Noise Management & Its Control
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Lecture – 23
Standing Wave Formation in a Closed Tube with Rigid Termination

Hello, welcome to noise management and its control. Today is the fifth day of this week and what we plan to do today is conclude our discussion on closed tubes, and maybe we have time we will start discussion on open tubes.

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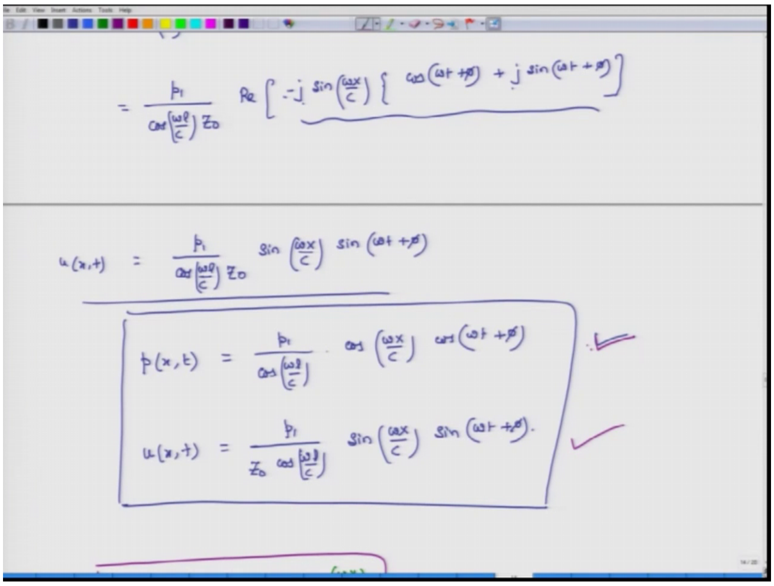

So, yesterday what we had discussed was we had developed these 2 equations, the first equation is for the pressure wave in a closed tube, and the second equation relates to particle velocity in a closed tube.

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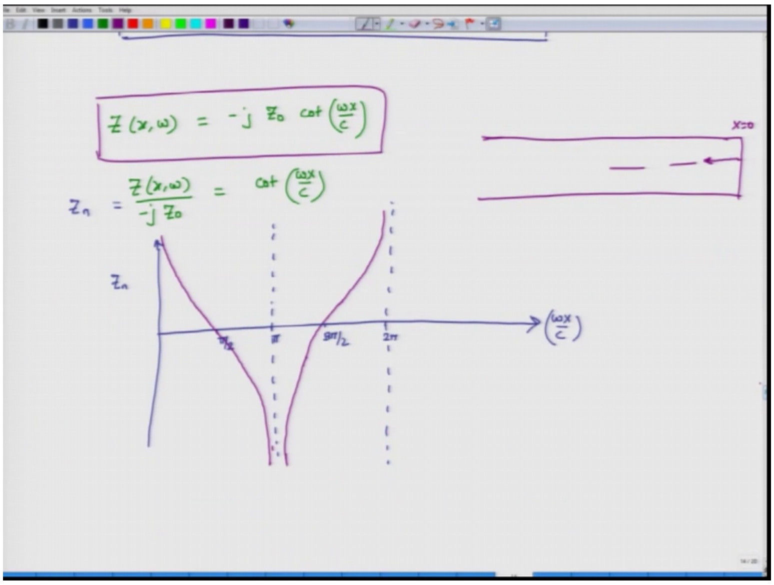

And then using a definition for a specific acoustic impedance, we found that specific acoustic impedance in a closed tube is minus j times z naught cotangent omega x over c and if I plot this function in an x y plane.

This is how my specific acoustic impedance curve is going to look like. Now the next thing I wanted to discuss in context of these waves is the concept of standing waves.

