Basics of Finite Element Analysis – Part II
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Lecture – 38
Assembly of 2 – D finite elements (Part – II)

We will assemble these equations and before we assemble these equations because, I do not want to go back and forth I will make this diagram one more time.

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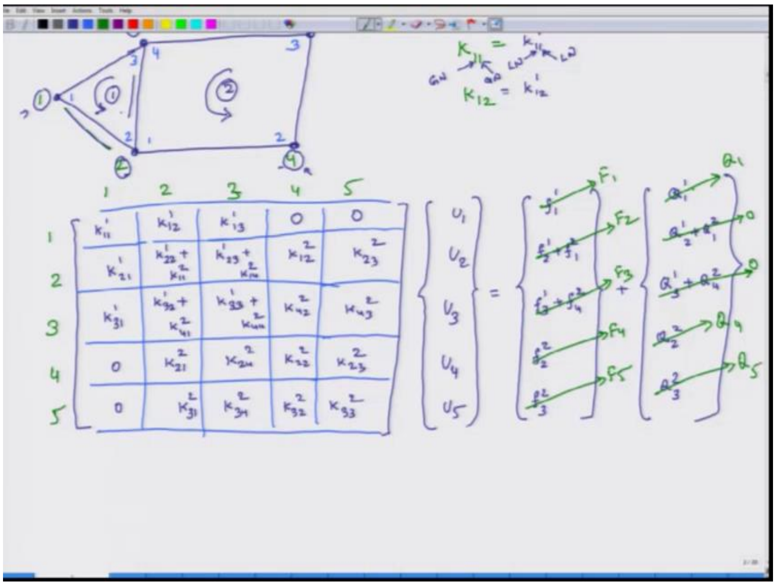

So, when I do the assembly how many equations will I get say 5 equations 7 equations will get compressed to 5 equations. So, I will have a key matrix which will be 5 by 5. So, let us say this is the box, we have to fill it in and my global displacement vector is u 1, what is the second displacement u two-third 1 u 3 u 4 and u 5. So, using connectivity matrix I can program the computer to automatically do the assembly, but now, when we are developing this connectivity matrix by hand we will learn slightly different trick which will give us physical inside into how to get assemble terms of a stiffness matrix.

So, this is global node 1 3 4 5 5. So, rows and columns correspond to 5 different nodes and these are global node numbers 1 2 3 4 5 now let us look at k 1 1, this is the global a stiffness term this equals. So, a k 1 1 global node 1 is connected to what elements is connected to only the first element. So, it will get contribution only from first element. So, that is equal to k 1 1 from the first element ok

Let us look at global node 2. So, k 1 2, this is equal to k 1 1 1 now look at k 1 2 k 1 2 is the line. So, 1 2 is this line right 1 2 is this line. So, k 1 to is the stiffness associated between global node 1 and global node 2 it is the cross stiffness cross stiffness both these. So, in this thing element 2 is not playing a role in this case. So, k 1 2 is equal to k 1 2 1. So, this is global node this is local node this is global node this subscript is local node.

It just happens this in this case in k 1 2 global node and local node are same numbers now let us look at k 1 3 again, global node and local node numbers are same and the contribution will come only from first element. So, it will be k 1 3 1 understood now let us look at k 1 4 for the assembly matrix this is my global node number and this is again global node this is capital this big K which is the assemble thing node 1 this node and node 4 are they physically connected they are separated by these elements, they are not physically connected.

So, this term will be 0, if there are 2 nodes belonging to 2 different elements then, they are not physically connected then the stiffness for them will be 0 cross stiffness now let us look at k 1 5 this is again global node and this is again global node is node 1 and node 5. So, this will be 0 understood. Now let us look at something more interesting. So, now, we want to do k k 2 1 for the assembly matrix this is global node and this is again global node. So, this is again this corresponds to this line and the role of second element is not here. So, I will write it has k 2 1 1.

