

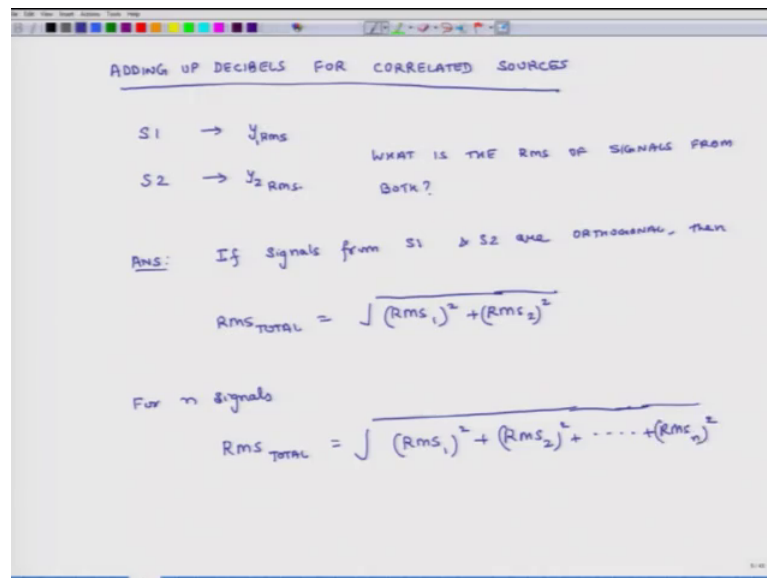
Noise Management & Its Control
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Lecture - 09
Decibel Scale - Part 3

Hello, welcome to noise measurement and its control. Today is a third day of the second week of this course, and what we plan to do today is, continue our discussion on adding up of decibels, and today is specifically we will learn how do we add up decibels, when sources are correlated to each other; specifically we will learn how to add up decibel levels for two correlated sources. And once we know that we can extend the same thought process to add up decibels, when there are more than two sources present.

So, that is what we plan to do today.

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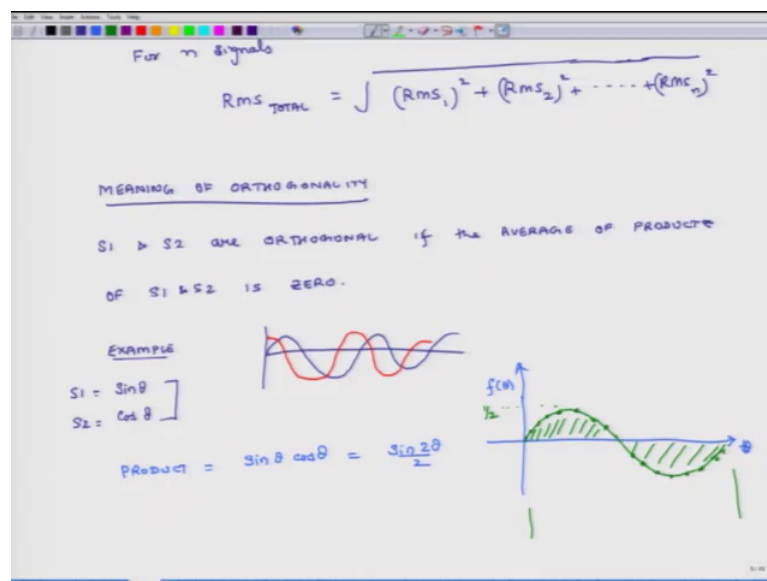


So adding up decibels for correlated sources, this is our theme. So, before we do that, we should also know how to add up, how to figure out the RMS value of two correlated signals. So, what do we do, by mean by that. So, suppose there are two signals S 1 and S 2 and the RMS of this signal is y_1 RMS, and the RMS of the signal is y_2 RMS, then the question is, what is the RMS of signal from both.

So, if both the signals are active, how do we figure out the overall RMS. So, here is the answer, and it comes with a qualifier with the condition. The condition is that, if signals from S 1 and S 2, and this is important orthogonal, and we will explain what does this mean orthogonal, but it is important to know that, if signal from S 1 and S 2 are orthogonal, they may be correlated, but they are orthogonal, then RMS total is equal to RMS 1 whole square plus RMS 2 whole square and if there are more than 1 or 2 signals for n signals. If there are n signals, and all these signals are mutually orthogonal, and I have still not explain what is orthogonal, but I will do that, but for n signal RMS total is RMS 1 plus RMS 2 and so on and so forth plus RMS n whole square.

So, if there are several signals, and there are each signal is mutually orthogonal, then there, then we can use this relation to compute the overall RMS of the system meaning of orthogonality.

