Advanced Composites
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 44
Finite Rectangular Plate

Hello, welcome to Advanced Composites. Today is the second day of the ongoing week which is the 8th week of this course. Yesterday we just finished our discussion on thermal effects in laminated composite plates.

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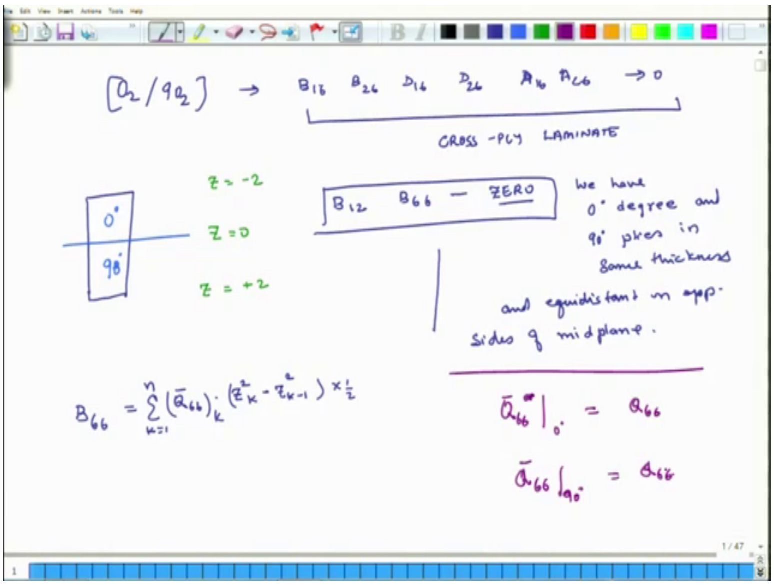

And while discussing these types of plates and also other semi infinite plates, we had stated that for the lamination sequence 0 2 90 2; this type of lamination sequence. We had stated that B 1 6 B 2 6 D 1 6 D 2 6 and A 1 6 and A 2 6 for these types of plates the all these terms are 0. And these are 0, B 1 6, D 1 6, B 2 6, D 2 6, A 1 6, A 2 6, because it is a cross-ply laminate, but we had also specified two additional terms that they will be 0.

We said that for this type of plate B 1 2 and B 6 6, they will also be 0. And these terms are indeed 0 but they are not, because it is a cross-ply it is not that these terms are 0, because it is a cross-ply. It is because the 0 degree and the 90 degree ply are in same thickness and they are at same distance located from the mid plane.

So, let me explain. So, these are this is 0, because we have 0 degree plies and 90 degree plies in same thickness. And equidistant on opposite sides of mid plane, why does that ensure B 1 6 or D 1 2 to be 0. So, let us look at it. So, this is the ply, this is the laminate, this is my mid plane. Let us say this is 0 degree ply, and this is 90 degree ply. And let us say there are two plies, each ply is thickness of unit thickness. So, at this point mid plane Z is equal to 0; here Z is equal to minus 2, and here Z is equal to plus 2.

So, let us compute B 6 6, so B 6 6 is equal to Q 6 6 bar times Z K minus Z K minus 1 square into 1 over 2, and this is also for the K th layer, and then I sum it from K is equal to 1 to n. Now, remember Q 6 6 bar for 0 degree, for 0 degree layer is same as; Q 6 6, because and Q 6 6 bar for 90 degree layer is also same as Q 6 6. And if you want to show get the proof of it, you please go back and check your earlier equations and you will see that it comes to be same as Q 6 6.

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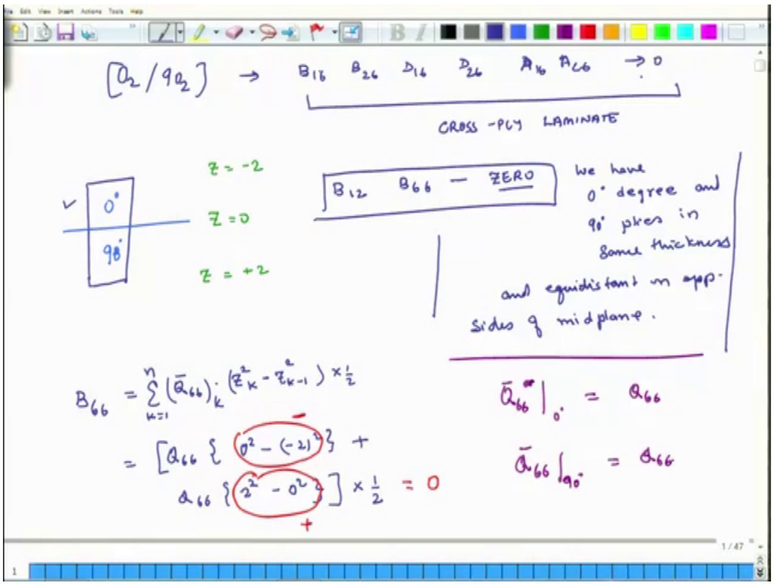

