Smart Materials and Intelligent System Design
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Lecture – 14
Modelling of Shape memory Alloys

Good morning everybody welcome to the course in the Smart Materials and Intelligent System Design.

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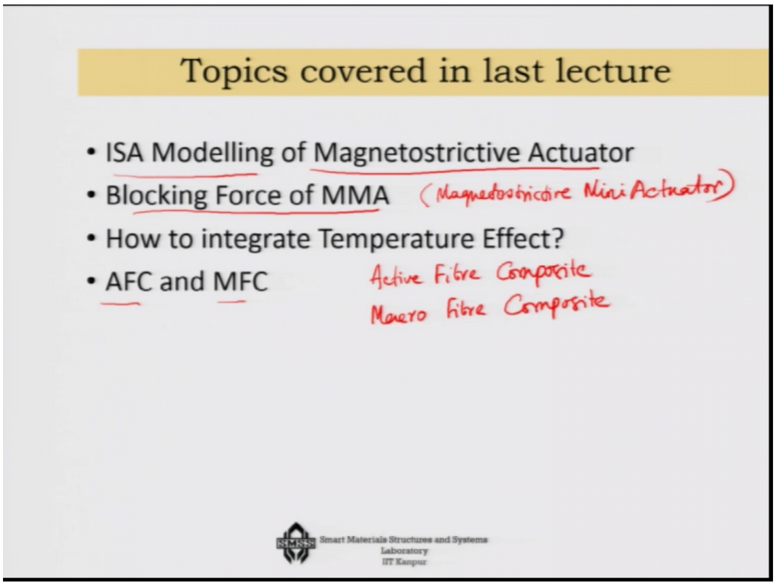

Now in the last lecture, I have covered the induced strain actuation modelling of magnetostrictive actuator. We have also shown that how to get the blocking force of magnetostrictive mini actuator MMA means Magnetostrictive Mini Actuator that is the acronym there are lot of acronyms in this course. And you have to be aware of them now also we have talked about how to integrate the temperature effect, which happens as a result of you know passing current through the magnetizing coil to generate magnetic field. And we have you know noted down that how that adverse effect may create problem unless we actually integrate it in the constitutive relationship and take care of it compensate this effect.

Also I have talked about two very similar you know kind of smart composites one is called active fiber composite AFC active fiber composite and the other one is macro fiber composite. These are of course, made of piezoelectric material, but they behave in a similar manner as you put them in the magnetostrictive actuator.

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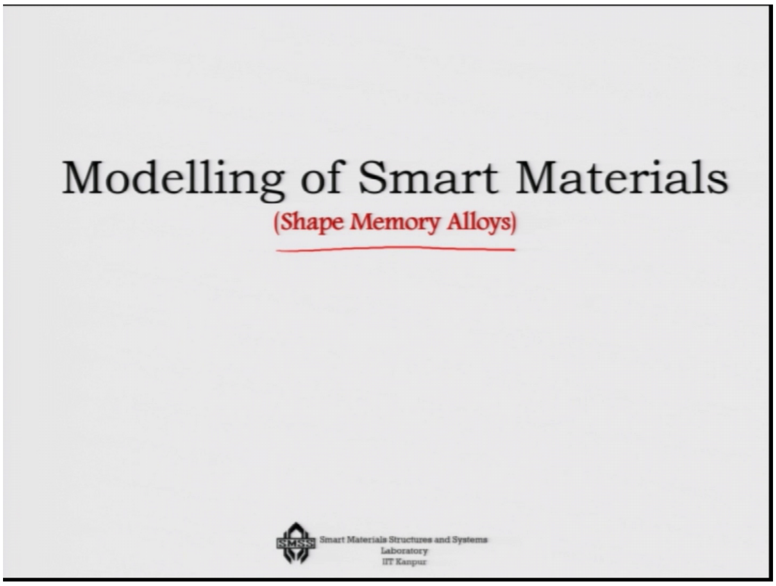

Now, in today's lecture I want to talk about the modelling of shape memory alloys ok, which plays a very predominant role among all the smart materials specifically because of its availability of large actuation strain, large actuation force and also robustness of the system. So, there are a many practical applications where shape memory alloys are used.

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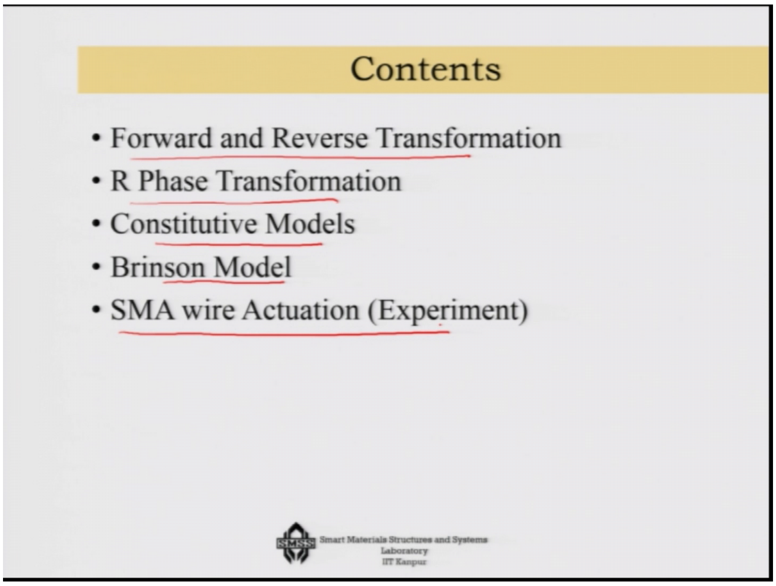

And hence we let us put a focus on them so we will talk about forward and reverse transformation in shape memory alloy, we will talk about R phase transformation constitutive models simple constitutive model, we will take up which is the Brinson model. And I will show you an experiment with shape memory alloy wire to kind of you know give the concept of that shape memory alloy waste control.

