Basics of Finite Element Analysis – Part II
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Lecture -03
Steps followed in FEA

Welcome to Basic of Finite Element Analysis course. This is the new course part 2, is the third day of the first week of this course. And in last lecture we reviewed the basic notion as to how the basic philosophy and the line finite element analysis, where complex problem is analysed, by breaking the domain into smaller elements and then we find solution for each specific element. And in the last class we did not do this exercises in context to solving ordinary or partial differential equations, but whatever we learnt in the last class or reviewed, we found that the accuracy of this kind of an approach increases if we increase the number of elements, if we increase the nature or order of polynomial functions which are used as interpolation functions, and also once we increase the computational accuracy.

So, what we will do in today's module is actually again review what are the different steps when we do finite element analysis. So most are time, when we talk about finite element analysis, that is in context of solving partial or ordinary differential equations so it is when we have to solve these equations and if we have hard time finding a closed form solution, then we use finite element method to find approximate or numerical solutions for the same equation. And do today is first get an over view as to how we go in this direction in on a step by step basis, and then maybe in the next couple of classes we will actually do some examples.

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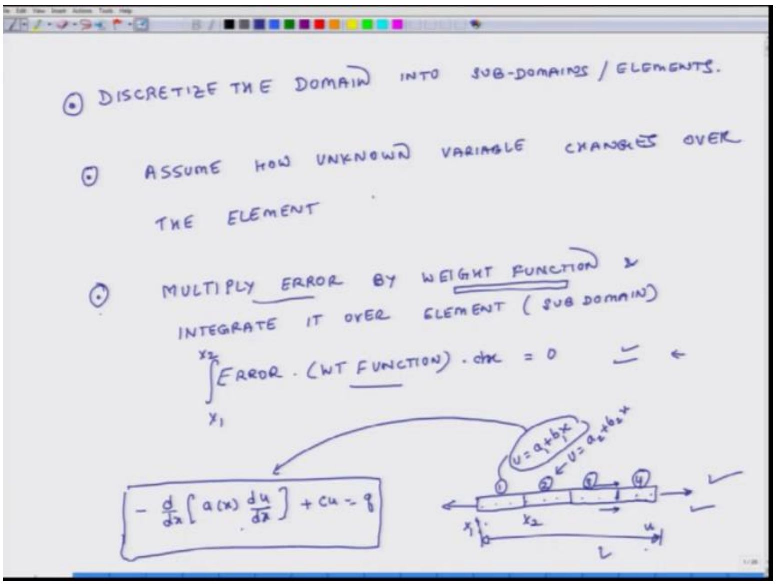

So, the first step when, so the steps in which are used as in the finite element analysis are the first step is that we discretize the domain. So we break the overall region in which we want to find the solution of the domain and we discretize the domain into sub domains. And these sub domains are called elements, so this is the first step. Second and we will learn the details of it maybe later today, is that once we broken the overall domain into sun domains, then over each element or each domain we assume how unknown variable changes over the element.

For example, suppose I have a rod, and am pulling this rod like this and I am also applying some traction on the rod in some other region. And what I am interested in finding out is the displacement in the rod at all these individual points. So my unknown is u which is the displacement in this direction in the direction of pulling of the rod. And I am interested in finding the displacement at all internal points in the rod. So the first step is the overall length of this rod is let say l, and I will break this overall domain into smaller sections. So in this case I broken it into 4 different elements. Element number 1, element number 2, element number 3 and element number 4. This is the first thing then the second step is that we assume how u varies over each element. We do not know the value of u at each point, but we assume how u varies over each element.

So, I can assume that u is constant, over element one it could be also constant. Over element 2 it could be constant, over element 3 on so on and so forth. Or I can assume that u varies linearly over element 1, 2, 3 and 4. Or I can assume that u varies quadratically over individual elements. I can also assume that u varies linearly an element 1 and 2 and it varies quadratically on element 3 and 4. That is also fine, but we assume that over each element how this unknown which is u it varies. And when we make that assumption basically we are assuming that if it is a linear thing then we assume that u is equal to a plus b x, over the length of an element right were a and b are unknown. Or if it is a quadratic variation then we assume that u is equal to a plus, b x plus c x square over each element and a b c are again unknown and they may change from one element to other element.

