho}{\partial t} = +\frac{\partial \rho U}{\partial x} + \frac{\pi d_1}{\Delta x}\rho U^* = 0$$

Now, we can assume that well S can be assumed to be constant across the distance x along the axis. So, you can take S out. So, this gets cancelled and you left with the familiar. So, we need to; we need to sort of simplify this thing further.

And so, for that end what we will do is that we will assume that rho is rho naught ambient density plus rho tilde ok. And let us assume like I said a stationary medium.

$$\rho_1 = \rho_0 + \tilde{\rho}_1$$

$$U = \widetilde{U}_0$$

and you know we can actually do one thing straight away and that is and we can do this something like ρ_1 and this is ρ_1, 1 because for the medium because this pertains to the medium 1.

And there is no mean flow in the duct let for simplicity we can assume that this is d_1 and what we can

$$\frac{\tilde{p}_1}{\tilde{\rho}_1} = C_0^2$$

Now, what is S c? It is nothing but

$$d(x)$$

$$S_C = \pi d.\Delta x$$

one can expand this out and

$$=> \quad \frac{1}{C_0^2}\frac{\partial\tilde{p}_1}{\partial t} + \frac{\partial(\rho + \tilde{\rho}_1)\widetilde{U}_1}{\partial x} + \frac{\pi d_1}{\pi}\frac{4}{d_1^2}\,\rho_0 U^* = 0$$

So, 4 goes in the numerator and $\tilde{\rho}_1$ you can sort of ignore because this will be multiplied by U*. So, I would just put this as U* and there is a cancellation here and this is here. So, this is 0.

$$= \frac{1}{C_0^2}\,\frac{\partial\tilde{p}_1}{\partial t} + \rho_0\frac{\partial\widetilde{U}_1}{\partial x} + \frac{4\rho_0}{d_1}\,U^* = 0$$

And once you simplify this equation further. So, you will get 1 by C_0^2 times this thing and again we drop the quadratic terms because the quadratic terms we have been shown, we have shown that they are small in the first week of this course. So, rho naught you can take to v common and this is Ux you know rho naught into this is U1 you can say like this and what about the other terms.

$$\tilde{p}_1 = p_1 e^{j\omega t}$$

Let us worry about the other terms as well. So, basically you get

$$\frac{j\omega}{C_0^2}\,\tilde{p}_1 + \rho_0\frac{d\widetilde{U}_1}{dx} + \frac{4\rho_0}{d_1}U^* = 0$$

So, if we assume time harmonicity.

You know so we get,

$$\tilde{p}_1 - \tilde{p}_1(x,t)$$

So, p 1 is nothing but *x* and t and

$$jk_0\,\tilde{p}_1 + \rho_0 C_0\frac{d\widetilde{U}_1}{dx} + \frac{4\rho_0 C_0}{d_1}\left(\frac{\tilde{p}_1 - \tilde{p}_2}{\rho_0 C_0}\right)\frac{1}{\zeta} = 0$$

So, we get this sort of a; we get this sort of a term, but further simplification perhaps also needs to be; needs to be carried out. And so, what is U* really? U* is nothing but you know I just right now just box this guy and say U* is nothing but

$$U^* = \frac{\tilde{p}_1 - \tilde{p}_2}{\rho_0 C_0 \zeta}$$

So, how about we plug this value straightaway here and that should simplify things a bit in the sense that we have got less responsibility. Now to remember terms we can just straight away use this thing and this goes away ok. So, this goes away. So, what do you; what do you get out of this? You get your 4 by I just do this you know you get this sort of a thing ok.

Now, one can sort of simplify this further

$$jk_0\, \tilde{p}_1 + \rho_0 C_0 \frac{d\widetilde{U}_1}{dx} + \frac{4}{d_1}(\tilde{p}_1 - \tilde{p}_2) = 0$$

$$U^* = \frac{\tilde{p}_1 - \tilde{p}_2}{\rho_0 C_0 \zeta}$$

$$\rho_0 C_0 \frac{d\widetilde{U}_1}{dx} + \tilde{p}_1 + \tilde{p}_1\left(jk_0 + \frac{4}{\zeta d_1}\right) - \frac{4\tilde{p}_2}{\zeta d_1} = 0 \qquad (1)$$

So, we get this sort of a thing. Now, the momentum equation in this duct will remain the same.

$$\rho_0 \frac{\partial \widetilde{U}_1}{\partial t} + \frac{\partial \tilde{p}_1}{\partial x} = 0$$

$$\rho_0\, j\omega\, \widetilde{U}_1 + \frac{d\tilde{p}_1}{dx} = 0$$

$$jk_0\, \rho_0\, C_0\, \widetilde{U}_1 + \frac{d\tilde{p}_1}{dx} = 0$$

So, basically we here ending up with the same momentum equation which can be written in the you know in the following form

$$\frac{d\tilde{p}_1}{dx} = -jk_0\left(\rho_0\, C_0\, \widetilde{U}_1\right) \quad (2)$$

We can just box this guy. Apparently I guess I realized that you know I have sort of made a mistake in the equation above. There is no $\rho_0\, C_0$ U term which I guess I have missed here. So, there was this term this guy sitting here and apologies for that I have kind of forgotten to write it down. So, this obviously, has to be written. So, this equation actually was something like this.

This is small mistake here which I am now correcting. So, this is ordinary derivative dU 1 by tilde by dx $\rho_0\, C_0$ this thing plus this guy. So, that is how you sort of arrange terms this is equation 1, equation 2. Now so far things have been quite you know clean they have been manageable clean derivation.