Now, look at k 2 2 this is global node and this is also global node for because. So, k 2 2 it will get a contribution from first element and it will also get because when, we look at it this global node 2 is being shared by element 1 and element 2. So, it will get contributions from 2 elements. So, it will have 2 terms it will have 2 terms, what are the 2 terms the first term will come from the first element which is equal to k 2 1 1 plus k 1 2 2. So, you will get k 2 2 1 plus k 1 1 2 understood. Now let us look at 2 3 k 2 3. So, k 2 3 corresponds to this line. So, this 2 3 is this 2 3 is these are global nodes again this line 2 3 line 2 3 you get contributions from, these are again global nodes I will write them in green you get contributions from both the elements.

So, k 2 3 is equal to k. So, contribution from the first element which will be, first element local nodes are 2 and 3 1 and in the second element line 2 3 is same as line 1 4. So, it is k

1 4 2 understood next 1 k 2 4. So, k 2 4 is k 2 and 4 here, the contribution is only from the second element. So, it is k 1 2 2 and then k 2 5 this is contribution comes only from the second element. So, it is k 2 3 2. Let us look at the third row. So, I get k 3 1 1. So, k k 3 1 is and then k 3 2 is a line which is common k 3 2 3 and 2. So, this line is common 2 nodes 1 and 2. So, I will get contributions from both and this will be k 3 2 1 from the first element plus k 4 1 2 from the second element and then finally, know not then I have k 3 3 which is related to the third global node itself.

Here I am getting contributions from again 2 elements. So, here I will get k 3 3 1 plus k 4 4 1 and then k 3 4 is k 4 2 2 k 4 3 2 is k 3 5 k 4 1 node 4 and node 5 they belong to different elements and not connected at all. So, this is 0 this is k 2 1 2 k 2 4 2 k 2 2 k 2 3 2 and then this is again 0 k 3 1 2 k 3 4 2 k 3 2 2 and k 3 there 2. So, this is how I develop the assembly for their stiffness matrix, if I have to do by hand and I can use the de matrix if I am; if I want to use my computer to program it, but if I have to use hand for this exam this course you may be ask to develop a stiffness matrix for 2 or 3 elements touching each other.

In that case using this thinking you can develop is specific terms. So, so this is my key matrix and that equals the force vector. So, this is f 1 1 this is f 2 1 plus f one-third 1 is f 3 1 plus f 4 2 fourth 1 is f 2 2 and fifth 1 is f 3 2 right and then, the q vector q vector will be q 1 1 q 2 1 plus q 1 2 q 3 1 plus q 4 2 q 2 2 and q 3 2 now. So, this excuse me this is same as f 1 this is same as f 2 this is same as f 3 this is same as f 4 this is same as f 5 and in last couple of classes we had said that the qs when, they are added up right because of a common boundary their sum becomes when we add up qs because, these are boundary terms these are boundary terms and when we add up boundary terms from adjacent elements their sum becomes 0s. So, this will be 0.

So, I do not have to compute even this q 1 2 and q 2 2 right this will again be 0 because of equilibrium force balance and these qs and this will be this is global q 4 and this is q 5. So, so If I have a 2, if I have just 2 elements total number of unknowns is u 1 u 2 u 3 u 4 u 5 and then what q 1 q 4 and q 5; 8 things unknowns are there. So, I have to know conditions on the boundary. Either I know, qs or I know three uses then I can solve these equations. So, that is what it boils down to, out of these eight unknowns 5 use and 3 qs either I know I have to no some 3 of these values through boundary conditions.

Once I know those then, I can eliminate in specific equations and I can solve further remaining unknowns. So, this is how it works out will do one more example and we will actually right down stiffness terms for another example.

