Smart Materials and Intelligent System Design
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Lecture – 08
Micromechanics and Macromechanics of Composites

Good morning, welcome to the course on Smart Materials and Intelligent System Design. And we are today at the round 3 of the composite materials where I would like to discuss mostly about the mechanics micromechanics of composite materials.

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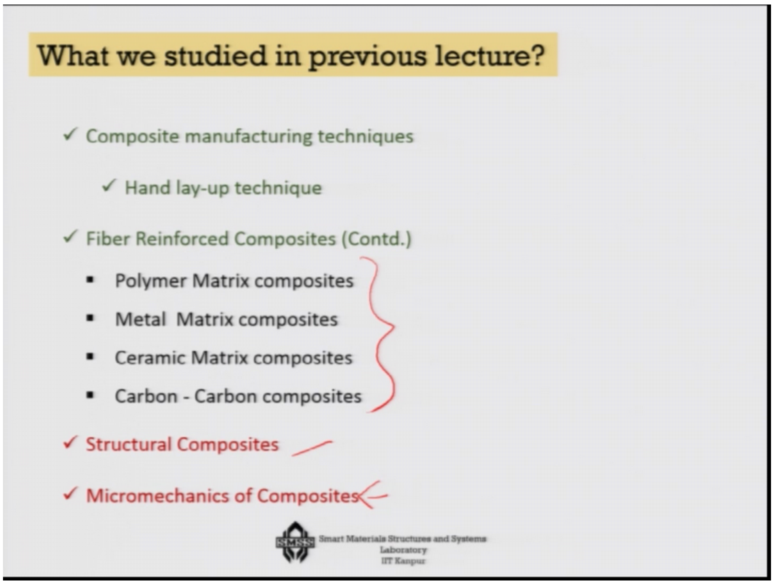

So, we have so far taken a quick good roundup of the composite manufacturing techniques, hand layup techniques. And all these different types of polymer matrix composite, structural composites and now we are here today at the micromechanics of composites.

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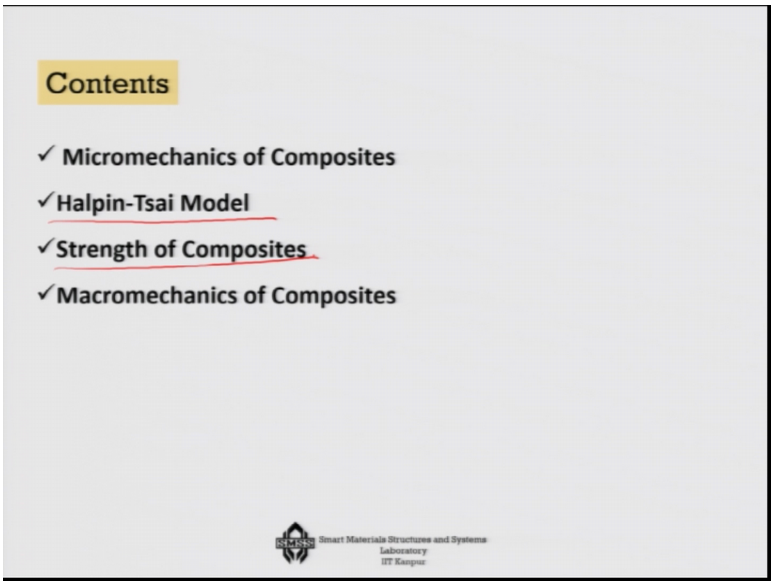

So, I would like to talk about the micromechanics, how the various fiber matrix interaction occurs in composites, how it affects the stiffness and the strength. There is a Halpin Tsai model, empirical model we would like to mention that the strength of the composites. And then from there we will take up gradually for the macromechanics of the composite.

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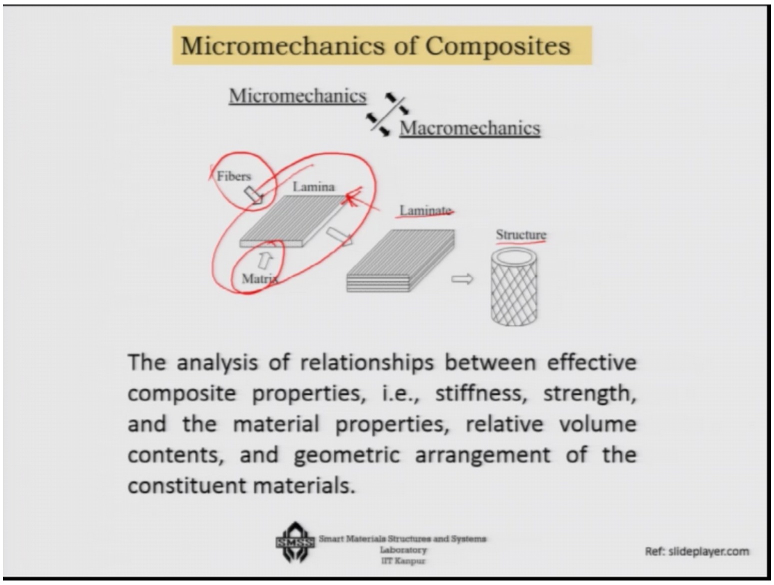

Now, what is this micro macro thing? Well, when we are talking about the composite in terms of a laminate, each laminate will consist of if you go back several lamina. So, first we have the structure, the structure will be made up of laminate, that laminate will consists of lamina. In while talking about micromechanics, we will focus ourselves in this domain. In the lamina we have the fibers we have the matrix, but generally all in one plane and then you know in the laminate we also stack them and then we make different you know structures out of it.

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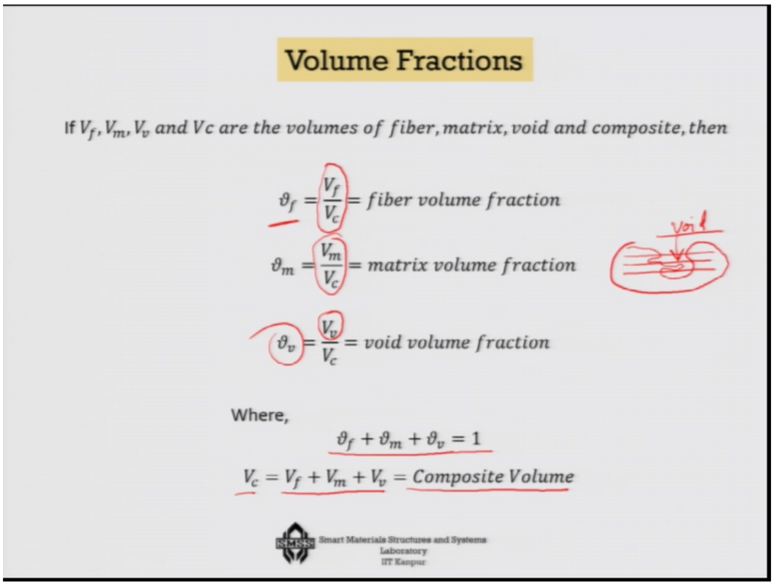

Now, one of the most important thing then we need to focus on is called the volume fractions, how much of fiber is there in the system. So, like V f which is a volume fraction, volume of fiber versus volume of composite or V m volume of matrix versus volume of composite. Suppose the composite is not fully impregnated by the resin; that means, suppose these are the fibers and if I look at the resin picture very carefully, I may see that this is the some of the areas are not actually weighted by the resins.