So, this is the second thing. The third step is that we find out the error, so in the differential equation once we make this assumption. So and we multiply this error by weight function and integrated over the domain. So and I will explain that later first let me just write it. So we multiply error by weight function and integrated over element or sub domain. What does that mean so suppose this problem is governed by an equation minus d by d x, a x, d u over d x is equal to q, both that is problem. Or to make it a little more general I will say plus c u is equal to q.

So, this is the governing differential equation and if we can solve this equation in context of this particular problem when we will have the solution, but suppose the solution is not easy or it getting a close from solution is not easy, then what do we do first step is that we broke this entire domain into 4 elements. Second is we assume that in first element let us say assume, to that first element u is equal to a plus b x, here also a 1 plus b 1 x in this case u is equal to a 2 plus b 2 x and so on and so forth.

Then the third step is we plug this, in this equation. And once we plug it if the solution is really true exact the left hand and the right hand side will be same, but because this is an approximate solution the left hand in the right side will not be same which means that there will be a difference in the values of lefts side and right side and that difference is called error. That difference is called the error, so that is the error we calculate and we multiply it by some weight function.

So that error so mathematically we will say that that error then we multiply by weight function and we will discuss what is the weight function later, so we multiply that by weight function and then we multiply and then we integrate it over the domain of the element. So this let say this here x 1, this is x 2 so I am integrating it over x 1 to x 2, and this I equated to 0. And the reason why we do it in all that stuff we have explained it in basics of finite element part one. So if you have doubt I will definitely encourage you to go back and review what are we talk about at the last class.

So, this is the third step and this step gives us and we will see that later it gives us good large number of equations at each for each element. So we will have this equation for element one will have us different equation of same nature for element 2 and so on so forth. And for each element we get several equations, we get several equations. So we get element level equations. So this third step it gives us element level equations.

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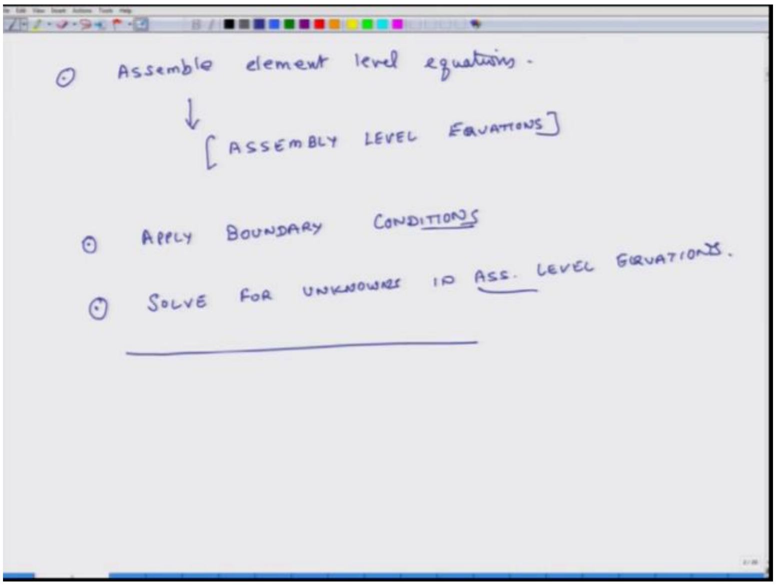

Then the fourth step is assemble element level, we assemble element level equations. And then these then what we get is assembly level equations.

Then the next step is that I apply boundary conditions. And we will again do all this through an example so we apply boundary conditions. So for instance in this problem if the boundary condition would be that at x equal to 0, there is the force on the left side this is the one boundary condition and at x is equal to l there is the force tensile force which is being applied at the and which is getting inserted in the right side, or in the positive x side those are the 2 boundary conditions. We can apply those boundary conditions and once we have applied the boundary condition then we solve for unknowns, in the equations in assembly level equations and this is how we get the solution, there is how we get the solution for the overall thing.