Now, mean flow velocity in the airway has not been quite considered, because we are considering a stationary medium to just to bring out the effects of this variable area duct and focus not probably focus on the mean flow effects. Other thing that I want to point out is that you know these two equations 1 and 2 appear to be the, are actually they are the same as the one derived for the CTR.

Now, the real difference lies in the outer duct. So, remember this configuration the outer duct has a gradually varying flair and this will show up in the momentum equation of the I am sorry the continuity equation of the duct.

So, what happens is pretty much you know is for the is pretty much the same. So, in such a case we have $\frac{\partial \tilde{\rho}_2}{\partial t}$, which is your density in the total density in the second duct and this thing

$$\frac{\partial \rho_2}{\partial t} + \frac{\rho_0}{S}\frac{\partial S U_2}{\partial x} - \rho_0\,\frac{S_C}{S}\,\frac{U^*}{\Delta x} = 0$$

So, once you do that you will get

$$\rho_2 = \rho_0 + \tilde{\rho}_0$$

and once you simplify the above terms you know you will get the following which is basically,

$$=> \frac{\partial \tilde{\rho}_2}{\partial t} + \frac{\rho_0}{S}\frac{dSU_2}{dx} - \rho_0\,\frac{\pi d \Delta x}{S \Delta x}\left(\frac{\tilde{p}_1 - \tilde{p}_2}{\rho_0\, C_0 \zeta}\right) = 0$$

Now S here is different from the one in the duct one. So, S is nothing but

$$S = \frac{\pi}{4}\{\Delta_2^2(x) - d_1^2\}$$

So, let us sort of quickly simplify the algebra. So, you get,

$$=> \; jk_0\tilde{p}_2 + \frac{\rho_0 C_0}{S}\frac{dS}{dx} + \rho_0 C_0 \; \frac{\pi d}{\Delta x}\left(\frac{\tilde{p}_1 - \tilde{p}_2}{\zeta}\right) \quad = 0$$

So, here there will be minus sign instead of plus because I mean because of the conventions that we have followed. Remember in the other one in the equation just derived for the continuity equation derived for the first duct here you had a plus sign here, but in this equation you have a minus sign.

We purely because for the convention whatever mass is leaving duct 1 is entering the annular region 2. So, you get all these expressions and finally, after some simplification what you will get, where S is given by the above equation and now clearly S is a function of x when, so, once you simplify further the final form of this equation would look something

$$\rho_0 C_0 \frac{d\tilde{U}_2}{dx} = \frac{\pi d_1}{S\zeta}\,\tilde{p}_1 - \tilde{p}_2\left(jk_0 + \frac{\pi d_1}{S\zeta}\right) - \frac{1}{S}\frac{dS}{dx}\,(\rho_0 C_0 U_2) \qquad (3)$$

So, you get this sort of a thing. So, let us call this equation (3), this is like this.

$$\frac{d\tilde{p}_2}{dx} = -jk_0\rho_0 C_0 V_2 \qquad (4)$$

So, we get this sort of a thing and so, when we are sort of putting the entire thing in the matrix form and you know notice carefully the difference that you would observe with respect to a CTR what happens to a CCTR, ok. So, this will become

.

$$\frac{d}{dx}\begin{Bmatrix}\tilde{p}_1\\ \rho_0 C_0\tilde{V}_1\\ \tilde{p}_2\\ \rho_0 C_0\tilde{V}_2\end{Bmatrix}=\underbrace{\begin{bmatrix}0 & -jk_0 & 0 & 0\\ -jk_0-\frac{4}{d_1\zeta} & 0 & \frac{4}{d_1\zeta} & 0\\ 0 & 0 & -jk_0 & -jk_0\\ \frac{\pi d_1}{S\zeta} & 0 & -\frac{\pi d_1}{S\zeta} & -\frac{1}{S}\frac{4}{d_1\zeta}\end{bmatrix}_{\Delta x}}_{[C]}\begin{Bmatrix}\tilde{p}_1\\ \rho_0 C_0\tilde{V}_1\\ \tilde{p}_2\\ \rho_0 C_0\tilde{V}_2\end{Bmatrix}$$

You know this particular thing if d_2 is not a function of x; that is outer radius is outer diameter radius is constant then d dx of S is 0. So, you would end up getting rid of this term ok. And only term that would basically S would still survive and S would be d_2 square minus d_1 square and there will be a factor of π by 4. So, 4 will go in the numerator and π will cancel.

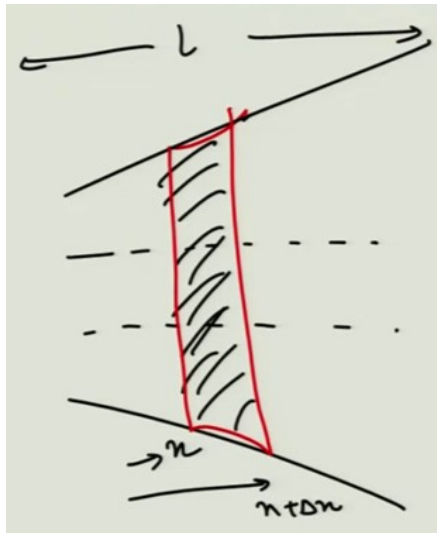
So, you will eventually get the same thing you know 4 well 4 d_1 divided by you know d 2 square minus d1 square into zeta and same thing here as well. So, basically you know this is a. So, this case reduces to that of a CTR when you have no variation none of the annular cross section area with respect to the distance x actual distance x. In case you have then you definitely end up with a variable system of ordinary differential equation with the variable coefficient.

