**Heat Treatment and Surface Hardening - II**
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Lecture – 30
Implication of Avrami's equation with example on excel spreadsheet

So, in today's lecture we will first start with having the look at the Avrami equation, which we evolved in the last class.

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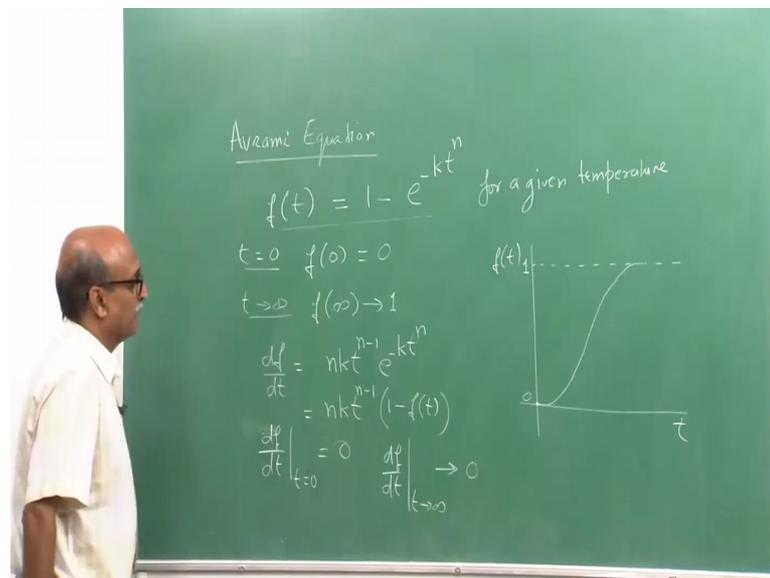
And the Avrami equation is given by the fraction transformed as a function of time is given by 1 minus e to power minus k t to power n. Where k and n are constants for a given temperature. So, first let us look at this function itself, and see how it varies between as a function of time. First of all on the left hand side this is a fraction transform which means the left hand side should vary between 0 and 1 as time increases.

So, let us see first what happened at let us look at the values of this function at t equal to 0. At t is equal to 0, fraction transformed is clearly 0. And when does the fraction transformed become equal to 1. Well, if you look at this function and look the exponential part when the exponent minus k t to power n tends towards infinity this exponent will go to 0 and this function will tend towards 1. Which means that as time tends to infinity the fraction transformed tends towards 1 or 100 percent transformation.

So, if you attempt to plot this fraction transformed versus time ft is equal to 0 f would be 0. And this function will vary between 0 and 1. So, I will put here 1 and out here 0. And at very large t this function should tend towards 1. Let us look in order to determine the shape of this curve. Let us look at the slope of this function as a function of time that is take a derivative of this function df by dt. So, the derivative of the function clearly would be n k t to power n minus 1 e to power minus k t to power n, or I can write this as n k t to power n minus 1 and this exponential minus e to power minus k t to power n from out from the avrami equation itself can be written as 1 minus f of t.

If I now look at this derivative at time t is equal to 0 let us look at what the derivatives becomes at time t is equal to 0 the derivative is 0. So, at t equal to 0 the derivative is 0. Which means the slope at t is equal to 0 of the function should be flat or 0. Similarly what happens at t tending towards infinity? This function should tend towards well as t tends towards infinity, If you look at this bracket term 1 minus f t since f t tends towards infinity a towards 1.

This term in the brackets would tend towards 0, and hence again f t tends to infinity the slope tends towards 0. So, I have slope 0 to start with and towards the end of the transformation again the slope should tend towards 0. So, if you attempt to plot this a possible shape could be of curve like this. So, here it is 0, and at t tends tending to infinity again the slope is 0. If you look at this function it tends to suggest that the slope initially starts to increase from 0, and then toward the end the slope is again decreasing. And In fact, we can also determine at which means if the slope is increasing in the in the beginning and slope is decreasing towards the end.

Which means this function the rate of transformation df by dt should go through a peak at some value of time. In order to find the position where this would to go through a peak you would need to take the second derivative of the avrami equation.