So, if that is the case, then what is B 6 6 it is equal to Q 6 6 for the 0 degree layer times. So, we will first do for the 0 degree layer for this layer, so that is 0 square minus minus 2 square plus Q 6 6 times for the 90 degree layer 2 square minus 0 square and this entire thing into 1 over 2. So, when I add this, this is a negative number; and this is this is negative and this is positive. And they are exactly equal in magnitude so they cancel out, so that sum comes out to be 0.

So, for this reason B 6 6 is 0, and for the same reason B 1 2 is also 0. So, I did not specify this very explicitly in last few classes, but I thought it is important for all of you to know that B 1 6 and B 2 6 and 1 6 terms are 0 for cross-ply, because all that layers are either 0 degrees and 90 degrees. So, Q 1 6 and Q 2 6 terms for cross-ply laminate for each of the cross ply layers is 0, but B 1 2 and B 6 6 is because they are 0 degree and 90 degree ply and they have the same thickness, and they are located on opposite sides of the mid plane. So, this is very important to understand. So, this is something as a point of clarification.

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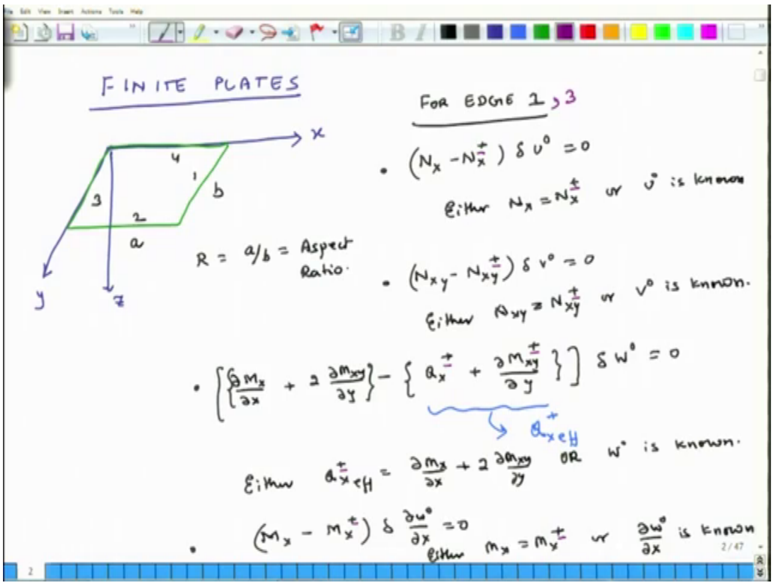

Now, we will move onto a new topic. So, we will start discussing finite plates finite plates. So, we have discussed infinite plate, now we will start discussing finite plates. So, first we will consider so that is my x-axis, this is my y-axis, and let us say my plate is like this so that is my x-axis, y and this is the z-axis. So, this dimension is in the y direction is b; in the x direction is a; and we say that R is equal to a over b, and that is aspect ratio.

Now, first what we will do is we will once again visit. The boundary conditions of these plates, because that is important stuff. So, let us call this edge 1 2 3 and 4, and we will just talk about boundary conditions on the x on edge 1 and similar type of bound and edge 1 and edge 2. So, for edge 1 so this is a recap of earlier stuff so, N x minus N x plus times delta U naught is equal to 0. So, this is the first boundary condition which means either N x equals known N x plus or U naught is known. So, each edge will have four boundary conditions.