That means there are voids here. So, then volume of void also plays a role here. So, you can also have a void volume fraction corresponding to this. So, all this things together should make up the volume of the composite and as a result, all these volume fractions together equals to unity. The volume of the composite is actually V f plus V m plus V v which is the composite volume.

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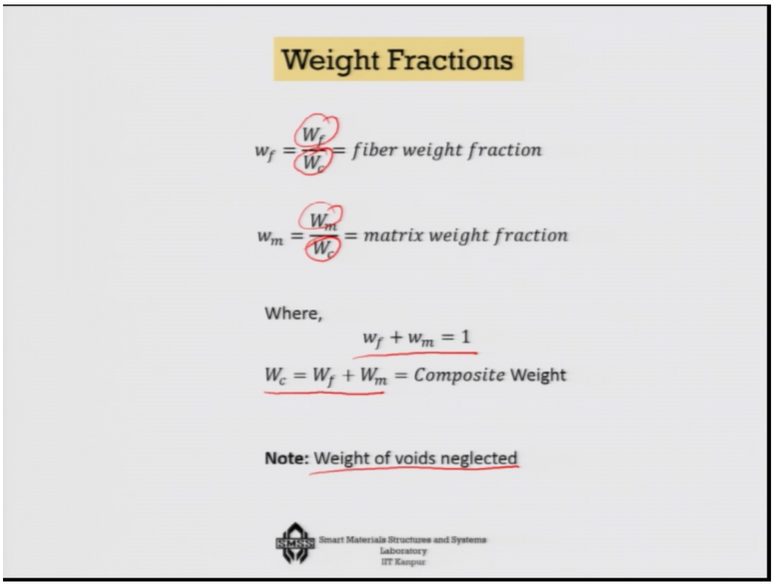

Now if you think of the whole thing in terms of weight fraction. Then you have a weight of the fiber versus weight of the composite that is W f, weight of the matrix versus weight of the composite that is W m and void has no weight. So, W f plus W m has to be equal to unity and weight of the composite is W f fiber total weight and matrix total weight.

Many times so, weight of voids is neglected here. Many times it is actually easier to measure the weight fraction and then we can try to get the volume fraction out of it.

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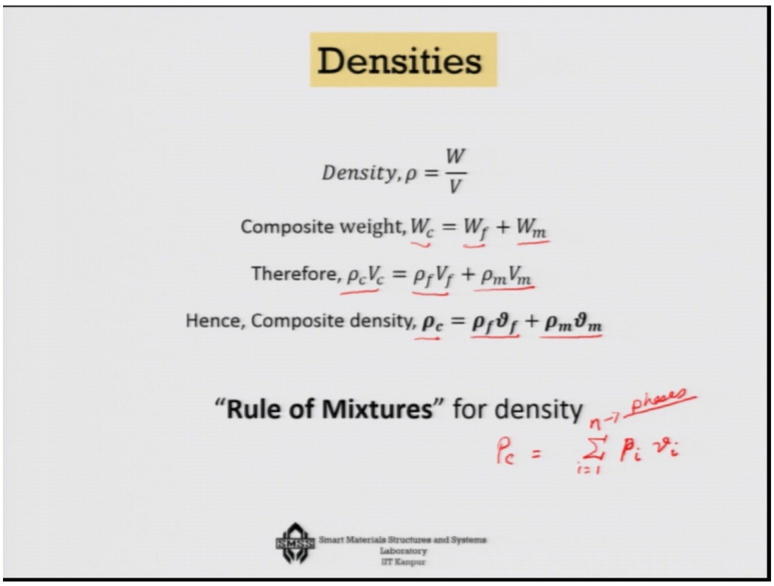

To do that first of all we have to know what is the density of a composite. Now the composite weight W c is W f plus W m. Now if I write W c in terms of rho c V c then this is rho f V f this is rho m v m. So, in terms of composite density rho c is nothing, but rho f V f plus rho m V m this is also known as rule of mixture.

In many cases we will try to extend this, because we will say that the property P c will be summation of i equals to 1 to n p I you know property; so P i and v i. So, each one of these you know there are n number of phases in the system. If it is simple one fiber one matrix then n equals to 2 if it is hybrid composite with 2 fibers one matrix then n equals to 3 and so on. So, we can use this you know extension of the rule of mixtures.

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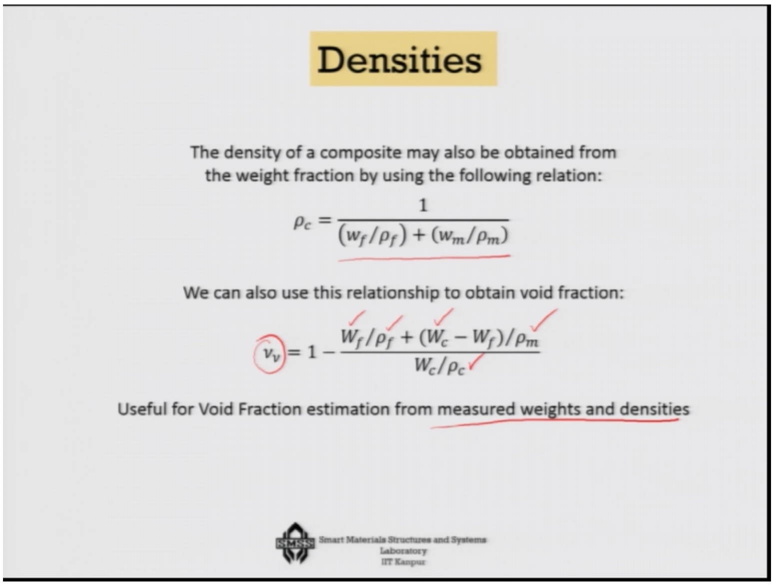

Now, next is that this rho c I can write in terms of the weight fraction right just by simple you know algebraic manipulation. I can also obtain the void fraction once I know weight of the fiber, which we can do from a (Refer Time: 05:00) test weight of the composite is known to us and density of the fiber known to us, density of the matrix known to us, density of the composite and is known to us then I can find out what is the void fraction.

So, in terms of the measured weights and densities we can actually get the void fraction of a composite material.

