Basics of Finite Element Analysis – Part II
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Lecture – 40
2 –D Heat transfer problems (Part-II)

Hello, again welcome to Basics of Finite Element Analysis Part II. Today is the 4th day of the 7th week of this course. What we did in the last class? We solved a heat conduction problem as represented through Poisson’s equation and we had solved it for re square plate and the problem was symmetric in h r along to 2 dimensions. What we will do today is something very similar, but we will extend that idea of symmetric further and we will see how we can exploit that other situation also.

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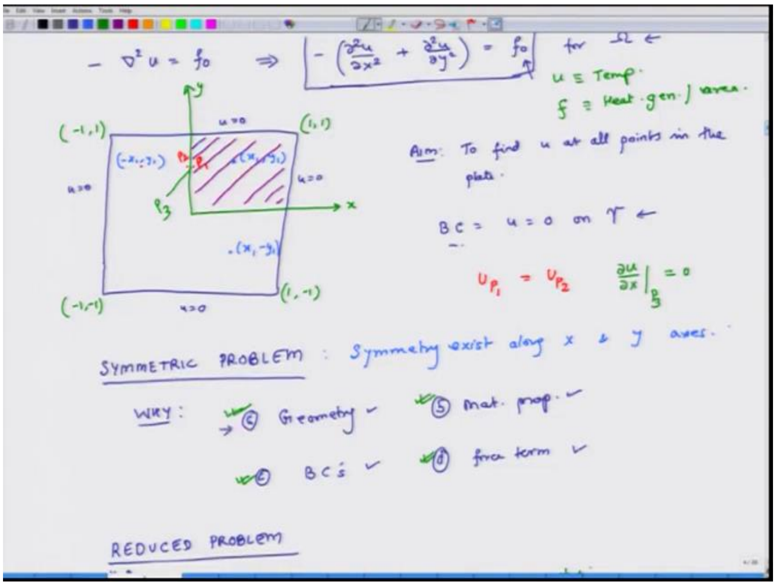

So, the problem we will solve is still the same identically the same which is that we have a plate, where we are having heat conduction u is 0 on all 4 sides and the coordinate system is located at the central of the plate. The size of the plate is 2 by 2 and we have symmetry of geometry, symmetry of material property, symmetry of boundary conditions and symmetry of the force term and because of this we had said that the problem is symmetric along the x axis as well as the y axis, but there is one more plane along which it is symmetric and what is that plane?

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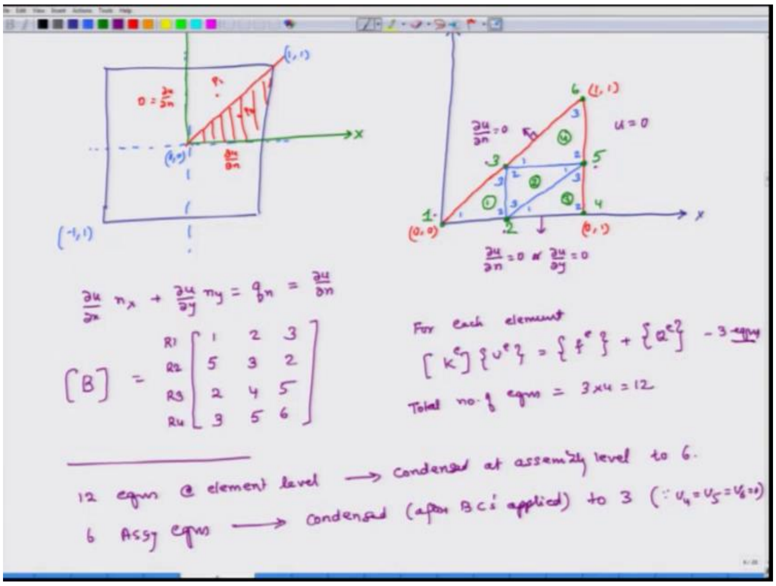

So, the other plane of symmetry will draw there separately just to make things clear, this is my original problem, these are my axis and we had said that the planes of symmetry this is 1 plane of symmetry, this is another plane of symmetry because along these 2 planes the geometry symmetric, the boundary condition is symmetric, the force variable is symmetric and the material properties they are symmetric all those 4 parameters are symmetric. So, because of that we had broken this plate into the first quadrant and we just solved the problem for that.