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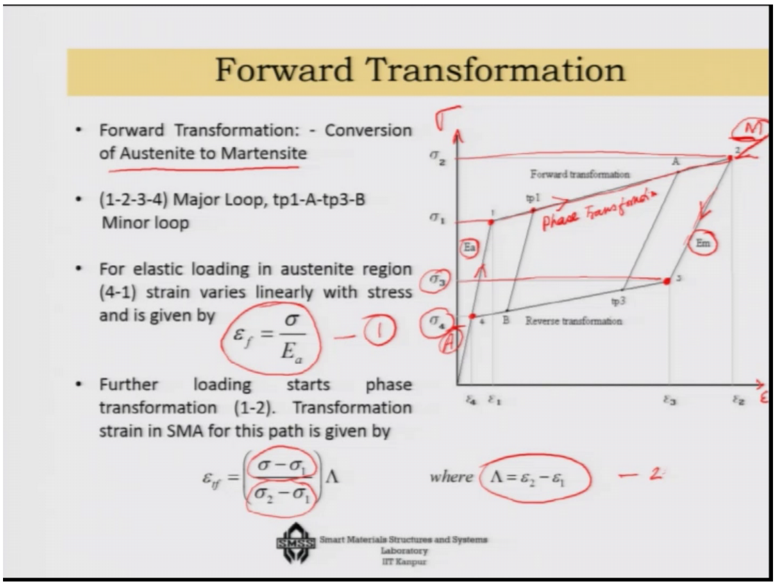

As I have discussed earlier that shape memory alloy base system they actually go through two very important transformations also there are other transformations like r transformation, but the major transformation is what we will call the forward transformation of the shape memory alloy.

So, forward in the sense that when it is getting converted from austenite to martensite. In this particular case of course, we will see that this is a stress induced austenite to martensite phase. So, in this phase we will start with austenite here actually in the temperature diagram we do the other way around and so that means, we will start from here. So, this will be our austenite this will be martensite for temperature diagram we can show it in with respect to high and low temperature, but this is a stress strain diagram as you can see that this is the stress versus the strain that we are plotting.

So that means, we are you know applying reversible stress in to a shape memory alloy base system and here we are starting then from a austenite phase or the parent phase ok. So, there is a loop that we have shown here, now what will happen if you know actually start to stress it at the austenite phase, you will see that the modulus of elasticity is E a at the austenite phase. So, initially the stress strain diagram pretty linear it will go like this until and unless it will reach this point 1 that is the starting journey where the stress induced martensite formation will start to happen into the system. So, this is where the phase transformation is occurring. So, this is also sometimes referred as pseudo elasticity because you get actually a large deformation here that looks like a plasticity, but it is not it will go back to the elastic region, so the phase transformation happens at this stage.

So, from one until I reach this 0.2 point beyond 0.2 phase transformation is over it will reach the martensite phase, but you know after this it will have its own stress strain diagram and let us say we are not interested on that. So, from this point let us say, I am actually reversing my stress. So, if I reverse my stress from here then I am coming in down in this direction and in these direction I have the elastic modulus which is E m the first of all in the major loop part one by one if we see in this particular region ok.

So, if the 4 to 1 let us say 4 is the point where we will be actually coming back finally, as we will be applying reversing the stress. So, from 4 to 1 the stress strain relationship is something like this, which is a very simple linear relationship that the strain will be proportional to the stress and the modulus of elasticity that will govern is E m.

Now, as we go to the next phase what will happen we have to make a shot of interpolation between 1 and 2. So, we the maximum change of stress is sigma 2 minus sigma 1 the sigma 1 is here and sigma 2 is here that is the maximum stress change ok, and what is the corresponding maximum strain change that is lambda that is the phase free strain that we call it. So, lambda over sigma 2 minus sigma 1 and now let us say I have anywhere inside this test point.

So, at that point if the stress is sigma then through interpolation I know, what is the transformation strain that is there in the systems so, that is between 1 to 2 that is the relationship between these region you know until the phase transformation is over anywhere in this region this will be the constitutive relationship in the system.

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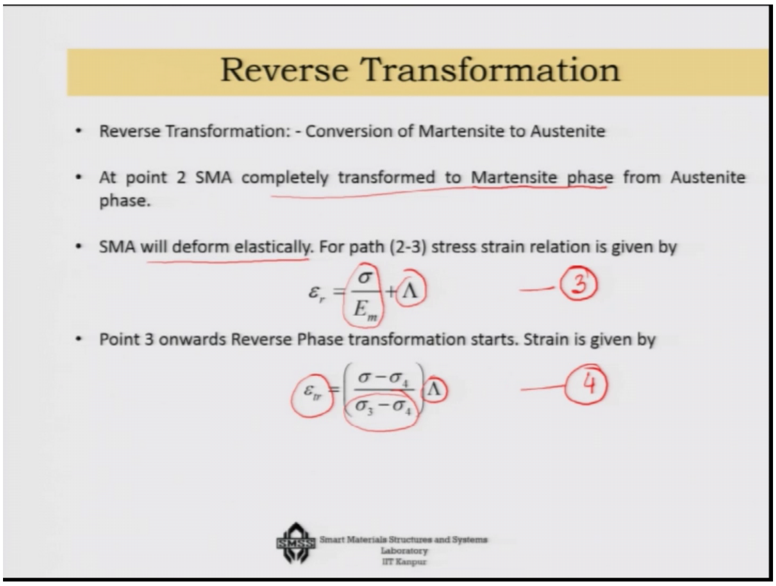

Now, if I go to the you know conversion reversion of stress, so at 0.2 there is a complete transformation to martensite phase that has occurred from the austenite state and then as I am bringing reducing the stress again first it will deform elastically. And so, at that point you have this relationship sigma over E m plus whatever the free strain has occurred into the system. Now depending on the position of sigma you will be having actually reduction of stress. So, epsilon r will come down and then point once you reach 0.3, so we will go back here once I reach 0.3 then that stress you know sigma three comes to the critical level which we will initiate the reverse phase transformation.

So, austenite will then start to form, so that is where austenite formation is starting and austenite so martensite to austenite transformation again a sigma 3 minus sigma 4 interpolation you have to do, so this is between sigma 3 and sigma 4. So, just like sigma 1 and sigma 2 we have to do this interpolation and based on lambda free strain we will find out what will be the transformational state at that stage.

So, for the this is the easiest one based on the stress strain pseudo elasticity the simplest constitutive relationship where temperature does not come into picture you have actually 4 phase of this transformational you know system. First phase is this epsilon f equals to sigma over a that is the first phase and then second phase, where you are getting the transformation austenite to martensite and then you have the third phase when you are getting we are in the martensite phase and finally, the fourth phase where you are back from martensite to the austenite phase.