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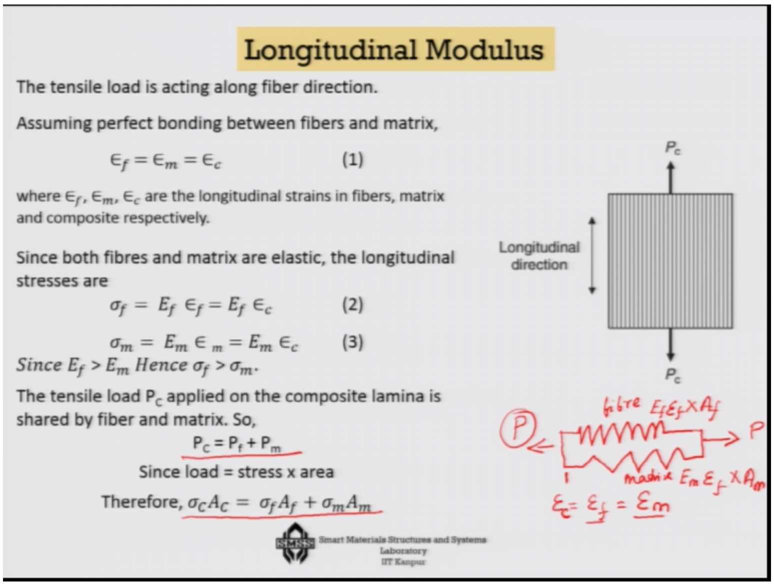

Now, one of the most important property that we invariably try to first measure is what is the composites longitudinal modulus. To do that you imagine that you have to have an assumption here, that suppose you have 2 springs here one spring is a little bit of a strong spring and the other spring is a bit weak spring ok. So, you have 2 springs here and you are applying the load ok. The stiffness of each of this spring if this is a stiffer spring, you may think that this is a fiber and this can be a matrix.

When I am applying the longitudinal load here P, the load is does not know that it has 2 different types of springs. So, the load goes to the entire system in a manner that the strain at fiber and the matrix ultimately has to be same. So, the strain that I am going to see here in that epsilon equals to epsilon fiber, and that equals to epsilon of the matrix and that is also sometimes the strain we call it as epsilon C.

So, if the strain is same; that means, what is the stress in each one of them? Well for this one it is E f epsilon f, and for this one it is E m epsilon. Once again I can write is f because epsilon f and epsilon m are same. So, that is what are the stresses multiply it by the characteristic area of each one of them A f and A m. So, you know how much of force is going to each one of them. Finally, the total load has to be taken by these 2 springs. So, P c has to be equal to P f plus P m. So, there if I now apply this relationship back I am going to get this relationship which you know I can further write it down a little bit of algebraic manipulation, I can write it down that sigma c is nothing, but sigma f times A f over A c sigma m times A m over A c.

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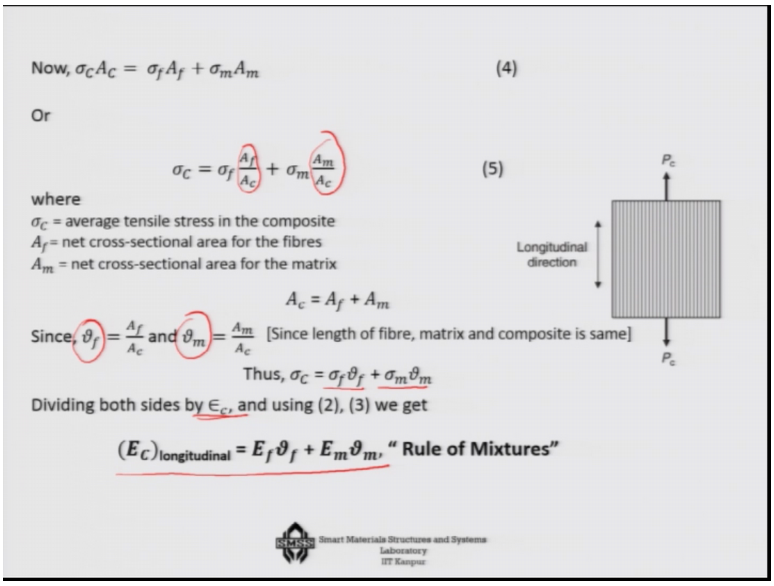

Now, what is this A f over A c and A m over A c? If you think of it that if it if I take the cross the width direction unit width, then it is nothing, but the volume. So, it is the volume fraction of the fiber and volume fraction of the matrix. So, I can actually write sigma c as sigma f V f plus sigma m v m here also you can see that the rule of mixture is coming into picture. Now if I you know divide both the sides by epsilon c which is same for fiber, same for composite, same for the matrix, I actually get E c as E f V f plus E m v m. Again we have the rule of mixture here just like we saw it for the density.

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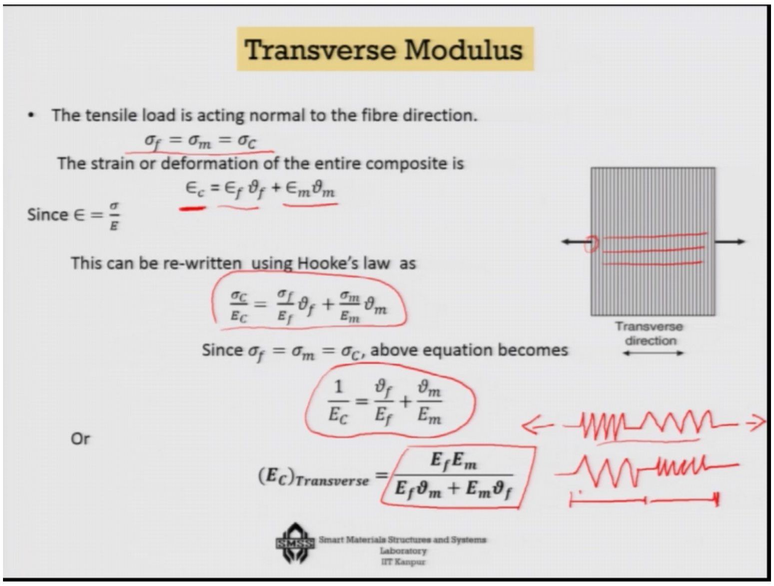

So, that is about the elastic modulus of the composite in the longitudinal direction. Now what happens? In the transverse direction because in the longitudinal direction we assume that there is a same strain, but in the transverse direction is the fiber, which is actually or the matrix one of them we will get directly interaction with the force. It is not that both of them together are getting you know in touch with the force, it is either the fiber or the matrix. So, in this case we will assume that the load will be transferred across these which will be same.

So, in this case the 2 strings that we have earlier shown, now this 2 springs are in you know depending on the condition they are in series. Either like this if it is fiber first or like this if it is matrix first, but they are in series. So, what it means is that, if I apply the load at the 2 ends here the same load is going to get passed. In the earlier case the deformation was same in this case it is the load which will be the same in both the things.

So, here uniformity will be coming in terms of that the stress at the fiber has to be equal to the stress of the matrix, and that has to be equals to the stress in the composite. So, on the other hand we also know that for the entire composite, the epsilon c is actually the deformation that we will see here plus the deformation that we will see here. So, it is epsilon f V f plus epsilon m v m. So, if I apply all these things you know these 2 things together, I get this particular way of you know writing the constitutive relationships and that shows us that in this case the composite modulus of elasticity is actually in a reciprocal relationship.

So, the E c transverse as per this model is actually like this ok. So, it is it will not follow the rule of mixture, but it is a reciprocal relationships that will be followed in this particular case.