But as we look at this problem is also symmetric along a third axis or third line and that would be this line. Again why is it symmetric? Along this line the geometry is symmetric right what about boundary conditions, on one edge, on this edge, on the vertical edge boundary condition is del u over del n is 0 and on the horizontal is del u over del n is 0 right. So, the boundary conditions are also symmetric, geometry is symmetric, what about force? If I take along this line if I take a point p1 and I take a point p 2, which is the mirror image of this, are the material property same. So, the material property is our symmetric and finally, for the same 2 points what is the value of F naught at all though at both those points, F naught is constant the across the whole plate and because it is constant the value of F naught at point p1 and at point p 2 it is same.

So, it is symmetric also along this plane right. So, I do not have to solve one fourth of the problem, but I can even use one eighth of the problem and solve it. So, in this case the complexity of the problem or the number of equations, which I will have to eventually solve it, will go down by a factor of 8 not just 4. So, that is what we will do? So, we will draw this problem again. So, again my coordinate system once I have identified planes of symmetry it does not change. So, this is x axis, this is y axis and I will draw my reduced problem. So, the coordinate of this is 0, 1, this is 1, 1, this is 0, 0 and now what I will do is I will break this up into different elements.

So, I break it up into 4 triangles or I can remove this and I can break it up into 52 triangles and a rectangle or I can make (Refer Time: 05:44) larger, but for this description or this coverage we will just break it up into 4 triangles. So, this is element 1, element 2, element 3, element 4 and I will write down my global node numbers. So, global nodes are indicated in green 1, 2, 3. So, these are global node numbers actually I have in my nodes as 4, this is node 3, node 5 and this is node 6. So, this is important as we have formulating, we should very clearly indentify global nodes and local nodes and then the local nodes are 1 2 3, 1 2 3, 1 2 3, 1 2 3.

Let us also write down the boundary conditions. So, what is the boundary condition on the right edge of the problem? u is equal to 0 right. What is the boundary condition on the bottom edge of the problem? Del u over del n is equal to 0, in this case the normal direction is y. So, I can also write it as del u over del y equals 0. Which means that the flux or the gradient of u which in this could be temperature is 0? Then what is the boundary condition on the edge 1,6 it is a still del u over del n equal 0, which means that the gradient of u in this direction normal to the edge is 0, that of the thing. What is del u over del n? We are written it del u over del x times n x plus del u over del y times n y this is equal to q n and this is also in our case it happens mean del u over del n ok.

So, del u over del n is 0 in this case. Very quickly will I develop the connectivity matrix for this? So, the b vector is how many rows? Number of rows corresponds to number of elements so it will have 4 rows and how many columns, 3 columns because the maximum number of elements in any maximum number of nodes in any column is 3 so the first row. So, this is row 1, row 2, row 1 is for element 1, row 3 is for 2 and row 4. So, row numbers correspond to elements. So, the entries in the first row will be 1, 2, 3. Entries in the second element will be 5, 2 and 3. Entries in the third row will be 2, 4 and 5. And entries in the fourth row will be 3, 5 and 6. For each element we will have an equation K e, u e equals f e plus q e. How many equations will have for each element 3 because each element has 3 nodes, if there was a rectangle then that for that particle element we will have four equations here we will have 3 equations. So, these are 3 equations. Total number of equations at element level, how many? Total number of equations, it will be 3 times 4, 12 equations right. These 12 equations will be condensed

12 equations at element level they get condensed at assembly level. How many equations? We get to 6 because we have total of 6 nodes in the global domain hm.

Further once we apply the boundary conditions one primary variable, these 6 assembly equations, they get condensed to how many equations? After bcs are applied to 3 why? Because u 4 is equal to u 5 is equal to u 6 is equal to 0.