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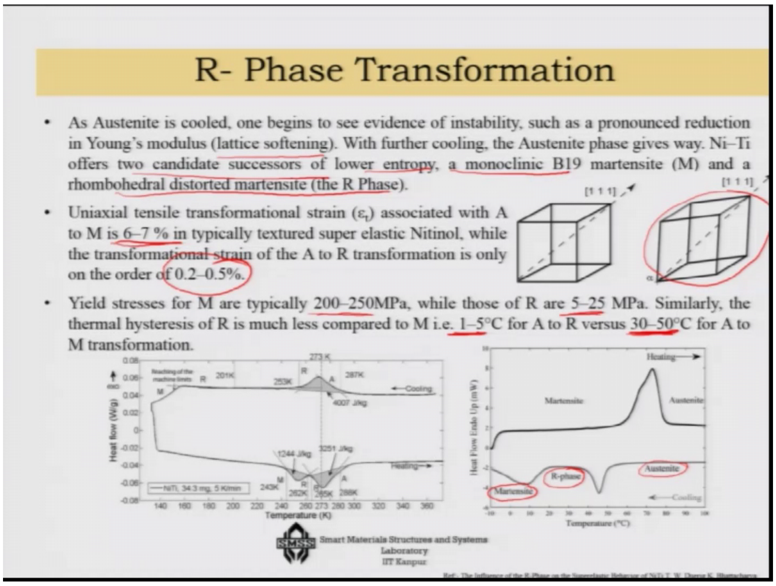

I told you that when you come back from the austenite to the martensite phase let us say you are doing it not using stress, but using temperature that means, you are simply cooling it ok. If you cool it then also you will come back to the martensite state, but there is a possibility that when you are actually cooling these you know what we call that from the austenite phase. You would see that there can be two candidate successors of lower entropy it can either go to the monoclinic B19 martensite phase or it can go to the rhombohedral distorted martensite which is something like these distorted martensite or the R phase.

So, there are these two possibilities that is there in the system, if you look at it say for example, in this particular case you are let us say you know you are at the austenite phase. So, you are somewhere here and you are cooling it phase transformation is occurring and you can get this R phase into the system and then you are getting to the martensite phase, so this is a smaller phase transformation.

In fact, austenite to full martensite phase transformation gives you the about 6 to 7 percent of free strain on the other hand the transformation still in the r phase is quite small it is about 0.2 to 0.5 percent. So, that is why it is not very significant, but today people are coming up with some good thermal sensing you know thermal sensor developments etcetera based on the R phase. The yield stress for the martensite phase is 200 to 250 where as at R phase is a lattice softening that is there.

So, it is only about 5 to 25 you know megapascal and thermal hysteresis however, in the R phase is much less is about 1 to 5 degree in comparison to A to R, which is about 30 to 50 degree centigrade for A to M ok. So, A to R is 1 to 5 degree and A to M is 30 to 50 centigrade, so it has a smaller hysteresis, but it is a softened stage soften stage its yield stress is less and the transformational strain is much less in the R phase transformation of the system.

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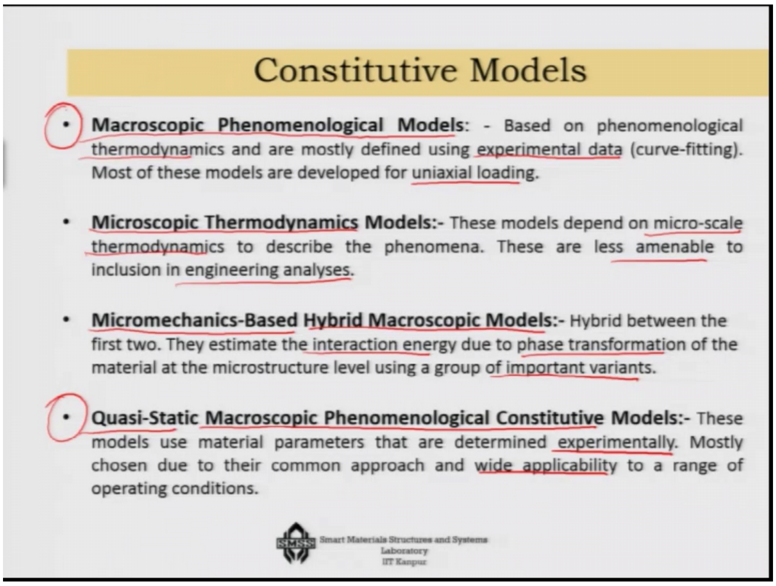

Now, having knowing all these things that is happening in the SMA let us try to actually develop a constitutive model. Let us try to see that what people have developed considering both stress temperature and strain there are various variations of this model one of the most popular ones are macroscopic phenomenological models. So, these are based on phenomenological thermodynamics and are mostly defined using experimental data ok. So, there is a heavy emphasis on experiment in this case and most of this models are developed for uniaxial loading.

Now, if you want to refine it further you can use actually microscopic thermodynamics model, which are based on micro scale thermodynamics that is phase transformations etcetera that is happening in the system. And they are however, so complicated that they are less amenable to include in the engineering analyses, some of this microscopic thermodynamic models may need more than actually 30 parameters. So, in a you know real life or a real time analysis or a control system it is quite difficult although it is a very accurate way of actually defining the constitutive relationship of the system.

There is also some micromechanics based hybrid macroscopic models. So, these are in between macroscopic and microscopic thermodynamic models, they estimate the interaction energy due phase transformation of the material at the microstructure level using a group of some important variants and then include that effect in the microscopic level. There is also another group, which is called quasi static macroscopic phenomenological constitutive model; this model seems material parameters that are determined completely experimentally, mostly chosen due to their common approach and wide applicability to a large range of operating conditions.

So, in our constitutive modeling we will focus on the macroscopic and the quasi static macroscopic phenomenological models in our, you know constitutive relationship that we will consider.

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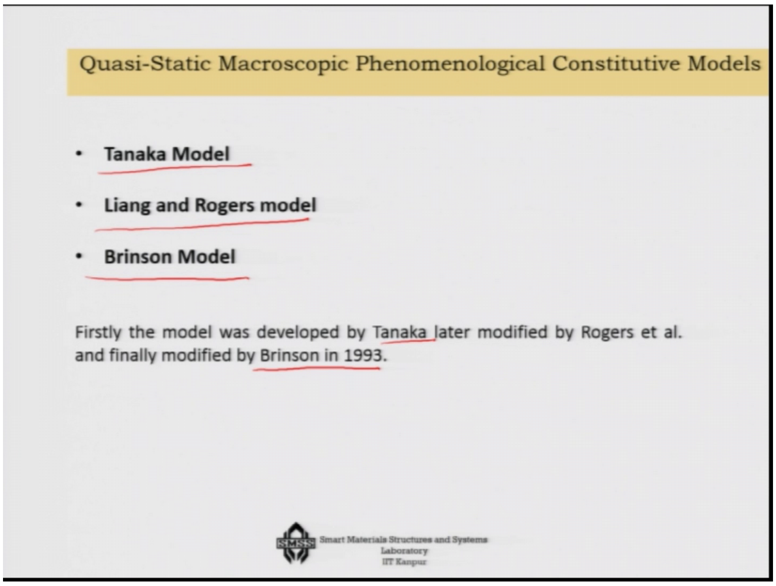

Now, in these quasi static stage we there are three models that are proposed by people Tanaka model and then slightly more refined by Liang and Rogers's model and further refined by Brinson model. So firstly, this model was developed by Tanaka and later modified by Rogers and finally, it is modified by Brinson in 93 and since then it became very popular particularly for the analysis of shape memory alloy wires.