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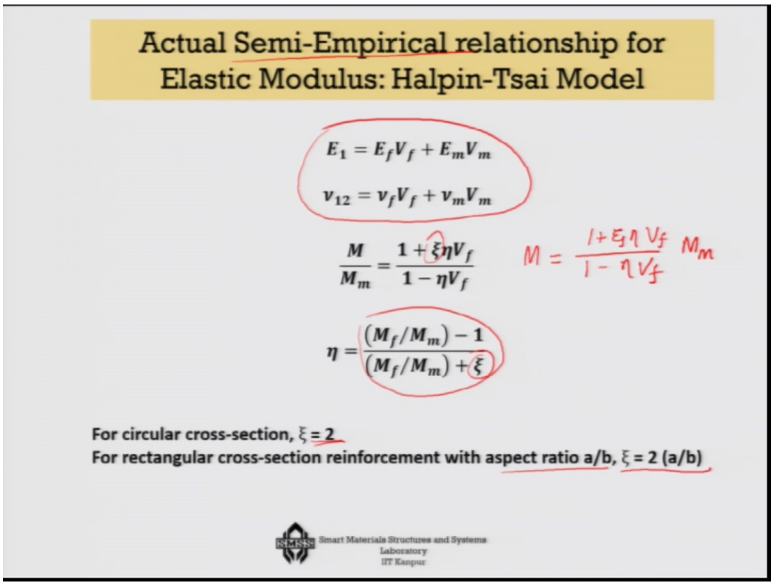

This is second expression many times actually would not match with the experimental results. The reason being that you can neither say that it is completely like this fibers first then matrix, it is actually interplay between the fiber and the matrix. So, from that point of view the rule of mixture theoretical part is used for longitudinal modulus of elasticity and Poisson's ratio whereas, for the transverse direction we use something which is known as Halpin Tsai model.

In the Halpin Tsai model, there are so many other factors here like the what is the cross section of the fiber for circular cross section it is 2 and also we have to know what is eta, and if you look at the expression of eta, then other along with all other things there is something called xi there, and that is you know that depends on the aspect ratio of the fiber and xi is generally considered as 2 times a over b. So, as a result you know this entire formulation gives us the M in terms of you know all these reinforcement factors 1 minus eta V f times M m as a multiplication factor with the matrix.

So, this is pretty good actually even though this is an empirical relationships semi empirical we call it, but this matches pretty good with the experimental results and this is popularly known as Halpin Tsai model.

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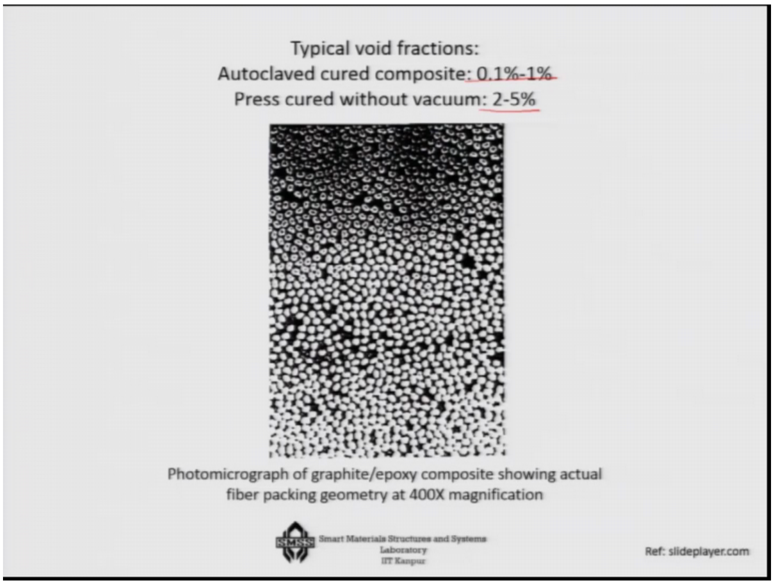

Now, you can see here that a typical composite which has you know autoclave which has been and it has void here typical void fractions, varying between point 1 to 1 percent and if it is pressure cured without vacuum, it may be 2 to 5 percent. That is why if you remember in the earlier lecture I have shown you how they were doing the vacuum curing. So, that they can reduce these you know void fractions in the composite. So, as a result you can get actually higher density you can get higher modulus of elasticity higher strains etcetera.

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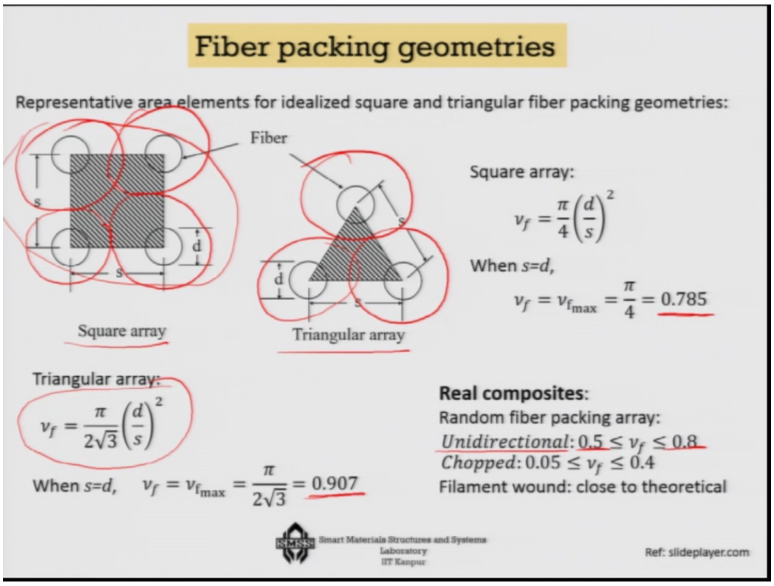

Also another important thing here is that what is the fiber packing geometry. So, you can have fibers in this manner placed where d is the diameter of the fiber, and s is the spacing distance and this is called as a square array formation. You can have a triangular array formation also and with a simple we talk geometry checking you would see that the fiber volume fraction can be expressed as in terms of these 2 parameters the diameter of the fiber and the spacing of the fiber. And you can also see that the extreme case when s equals to d means each fiber is so big that they are actually touching each other, that is the extreme case then the maximum volume fraction from triangular array will come out to be about 0.907.

Whereas for square array when each one of them at you know touching each other extreme case, that all of them are touching each other there you will get a maximum volume fraction of about 0.785. So, we can conclude that triangular arrangement although it is difficult, but it will give you a better you know fiber packing actually. The other point is that if we go for you know unidirectional packing, you get a volume fraction anywhere between 0.5 and 0.8. Whereas the moment you chop it actually comes down. So, chopping the fiber would also reduce the packing volume of the fibers.