So, you can still use; now the clever thing is that you know if you have the if you have 0 variation of course, you have a constant coefficient differential equation and the solution of that would be easily given by you know as we discussed

$$\begin{Bmatrix}\tilde{p}_1\\ \rho_0 C_0\tilde{V}_1\\ \tilde{p}_2\\ \rho_0 C_0\tilde{V}_2\end{Bmatrix}=expm\ \ ([C]\Delta x)\begin{Bmatrix}C_1\\ C_2\\ C_3\\ C_4\end{Bmatrix}$$

So, this would be something like exponential expm matrix you can use this command in MATLAB to figure out thing then say the length is l ok and this is the constant C_1, C_2, C_3, C_4. So, your life is pretty much sorted out, but then once you have variable differential coefficients with variable differential equations things are little more tricky. So, what we need to do is that follow the matrix and approach.

So, matrix and approach what does it do? Basically you know over a small segment delta x distance. So, whenever you are you know getting this sort of a solution you assume that this l is replaced by delta x and eventually what you would do? You know this entire thing let us say I am drawing a very exaggerated view of this coupled system.



So, you have this kind of a thing. So, let us see you consider a small segment here, you divide this an exponential kind of a duct ok. You divide this in with an exponential duct something like this and you relate things at x those occurring at $x + \Delta x$. So, eventually what we will do is that let us say you know so L is divided notionally into n number of parts each segment being Δx thing.

$$\frac{L}{x} = \Delta x$$

$$\frac{1}{S}\frac{dS}{dx} = \propto$$

So, over this part we are assuming that 1 / S into this thing remains constant. It can be shown that it is actually an exponential profile because you know we can just

$$\frac{1}{S}\frac{\Delta S}{\Delta x} = \alpha$$

and in the limit of

$$\frac{\Delta S}{\Delta x} = \alpha \, \Delta x$$

$$S(x) = Ae^{\alpha x} \log \; S = \alpha x$$

So, you would get you know S over the small distances e to the power alpha x and there you go with a constant, you can add a constant here as well.

So, basically what it means the moment you assume that over a small distance this guy is constant that is the particular thing is constant you would see that this thing is actually exponential ok. You see this is actually exponential in nature ok.

So, the idea is that you know over such a small segment you would go

$$\begin{Bmatrix} \tilde{p}_1 \\ \rho_0 C_0 \widetilde{U}_1 \\ \tilde{p}_2 \\ \rho_0 C_0 \widetilde{U}_2 \end{Bmatrix}_x = \left[e^{-Cx_0 + \frac{\Delta x}{2}} \right] \begin{Bmatrix} \tilde{p}_1 \\ \rho_0 C_0 \widetilde{U}_1 \\ \tilde{p}_2 \\ \rho_0 C_0 \widetilde{U}_2 \end{Bmatrix}_{x+\Delta x}$$

$$\begin{Bmatrix} \tilde{p}_1 \\ \rho_0 C_0 \widetilde{U}_1 \\ \tilde{p}_2 \\ \rho_0 C_0 \widetilde{U}_2 \end{Bmatrix}_x = \left[e^{-Cx_0 + \frac{3\Delta x}{2}\Delta x} \right] \begin{Bmatrix} \tilde{p}_1 \\ \rho_0 C_0 \widetilde{U}_1 \\ \tilde{p}_2 \\ \rho_0 C_0 \widetilde{U}_2 \end{Bmatrix}_{x+2\Delta x}$$

So, basically this is only over a short segment. So, we will soon go to the code. So, C matrix mind you I made a small typo here I mean not to get not to make you guys confused that this let us call this as a C matrix.

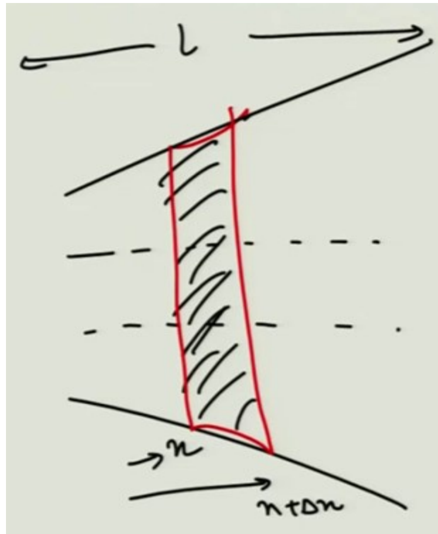
$$\frac{L}{x} = \Delta x$$

$$\frac{1}{S}\frac{dS}{dx} = \alpha$$

$$\frac{1}{S}\frac{\Delta S}{\Delta x} = \alpha$$

$$\frac{\Delta S}{\Delta x} = \alpha \, \Delta x$$

$$S(x) = Ae^{\alpha x} \log\ S = \alpha x$$

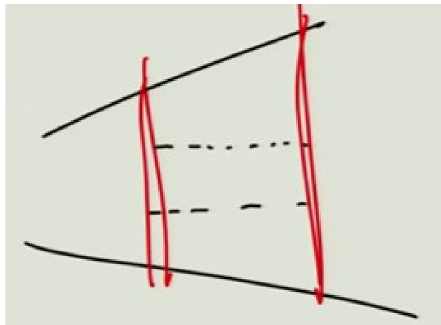


And this constant can be something else you know this can you can see this as capital A into e to the power alpha x not to have any confusion. The main idea is calling this entire matrix a C matrix C of course, is a function of x as you can very well see because S is a function of x especially when you have you know inner pipe also with the variable area then you can have the you know terms like this one, this one here everything will be function of x.