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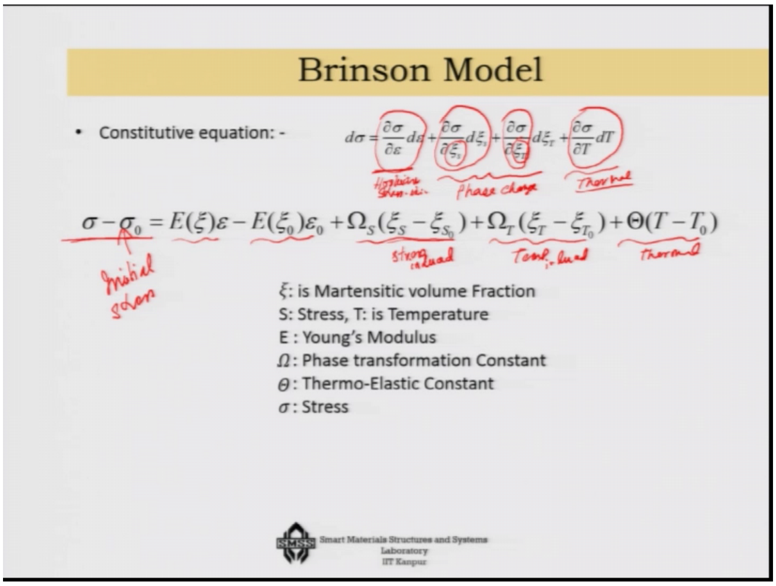

Let us look at the Brinson model, so if you look at the Brinson model you will see that the stress strain relationship here is given in a differential shear form ok. So, now, in that as you can see that the first part is the Hookean part, that is what is your you know combined stress strain you know relationship that that occurs every take time ok. Then you also have another part, which is related to the prestresses that happens in the system which can actually you can include that in this and then you have the second part here d you know is dou sigma dou psi S and psi S here, a that this you know psi S and psi T these are actually related to two parts of stress induced and temperature induced you know volume fractions in the system.

So, psi S takes care of the stress induced change and psi T takes care of the temperature induced changes and finally, there is of course, the thermal stress that is there in the system. So, this part we would say is like a normal stress strain relationship ok, so it is a common stress strain relationship something like Hookean relationship. And these two parts are related to actually phase change and the phase change has two part in it to one is the related to you know stress induced chain another is related to temperature induced change and this part is actually thermal.

Now, the effect of this constitutive relationship can be also expressed you know if we consider these dou sigma dou epsilon to be kind of a slope actually at then instead of differential shell form if you want to actually use it in the form of a algebraic relationship. Then you will get first of in terms of d sigma would become your sigma minus sigma 0 where sigma 0 is the initial stress. So, this is the initial stress ok, initial stress play a very important role I told you in the shape memory alloy base system, the slightly at you know enhance initial stress will give you better phase transformation.

Now, that sigma minus sigma 0 then it is related to first of all E epsilon and then the initial stress part minus E epsilon 0. So, that is the Hookean stress strain relationship part then you have the space transformation and the first part of phase transformation is related to what you call stress induced transformations, so this is stress induced.

In fact, this is the contribution of Brinson otherwise earlier it was simply omega times psi itself the phase total phase transformation factor, but then psi minus psi at any stress and temperature minus psi at 0 stress condition, but then it is Brinson who actually divided this into two part one is the stress induced and another is the temperature induced. And finally, just like that last expression you have this thermal part, so that is what is the you know constitutive relationship that we will be using for modelling of the system.

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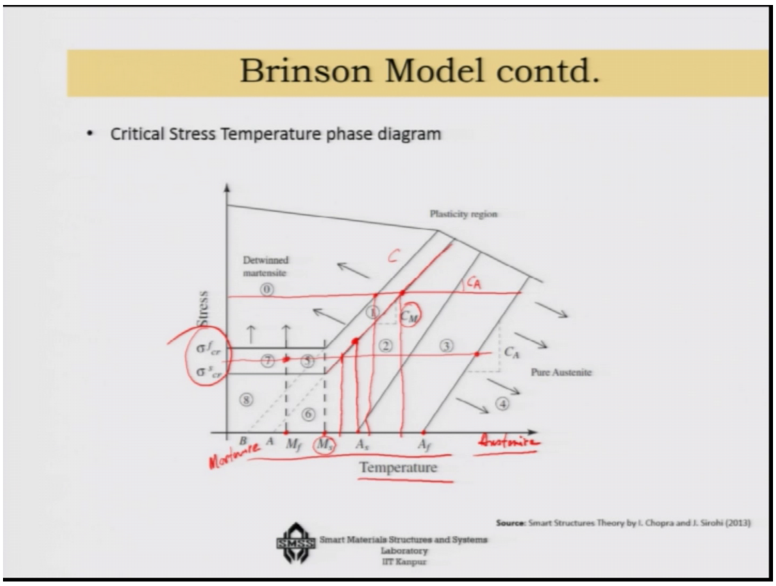

Now, how this constitutive relationship you know we can first of all visualize this whole thing ok. So, if we consider the stress temperature plot earlier we have seen the stress strain plot, now we are considering this stress temperature plot. So, at the high temperature phase what is the state we are supposed to see the austenite state? So, this part is austenite right, and as I am let us say this is the austenite and as I am reducing the temperature I will come first to this particular point which is the martensite stage ok. So, we come to the martensite stage and then martensite start and then martensite finish.

So that means, beyond this stage if I cool it with no additional stress condition on this line I will come back to martensite stage remember, that martensite stage side stage may have various variants there are many variants of martensite stage. Now if I start to heat it up what will happen I will actually, so cooling is between these two. If I start to heat it up I will actually start transformation from austenite start and beyond austenite finish, I will be getting a fully austenite stage, what will happen if I now introduce stress into the system. Well so you can see that these curves ok, will remain you know similar, but from this point onwards there is a different behavior that you will expect say for example, if I take any, any situation here, here we have a sigma S critical that is there. So that means, the stress induced transformation that has also started to take place ok.