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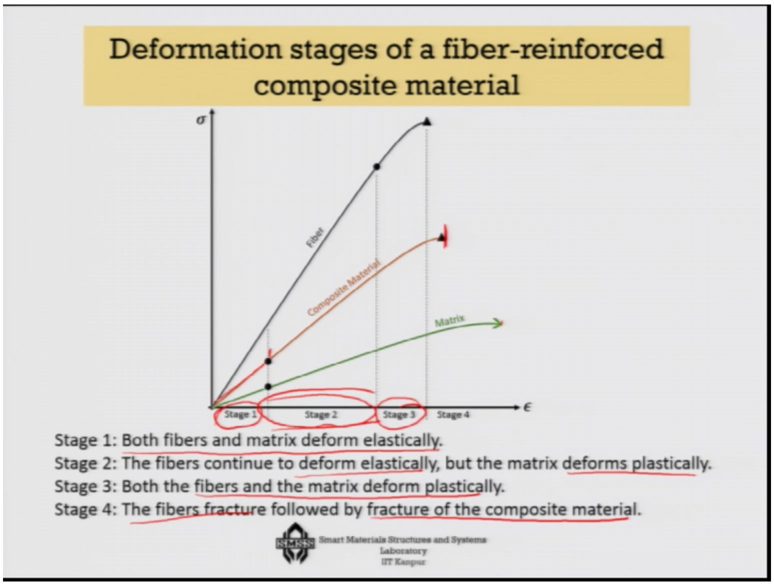

Now, we have talked about the strength of the stress strain relationship of the composite. And in terms of you know the stress strain deformations if you look at very carefully then, you will find that there are four stages actually need. Stage 1 where both fibers and matrix deform elastically till this region elastic deformation of the two. Stage 2 fiber continues to deform elastically, but the matrix deforms plastically. Matrix starts to go into plastic deformation at a very early stage. So, that is the stage 2 which is a very predominant stage here.

Stage 3 both the fibers and the matrix deform plastically this is a very brief stage most of the cases because fibers cannot deform much plastically, they are generally very brittle material. So, you get a smaller stage 1 and a smaller stage 2 and a very you know broad stage two. Stage 1 and stage 3 are big and stage 4 fibers fracture followed by the fracture of the composite material. So in fact, little more the fibers of fractured matrix is still there as you can see, but we will consider these to be failure because after all matrix except metal matrix composite matrix are not supposed to take the load.

So, whenever there is a failure of the fibe, we have to assume that the purpose of the composite is got the composite has failed. So, that is the point of failure of the composite.

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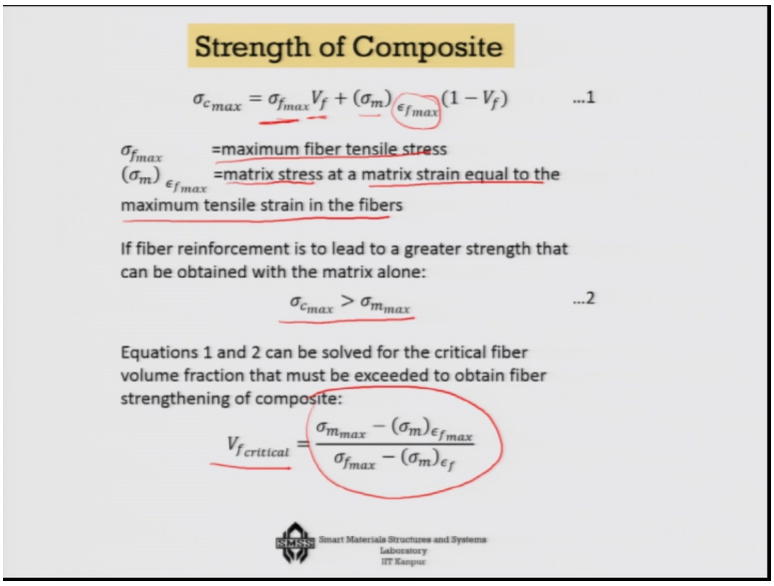

How do you find out then the strength of a composite? Well then it would mean that the strength of sigma f max will dominate. So, sigma f max failure point till the failure point strains time V f plus sigma m at epsilon f max that is what is very important; that is the maximum strain beyond which even if the matrix would have allowed we will not consider that part that will be the reserved part if at all. So, sigma f max is our maximum fiber tensile stress and sigma m at epsilon f max is matrix stress at a matrix strain equal to the maximum tensile strain in the fiber.

In a fiber reinforced composite you know we always try to get sigma c max to be greater than sigma m max. So, that is way otherwise why do we have a composite? So, we need to know how much of volume fraction is needed at all to achieve this? Well the V f critical is actually this ratio of the 2 which will tell us that yes we have achieved this strength of the composite. So, that is where the critical volume fraction will come into the picture.

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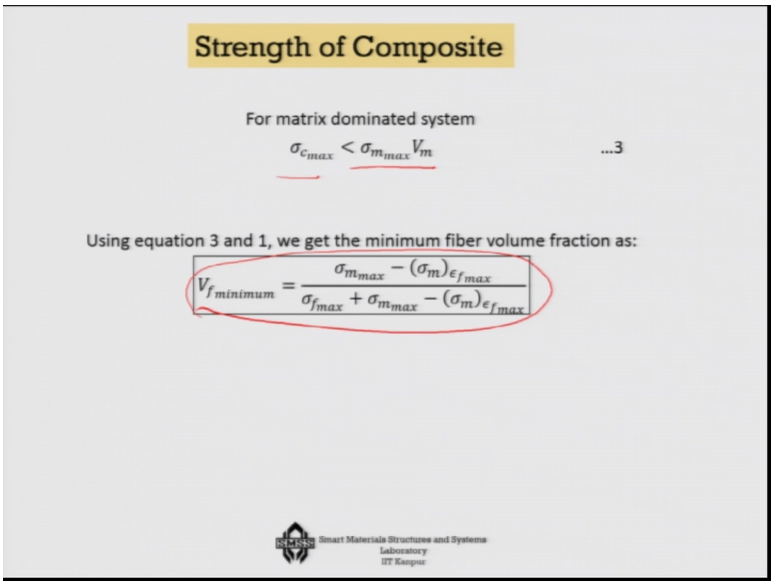

Now, for a matrix dominated system let us say for a metal matrix composite, sigma c max will be less than sigma m max time V m. So, that will tell us using this equation in the first that what is the V f minimum in order to avoid matrix domination, you need to know; what is sigma m max. What is sigma m at epsilon f max and in this expression we will tell you that; what is V f minimum that you have to give in order to avoid the domination of the matrix in the system; so basically these 2 things guides us towards the volume fraction of the fiber that we have to have in the system in order to get improvement in terms of the strength of the composite.

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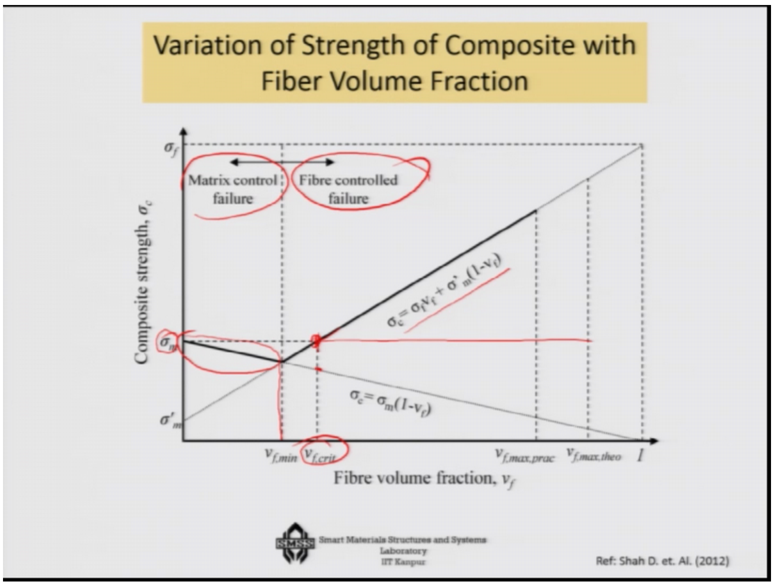

So, here is a. So, you know small example to actually graphically illustrate the same thing. So, if you look at it, that this is the composite strength that we are plotting with respect to various fiber volume fractions. Now till this particular point that is; what is our V f minimum, it is the matrix strength which is dominating fiber is no where into picture. Beyond this point till this point if you add fiber then you can see that the matrix strength is coming down fiber strength is picking up the strength that is contributed by the fiber. But still it is not as high as sigma m, it is beyond this particular point V f critic and I have shown you earlier the V f critical that is V f minimum this is the V f critical.