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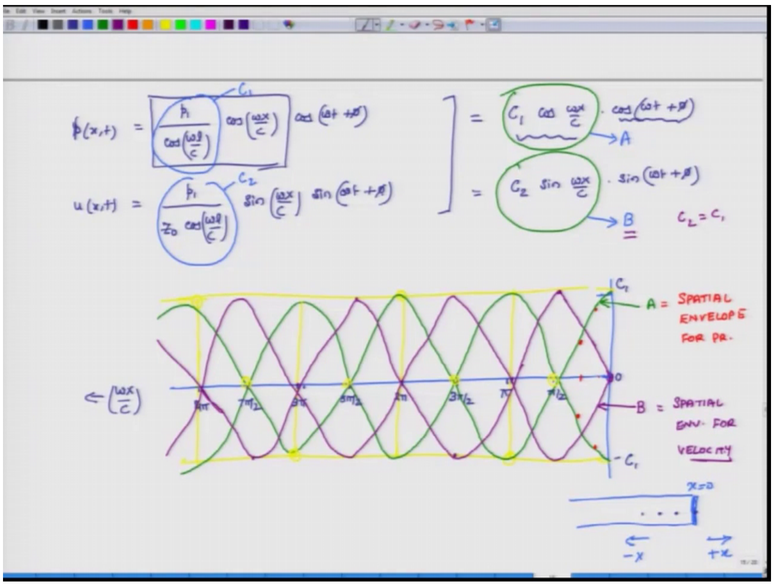

So, before we do that we will once again write down the equations for pressure and equations for velocity. So, for a closed tube our equation for pressure is p 1 by cosine omega l over c, cosine omega x over c, cosine omega t plus phi and my particle velocity equals p 1 divided by z naught cosine omega l over c, sin omega x over c, sin omega t plus phi these are the 2 equations. Now what we will do is first we will make a plot and then we will explain; what is the meaning of that plot. So, what we are going to plot is this term. So, what we are going to plot. So, let us. So, we are going to plot this term and lets call this term in light blue as some constant C 1 because p 1 is constant, omega is constant l is constant, c is constant.

So, this entire term is some constant C 2 C 1 similarly, this is another constant C 2. So, I can rewrite these equations as C 1 cosine omega x over c, times cosine omega t plus phi and this is C 2 sin omega x over C times sin omega t plus phi. Now what we are going to do is we are going to plot this term in green these 2 terms in green. So, that is my x axis on the y axis I am going to plot the terms in green x is equal to 0 at origin and if you remember our coordinates system was such that the closed end of the tube this is the closed end x is equal to 0, this is negative x and this is positive x. So, let us call this entire term as a capital A and let us call this entire term as B. So, at x is equal to 0, x is equal to 0 cosine of omega x is 1. So, the value will be C 1.

So, I am just going to do some construction and then I am going to actually plot this. So, please bear with me for a movement. So, this is plus A and this is minus A and on the x axis actually I am plotting omega x over c like this. So, at x is equal to 0. So, here this is 0 radiance then I have pi over 2, pi 3 pi over 2 2 pi 5 pi over 2 3 pi and 7 pi over 2 and then this is 4 pi and again I will just draw some construction lines. So, that we develop a good graph yeah. So, this green curve is for function A and these things and the maximum value of this function A is going to be C 1 and I incorrectly wrote this earlier it should be positive C 1 and negative C1.

So, if I plot this function A which is C 1 cosine omega x over c, it will look like this green curve it will look like green curve at x is equal to 0 its max values going to be maximum, at when omega x over c is pi by 2 its value is going to be 0, when its three by 2 again it will be 0 and so on and so forth what; that means, is that for different values of x the maximum pressure the value of maximum pressure changes, at x is equal to 0 the maximum pressure is c 1, at x is equal to some location when omega x is pi by 2, the maximum pressure at that location is 0, then if you move further deeper into the tube. So, may be the pressure here is maximum then pressure here becomes 0, then pressure maximum pressure here becomes again C 1 then again pressure becomes 0.

And. So, this is how the amplitude of the pressure is now of course, this is also multiplied by cosine omega t. So, at a given point the pressure fluctuate. So, for instance if we are looking at this location of x, then the pressure fluctuates between these 2 limits corresponding to this value of x the pressure fluctuates between this limit and this limit. At some other location the pressure fluctuates between some other limits. So, the pressure amplitude of pressure fluctuates between 2 specific limits and those limits are defined by function A, that is why this function A is known as spatial envelope for pressure. It is the spatial envelope for pressure and because the tube is closed the spatial envelope for pressure has a maximum value at the closed end of the tube where x is equal to 0 and this is how it behaves ok.