So, at this stage if I am at austenite stage then I am actually cooling it, but there is some stress in the system. So, much before the martensite state at 0 stress condition somewhere here your martensite will start to form and then you know the martensite will be finished ok. So, if I you know continue with this line beyond this point it will be finished. In fact, it will martensite formation will be enhanced if you come somewhere here. So, you can see that you know martensite starting has further started from this point and martensite finish is over in this point, so the entire transformation is happening much earlier than in comparison to the 0 stress condition.

Now, what is happening when I am going back when I am heating up the system then also this curve the slope of this curve is telling us that you are going to get the austenite starts, let us say for this particular line the austenite start is happening at this stage ok. So, that is where is my austenite start ok, so with this particular stress condition if I actually you know kind of enhance the stress further beyond sigma f critical, then what I am going to see is that that the austenite state will start at a much, much you know earlier condition itself.

So, then this particular thing is then telling us that if I in addition to all other things I have to also know the slope of this lines this two lines are approximately parallel. So, that is defined by CM and also this other one is defined usually by CA, so that is CM and

CA that is the austenite line that is defined usually by CA. So, here we get a straight line, but here you know the relationship has these actually slope lines here and the slope lines modify this temperatures because of the presence of the stress in the system. So, that is one feature that Brinson model has taken into consideration.

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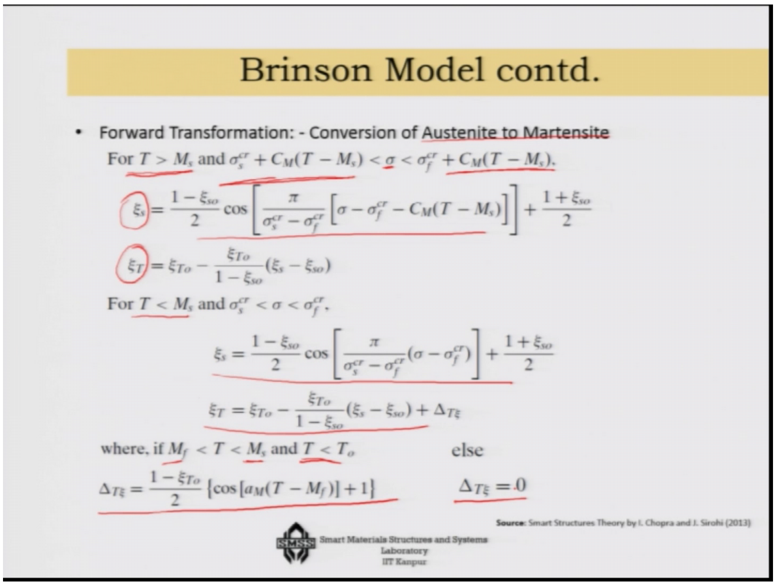

Keeping all these points in mind then when the you know when it is a condition where you have both stress and temperature then for temperature greater than martensite start ok.

So, what you will see is that the forward transformation relationship when I am converting from austenite to martensite ok. And at that phase this is the relationship that sigma here is in between these CM into T minus Ms and here it is sigma f critical plus CM into T minus Ms. And where your psi M and psi S and psi T the stress and temperature induced transformations are given these two functions. Now when T is actually less than Ms, then you have a different relationship here. Here this is the psi S and this is the psi T relationship just simply based on these curve you know if you actually work on you will be able to get this relationships.

Now, when temperature is in between M f and M s between martensite start and martensite you know formation finish is happening and T is less than that T 0. Then you have this temperature relationship that we will find in the system otherwise this delta T zeta will be 0.

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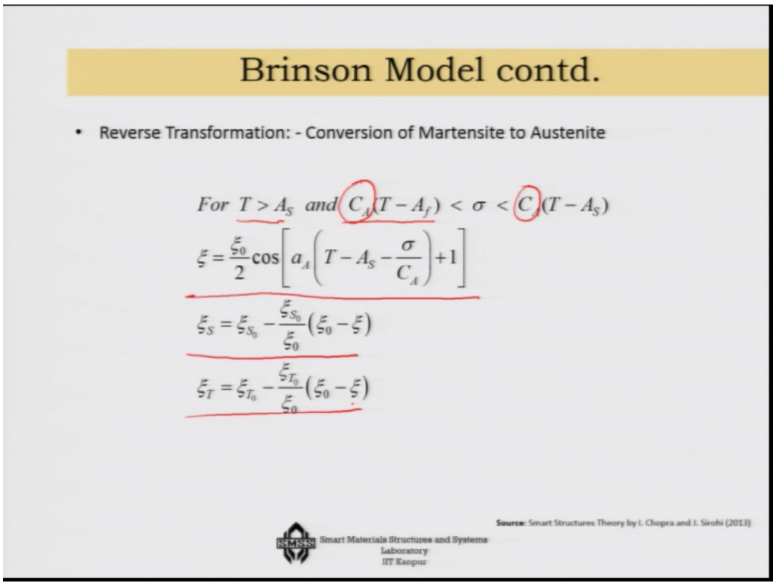

And fro the rivers reverse transformation for T greater than As here it is C at that this slope will come into picture CA, CA T minus As. And here the stress psi in the phase transformation psi is related to psi 0 with this relationship and psi has two part psi S and psi T, so that is for the reverse transformation of the Brinson model.

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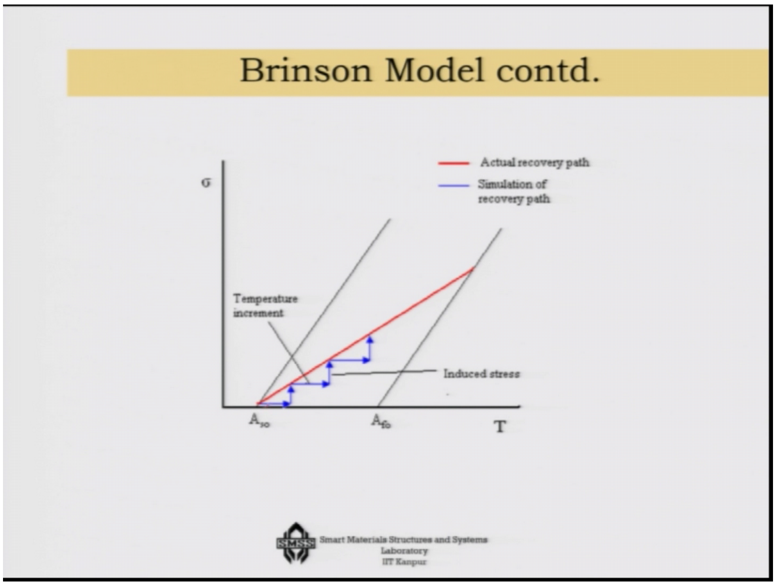

What you know when we actually try to model such a system and you have the CA and C Ms etcetera transformations what is important is that we do not actually model them along the a particular slope. Suppose, this is the actual part we model them by actually taking little, little increments in terms of the temperature first and then the stress and again temperature and then the stress that is the way we actually algorithmically try to follow the recovery part.