So, if I put V f critical which is higher than V f minimum, then the fiber is going to dominate in the sense that matrix strength is already far low and then you know it will never be the if we you would have only used the matrix, I would have never achieve that same strength. And then this becomes the picture which will the rule of mixture will start to come into the picture until the failure of the fiber. So, that is the fiber controlled failure region and before this there is a matrix controlled failure region.

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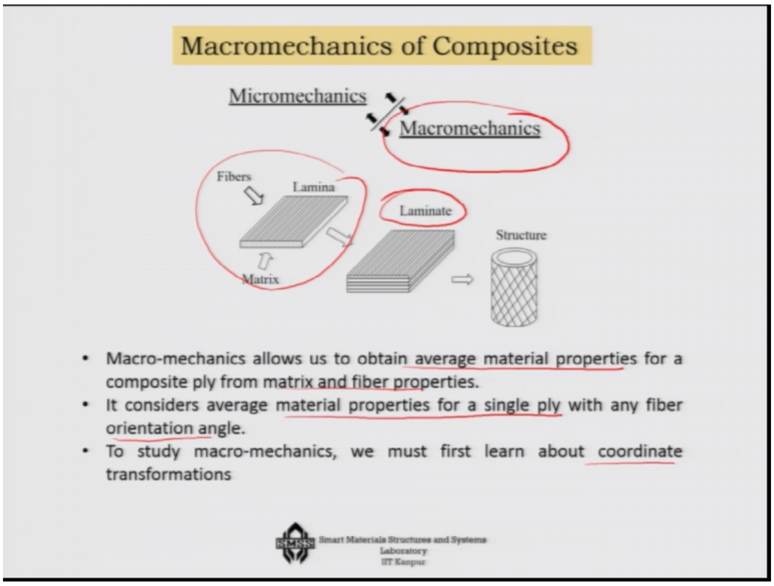

Now, from here we have to you know once we know the micromechanics of the system like the fibers matrix and the lamina interplay, then we can actually stack them in order to form the laminate ok. And once we come to laminate, we actually change our domain from micromechanics to macromechanics. So, this will allow us in the macromechanics domain, to obtain average material properties for a composite ply from matrix and fiber properties. And it will also consider average material properties for a single ply with any fiber orientation angle, and to study this we have to learn about the coordinate transformations ok.

So, let us look into the coordinate transformations.

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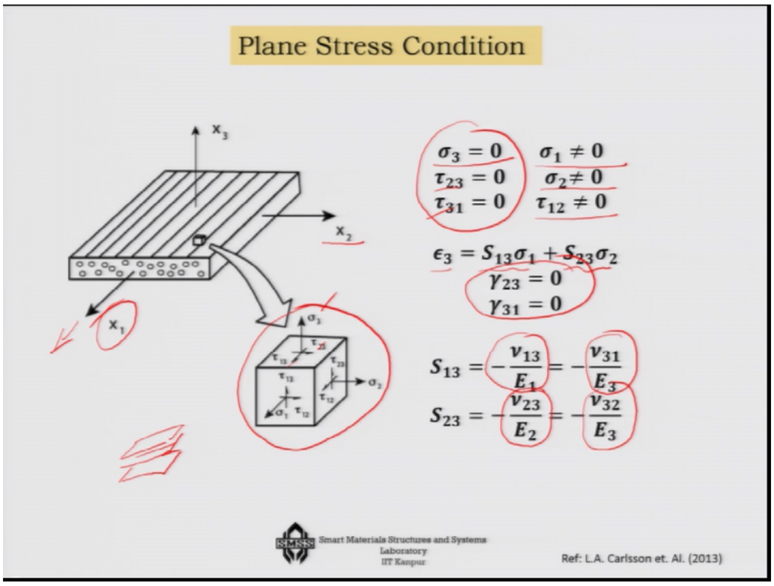

So, as you know that the fibers could be at various angles ok. So, the angle of the fiber if it is you know aligned to one of the coordinate axis fundamental directions. So, then you know we can talk about it in terms of the principal components of the stresses. So, first of all we have a plane stress condition as we can see, and the fibers are aligned along this direction this is the transverse direction and this is the out of plane direction.

Now, if we look at the state of stress of any small cube here, you know that what are the various components of the stresses, but in the plane stress condition some of the state of stress are 0 like sigma 3 is 0 tau 2 3 and tau 1 3 they are 0. Why is it so, because we are essentially making a composite out of some of these thin sheets. So, in each one of these sheets they are actually planar in nature. So, that is why you know you get this out of planes to be 0. And if we have a substantially thick composite, then actually you cannot neglect it, but in most of the cases the composite thickness will not be too high and you can consider it as a plane stress condition.

So, what is non zero here sigma 1 is non zero the stress along these direction and the direction of the fiber sigma 2 transverse to it and tau 1 2. Now what is the epsilon 3 for us? Well that can be expressed in terms of epsilon 1 complained why did you know the complained modulus and you know S 23 and you can express that.

Shear strains gamma 23 and gamma 3 ones are 0 and the S 13 you will can be expressed either in terms of nu 13 or in terms of nu 31. And S 23 can be expressed in terms of either nu 23 or nu 32. So, this is in a plane stress condition that stress you know and the modulus.

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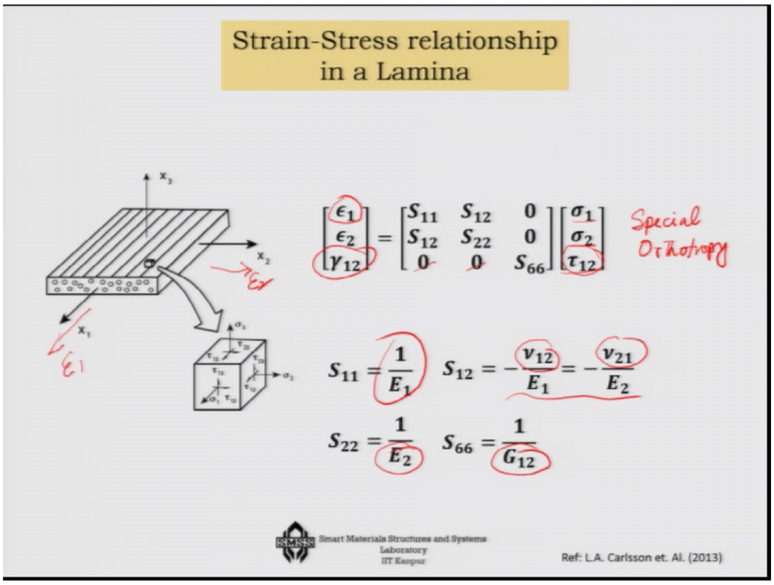

So, what is the constitutive relationship for us now? Well, we are considering only 3 strain components here right epsilon 1 that is the strain along this direction epsilon 1, epsilon 2 strain along this direction and there is a shear strain gamma 12 correspondingly you have sigma 1 sigma 2 and tau 12. So, as you can see that the shear strain is not related to this sigma 1 and sigma 2.