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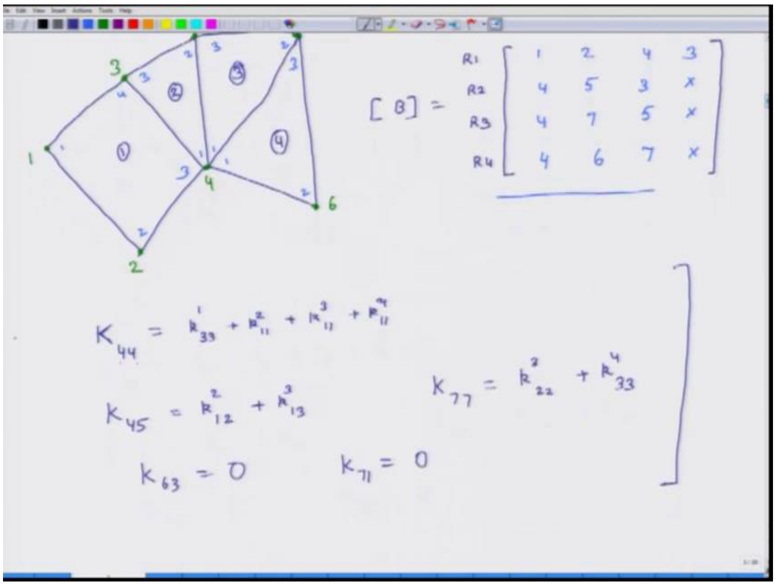

So, let us looks at another set of elements. So, this is element 1 2 3 and 4 and the local nodes are 1 2 3 4, 1 2 3 1, 2 3 1 2 3 and the global nodes are 1 2 3 4 5 6 7. So, these are seven. So, the first question is that if, I have to develop a b matrix for this how many rows this matrix is going to have if I have to develop a b matrix it will have 4 rows because they are total number of 4 elements.

So, it will have row 1, row 2, row 3, and row 4 and how many columns it has going to have 4 because a maximum number of elements in maximum number of nodes in any element is 4 there as some elements which a 3 nodes there some elements which a 4 nodes that maximum number of nodes is 4. So, it will have. So, in this case it will be a 4 by 4 matrixes, but the b b matrix can be of any order, can be of any order now with that understanding. So, this is corresponding to first local node second local node third local node and fourth local node.

Now, what are the entries in the first row 1 what are the entries it will be 1, 2, 4 and 3 agreed because the first local node is same as it its number is 1 global number is 1 second local nodes global number is two-third local nodes global number is 4 and fourth local nodes global number is 3 understood second, 1 is 4. So, this is my second element.

So, it will be 4 5 and 3 and there will be x here, third row it will be 4 7 and 5 another x here and the last one will be 4 6 and 7 x here understood.

So, with this we will just write down some specific stiffness matrix terns. So, suppose we will do 2 or 3 examples and then, we will move on suppose I have to write down what is the value of global. So, this is my global 4, 4 what this is equal to. So, first thing is how many terms it will have this Globa. So, this is global node number 4 and this is also global node number 4 is being shared by 4 different elements. So, it will have contributions from 4 different elements. So, it will have 4 different individual terms. So, the first contribution from first element will be 3, 3, 1.

From the second element from the second element what will be the contribution 1, 1 from the third element contribution will be 1, 1 from the fourth element contribution will be 1, 1 understood we will do 1 or 2 more examples. So, that we become comfortable because then when, we look at any complex mesh you can very quickly add up just by looking at the picture k 4 5 k 4 5 is what do we see here k 4 5. So, 4 5 is a line and that is being shared by elements 2 and 3. So, it will have how many terms 2 terms. So, the first contribution will come from element 2 and from element 2, it will be 1 2 from the second element what is the second element number 3 the term will be 1 3 understood.

One more example and then, we will move on k 7 7 k 7 7. So, this seventh node global node is being shared by 2 elements element 3 and element 4. So, it will have a contribution from third element plus a contribution from fourth element the contribution from third element will be 2 2 and contribution from fourth element will be 3 3, last thing what is k 6 3 sixth node and third node they, do not belong to the same element. So, they are not because, they are physically connected this number is going to be 0 k 7 1 same thing it will be 0.

So, using this third process by looking at the picture and how things are connected you can develop expressions for k matrix, as well as f and q matrix vectors and if you have to equations by hand using this approach you can write down the equations very quickly, but it is important that. So, while you are doing it is important that you clearly label out global nodes and local nodes very clearly and do not get confused, between those to otherwise you may end up writing in correct expressions.

So, this completes our lecture for day will continue this discussion tomorrow, which will be the third day and what we will do is we will quickly solve problem for heat transfer and once that is one we will move on to some other topics.

Thank you very much and have a great day bye.