The second boundary condition so, this first boundary condition from comes from the first equilibrium equation which is del N x over del x plus del N x y over del y equals 0. From the second equilibrium equation, we get the second boundary condition and that is N x y minus N x y plus times del V naught equals 0, which means either N x y equals a known entity N x y plus or V naught is known; on the edge V naught is known on the edge.

The third boundary condition is del M x with respect to x plus twice of del M x y with respect to x, no I am sorry this is y minus Q x plus plus del M x y del y times del W equals 0. And if you remember, we had said that this term is called Q x effective, and it is acting on the positive edge. So, it is putting a subscript a superscript plus so, what it means is that either Q x effective equals this thing partial of M x with respect to x plus twice of del M x y over del y or W naught is known.

And fourth one is M x minus M x plus partial of del W naught over del x is equal to 0, which means either M x is equal to M x plus or del W naught over del x is known. So, this is for edge 1, I can use the same boundary conditions for edge 2; no edge 3 the only thing is that instead of plus it becomes negative because that is all, because that edge 3 represents the negative side. So, I just put negative everywhere.

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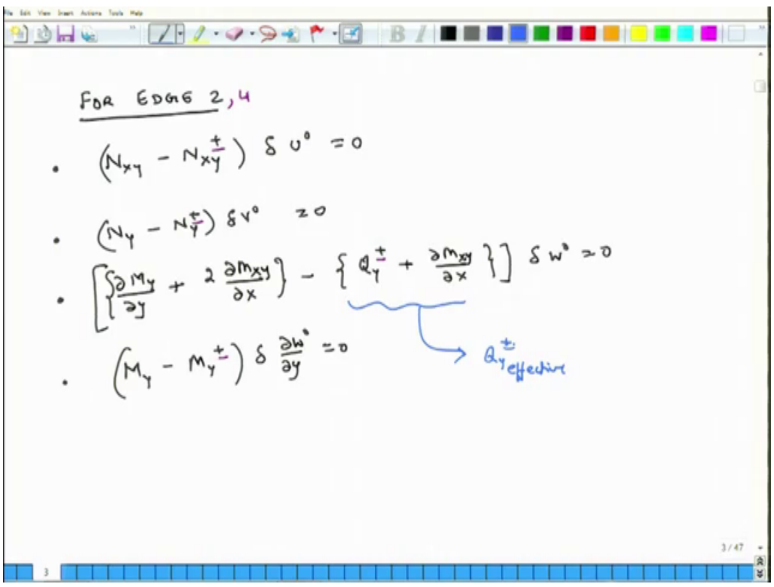

And then for edge 2 and 4; for edge 2, what would be the boundary conditions so I will just not explain, but the boundary condition equations are N x y minus N x y plus del U naught equals 0, N y minus N y plus del V naught equals 0. Then we have del M y with respect to del y plus twice of del M x y divided over del x minus Q y Q y plus plus del M x y with respect to del x times del W naught equals 0.

And, the fourth boundary condition is M y minus M y plus variation in del W naught with respect to del y equals 0. And similar boundary conditions in form are also for edge 4. So, I will just put also a negative sign as a subscript ok. And this thing is Q y effective, Q y effective. So, these are the boundary conditions.

Now when we solve for these finite plates, the first thing we should always try to do is try to see if we can make some simplifications, and what are the simplifications we can look forward to; for instance in case of semi infinite plates, we made a lot of simplifications. We made one simplification by assuming, because we said that a is very large than v. So, all the partial derivatives with respect to y are 0. And that also made another simplification, let to another simplification that partial derivative of any entity with respect to x equals d over d x. So, these types of simplifications we should always make.

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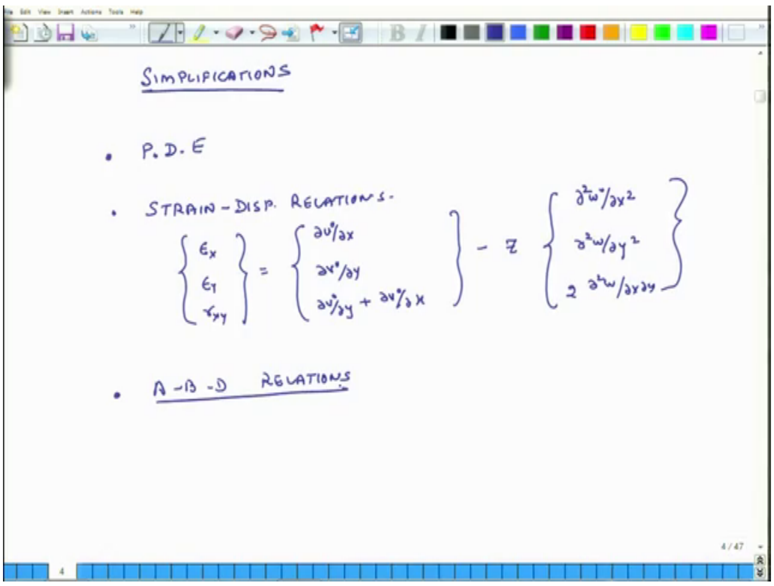

