

Fundamentals of Acoustics
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Lecture – 50
Designing of Reactive Mufflers

Hello, Welcome to Fundamentals of Acoustics; today is the second day of the 9th week of this course. Yesterday we started discussion on modeling and design of expansion type of mufflers another name for which is reactive mufflers.

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The image shows handwritten notes on a whiteboard. At the top, there is a diagram of an expansion chamber. It consists of three sections with cross-sectional areas A_1 , A_2 , and A_3 from left to right. The chamber has a length L . The pressure and density in each section are denoted as p_1, ρ_1 , p_2, ρ_2 , and p_3, ρ_3 respectively. The word "EXPANSION" is written above the diagram, and "CHAMBER" is written below it.

Below the diagram, the transmission coefficient T is given by the equation:

$$T = \frac{4 \sigma_{13}}{(\sigma_{13} + 1)^2 \left[1 - \frac{(\sigma_{23} - 1)(\sigma_{12} - 1) \sin^2(k_2 L)}{(\sigma_{13} + 1)^2} \right]}$$

Below the equation, the assumptions are listed:

ASSUMPTION: $A_1 = A_3$ $A_2/A_1 = m$

To simplify $T \rightarrow$ we need to find σ_{13} , σ_{23} , σ_{12} .

FROM LAW OF MASS CONSERVATION

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$ $\rho_1 \rightarrow$ Total density.
 But $v_1 = v_2$
 $\rho A_1 = \rho_2 A_2$
 $\rho_2 = \rho_1 (A_1/A_2) = \rho/m$ $T \text{ (2)}$

And what we had show yesterdays that if we have a muffler with a topology like here a shown in this figure, then the relations between pressure in section 3 and section 2 and section 2 and section 1 and similar relations for density are given here. So, with this understanding now let us calculate the density in each of these sections.

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$$P_1 V_1 = P_2 V_2 \rightarrow P_2 = \frac{V_1}{V_2} P_1 = P_1 \frac{A_1}{A_2} = \frac{P_1}{m}$$

Similarly $P_3 = P_2 m$

SUMMARY: $P_3 = P_2 m$ $P_2 = \frac{P_1}{m}$ $P_3 = P_2 m$ $P_2 = \frac{P_1}{m}$

$$C = \sqrt{\frac{\gamma P}{\rho}}$$

$$C_3 = \sqrt{\frac{\gamma P_3}{\rho_3}} = \sqrt{\frac{\gamma P_2 m}{m \rho_2}} = C_2$$

Similarly $C_1 = C_2$

$$C_1 = C_2 = C_3$$

$$r_{23} = \frac{\rho_3 C_3}{\rho_2 C_2} = m$$
 $r_{12} = \frac{\rho_2 C_2}{\rho_1 C_1} = \frac{1}{m}$ $r_{13} = 1$

So, density we have already calculated, so next we will do is compute velocity of sound. So, we know that C equals gamma over all pressure, divided by over all density and then we have to take the square root of this. So, we can write that C 3 which is the velocity of sound in the third section equals gamma times P 3 divided by rho 3, and what is P 3? This is equal to gamma P 2 over rho 2 and I have to multiplied by m on both sides because P 3 is equal to rho 2 m and rho 3 also equal to rho 2 m; so essentially what is happening that density increasing and also the pressure is increasing in the third section. So, this means that C 3 is equal to C 2.

Similarly, we can show C 1 is equal to C 2. So, essentially what that means is that C 1 equals C 2 equals C 3. So, the speed of sound throughout the muffler its does not change. And the reason it is not changing is that wherever the pressure goes up, the density also goes up, so this ratio p over rho remains constant. So, pressure is changing rho is changing, but the ratio does not change and because of that the speed of sound does not change throughout the muffler.

