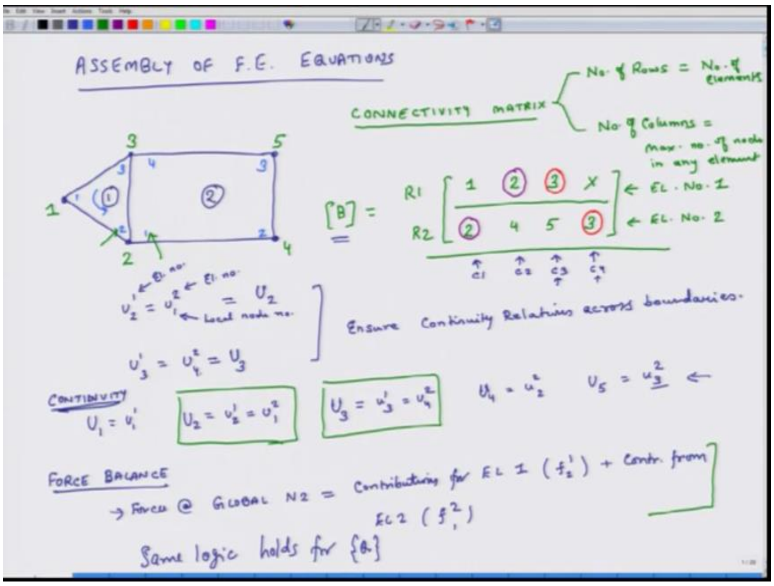
Basic of Finite Element Analysis – Part II
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 37
Assemble of 2- D finite elements
(Part-I)

Hello. Welcome to Basics of Finite Element Analysis Part II, this is the seventh week of this course and in this course, what we are doing is essentially an extension of what we did in the first course, which was part one. So, this week what we plan to do is continue that discussion of single variable 2 dimensional problems and specifically what we will learn in this week is, how we go around assembling the element level equations, that we can build a unified overall system of equations for the entire problem.

So, that is one thing we will do and the other thing we will touch upon is what do we do? Numerical integration in 2 dimensions in one dimension, we had seen that we can do numerical integration using methods like Gaussian codration and the we will use similar concept or same concept, but we will extend that understanding to 2 dimensions once we have that then extension to three dimension or three dimensions problems is not difficult at all.

(Refer Slide Time: 01:31)



So, we will start with assembly. Assembly of finite element equations let us consider 2 elements. So, let us say this is element 1 this is element 2 and. So, the first element is a first order triangular element and the second element is a 4 nodded quadrilateral in this case, it looks like a rectangle and what I am going to do is I will specify element specific node numbers in light blue colors. So, for element 1 my node numbers are one 2 and three and for element 2 it is 1, 2, 3 and 4.

And then I am also going to specify the global coordinates not global coordinates global node numbers that is the specified in green; so 1, 2, 3, 4 and 5, now the first thing when we do assembly for these types of problems. So, here what we have to do is what are aim is we will have one set of equation for the triangular element another set of equations for the rectangular element and we want to assemble these 2 equations together and we want to understand how do we go around doing that. So, the first thing, we do is we write down a connectivity matrix we write down a connectivity matrix. So, let us call this connectivity matrix as matrix b and what does that matrix mean, we will explain that. So, this connectivity matrix has certain numbers of rows and certain numbers columns. So, number of rows equals numbers of elements.

So, in this case we have 2 elements. So, this connectively matrix will have 2 rows and numbers columns this correspond to maximum number, maximum in any element what does it mean. So, in this triangular element, we have how many 3 elements, a 3 nodes and in rectangular element we have 4 nodes. So, how many columns we will have we will have 4 columns because that in. So, because 4 correspond to the maximum numbers nodes and any element. So, in element 2 you have larger numbers of nodes. So, we will have 4 columns.

So, now the connectivity matrix is very simply. So, it will have 2 rows row one row 2 and. So, in the first row, will call as correspond to element number one and what we will do is we will write down in the global node numbers the global node numbers of all the nodes of the triangular element. So, the first global node number corresponds. So, for we look at the first local node number which is one corresponding to that first local node number global node number is capital one or one. So, that is that goes here then we look at the second local node number of element 1 it is 2 and corresponding to that the global node number is two. So, we write down 2 then, we look at the third local low node in triangular element which is three and the corresponding global node is three the green.

And because this triangle has only three nodes, but the matrix has how many columns 4 columns. So, the fourth entry in row one is empty. So, I just up at an x, now look at the second row. So, second row corresponds to element number 2 here the first local node is number one and the corresponding global node is 2. So, we write down 2 the second local node is 2 corresponding global nodes 4. So, the second one is 4 the third local node is three oh I am sorry local node number is 3 and the corresponding global node number is 5 and the fourth local node number is 4 and the corresponding global node number is 3.

So, this compare the this connectivity matrix is set to the computer because when, we are developing finite element equation we have to figure out to how do we automate the assembly process also. So, this connectivity matrix is used by the computer to assemble all the finite element equations for different elements. So, what does this connectivity matrix tells tell us, it tells us that node 2 is common to element 1 and element 2 and it also tells us that node three is common to element 1 and element 2 it tells us something more it say that the second node in element 1 is same as the first node in element 2. It also says that the third node in element 1 is same as the fourth node in element two. So, this gives us an idea how we enforce equality of displacement or primary variables. So, so the global matrix this connectivity matrix it tells us that second node second node is common to second node of the element 1 and first node of the element 2 are same which means if, I am talking about primary variable u then in that case this is the primary variable associated with the first element which is the superscript and second node this equals u 1 of 2.