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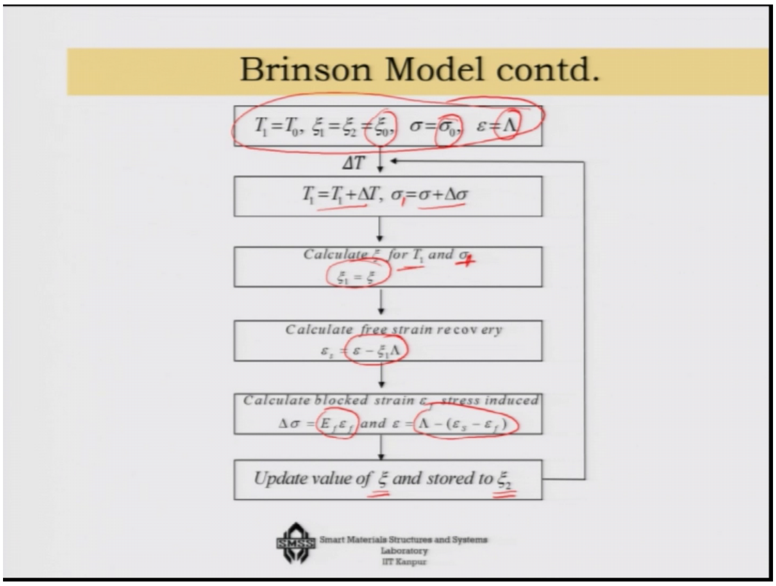

In fact, that is what you know is the application of the Brinson model let us say I am starting from a temperature which is T 0. And I am starting from a initial phase transformation that is there in the system and initial stress sigma 0 and epsilon, which is free strain.

And then I increase the temperature and then the stress del sigma I calculate the psi for this new temperature and thus new stress, so we should call it sigma 1 here. So, for that I calculate it and then that psi would become the new psi; psi 1 would become psi now. I calculate what is the free strain recovery based on wherever we are at the psi 1. And then I can calculate what is the blocked strain that means, if I try to stop this whole thing, what is the blocked strain that is coming into the system. And then I update the value of psi and store it to psi 2 that is the loop in which we will work until we follow the recovery path.

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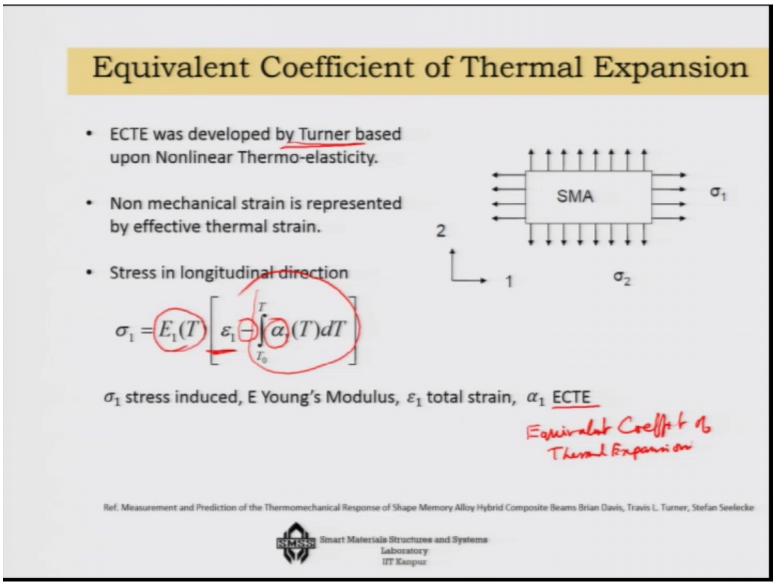

So, there is another way in which this whole thing is done also it is a simpler way in terms of applying this system in a finite element model this was initially developed by turner based on non-linear thermo elasticity. So, what turner has shown is that you can do you know describe these relationship, in terms of two parts where first of all your modulus of elasticity is temperature based and then you have this stress strain relationship and you have an addition of thermal like relationship. Only thing there is a negative sign here because you remember that from austenite to martensite your system is actually shrinking during the phase transformation ok.

And from martensite to austenite as you are a heating it up, it is shrinking and hence you know the temperature you are increasing, but the coefficient of thermal expansion is considered negative, so that it is actually shrinking from martensite to austenite. So, by using this kind of an alpha, which is known as equivalent coefficient of thermal expansion ECTE equivalent coefficient of thermal expansion using this ECTE you can actually model the relationship. So, that is another easier way, but this alpha you have to determine experimentally.

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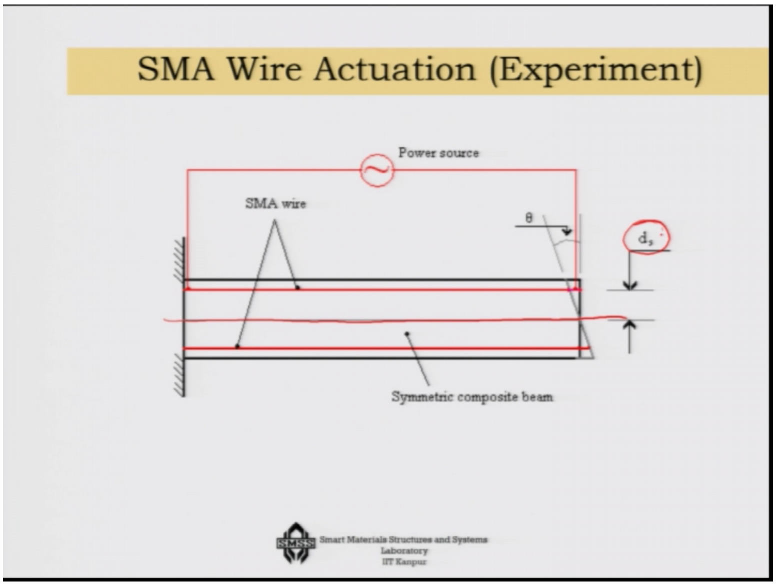

Now, I will show you one such experiment, so here you have the SMA wires and you have a beam and let us say you are applying power to the SMA wire, SMA wire can be inside the system or it can be outside the system ok. So, and let us say the offset here in this particular (Refer Time: 28:33) from the geometric meet point let us consider a symmetric beam that is about ds.