Now, let us look at function D. So, we are going to plot function B in purple color and the function B is going to look somewhat different. So, at x is equal to 0 function B where because x is equal to 0 its value is going to be 0. So, we starts from 0. So, here C 2 we assume that C 2 is equal to C 1 we just assume numerically otherwise they may be different numbers C 1 has a different unit C 2 has a different unit, but one plot we have to plot something. So, we are just assuming that C 1 is equal to C 2 that is all. So, if I plot this at x is equal to 0 it is 0 and then when pressure is maximum pressure amplitude is minimum velocity amplitude becomes maximum, it goes like this.

Now, these are sinusoidal curves they may have I may not have drawn them well enough, but please note that function B. So, this is the purple is function B and what we see from this graph is that wherever the amplitude for pressure is maximum, at that location the amplitude of velocity is minimum and vice versa. So, the plot for B is called is spatial envelope for velocity.

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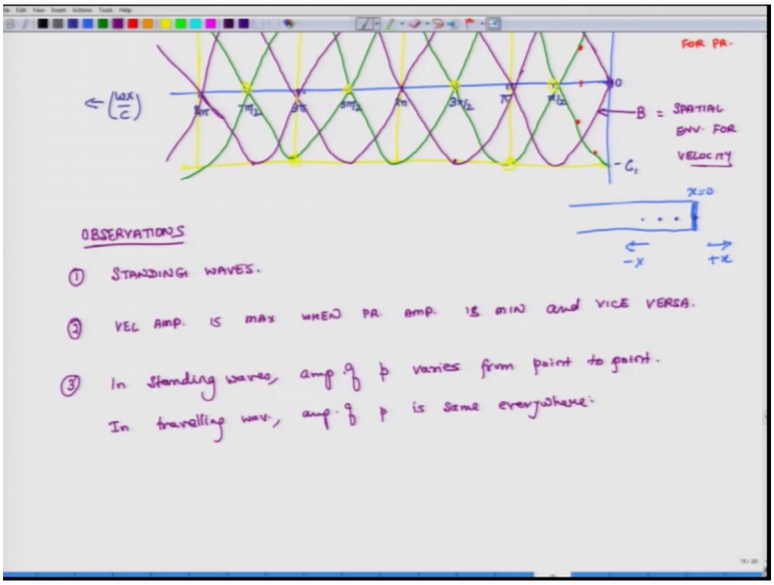

That is spatial envelope for velocity. So, we make couple of observations, one these types of waves in a closed tube the pressure fluctuates between 2 specific limits and these 2 limits vary from place to place. These limits are fixed and, but they vary from place to place and these types of wave because of that these types of waves are known as standing waves they are known as standing waves here for instance.

Good example which you can visualize what is a standing wave is that if you tie a rope to nail which is inserted into a wall and then you move the other end of the rope up and down, there is wave motion in the rope, but when you look at the pattern of the rope it appears fixed and it appears something similar to the a spatial envelope for velocity and it appears fixed and it looks like the wave is not travelling. So, that is why they are known as stationery waves or standing waves. The second observation we make from these graphs is that velocity amplitude is max when pressure amplitude is min and vice versa. Third thing is that in standing waves in standing waves amplitude of pressure varies from point to point depends on the position.

But in travelling waves and an example of travelling wave would be a wave travelling in an infinitely long tube it does not get reflected and in a travelling wave you do not get these kind of equations you know you do not get these type of equations. So, in a travelling wave amplitude of pressure is same everywhere. If we have if we can figure out the solution for a travelling wave we go back to couple of lectures earlier see this is the equation for a travelling wave.