So, these components are actually 0 similarly the normal stresses are not actually going to affect the shear strain or the shear stress is not going to affect the normal strains ok. So, that is what we call as a special orthotropy it is a special case of a orthotropic system special orthotropy. And S 11 is reciprocation kind of E 1 S 12 can be described either in terms of nu 12 or in terms of nu 2 1 S 22 is similarly s inverse of E 2 and S 66 is inverse of the shear modulus G 1 2 of the system.

So, this is the stress strain relationship in a lamina where the fiber are aligned with respect to the axis.

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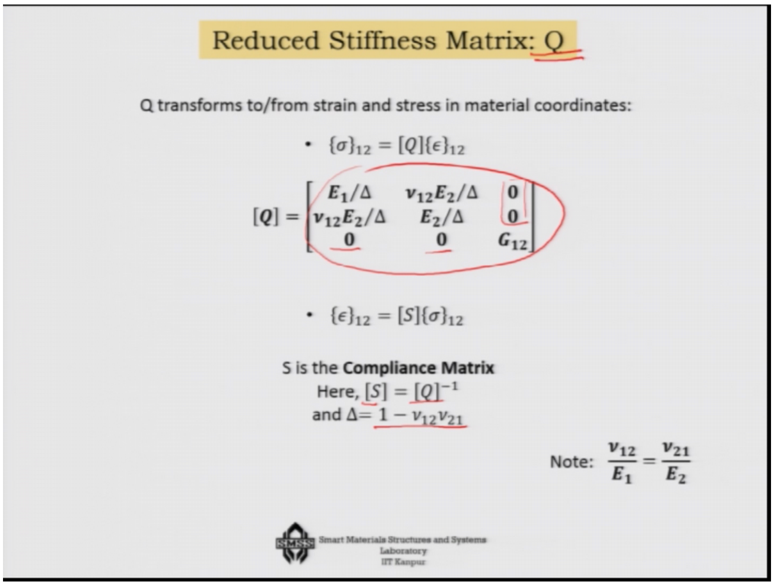

If the fibers are not aligned then we have to have an angle condition to it now we will come to that. So, what is the in the principal direction, then what is the constitutive relationship stress versus strain? The earlier relationship was strain versus stress. So, strain was expressed in terms of stress. If I express stress versus strain then that S compliance matrix now becomes Q matrix and this becomes the relationship of the Q matrix. The important fact is that still these zeros tell us that the shear stress shear strain is actually decoupled from the normal stress normal strains. So, S is essentially Q inverse and delta here is 1 minus nu 1 2 nu 2 1.

So, this is we call it as a reduced stiffness matrix. So, we have either the compliance matrix or we have the reduced stiffness matrix with us.

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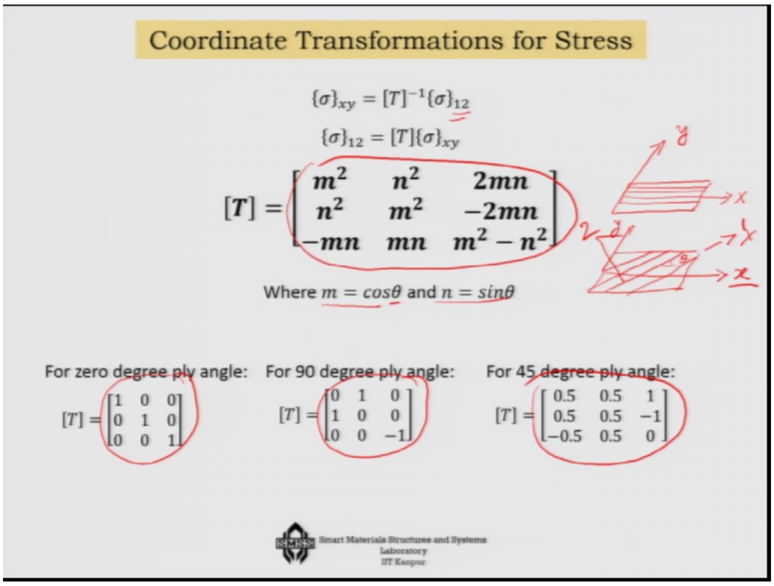

You know fibers I told you that suppose this is what is my x direction and the fibers are aligned to it, then I can use straight forward this particular relationship ok. So, this is my x and the y. But suppose the fibers are having some angle with this. So, this is my x direction and this is my coordinate system and the fibers are at an angle with this let us say this fiber angle is theta you also call it as ply angle. If there is a ply angle there then the stress strain relationship if I am not interested in these direction, but in these direction along x and y strain, then I have to transform this stress strain relationship in the xy direction from the 1 2 direction.

This was my 1 2 direction right along the direction of fiber and transfers to it, but I want to transfer me to xy direction. So, I have to apply a transformation matrix to do it and this is what is the transformation matrix here, where m is cos theta and n is sine theta just keep in mind that this is orthogonal transformation. Now this if I put various values of theta like if I put theta equals to 0, the same matrix we will see it will take this form apply theta equals to 0 ok. If I put theta equals to 90 it will take this particular form.

If I take theta equals to 45 it will take this particular form. Sometimes it is the good idea to keep some of the t matrix in your mind that for particularly 0 degree 90 degree and 45 degree because this helps us to very quickly transform the stresses from the principle direction to the direction along which we want to carry out the analysis of the system.