So, now what we will do is, we will compute r 23. So, r 23 is equal to rho 3 C 3 over rho 2 C 2 and we know that C 2 is equal to C 3. So, they will cancel out, C 2 and C 3 cancel out and rho 3 and rho 2 the ratio of rho 3 and rho 2 is m; the next parameter we are going to compute is r 12, and that is equal to rho 2, C 2 divided by rho 1 C 1 and once again C 2 equals C 1. So, I am going to cancel them out and the ratio of rho 2 and rho 1 is 1 over

m and finally, r_{13} equals 1. So, we will plug these relationships into our expression for T which is here and we will compute the transmission coefficient.

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$$c_1 = c_2 = c_3$$

$$r_{23} = \frac{p_3 - p_2}{p_2 + p_3} = m \quad r_{12} = \frac{p_2 - p_1}{p_1 + p_2} = \frac{1}{m} \quad r_{13} = 1$$

$$T = \frac{4r_{13}}{(r_{13}+1)^2 \left[1 - \frac{(r_{23}-1)(r_{12}-1)}{(r_{13}+1)^2} \sin^2(k_2 l) \right]}$$

$$T = \frac{1}{\left[1 - \frac{(m^2-1)(1/m^2-1)}{4} \sin^2(k_2 l) \right]}$$

So, T equals. So, I will first write that expression again and then we will plug those values. So, it is equal to $4 r_{13}$ divided by $r_{13} + 1$ whole square, $1 - r_{23} - 1$ r_{23} square minus 1 times, r_{12} square minus 1 divided by $r_{13} + 1$ whole square, $\sin^2 k_2 l$. Now r_{13} what is r_{13} ? We have calculated that it is equal to 1 , r_{23} is equal to m and r_{12} is equal to 1 over m . So, we will make these replacements in the relation and what we get is that transmission coefficient equals. So, this $4 r_{13}$ divided by $r_{13} + 1$ whole square they cancel out, because $4 r_{13}$ is and $4 r_{13} + 1$ whole square is 4 also. So, this is 1 divided by $1 - m$ square minus 1 times 1 over m square minus 1 divided by $4 \sin^2 k_2 l$.

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The image shows a whiteboard with handwritten mathematical equations. At the top, there is a toolbar with various icons. The main content consists of two equations. The first equation is:

$$= \frac{1}{1 - \frac{(m^2 - 1)(1 - m^2)}{4m^2} \sin^2(k_2 l)} = \frac{1}{1 + \frac{(m^2 - 1)^2}{4m^2} \sin^2(k_2 l)}$$

The second equation is:

$$T = \frac{1}{1 + \frac{(m^2 - 1)^2}{4m^2} \sin^2(k_2 l)}$$

At the bottom right of the whiteboard, there is a small text "6 / 30".

And this I can write it has 1 over 1 minus m square minus 1, divided by 1 minus m square by 4 m square sin square k 2 l.

So, m square minus 1 times 1 minus m square is same as m square minus 1 the whole thing were times minus 1. So, this I can write it has 1 minus m square minus 1 whole square, sin square k 2 l divided by 4 m square m because I made m square minus plus 1 whole square I have to change this negative sin to positive. And finally, we bring it into its final form. So, this is equal to 1 over 1 plus m minus 1 by m square by 4 sin square k 2 l. So, that is my transmission coefficient. Now what we are really interested in is not how much sound is getting out of the muffler, but how much sound is getting reflected back. So, what we are interested is not the transmission coefficient, but transmission laws.

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$$TL = 10 \log_{10} \left(\frac{1}{T} \right) = 10 \log_{10} \left[1 + \frac{(m-1)^2 \sin^2(k_2 l)}{4} \right]$$

STATE: TL will be max for a given value of 'm' when $\sin^2(k_2 l) = 1$

$$\Rightarrow k_2 l = \frac{\pi}{2} \Rightarrow l = \frac{\pi}{2 k_2} \quad k_2 = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \frac{\omega}{c}$$

In cars, engine runs between 1500 and 4000 RPM.