The key idea behind the matrix and approach we have done only segment week 6, but now we is you know it is a good idea to talk about the exponential you know matrix approach as well matrix and approach. So, we assume this matrix C matrix to be constant over a small distance and hence we integrate to relate things upstream with those at downstream ok.

And now once we have done that we can similarly you know do now I guess had it been a you know class which is to be delivered in person, I would have stopped here and ask 1 of you guys to come and derive it for me. But this being an online class I will give you a solution which basically tells you how to relate how mean how to basically keep multiplying and traverse the entire length of the duct. So, that you can relate things upstream with those at downstream ok.

And c x_0 and this is let us say this is done at you know 3 into delta x/2 into Δx. So, this is what it is.



You know you keep the C matrix as a value what it is going like this and eventually what you will get is something like this.

$$\begin{Bmatrix} \tilde{p}_1 \\ \rho_0 C_0 \widetilde{U}_1 \\ \tilde{p}_2 \\ \rho_0 C_0 \widetilde{U}_2 \end{Bmatrix}_{x=0} = [e^{-C}]_{x_0+\frac{\Delta x}{2}} [e^{-C}]_{x_0+\frac{3\Delta x}{2}} \cdots\cdots \; [e^{-C\Delta x}]_{x_0+(n-1)\frac{\Delta x}{2}} \begin{Bmatrix} \\ \\ \end{Bmatrix}_{x=L}$$

And so on and $x_0 + (n-1)\frac{\Delta x}{2}$ this evaluated the C matrix at that point. And finally, you know what you will get? This is let us say this is at x =0 this entire thing will be at x =L.

So; that means, what essentially you have done is that you have related things across this section to go about this section. And then it is a pretty much a straightforward thing that you apply boundary condition. So, you know you we will demonstrate some simple MATLAB codes in which basically you know going back to the configuration here.

Let this that be partially perforated only. So, the boundary condition recall the discussions that we had in week 7 at this section the acoustic impedance is something like this

$$\frac{p_2}{\rho_0 C_0 \widetilde{U}_2} = -j\cot\, l_2$$

But actually this would be different now this would not be just this term because this part is no longer a uniform duct of length l_2 or l_1.

You know it is really variable area duct. So, I would replace I would the idea the problem is still quite simple in the sense that this is a much simpler problem to solve for the impedance here when you have a solid pipe here non perforated pipe here and a rigid wall here. You can apply your segmentation approach to get some impedance expression Z_1 similarly you can apply your thing here

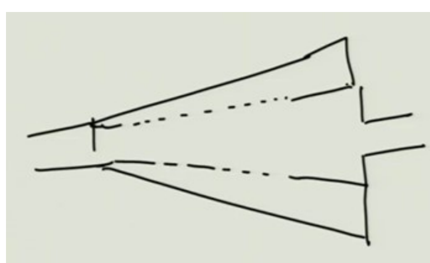
$$\frac{p_2}{\rho_0 C_0 \widetilde{U}_2} = Z_1$$

$$\frac{p_2}{\rho_0 C_0 \widetilde{U}_2} = Z_2$$

with these boundary conditions you can go about analyzing this. So, in the most simplest cases when it is a fully perforating U where U_2 is 0 here and U_2 is 0 here also.

```
function [] =transmission_loss_plot()

R20 = 50/1000;
R21 = 75/1000;
dp = 50/1000;
L = 400/1000;
L1=0.5*L; % + 5/1000;
L3=0.25*L;% - 15/1000;
L2 = L - L1 - L3;
sigma=30/100;  %%% sigma is convetred into perce
 %%%% sigma is enterd in the file as a number be

frange1=5;
```

So, what we will do quickly is that we will go to MATLAB code which has already been written by me. As usual we have transmission loss plot thing and let you know R2 naught means the radius at x = 0 which is 50mm, it is 75mm, diameter is 50 mm.



So, we are basically having a configuration something like this. So, this diameter is same as the radius here. So, once we do that once we go to MATLAB we this is all in our hands, length we have chosen.

```
Y1=c0/(pi*(dp/2)^2);
Y2=c0/(pi*(dp/2)^2);  %%%% impedances of the inl
 elseif choice==2
  Y1=c0/(pi*(dp/2)^2);
 Y2=c0/(pi*(0.04/2)^2);  %%%% impedances of the
 end

[Tf]=uniform_inner_pipe_conical_outer_pipe(R20,R
%%% computing Transfer matrix between inlet/outl
v=abs( sqrt(Y2/Y1)*(Tf(1,1)+(Tf(1,2)/Y2)+(Tf(2,1
Tl=20*log10(v/2);


```

This the reason why let me just; let me just comment it out and put this thing here and porosity is 30 percent everything else is the same you see and we using something to calculate the transmission loss once you get the overall transfer matrix. So, we are giving choice 1; choice 1 means the inner pipe is uniform across its length outer pipe is varying and then we get this kind of a thing.

```
function [Tf_final]=uniform_inner_pipe_conical_
j=sqrt(-1);
c0=343.1382;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
alpha=(R21-R20)/L;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
n=50;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
t=1/1000;
dh=3/1000;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

if choice==1 %%% inlet and outlet ports situate
```