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So, at least in context of today's lecture what does. So, two signals S 1 and S 2 are orthogonal, if the average of products product of S 1 and S 2 is 0. So, two signals are mutually orthogonal, at least in context of this discussion if. So, how do I figured whether the signals are orthogonal or not. Take two signals, I multiply them, I plot them on a graph and see whether the average of that gas is 0 or not example. So, example could be sin theta and cosine theta ok.

So, sin theta is this cosine theta is like that, it is off, but if i, but I do not know whether they are orthogonal or not. To see whether they are orthogonal or not what do I do? So, this is S 1, this is S 2 take their product. So, product is equal to sin theta, cosine theta, sin theta, cosine theta. I can also express this sin theta cosine theta is what sin 2 theta over 2, and I plot this. So, I plot it on the. So, x axis I am plotting x axis, I am plotting theta y axis, I am plotting the function f theta, and how will this function look like. it will look like this sin 2 theta by 2 right.

So, this is 2 theta. So, and this will repeat it itself right, and the magnitude of the signal will be 1 by 2. It is the amplitude 1 by 2, maximum value will be half. What is the average of this signal this, and this, the cancel, it cancel out each other. So, if I add up all the points. If I add up all the points on this signal, there is a positive half, there is a negative half and each positive half cancels exactly each negative half. So, because the average of the. So, if I add up all these and divide by number of points, why, what I find is, average of points average of S 1 and S 2 is 0. It means that sin theta and cosine theta are mutually orthogonal.

So, any two signals which are mutually orthogonal, there will be for them. The RMS could be computed using this relation. So, with this background we will go back to the correlated sources..

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DB FOR CORRELATED SOURCES

$$y_1 = a \sin \omega t$$

$$y_2 = b \sin (\omega t + \phi)$$

} → y Represents pressure -

$$y_{\text{total}} = a \sin \omega t + b \sin (\omega t + \phi)$$

$$= a \sin \omega t + b \cos \phi \sin \omega t + b \sin \phi \cos \omega t$$

$$= \underline{(a + b \cos \phi) \sin \omega t} + \underline{(b \sin \phi) \cos \omega t}$$

So, decibels for correlated sources, let say y_1 is $a \sin \omega t$, and y_2 is $b \sin \omega t + \phi$. Now these two signals $a \sin \omega t$ and $b \sin \omega t + \phi$, they are not necessarily orthogonal, because if I multiply y_1 and y_2 and I plot it, I may not necessarily get or maybe I will get, but we have to see it carefully ok.

So, what I will do is? I will do some mathematical manipulation, and then I will see that they are actually orthogonal. So, we have started that if the signals are correlated and this y represents, what, why represents pressure, then for such a signal the overall decibel level has to be computed by adding up the pressures, not the energy, because the signals are correlated. So, y_{total} is what $y_1 + y_2$. So, it is $a \sin \omega t + b \sin \omega t + \phi$. So, I do. So, I get $a \sin \omega t + b \cos \phi \sin \omega t + b \sin \phi \cos \omega t$.

So, I am getting $a + b \cos \phi \sin \omega t + b \sin \phi \cos \omega t$. Now this and this signal $a + b \cos \phi \sin \omega t$ and $b \sin \phi \cos \omega t$. They are mutually orthogonal. How do we check if you multiply these two plot them on an x axis, and you take the average. You will find that their average actually comes to 0, using exactly the same thing $\sin \theta \cos \theta$ here. The only thing see $\cos \phi$, what is ϕ ? ϕ is the phase, is it changing with time? It is not changing with time.

So, this thing in a bracket is a constant number, same thing $b \sin \phi$ or $b \sin \phi$ is also a constant number. So, the only thing which is changing with time is, $\sin \omega t$ and $\cos \omega t$, and we know that \cos and $\sin \omega t$ are mutually orthogonal, and they are multiplied by two different constants, and when you actually do the math, you will see that they are actually orthogonal. And if they are orthogonal, then what is the. So, what is our goal? Our goal is to find their RMS value. Now we have added up the signals. Now we have to find the RMS value.

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The image shows a handwritten derivation on a whiteboard. At the top, it defines two sine waves: $y_1 = a \sin \omega t$ and $y_2 = b \sin(\omega t + \phi)$. A bracket indicates that y represents pressure. The total signal is given as $y_{\text{total}} = a \sin \omega t + b \sin(\omega t + \phi)$. This is then expanded using the angle addition formula: $y_{\text{total}} = a \sin \omega t + b \cos \phi \sin \omega t + b \sin \phi \cos \omega t$. The terms are grouped into two boxes: $(a + b \cos \phi) \sin \omega t$ and $(b \sin \phi) \cos \omega t$. The RMS of the total signal is then calculated as $y_{\text{RMS NET}} = \sqrt{\frac{(a + b \cos \phi)^2}{2} + \frac{(b \sin \phi)^2}{2}}$. Finally, it is simplified to $y_{\text{RMS NET}} = \frac{1}{\sqrt{2}} [a^2 + 2ab \cos \phi + b^2]^{1/2} = (P_{\text{RMS}})_{\text{overall}}$.