So, transmission loss we write it has T L and this we calculate on log scale. So, because T relates to the intensity of sound which is get in transmitted, if I take its 10 log and f t is high, f t is large then transmission laws will be less and transmission and if t is large then transmission loss will be high, so f t is high then T L will be low, f t is low then T L will be high. So, do that we take 1 over t.

So, this is equal to 10 log 10 of 1 plus m minus 1 by m whole square sin square k 2 l by 4, this is our final expression. So, this is transmission loss and we would want that this transmission loss for muffler should be as high as possible. So, what we see is that this transmission loss depends on 2 parameters, 1 is this parameter m minus 1 over m; if this m minus 1 over m parameter is large then transmission loss is going to be large. The second parameter is sin square k 2 l if sin square k 2 l is large, then transmission loss will be large otherwise it will be small. So, what we do is, we state we can say that T L will be maximum for a given value of m, when it is going to be maximum? When sin square k 2 l equals 1, because that is the maximum possible value this means k 2 l equals pi over 2 radian; it implies that l is equal to pi over 2 k 2 and k 2 equals 2 pi over lambda and what is lambda it is equal to C divided by f and this is equal to 2 what is 2 pi f? It is equal to omega over C; will do some math.

So, let say that we want to reduce the sound coming from an IC engine which is connected to our car; so in cars most of the time the engine runs, it runs between what? It runs between 1500 and 4000 RPM. So, 1500 RPM corresponds to 25 hertz.

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$\sin(k_2 l) = 1$
 $\Rightarrow k_2 l = \frac{\pi}{2} \Rightarrow l = \frac{\pi}{2 k_2}$ $k_2 = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \frac{\omega}{c}$

In cars, engine runs between 1500 and 4000 RPM.
 $\hookrightarrow 25 \text{ Hz}$ $\hookrightarrow 67 \text{ Hz}$.

Our silencer should work in the the limit 25 - 67 Hz.
 Since not much sound is heard below 40 Hz,
 WORKING RANGE of muffler should be 40 - 67 Hz.

At 40 Hz. $l = 1.37 \text{ m}$
 At 67 Hz $l = 0.82 \text{ m}$ $c = 345 \text{ m/s}$

And this 4000 RPM corresponds to 67 hertz. So, what does that mean? That our silencer or muffler should work in the limit 25 to 67 hertz. So, we have to design a muffler in such a way that it reduces the sound between 25 hertz and 67 hertz. Now even though are audible sound range is from 20 hertz to 25000 hertz, most of us can dearily heard 20 hertz sound, we really start listing sound above 40 hertz or so. So I will reduce this frequency further. So, since not much sound is heard below 40 hertz working range of muffler should be 40 to 67 hertz.

So at 40 hertz what we do? We compute l using this relationship and we find that l equals 1.37 meters. How do we compute? I first compute k 2 which is omega over C and then divide pi over 2 by k 2 and at 67 hertz, l is equal to 0.82 meters. So, this is assuming C is equal to 345 meter square per second. So, what does that mean?

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WORKING RANGE of muffler should be 40 - 67 Hz.

At 40 Hz. $l = 1.37$ m
At 67 Hz. $l = 0.82$ m

$c = 345$ m/s.

l

The diagram shows a rectangular muffler with two horizontal lines representing the inlet and outlet pipes. The length of the muffler is labeled as l .

What that means is that if I have a silencer of this type then this parameter l should be if I want to kill 40 hertz sound very effectively, then the length of this parameter the value of this parameter l should be 1.37 meters and if it shift away from 1.37 meters sound will get reduce, but it will not be that effective. If I want the design to be such that 67 hertz sound is reduced, which corresponds to engine running at 4000 RPM then l should be 0.82 meters.

So, this is the conclusion for today's lecture. Tomorrow will continue this discussion on mufflers and we learn more about this particular relation which we have developed. And we will also discuss somewhat about dissipative mufflers. So, with that have a great day and we will meet once again tomorrow.

Thank you.