So, we are dividing the length l the gradually varying area duct into say 50 segments of small small lengths over which dd1 by s into d dx of S will remain constant thickness and diameter of the perforated pipe and choice 1 we are giving.

```
if choice==1 %%% inlet and outlet ports situate


    A=eye(4,4);  %%% A is the matrix to be inte
    R2s1 = R20 + (alpha*L1);
    R2s2= R20 + alpha*(L1+L2);
    L3=L- (L1+ L2);

    %ec1= 0.6*(dp/2)*(1- 1.25*((0.5*dp)/R2s1) )
 % L2=L2-ec1-ec2;  L1=L1+ec1; L3=L3+ec2;

    dz=L2/n;
    zeta= ( 0.00734 + j*k0*( t + (0.85*dh) ) )/
```

```
    L3=L- (L1+ L2);

    %ec1= 0.6*(dp/2)*(1- 1.25*((0.5*dp)/R2s1) )
 % L2=L2-ec1-ec2;  L1=L1+ec1; L3=L3+ec2;

    dz=L2/n;
    zeta= ( 0.00734 + j*k0*( t + (0.85*dh) ) )/

for i=1:1:n
        mat=zeros(4,4);
        R= R2s1 + alpha*( ((i-1)*dz) + (dz/2) )

      mat(1,2)=-j*k0;
```

```
    zeta= ( 0.00734 + j*k0*( t + (0.85*dh) ) )/

for i=1:1:n
        mat=zeros(4,4);
        R= R2s1 + alpha*( ((i-1)*dz) + (dz/2) )

      mat(1,2)=-j*k0;

      mat(2,1)=-j*k0 - (4/(dp*zeta));  mat(2,3)

      mat(3,4)=-j*k0;

      exp1=( R^2 - (dp/2)^2 );
```

```
        A=A*expm(-mat*dz);
        clear mat R;

    end

    T1=A;

    clear A dz

    S1u=(pi/4)*(dp^2); S1d=(pi/4)*(dp^2);
```

```
T1=A;

clear A dz

S1u=(pi/4)*(dp^2); S1d=(pi/4)*(dp^2);

S2u=pi*(R2s1)^2 - (pi/4)*(dp^2); S2d=pi*(R2s2)^

T1(1,2)=T1(1,2)*(c0/S1d);   T1(1,4)=T1(1,4)*(c0/

T1(2,1)= T1(2,1)*(S1u/c0);     T1(2,2)=T1(2,2)*(
```

```
T1(1,2)=T1(1,2)*(c0/S1d);   T1(1,4)=T1(1,4)*(c0/

T1(2,1)= T1(2,1)*(S1u/c0);     T1(2,2)=T1(2,2)*(

T1(3,2)=T1(3,2)*(c0/S1d);   T1(3,4)=T1(3,4)*(c0/

T1(4,1)= T1(4,1)*(S2u/c0);     T1(4,2)=T1(4,2)*(
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% coding for the resonator at the inlet....

dz=L1/n;
A=eye(2,2);
```

And so, eventually you know these are some of the parameters that we play around with, do not worry about the end corrections just yet. We just divide this thing into number of parts here and then you know define the transfer matrix for each segment it is initialized with 0 every time when enters the loop.

And what it does is basically calculates this matrix and you know keeps multiplying the A matrix. And finally, at the end of the code we get this guy and then these are the these are the area of the cross section or the inner pipe at upstream and downstream and this is also for outer pipe upstream and downstream.

And then you know once we do that then we basically have to figure out the transfer matrix get into the p v form rather than p rho C naught U form.

```
T1(4,1)= T1(4,1)*(S2u/c0);     T1(4,2)=T1(4,2)*(
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% coding for the resonator at the inlet....

dz=L1/n;
A=eye(2,2);

for i=1:1:n

    r=R20 + alpha*( (i-1)*dz + (dz/2) );
    Sr=pi*r^2 - (pi/4)*(dp^2);
    Yr1=c0/Sr;
```

And then we need to code write an algorithm for the resonator at the inlet of length L1 by n.

```

Z1= -A(2,2)/A(2,1); %%% direction of particle v

clear A dz Tf
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% coding for the resonator at the outlet....

dz=L3/n;
A=eye(2,2);

for i=1:1:n

    r=R2s2 + alpha*( (i-1)*dz + (dz/2) );
```

And similarly coding for the resonator the outlet that is L_3 by n. So, basically L_1 and L_3 were defined to be something like this. You know and let me comment it out sorry put a colon and L_2 is the thing that is in between.

```
Tb(1,1)=T1(1,3); Tb(1,2)=T1(1,4);
Tb(2,1)=T1(2,3); Tb(2,2)=T1(2,4);

Tc(1,1)=Z2;
Tc(2,1)=1;

Td(1,1)=T1(4,1) - (T1(3,1)/Z1); Td(1,2)=T1(4,2)

denom= ( (T1(3,3)*Z2 + T1(3,4))/Z1 ) - (T1(4,3)

Tf_final= Ta + (Tb*Tc*Td)/denom;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
Td(1,1)=T1(4,1) - (T1(3,1)/Z1); Td(1,2)=T1(4,2)

denom= ( (T1(3,3)*Z2 + T1(3,4))/Z1 ) - (T1(4,3)

Tf_final= Ta + (Tb*Tc*Td)/denom;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

elseif choice==2  %%%% inlet port situated on t
    zeta= ( 6*(10^-3) + j*k0*( t + (0.75*dh) )

A=eye(4,4);  %%% A is the matrix to be integrat
    R2s1 = R20 + (alpha*L1);
```