So, this tells us element number this is element number and this is local node number. So, by looking at connectivity matrix I can write down this letter this expression and this equals the global degree of freedom which is capital u times 2. Similarly it also says that for the first element the third degree of freedom which is u one three is equal to for the second element the fourth degree of freedom u 2 4 right this is C4 C3 C2 and C1 these are columns right. So, three correspond to this column 4 corresponds to this column. So, u one three is equal to u 2 4 and that equals the global degree of freedom associated with global node number three which is capital u 3.

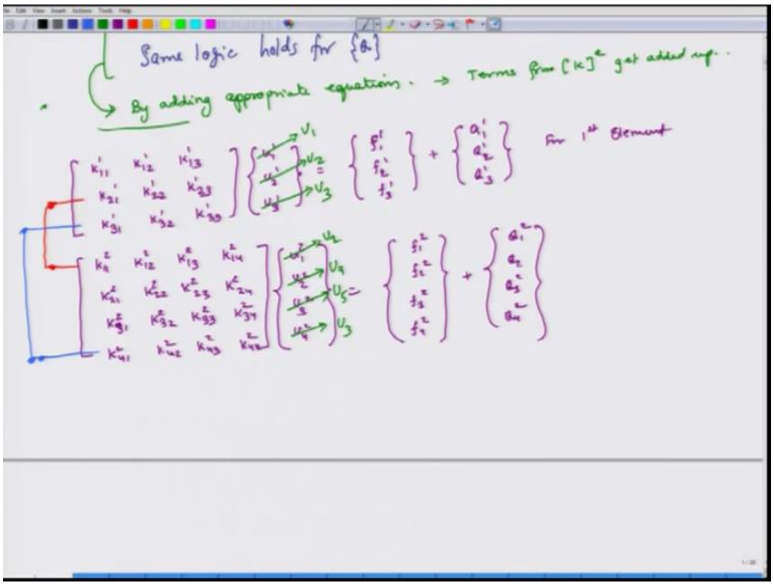
So, the movement you define a connectivity matrix from that connectivity matrix you can enforce these displacement continuity relations across the boundary of elements, where ever 2 element meets you can enforce continuity of primary variables because, we know that primary variables have to be continuous during assembly we have to ensure that so, in this way we have been ensure continuity relation across boundaries. So, what does that mean? So, now, I will write down expirations for all the global node numbers. So, what is u 1 this capital letter is the global degree of freedom which is capital u 1 this equals u 1 1 global with degree of freedom u 2 is common to first and third second element and that equals u 2 1 equals u 2, u 1 2 then u 3 equals u 3 1 equal u 4 2 then, u 4 global degree of freedom is same as u second degree of freedom for the second element and then u global degree of freedom 5 equals third degree of freedom for second element.

And all these relations can be reduced from the connectivity matrix. So, using this logic we can program develop a program in computer to ensure this continuity of primary variable across a boundary. So, this one, this about continuity the second thing when do assembly we learnt in our part one f e is that we also have to do balance of forces balance of forces. So, so what do we see the same connectivity matrix also tells us that force at global node 2 is equal to contributions from element 1 it is due to contribution from element 1 and that is due to f 2 1 plus contribution from element 2 and that is f 2 1 excuse me this f 2 1. So, this is about force. So, I can write similar expirations for all forces, force members of the force sector and same thing same logic holds for members of this q matrixes so, I can do the force balance also and then finally, what it tells us that. If I write equation for element 1 I will get how many equations just for element 3equations right because there is 3 degree of freedom in this triangle I will get 3 equation for element 2 I will get 4 equations total numbers of equations will be 7, 4 plus 3, 7 right.

So, at element level we will have 3 equations for element 1 4 equations for element 2 total number of equations will be 7. When we do assembly we will enforce this equality and we will also enforce this equality not only that, we will also add up contributions right. So, what that adding up contribution means is that I have to add up that equation associate with this degree of freedom to equation related to this degree of freedom?

So, the same global matrix tells us which equation have to be added up right because then I have to add a forces on air on the right side of the equality sign and if I have to add a forces this connectivity matrix tells us that, you add up the equation associated with second degree of freedom for element 1 to first degree freedom associated element 2 similarly it says that add up equations associated with third degree of freedom of element 1 with 4 degree of freedom associated with equation 2.

(Refer Slide Time: 17:52)



So, the using connectivity matrixes we learn how to do continuity enforcement, how to add a force I mean the force balance and this thing is done by adding up appropriate equations by adding up appropriate equation.

So, once we add up this appropriate equation terms in the k matrixes at element terms from I will say get added up terms from k matrix get added up and once that happen, we get a global k matrix and the global force sector. So, this is what we will do now? So, we will write down the equations for. So, these are the equation for which element for first element and then, we have a larger number of equations for second element and because these are associated with the second element I will put a superscript 2 on each of this.

So, the first thing we do when we are doing assemblies we replace this local degrees of freedom with global degrees of freedom. So, this became u 1 u 2 u 3 this becomes u 2 what does it become u 2 2 u 4 u 2 3 it becomes u 5 and this becomes u 3, there is the first thing we do then, we assembling the equation and again the connectivity matrixes helps us assemble them it is a guide to assemble these. So, what the connectivity matrix tells us it that you assemble this equation the second equation for element 1 second equation for element 1 which is rounded in purple in connectivity matrix with the first equation in element 2. So, we add up these 2 equations.

And we also add up the third equation in element 1 with fourth equation in element two. So, we add up third equation in element 1 with what is it fourth equation? Fourth equation we had these 2 together. So, the connectivity matrix is a very important tool and once we have connectivity matrixes. I can use this method to do assembly at for one dimensional equation 2 dimensional and also three dimensional equation same approach holds because it is a matrix which tells how thing are related to each other.

So, this completes our lecture for today will continue this discussion tomorrow.

Thank you very much and have a great day.