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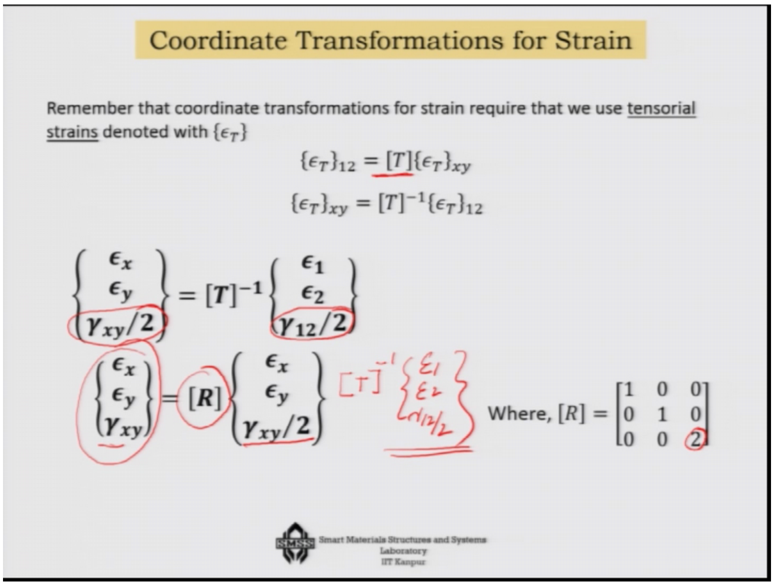

Now, once we have transferred the stress, can we not transfer the strain in the same way well in order to do that that we have to keep in our mind that we use the tensorial strains here. So, if I want to keep the same transformation you know for system, then I have to use the tensorial strains and that means, the last term has to be gamma one 2 by 2 ok. Otherwise, nothing else will happen, but this t will become another you know transformation matrix that is all. But if you want to preserve the same transformation matrix then you have to keep in your mind that the shear strains will be actually half the relationship will be between the half in both the sides.

So, in terms of the full shear stress shear strain or the full strain vector epsilon x epsilon y gamma xy, you have to multiply this with something called R; where R is simply a diagonal matrix with the last term s 2 because this is gamma xy this is gamma xy by 2. So, you have to multiply it by 2 that is all. So, you need an R and then that R will be followed by T inverse that is what is this part and that will be multiplied by epsilon 1 epsilon 2 gamma 1 2 by 2.

So then epsilon x epsilon y gamma xy will be related to the tensorial strain you know in the principal axis direction.

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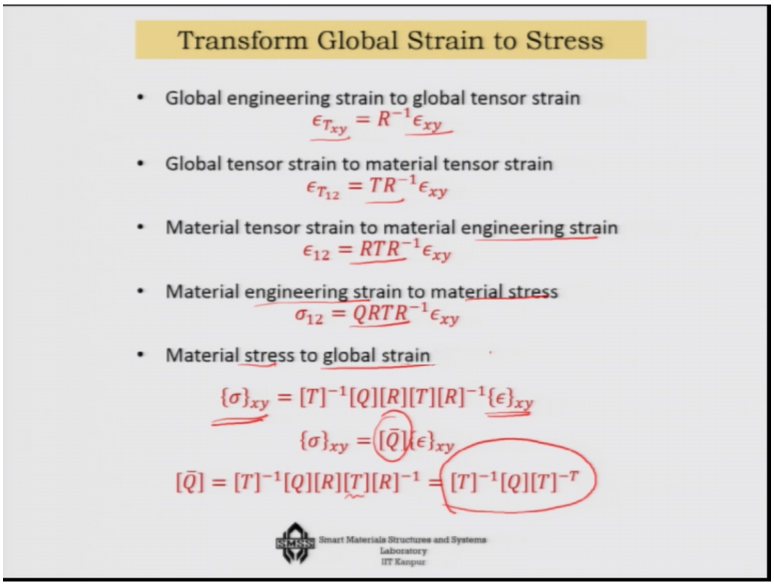

These are the following easy transformation, you want to convert between engineering strain to global strain a tensorial strain R inverse, global tensorial strain to material tensorial strain TR inverse, material tensorial strain to material engineering strain it will be RTR inverse. And if I want to have a relationship between material engineering strain to material stress multiply it by Q, it will be QRTR inverse. If I want material stress to global strain then this sigma xy to then it will be sigma xy which will come out with respect to epsilon xy.

So, the multiplication you know that matrix will be now Q bar which is nothing, but this T inverse Q T minus which will contain you know the Q along with this you know this T minus T which will have that transformation part in it. So, basically step by step with the help of R and T you can get a relationship between sigma xy and epsilon xy ok. So, that is the global stress strain relationship that you are going to get through this way.

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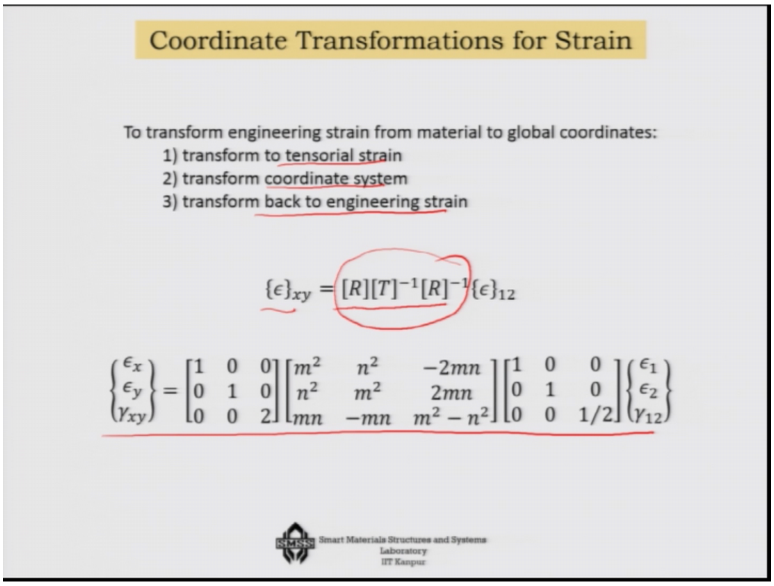

Coordinate transformations for strain if you want to do, then epsilon xy can be also written just by the same way that transform to tensorial strain first, transform coordinate system, then transform back to engineering strain. So, you will get a relationship as simply RT inverse R inverse times epsilon 1 2. This is just from the basic definition you do it one by one, you will get this relationship with you ok. So, that is coordinate transformation for the strain of the system.

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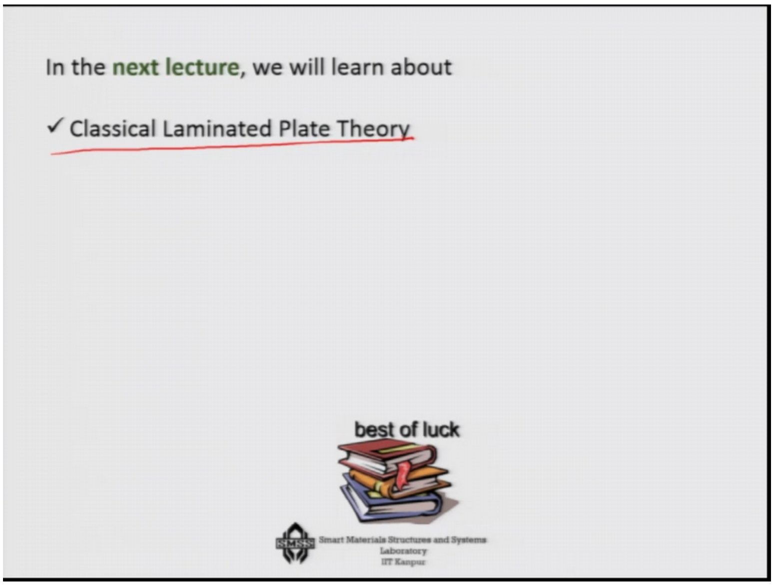

So, this is where we will put an end. And in the next lecture we will talk about with all this knowledge will build up a classical laminated plate theory.

Thank you.