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$$\frac{d^2f}{dt^2} = n(n-1)kt^{n-2}(1-f(t)) - (nkt^{n-1})^2(1-f(t))$$

$$= \left[(n-1) - nkt^n\right](1-f(t))nkt^{n-2} = 0$$

$$(n-1) - nkt^n = 0$$

$$kt^n = \frac{n-1}{n}$$

$$\boxed{f = 1 - e^{-\frac{n-1}{n}}} \Rightarrow \frac{df}{dt} = \text{maximum}$$

$$n = 4 \Rightarrow f = 0.53$$
$$n = 3 \Rightarrow f = 0.49$$
$$n = 2 \Rightarrow f = 0.39$$

So, if I take the second derivate of the avrami equation d square f by dt square. Then this would be n times n minus 1 k t to power n minus 2 times 1 minus ft. Minus n k t to power n minus 1 squared 1 minus ft. This is a pretty straight forward differentiation of this function. So, you can try this yourself that this is what you would get, and we can sort of simplify this a little bit and take some common terms out of this and one can write the second derivative as n minus 1 minus n k t to power n times 1 minus ft. So, I am taking out the common terms. So, 1 minus ft is common and I am can take n k t to power n minus 2.

Now, peak in the rate of transformation bf by dt or the would be given by when d square f by dt square the second derivative goes to 0. And if I look at it now how can this a function go to 0? Except for t tending to infinity of course, it goes to 0, but then we are not interested in that particular value. We are interested in somewhere in intermediate where a df by dt goes through a maximum. That could be when this square bracketed term goes to 0. So, this function d square f by d t square will become 0, when n minus 1 minus n k t to power n goes to 0. From this I can write k t to power n is equal to n minus 1 upon n. Now what can we get out of this? Well, I can substitute that when this condition is met substitute that df by dt is a maximum.

So, I can substitute k t to power n equal to n minus 1 upon n in the avrami relation and I can write that when the fraction transformed f is equal to 1 minus e to power n minus 1 upon n. Then df by dt is a maximum. So, if I put that means, df by dt is a maximum is independent. The position of this maximum is independent of the value of k it only

depends on n. And hence if I look at few values of n for example, at n equal to 4, I substitute in this I get f equal to 0.53 or of 53 percent after fifth at 53 percent transform product the rate of transformation at that point is a maximum.

And beyond that the rate will start to drop and for large time t the rate will become very small. At n is equal to 3 the fraction transformed is 0.49 roughly 50 percent transformation. At n is equal to 2 f is v 0.39 40 beyond 40 percent transformation the rate of transformation starts to drop. We can have a detailed look of this particular avrami equation, but plotting this equation on a spreadsheet. So, let us take a look at and look at how the avrami relation behaves, as a function of time for some different values of the parameters n and k.

So, let us have a look here. So, here we are we will examine the avrami relationship on a spreadsheet.

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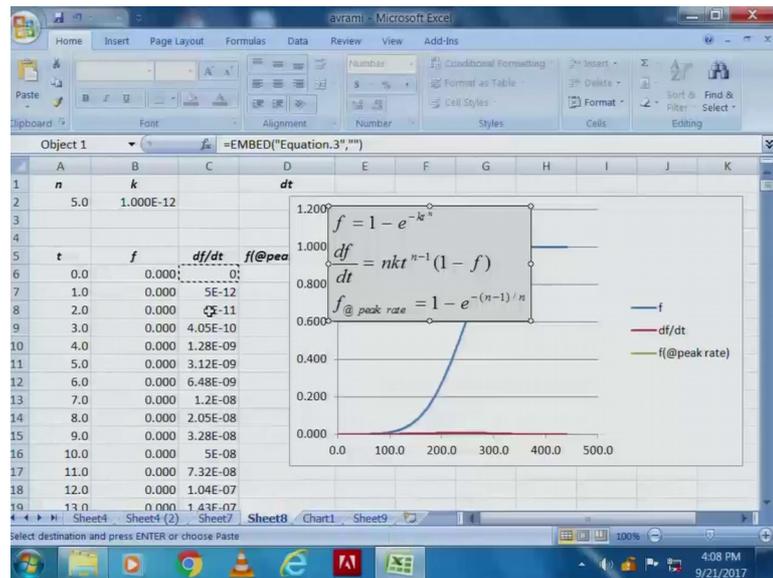
So, let me put some values for the avrami parameters n and k, in the spreadsheet let us say that n is n takes a value of 5. And k usually has a small value and let me a assume that case of the order of 10 to power minus 12, and time is in seconds. I will also take a path value of the time increment dt. So, we will be calculating the fraction transformed as a function of time in increments of one second.