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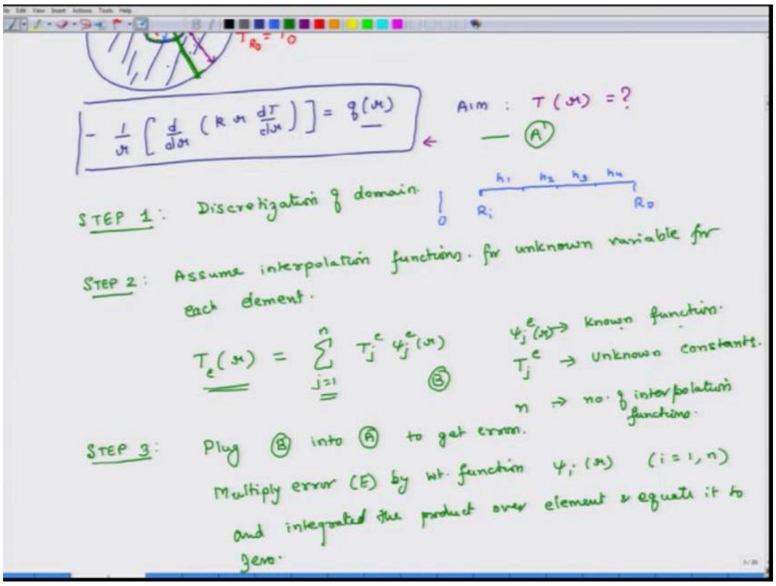

So, what we will do is, we will again the next 10 15 minutes, will illustrate this methodology, will go through an example. So suppose I have a hollow cylinder and so this is all material and suppose since this hollow cylinder heat is flowing. How could heat be flowing? Suppose, this is the metallic hollow cylinder and I am sending current though this, and is a long cylinder so this current times i square time the resistance of the cylinder will generate heat at every point in the cylinder so heat is getting generative.

And what I am doing is for using some method at, so this is outside radius, this is inside radius r i and at some other location, which on the radius is r. Which varies r, this is an access symmetric problem. And what I am also saying is that on the outside radius t at r 0 is equal to t nought. So suppose I am immersing the cylinder in say ice. So the outside temperature is maintain that 0 degree, centigrade or I am immersing it in boiling water outside temperature is maintained at 100 degrees centigrade, this is one thing. And the inside radius what I do is I put some thick layer of an insulator some perfect insulator. Which means so what are the boundary conditions in this case? Boundary conditions that t at r o is equal to t nought and no heat transfer at r i.

Now, the governing differential equation for this kind of a system is something like this. Minus 1 over r, d by d r, k r, d t over d r is equal to q r. Here k represents the thermal conductivity of the material, t is the temperature at any point in the medium, r represents radius and because it accesses symmetric problem for same value of r temperature will be same and q is the amount of heat which is getting generated, so rate of energetic radiant in the medium that is q.

So to find r, so our aim what is the aim, aim is to find r for different values of r between so, this is the entire range of r, r is starting from r i to r o, at different values of r, what is the value of temperature so we want to find temperature as a function of r, this is our goal and we can solve this problem by solving this equation and applying boundary conditions. Now what we will do is we will generate a finite element formulation for this method. So this is an access symmetric problem, so what I will do is, I will look at only radial line. So the first step is what we discuss was that we break the entire domain into small elements.

So, step one, discretisation of domain. So my domain is this r i r o. Let us say this is my centre origin; so the overall length of the domain is r o minus r i and I can break it up into smaller elements. So this first element is h 1 long, second element is h 2 long, third element is h 3, long h 4 long. And if I want I can break it up into whatever number of elements I want, so this is the first step. The second step, is step 2, assume interpolation functions or approximation functions for the variable for each element.