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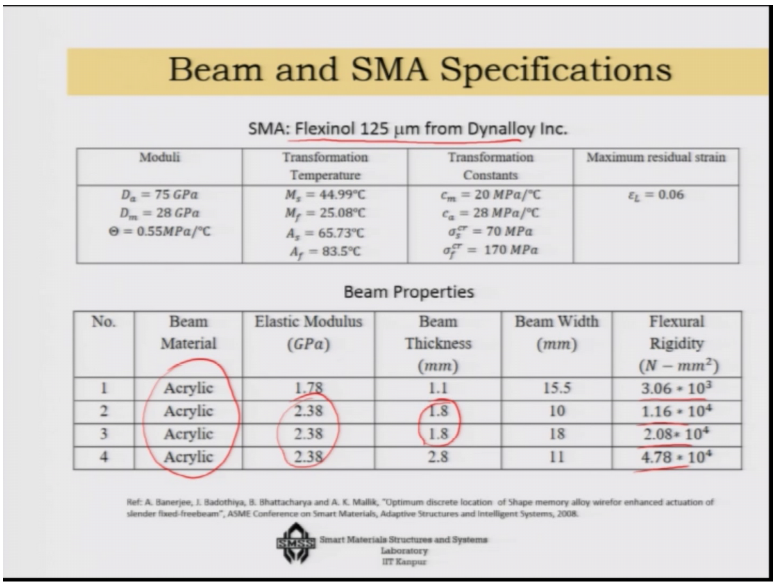

Then we have considered here say various types of you know conditions here, so beam is of acrylic material elastic modulus is changing this little higher in this case as beam thickness is also different 1.1, 1.8 and 2.8 beam width. And accordingly the flexural rigidity is are changing ok, now what we are doing is that we are using this flexinol 125 micron wire. And we are studying how the offset is changing how with respect to the change of offset the strain is changing in the system and the deformation and the end point deflection is changing with the system.

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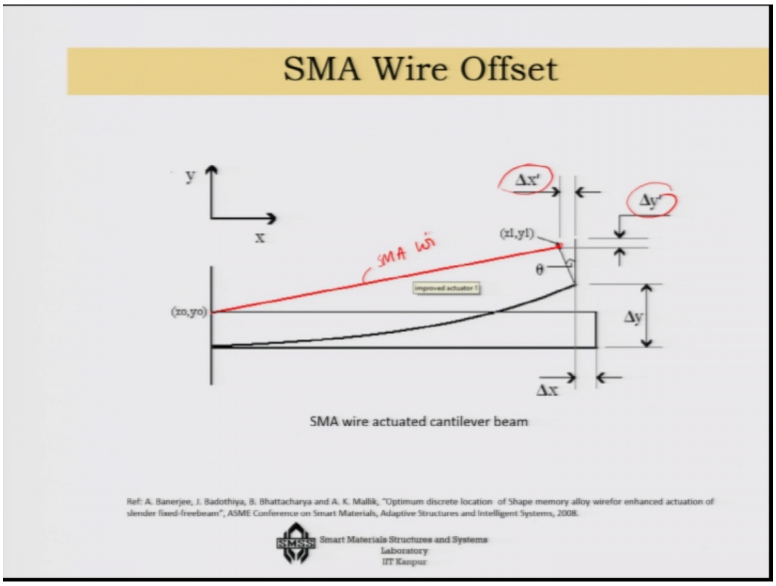

So, as you can see here that in this case you know the deformation there are two of them x deformation and y deformation of the tip point that is happening as I am applying these you know power to the SMA wire, so this is the SMA wire, which I am actually actuating.

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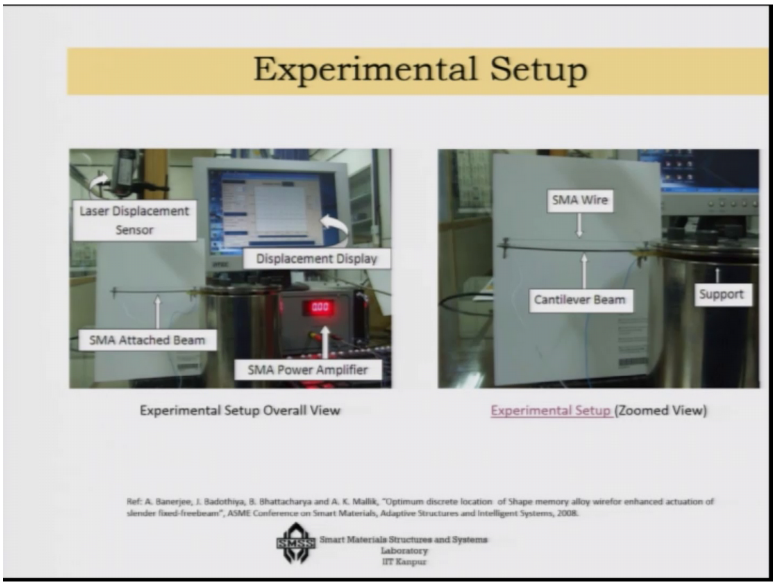

Here I can show you that this is the experimental setup as you can see that the SMA wire is attached to the beam and we have a laser displacement sensor which is sensing ok, that the tip displacement of the point, you can see this behavior in a small experimental setup.