So, we will start with 0 seconds at the start of the transformation, calculate what is a f and of course, which should be also be 0. At I will input the formula for the fraction transformed in the cell B 6. The formula starts with an equal to sign followed by 1 minus the avrami equation which is 1 minus exponential minus k times and value of k is a cell B 2, and we have to encourage by putting dollars in front of B as well as 2 so that when I copy this formula to others cells this cell will not change it will always be pointing to the value of k in cell B to multiplied by the time, time is here, in cell a 6 to power n and n the value of n is in cell a 2 and just like k we would also need to anchor this cell as we do not want this cell 2 also change.

When you copy this formula to other cells and as expected the fraction transformed is 0 against time of 0 seconds. The next time that we would take would be we will add dt to the previous time value in cell a 6 plus dt which is in cell d 2 having a value of one second and I will also anchor this cell as well. Now this puts a time of one second in cell A 7. And now I will just copy this formula on to the cell B 7. Now this has given me a value was which is showing 0, but it actually should be a non zero, but let us first copy all of these values. And let me copy all this formulae so, that I will get now the fraction transformed as a function of time, and I will paste it here. See it is showing only up to 3 decimal places and after 3 decimal places I even after 2 seconds or even after 9 seconds or 10 seconds it is 0, but eventually.

We start getting some fraction transformed and as we yes time is increasing as you can see the fraction is moving towards 1. And finally, it is stabilizers to a value of 1. Well after 3 decimal places it is rounding off. So, that is why I initially you have values of 0. Now if I plot this function using this spreadsheets chart function. So, I go to insert go to charts go to scatter plot and take lines I want a scatter plot with joining the points through straight lines. And I will get a plot here let me move this formula box outside here.

So, that we can see the we can see the plot clearly. So, here now you can see the plot, it shows just the expected behavior that I had drawn on the board. Initially a very low rate of transformation and this transformation rate keeps increasing and that eventually towards the end again the transformation rate starts to slow down, as it is moving towards a 100 percent transformed.

Now in this I will also now let us also before I should do anything else let me also calculate in another plot df by dt that is the rate of transformation. Now the rate of transformation as we have seen let us look at the formula for the rate of transformation, and here it is this is the formula for the rate of transformation df by dt is equal to n k t to power n minus 1 times 1 minus f this we have derived just now on the board, if I plot this. So, I will put an expression for this. So, this is n anchorite multiplied by k anchorite multiplied by time is 6 cell to the power n minus 1. So, n again anchorite minus 1.

So now becomes a 6 to power a 2 minus 1 which is basically t 2 power n minus 1, another bracket closed and then multiplied by the last term in the formula 1 minus f which is simply 1 minus f I have already calculated in column B, certificate of from B 6 brackets closed. And it is essentially telling me the rate of transformation is 0 actually exactly at the time t is equal to 0. Now I copy this formula as well paste it and suppose it didn't copy the whole thing let me just repeat this copy somewhere I lost the thing copy and paste it. And now we have the formula copy here.