So, once you get this coding you get the impedance expression I was talking about and then all these manipulations are very same very similar to what we saw in the week 5 lecture, ok.

```
L3=L3+ec2;


t + (0.85*dh) ) )/sigma; %%% GRAZING FLOW....



1)*dz) + (dz/2) ) ;




*zeta));  mat(2,3)= (4/(dp*zeta));
```

```
% L2=L2-ec1-ec2;   L1=L1+ec1;  L3=L3+ec2;

    dz=L2/n;
    zeta= ( 0.00734 + j*k0*( t + (0.85*dh) ) )/

for i=1:1:n
      mat=zeros(4,4);
      R= R2s1 + alpha*( ((i-1)*dz) + (dz/2) )

     mat(1,2)=-j*k0;

     mat(2,1)=-j*k0 - (4/(dp*zeta));  mat(2,3)
```

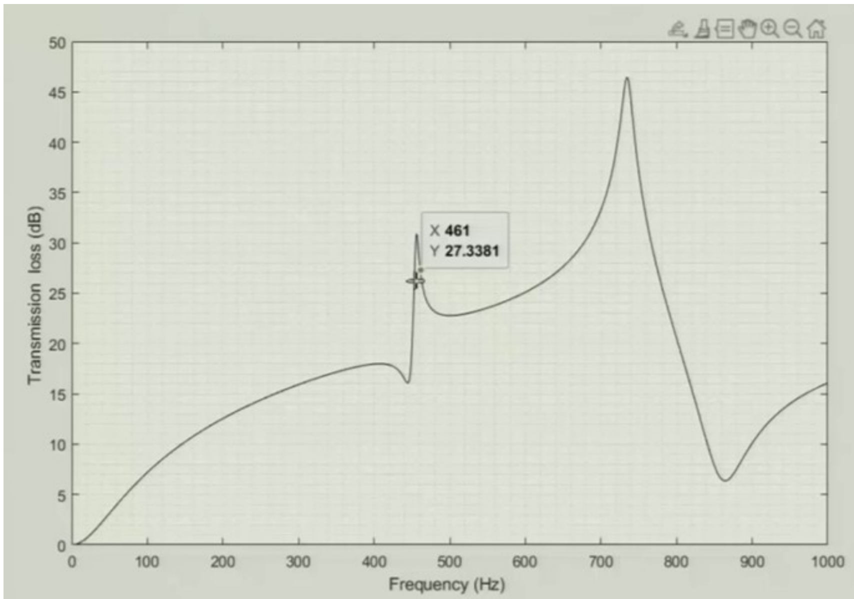
So, in the perforate impedance expression I am assuming for a stationary medium as somewhere mentioned somewhere here you know for a grazing flow you know things would not change too much as we probably would know if there is no kind of a flow. So, what I will do is that I will just run the code and you know do some parametric studies.

What we will do is that; we will you know divide the length L1 into say the resonator length can be 0.5 times L and L3 the resonator length at the outlet can be 0.25 L. So, this is what it is. So, the reason is that because in CCTR or CTR configuration I am sorry we usually you know to find out the tune length to we start off with 0.5 L and 0.25 respectively.

And then we basically because of the lumped inner holes you see the performance does not quite get tuned at L or 0.25 L. You need to relatively a bit shorter length of the inlet and outlet.

```
Command Window
New to MATLAB? See resources for Getting Started.
i =

   995


i =

   996

Elapsed time is 11.582714 seconds.
fx
```

So, let us do one thing; let us run this code for exactly 0.5 L and 0.25 L and see what we are getting. Most likely we will be getting a dip at the first and second actual resonance peaks.



So, we are you know this is not so bad. This is really a here and there is a mismatch in the tune tuning frequency to the configuration here. So, what happens is that the trough that you are seeing here because you are making this as at relatively larger length l by 2 you know whatever resonance peak you are getting here it is not exactly able to nullify the trough that you are getting here; that is why there is a mismatch.

And there is a large mismatch between the resonance offered by the resonance peak due to the second annular cavity here and the second actual resonance peak here. So, this should have really been occurred somewhere here, this is occurring somewhere here. So, what we do is that there are number of ways in which we can mitigate the problem; first one is of course, we will go with only one the first approach is basically do an iterative approach rather than an analytical solution because that would be very tedious.

```
function [] =transmission_loss_plot()

R20 = 50/1000;
R21 = 75/1000;
dp = 50/1000;
L = 400/1000;
L1=0.5*L + 15/1000;
L3=0.25*L - 15/1000;
L2 = L - L1 - L3;
sigma=30/100;  %%% sigma is convetred into perce
 %%%% sigma is enterd in the file as a number be

frange1=5;
```

What we do is that you know let us comment out this thing. Let us you know remember let us go back to this figure you will realize that you know the effective length of this cavity will be shorter than l by 2 because it is a gradually varying area duct, it is no longer l by 2 it is actually shorter ok.

So, what it means is that you know this peak was occurring before after the trough and if it was CTR configuration this peak would have occurred you know this peak would have occurred before. So, what really this is a, this is an important concept here. What it means is that in this we must add a little length let us say let us start with say I have written 5 let us start with 15.