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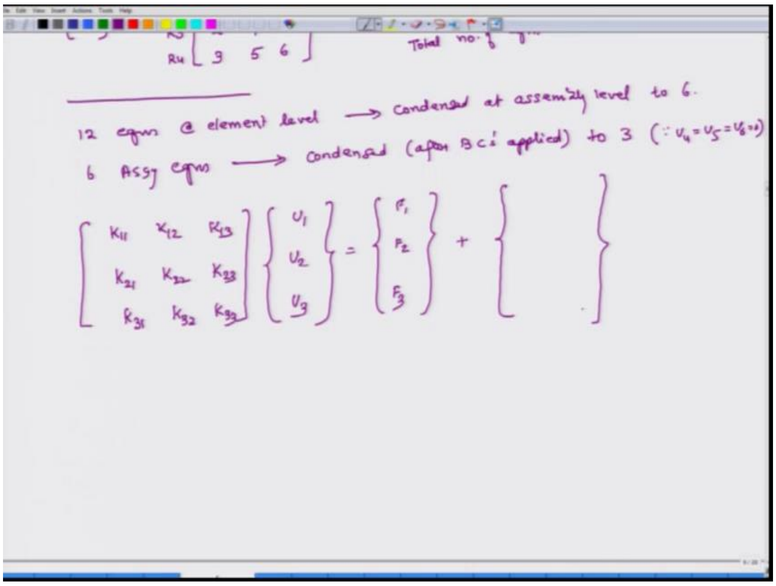

So, these 3 degrees of freedom because they get eliminated, we are left with only 3 equations. So, we will just quickly write down those 3 equations. So, what are the degrees of freedom left? 4, 5, 6 are gone because of the condition u is equal to 0. So, the only degrees of freedom left are 1, 2, 3 and we will f 1, f 2, f 3 plus q vector ok.

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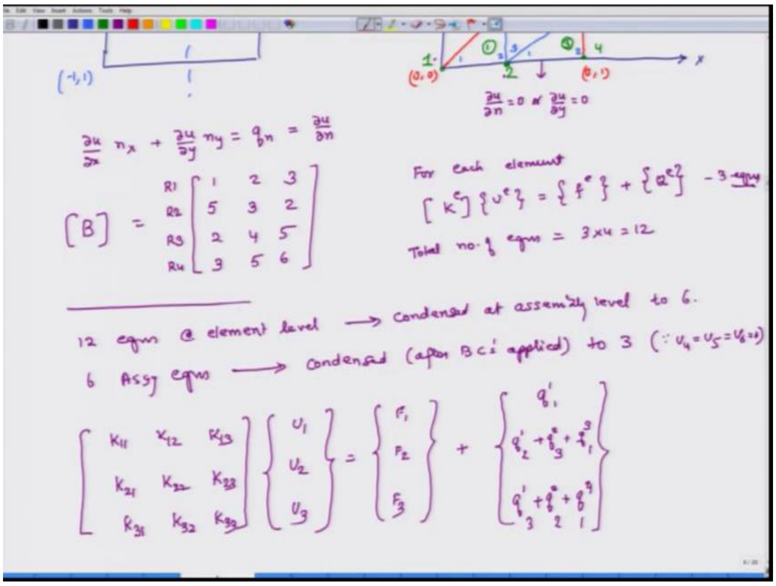

Now, let us look at the q vector. At node 1 how many components we will get in the q vector? Only one as so I will get q 1 1. At node 2 how many components do I get? I get 3 components. So, I get component from 1, component from second element, component from third element and the nodes are 2, 3, 1 agreed and at node 3 how many components do I get? I get again 3 components from element 1 plus from element 2 plus from element 4 and the nodes are 3, 2, and 1 ok.

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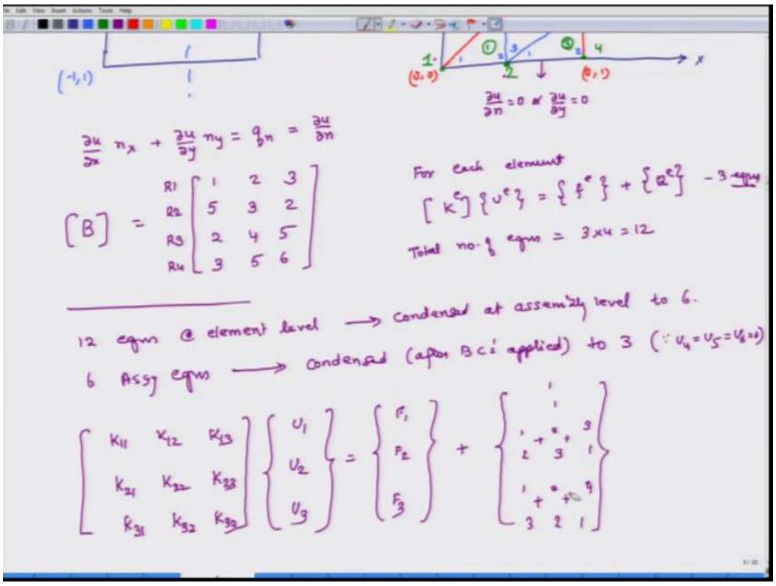