Now this is actually the plot showing me the df by dt by the red color graph except that the range for df by dt is very different from the range for f. And hence we it is a very compressed plots. What I will do is let me make a copy of this chart and paste it here. And let me in the second copy I will simply delete the fraction transformed graph.
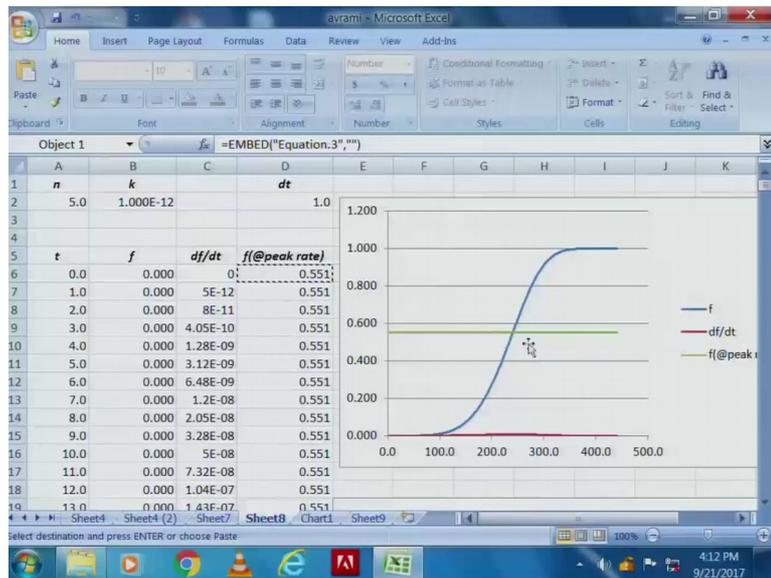
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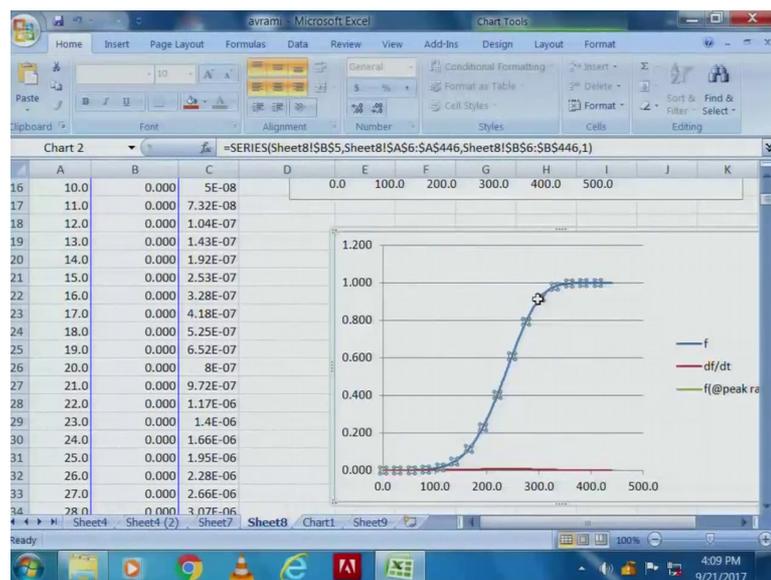
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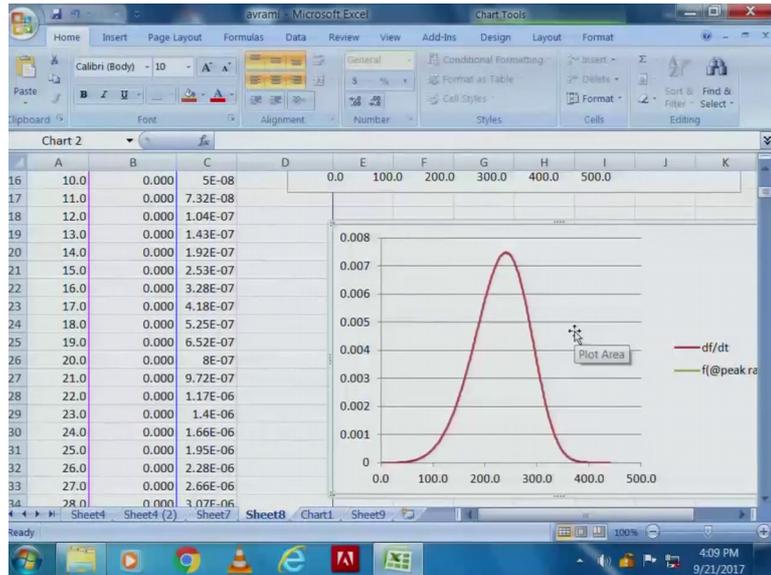
And now, you can see just as we had an expected and from our analysis, that there is a peak at the rate of transformation at some point in the transformation.

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Let us also examine the location of the peak, and we can easily examine the location of peak as well by looking at this formula over here in the box, that what is the fraction transformed at the peak rate, this is what we want to know. And I will calculate this fraction transformed at the peak rate by this formula 1 minus exponential minus n minus 1 upon n. So, let me write this term 1 minus exponential minus n minus 1 upon n, and that should do the trick except that I can not put n I need to put cell containing the value

of n which is a 2. So, an anchorite same thing I will have to do to this particular n click on cell a to anchorite and that should do the job.

So, actually it is giving me that this peak should occur when the fraction transformed is about 55 percent. So, let me copy this formula as well, paste it and here we go. Now let me move this box out of the way. So, that we can see the figure clearly. What this figure is showing me is that this is the behavior of the avrami equation, this green line indicates where the peak is expected as I said around 55 percent, a transformed and I can delete this particular graph from here So, that I can see the peak rate clearly.