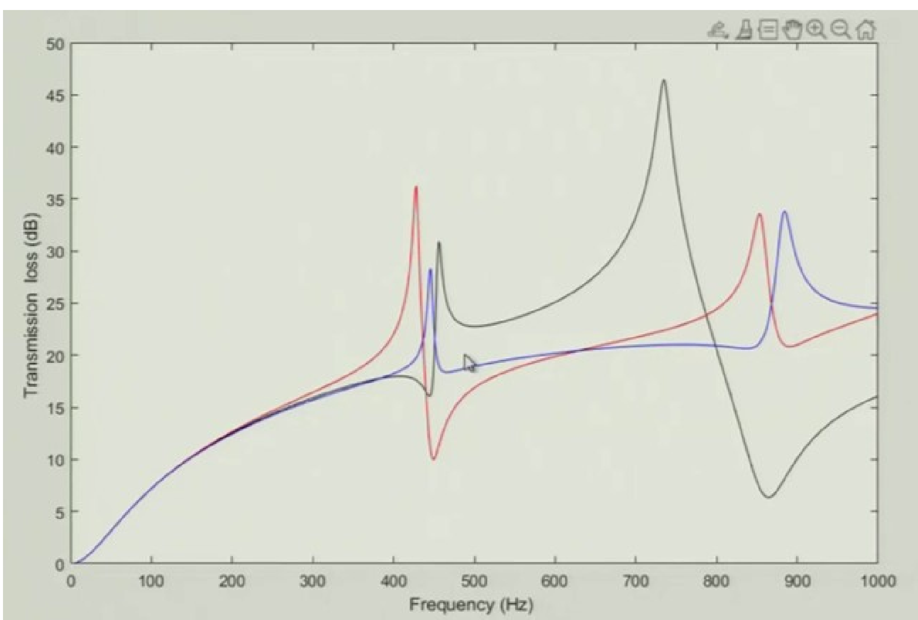
And in this the other thing we must subtract a little bit length because by that time you know at this thing it is almost behaving like a annular area with the uniform cross section.

```
Command Window
New to MATLAB? See resources for Getting Started.
    995


i =

    996

Elapsed time is 11.582714 seconds.
>> figure(1)
>> hold on
>> transmission_loss_plot()
```

So, basically we can sort of subtract a little bit length here and see what we are getting we can do figure 1 on hold on and plot this guy again and see what we are getting.

```
        Tl(i)=transmission_loss(R20,R21,dp,L,L1,L2,s
          i
    end
    ch='r';
    plot(f,Tl,ch);
    grid minor
    xlabel('Frequency (Hz) ');
    ylabel('Transmission  loss (dB) ');
    toc;
```

```
Command Window
New to MATLAB? See resources for Getting Started.

     995


i =

     996

Elapsed time is 11.101761 seconds.
>> grid minor
fx >>
```



So, we are getting this sort of a thing. So, things are a little better here they although you know with a little bit fine tuning we can definitely get rid of this peak I am sorry of this trough we will see soon and by subtracting certain length of this pipe we are able to get this thing this guy closer to the actual resonance second actual trough.

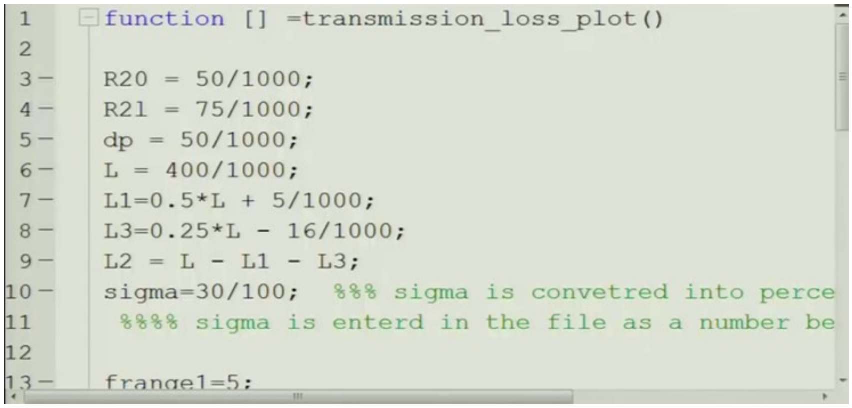
So, we will keep this thing as it is and what we will do is that we will go back to this thing and let me use a blue color thing and let us say reduce this by to 5.

```

R20 = 50/1000;
R21 = 75/1000;
dp = 50/1000;
L = 400/1000;
L1=0.5*L + 5/1000;
L3=0.25*L - 18/1000;
L2 = L - L1 - L3;
sigma=30/100;  %%% sigma is convetred into perce
 %%%% sigma is enterd in the file as a number be

frange1=5;
frange2=1000;
```

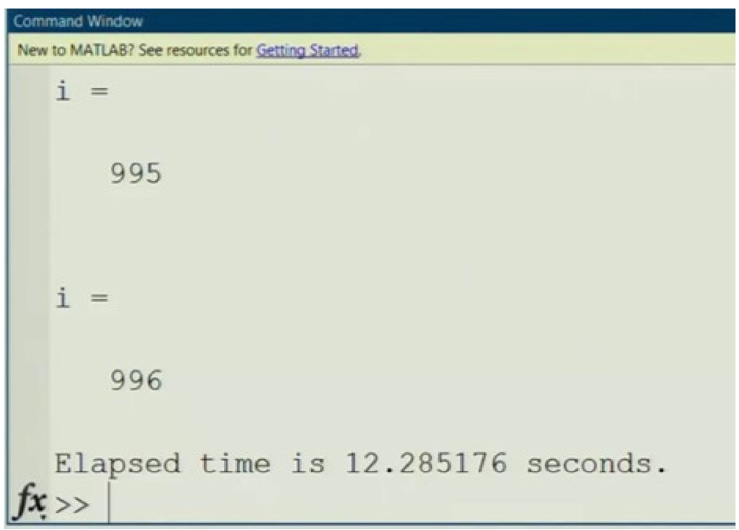
And let this guy be probably this is getting at 15 ok, so, minus 15. So, because I have subtracted a significant length so, the peak was delayed. And if I were to you know subtract this a bit more perhaps. Let us see make it 18, let us see what we are able to achieve.