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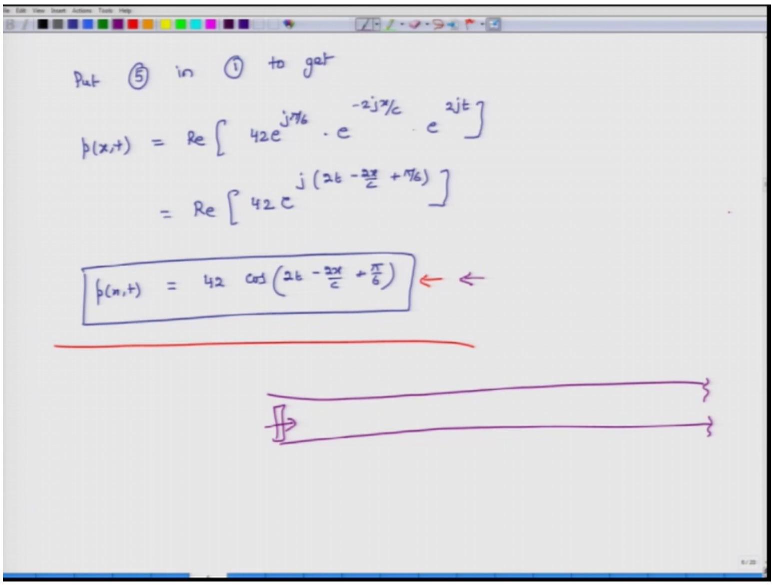

Which we had developed this is for a wave which is travelling in an infinitely long tube and the wave is being generated from this end and here you have 42 cosine 2 t minus 2 x by c plus pi over 6. So, here the amplitude is 42 and it remain same for all values of x and t, right.

But in case of standing waves, the cosine the term in x and the term in t they separate out and that is why and the term in x is a cosine function of x. So, because of this the amplitude of the wave which is designated by function A or the amplitude of velocity wave which is designated by function B it varies from for position to position some terminology.

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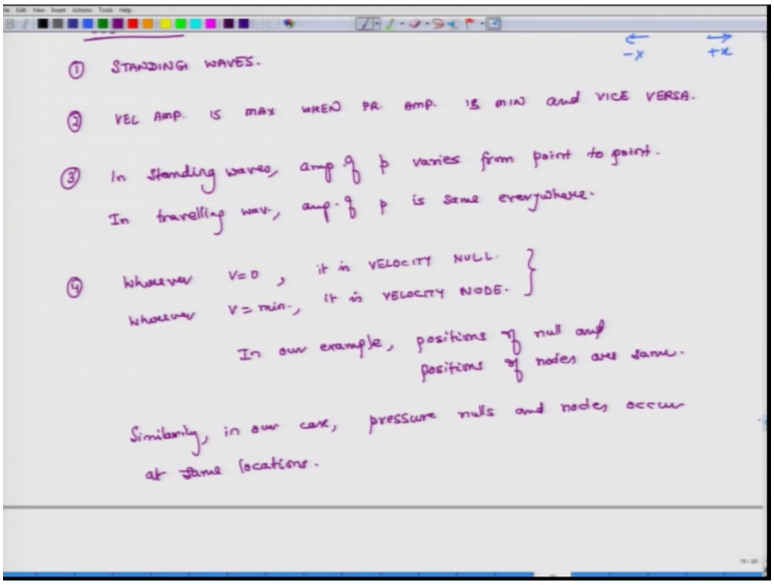

So, wherever velocity equals 0 we call it a; it is velocity null wherever velocity is minimum it is called velocity node. Now in our case the location where velocity is 0 and the location where velocity is minimum is same. So, in our case in our example positions of null and positions of nodes are same, but there may be some wave the reason why it is happening is because we have a rigid termination.

So, whatever velocity is coming that energy is getting reflected 100 percent, but in some waves where reflection is not 100 percent the place where velocity is minimum and the place where velocity is 0 they may be different. So, this they may be different. So, similarly in our case pressure nulls and nodes occur at same locations. So, that is the overall picture for standing waves in context of closed tubes. So, I presume that this concludes our discussion on standing waves in closed tubes, tomorrow is our last lecture for this week and we will close the discussion tomorrow by having some detailed description of how do sound waves one dimensional sound waves that travel in tubes which are not closed, but they are open. So, that is a totally different category and with that we will conclude the discussion on transmission line equations over this week.

Thank you very much and we will meet tomorrow, bye.