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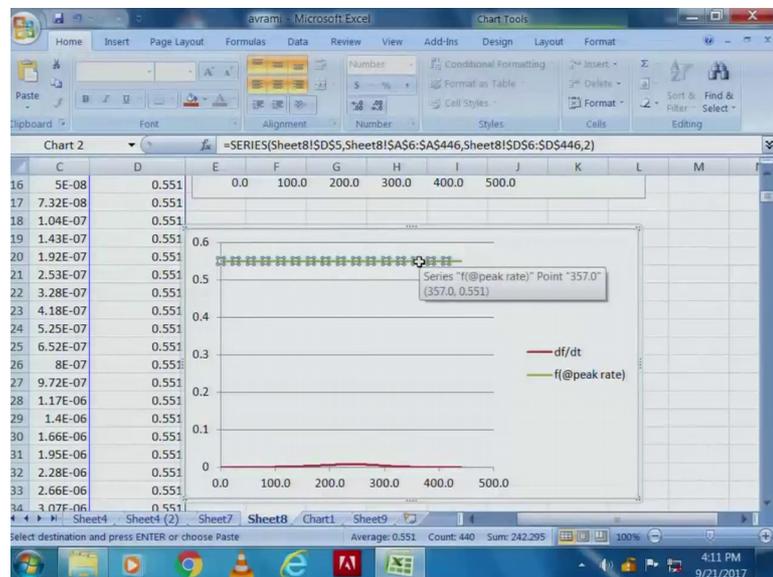
Now we can not now that we have all the graphs in place and I can move this formula actually out of the way completely. And let us look at how this graph change when I change the values of n and k. Let us see what happens if I change the value of k let us say to value a little high in higher order of or a lower order of magnitude which is at set instead of 10 to power minus 12, let me change it to 10 to power minus 14.

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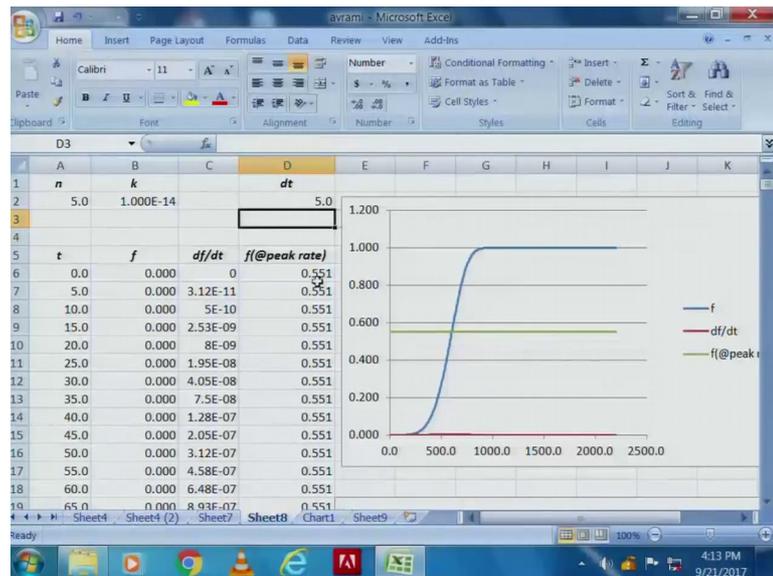
Now obviously, what it is meaning is that the transformation has slowed down and as you can see that we are not even reached the peak time. And incidentally the peak rate has not changed by changing k and that is to be expected.

Because a the fraction transformed at the peak at the peak rate is still 55 percent, and that is because is only purely a function of the exponent n. Now to get the full curve let me change this increment let us change it to instead of one second let me change it to 5 seconds. So, that I can cover longer time and as you can see. Now I have the full curve again the shape of the curve is the same sigmoidal curve for the avrami equation, and the peak has change to a different time, but the fraction transformed at that time is still 55 percent.

If I change the value of k let us say to 10 to the power minus 10 I am increasing the value of k by 2 orders magnitude as compared to 10 to the power minus 12, and as you as you can see the transformation is very fast and you cannot. So, in order to see this graph better let me go back to an time increment of one second. So, that you can see this equation a little better. Let me go back to the old value of k now and look at what happens if I change the value of n.

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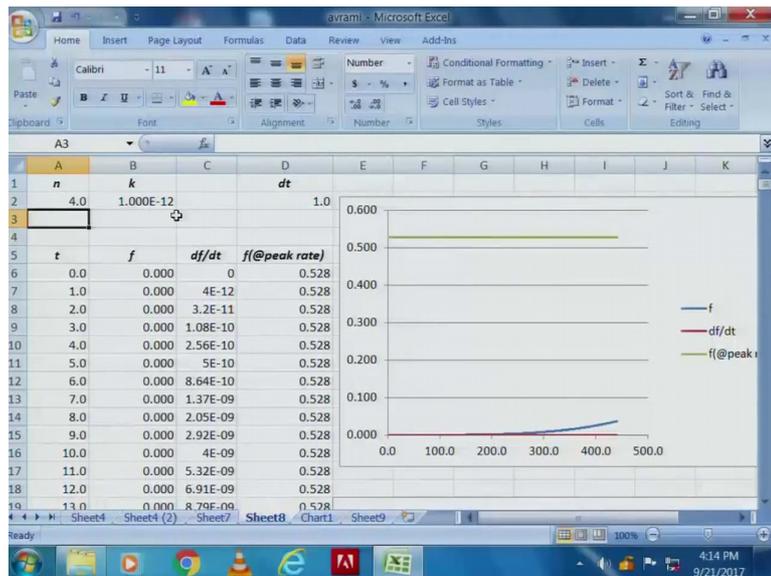
Let us say change the value of n to 4. Again by changing by reducing n I slowed down the transformation and Let me change the time increment to 5 seconds so, you can see the.