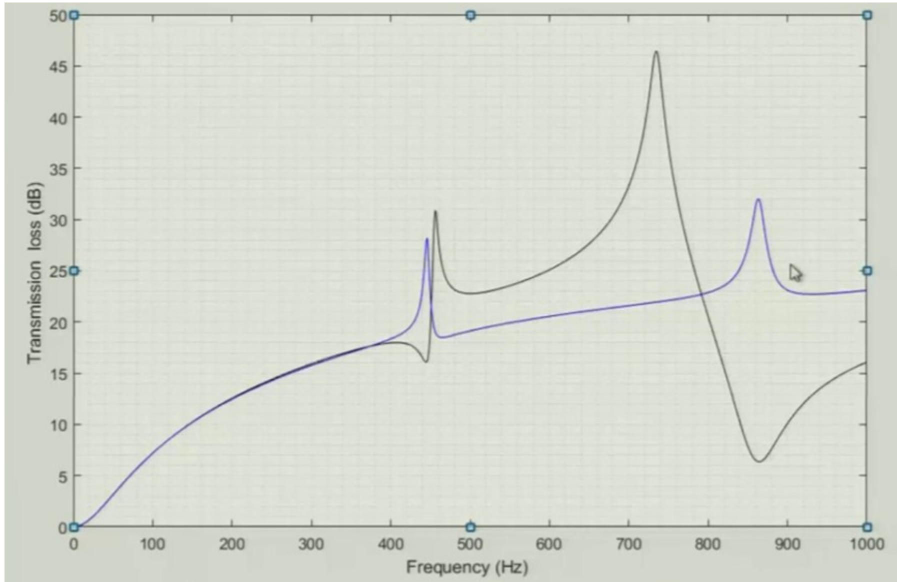
```
Command Window
New to MATLAB? See resources for Getting Started.
  i =

     995


  i =

     996

  Elapsed time is 10.251051 seconds.
fx >>
```

I hope the second thing would be sorted out is just an experiment that I am doing really the first is definitely killed. So, I will get rid of this red colored curve it is a little bit of distraction and this has definitely you know taken out taken away the first actual resonance and this one almost well this has almost sort of killed this thing.



So, you know roughly between sorry roughly between 15 to 18 mm you can find out, you can still you know you can still get a fine tuned value you can put it 16 and compute this guy again.

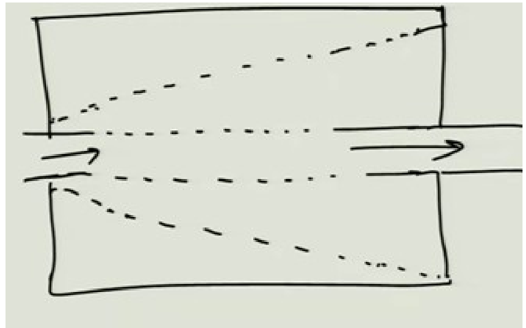




And let us see the value what we are getting, we are getting almost a perfect tuning ok. So, the point I am trying to make is that even for a CCTR you can you know find out the differential length by fine tuning them. Basically differential length is the difference between the length of the configuration in which you do not have a perforated bridge.

And find out theoretical based on plane wave limit exactly the frequency at which the first and second actual resonances would kill the first and second troughs or the actual resonance peaks actual resonance troughs of the chamber. And then from that thing, but then when you put a perforated bridge these perforated hole have a inertance effect and that would basically kind of you have to you know do some sort of a iteration and to figure out how much is the effect and all that sort of thing.

So, once you do that we can sort of find out the exact length. So, with this particular demonstration I would sort of end my presentation end my lecture for week 10 like this we can keep analyzing. You know much more complicated configurations. Let me just draw 2 3 things for you. We need not analyze them just straight away. Let us say we have something like this sort of a thing. This pipe is also having a variable area thing you know something like this.



Or you could probably have things like ok. So, you know you can have multiple interacting ducts with perforates and one can do a lot of things analyze these kinds of things. So, with this I will take a break here and I will see you in the week next week that is week 11 where you know week 11 is going to be important although we have had an extended session for perforated components because it is very important. Although we have been dealing only with planar wave analysis so far.

We for the in week 11 we will go for the first time for 3D analysis only of reactive mufflers in which they are no perforates ok some simple empty chambers like rectangular sections, circular sections and with different orientations of pipe. And I will introduce you to you know scattering matrix concept and impedance matrix concept with a little bit of more reinforcement of our fundamentals and derive relation between impedance and scattering matrix.

And how to find out transmission loss in terms of such parameters how to characterize you know such chamber single inlet single outlet chamber of rectangular cross section or circular cross section using you know a piston driven approach or a point source approach and try to side inlet side outlet. Number of such configurations will be the focus of next week's lecture. And what it does is basically by optimizing the location of ports you can definitely kill the resonance I mean basically get a broadband attenuation.

So, all this will be a focus of week 11 and let us see how far we can go up to for 3D analysis of such things. One can avoid a finite element analysis if we have already have 3D analytical codes, but again it is performance is still restrictive or simple configurations. Let me tell you something. We will see more of that in week 11. So, till that time stay tuned.

Thanks.