So, y_1 second y RMS net equals. Now this is a sin function, and this is a cosine function, and what is the RMS of a sin function the amplitude divided by square root of 2 right.

So, this is equal to $a + b \cos \phi$ square divided by 2 plus $b \sin \phi$ square divided by 2, this thing. This is the RMS of the total signal. First we have added up the individual signals, and to find the overall decibel level. I have to find it is RMS, then only I can plug this into the formula for this sound pressure level right. So, this equals, because $b^2 \cos^2 \phi$ and $b^2 \sin^2 \phi$, they add up to become b^2 . So, this is what $p_{\text{RMS overall}}$ and using this relation now I can plug in to by relation for what, for dB spl, you know, and I will be able to calculate the sound pressure level ok.

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IF $a = b$ (special case):

$$p_{\text{RMS overall}} = \frac{a}{\sqrt{2}} [2 + 2\cos\phi]^{1/2} = a (1 + \cos\phi)^{1/2}$$

RMS OF S1 $\rightarrow \frac{a}{\sqrt{2}} = p_{\text{RMS}}$

$$\frac{p_{\text{RMS overall}}}{p_{\text{RMS}}} = \sqrt{2} (1 + \cos\phi)^{1/2}$$

Now, we will make some simplification if a is equal to b, as a special case, if a is equal to b, then p RMS overall equals a by root 2 into 2 plus 2 cosine theta, and this becomes a. Excuse me 1 plus cosine theta 1 by 2 right. So, this is the overall RMS value of the pressure, when both the signals are active cosine phi. I am sorry. So, it is not be theta, it should be phi. Now we will make a small table and see what is going to happen for different values of phi. So, this is the overall RMS value of the pressure.

Now, what was RMS of S 1, what was the RMS of S 1, what is signal 1 a sin.

Student: A sin omega t.

Omega t. So, what is it is RMS.

Student: a by root 2.

A by root 2. So, let us call this p 1 RMS, then p RMS over all by p 1 RMS is equal to what do I get. I get root 2 1 plus cosine phi. The whole thing under square root right a. So, the overall RMS pressure is root 2 times 1 plus cosine phi square root times RMS of the first signal; that is what it means.

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RMS OF S1 → $\frac{1}{\sqrt{2}}$ RMS

$$\frac{P_{\text{RMS overall}}}{P_{\text{RMS}}} = \sqrt{2} (1 + \cos \phi)^{1/2} = \text{RATIO}$$

dB. ✓
dB

ϕ	RATIO	CHANGE IN dB
0°	2	+6
90°	$\sqrt{2}$	+3
120°	1	0
180°	0	-∞

Now we will make a table. So, here, I will call this ratio. So, second column is ratio and the third column is change in dB. Actually I should say that, when we write dB and I should have said that earlier, the first letter is small d and B is always capital, earlier maybe at some places I have written this. This is actually wrong, it should be this ok.

So, we will make this table fill up this table, when phi is equal to 0 degrees. What is the value of ratio phi is 0 cosine phi is 1. So, this becomes 2, which means that the overall pressure in for both signals, when both the signal are active is double, and when the pressure doubles then what happens to decibels.

Student: 6

It goes up by 6 dB, when theta is or phi is 90 degrees, then cosine phi is 0.

Student: 0.

What is the value of ratio?

Student: (Refer Time: 19:24).

Root 2, value of ratio is root 2, and what does, what happens to decibel level. It goes up by 3 decibels, when this phi is 120 degrees cosine of phi is minus half cosine of phi is minus half. So, 1 plus cosine phi is half. So, this is same one, and change in decibels is.

Student: 0.

Zero and when it is 180 degrees, then $1 + \cos \phi$ becomes 0. So, this is 0, change in decibels is negative infinity becomes negative infinity. So, this is how you deal with correlated sources. So, we have learnt how to add up decibels in presence of correlated sources, and we have learnt how to add up decibels in presence of uncorrelated sources, and it will be important to go back to your rooms and homes, and review whatever we have discussed, because this is very important information which I have explained, because when you go to your field at your work, and you see that there are several sources of sound or noise, and you want to add up the influence of both, all the sources. This is the approach you are going to use to figure out the overall sound pressure level, sound power levels and so on and so forth. With that we conclude for today, and we will once again meet tomorrow.

Thank you.