Now the fraction transformed peak rate is different it is actually now about 53 percent it is come down bit, but it is in that same order of a magnitude. Let us look at value of n as 3. I will have to again change the time increment and his time let me change it to 50 I can see this clearly.

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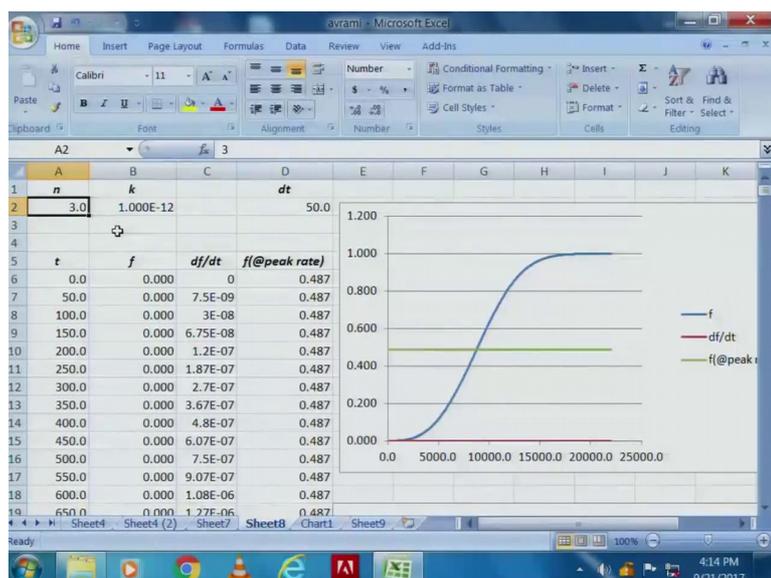
And now the peak that is a occurring occurs now at the fraction traction when the fraction transformed is about 48, 49 percent. Similarly I can also examined the value of n equal to 2, and let me change this dt to still larger number I have to take that is not good enough. So, let me take it to thousand seconds increments. In fact, that is also not good enough. So, 5000 seconds and that is good enough for me to see the peak.

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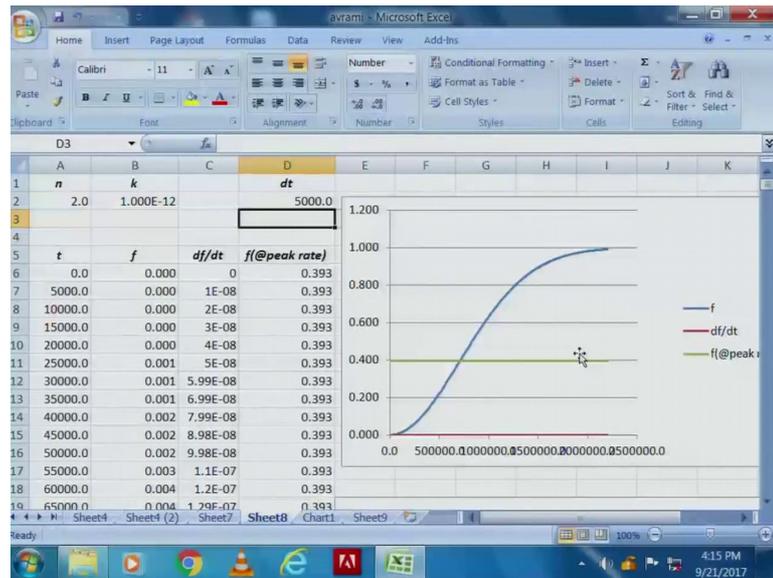
But now the transformation is taking a much longer time as compared to n is equal to 5. So, with this I have showed to you how the avrami equation how the avrami curve or the sigmoidal curve will behave for different values of n and k. And with this we come to a close of this lecture.