So, we should always look whenever, because these plates which are finite in length their solution procedures are more complicated. So, we should always look for simplifications and see what are possible and realistic. So, we should always look for simplifications in partial differential equations, shall we see partial differential whether it is possible or not; what kind of simplifications are possible. Most of the times for finite plates simplifications in plates of simplifications in these partial differential equations, which dictate the equilibrium of the plate these simplifications are not that much possible, but we should still explore if it is possible.

Then we should also look for simplifications in strain displacement relations. So, what do I mean by strain displacement relations; epsilon x, epsilon y, gamma x y is equal to del U naught over del x, del V naught over del y, del U naught over del y plus del V naught over del x minus z times del 2 W naught over del x square del 2 W naught over del y square and twice of del 2 W naught over del x del y.

So, we should also see if anything in these strain displacement relations can they be simplified, and once again in context of finite plates most times we cannot simplify these relations also. The third area to look for it forward is A-B-D relations, A-B-D relations what kind of simplifications can exist in these things. So, if the plate is symmetric, so this we will quickly cover some of the scenarios for symmetric plates.

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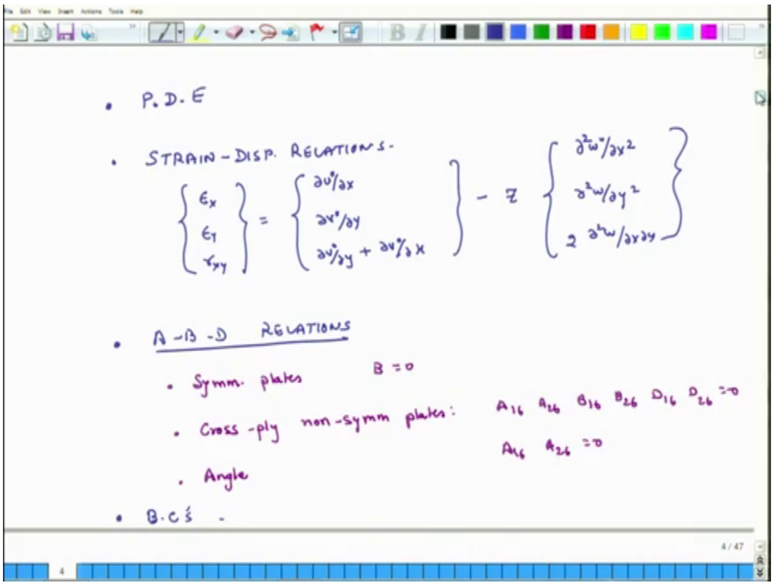

B is equal to 0 for cross-ply plates. Cross-ply, but non symmetric plates A 1 6, A 2 6, B 1 6, B 2 6, D 1 6, D 2 6 are 0. For angle ply laminates, again A 1 6, and A 2 6 is equal to 0 for angle ply laminates, but B 1 6, B 2 6 and D 1 6, D 2 6 may or may not be 0 you have to see exactly, what is the stacking sequence, but if it is an angle ply for every theta there is a negative theta, and they cancel out each other and so on and so forth.

So, we have to look at these relations also, and see what can happen. And then finally, we should look for simplifications in boundary conditions, this is another area. So, the first step in making this step in solving the problem is to see what kind of simplifications we can do, because then we can handle the problem in a less complicated way; and then we actually go around and start solving the problem. So, this is what I wanted to cover today. Tomorrow we will continue with another topic, and also in the case we will start discussing finite plates in much more a depth and detail, until then I look forward to seeing you tomorrow.

Thank you.