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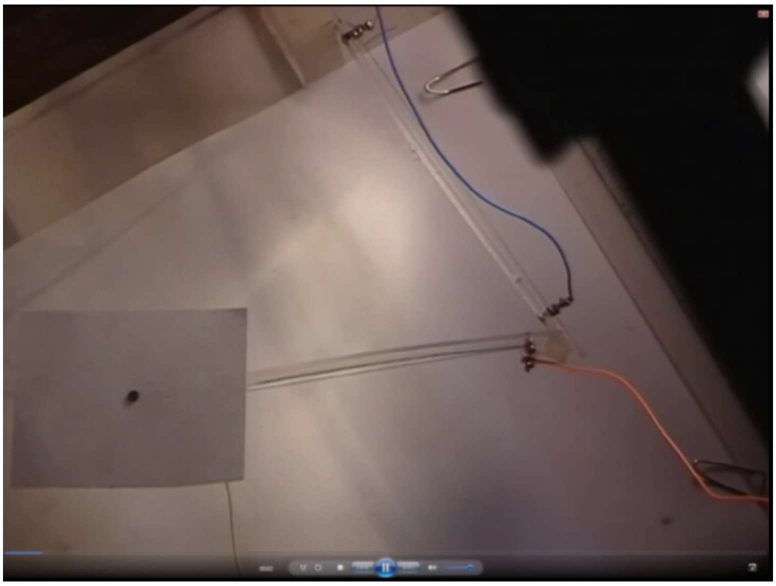

So, if I just you know show you can see that in this case there are two links in which we have this SMA wires in offset. And as I am increasing the temperature SMA wire is getting shortened and that force is actually the blocked force is actually bending these links, and as a result this tip point which we are tracking. In fact, you can track it and you can create many different trajectories as a simple too link flexible manipulate a system. So, this is how you can see that you know you can actually actuate such a system and you can control the deformations. And as a result you can control the trajectory of any such point it is a simple mechanism that we have developed, which shows the application of SMA wire.

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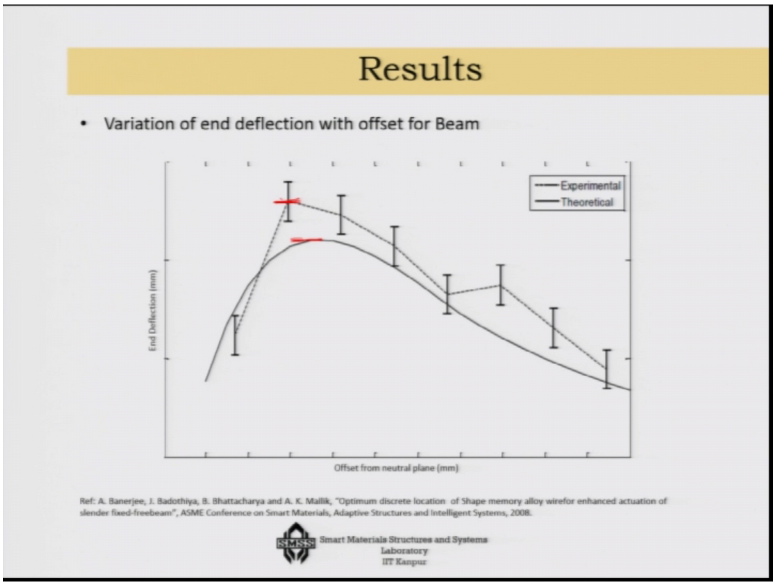

Now, what we will be finding out is that as the offset is increasing you are actually getting more and more displacement ok. Both experimentally and theoretically we have seen that this is what is going to happen experimentally this is the peak theoretically this is the peak, there is the bit of difference between the two, but it is true that with increase of offset end deflection increases. However, as the you actually reduce as you further increase the offset the tip deflection will come down why do you think this is happening.

That the offset you know here as you are increasing this offsets more and more beyond a certain point, what happens is that you need more stress and that stress actually creates stress induced transformation and has a result you know you do not get enough active strain and hence the tip deflection is coming down in the system.

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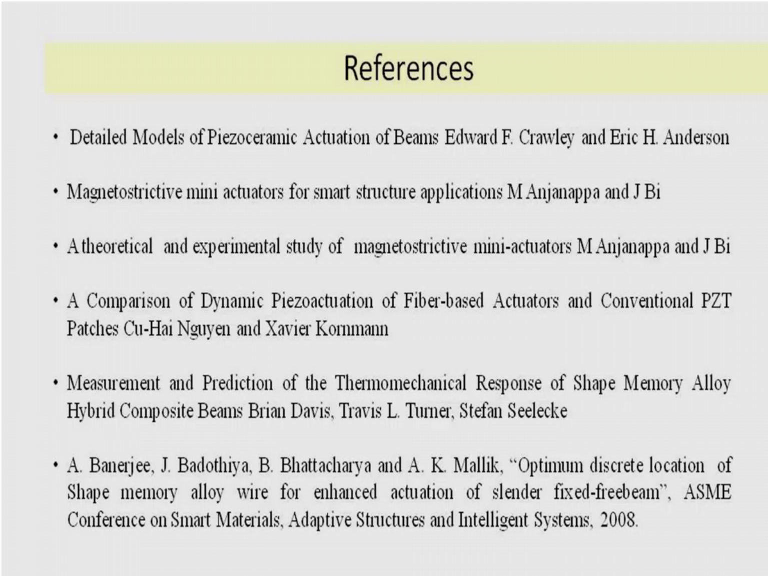

This is an interesting part these are the references that, you can use for your further you know study in this course.

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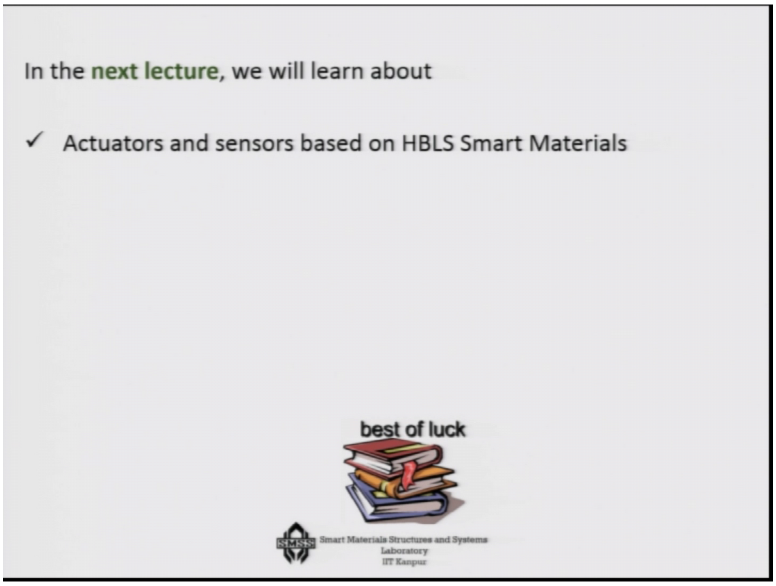

So, this is where we will put an end and we will learn about in the next lecture about the actuators and sensors based on high bandwidth low strains smart materials.

Thank you.