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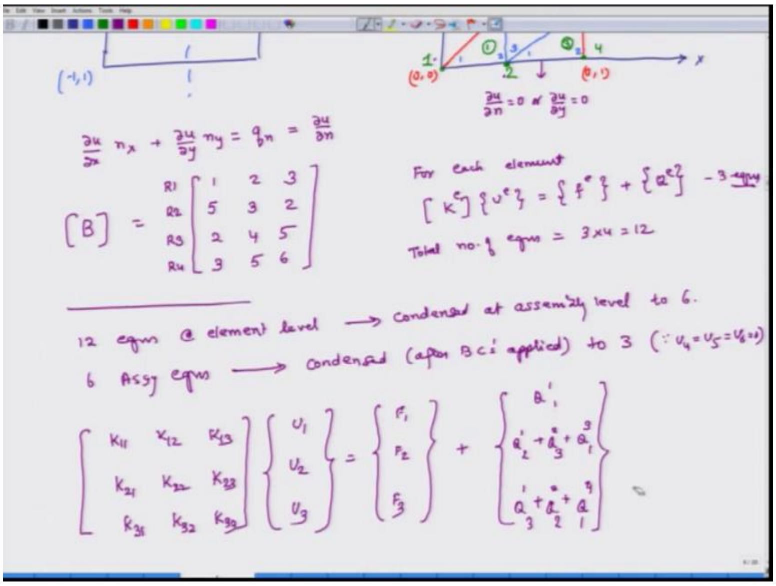

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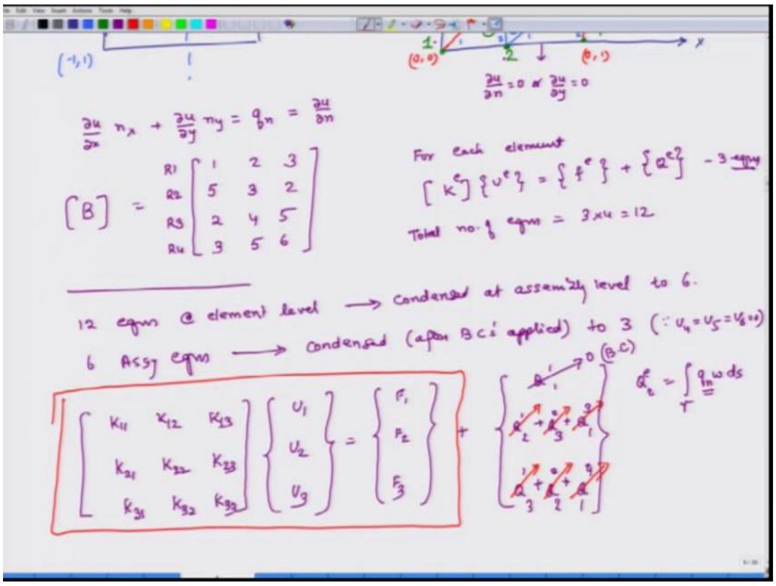

Now, what is the value of q 1 1. Q 1 1 is again actually I should have labled these as capital Qs. We know that QI for the eth element is what q n w d s integrated over the domain. So, q 1 1 is the value of q 1 of q n, what is the value of q n at node 1? It is 0 because del u over del n is 0 at right. So, this is 0 because of BC.

So, we have to know what about q 2 1? There all 0, individually because of the boundary condition. Same thing is true in this case. So, because everything comes to 0, we do not worry about this and we solve these equations and we get our solutions. So, what we have seen in this class and also in the last classes that we can exploit this problem of symmetry of the feature of symmetry to reduce the number of to reduce the complexity of the problem and using that we can reduce the size of the problem and make our computation faster.

So, whenever you encounter a problem which is symmetric in nature, and by symmetry I imply four things the geometry should be symmetric about the reference frame. So, if in a if in your f a code the reference frame is not located at the point of symmetry you have to shift here, reference frame to the point of symmetry only then you can app apply the condition of symmetry. So, about their point of plane of symmetry, is geometry is symmetric, material property is a symmetric, boundary conditions as symmetric, and the forcing term as symmetric is symmetric. Then you can use that feature to reduce the size of a problem. So, that concludes our discussion for today and tomorrow we will start a new topic which is related to numerical integration.

Thank you very much and have a great day bye.