So, what is the variable in this unknown variable in this problem, t. So for each element we assume how t is going to vary. So we will say that t, it is the function of r and for the eth element I will call it t e, for the eth element I will call it actually subscript t. So temperature is varying as a function of r for eth element, to the value of e will be 1 for first element, it will be 2 for second element, it will be 3 for third element; so t e is equal to now am assuming a function interpolation function. So I am saying this is equal to t j, psi j, r e and j is equal to 1 to n. Now here psi j e is a known function. It is a known function. An example of this function could be 1. It is a constant right psi j is the known function, another example of this function could be a plus b x. It is a known function, but here a and b they are unknown constants.

So, this a known function and t j e these are unknown constants. So suppose the overall known function is a plus b x. Then in this case it is some of 2 is smaller functions. First function is a and the second function is b x, right. In a is unknown, that is same as t j and a times 1, so 1 is the known function which is psi j. The second function is b x, there b is unknown and so that is t j, that represents t j and psi j is x which is the known function. So that is there, so in that in this case it would not be a plus b expect maybe a plus b r, because the variable is r.

So, I am saying that temperature over element e is a sum of products of t js, and psi js, psi js are known function, t j is an unknown constant. And I will have, I can some several of these, so n is what is the number of interpolation functions. So let us called this governing equation as equation a and let us call this equation as equation b. So the third step is step 3, step 3, is plug b into a. So plug this equation b into a, and what will be get because this is an approximation of the thing, so we will get an error.

Plug b into a to get error and then the other part of the step 3 is multiply error I call it e, by some weight function. Now how do we choose weight function, we will see that later and we have also explain that in f a 1, so we multiplied by weight function and here we will just take weight function as, another functions psi I and again it is a function of r, weight function r. And the value of I could vary from one to n. So multiply error by weight functions psi i and integrate the product over element and equate it to 0.

So, what are we doing? We are what essentially in this step we are doing is, we are calculating the error, assigning a weight to the error because we are multiplying it by a weight function, and integrating this overall multiple of weight function over an error over the domain and equated to 0. So in this sense the weighted integral of the error is 0. So that is why this is the weighted integral that is dual approach, so weighted that is dual approach, in finite element analysis. So error is not 0 at ever point, but over the entire domain if I integrate the error, then it is in a 0 in a weighted sense, into 0 in a weighted sense, the size of the element essentially as I keep my element size smaller and smaller.

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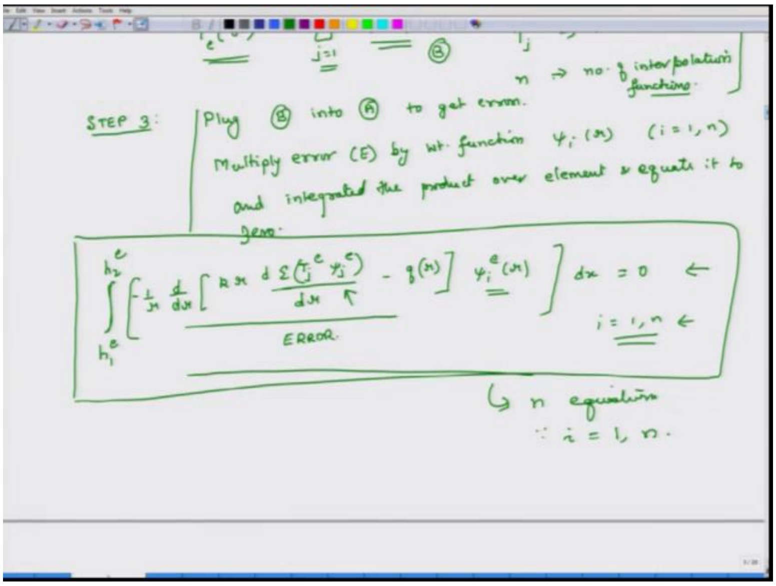

Essentially when I do this step, I go towards a point that error is approaching 0 at every point, because over the entire domain it is between 0. And as I in string the size of the element that element is increasingly becoming more and more like a point.

So, the error is between 0 at every point over, but again in an approximate sense. So when I do this what is the equation I get, so this is what I get, minus 1 by r, d by d r, k r d minus; so this is the error, right this is the error. And what am I doing? I am multiplying this error; see here what have I done. I have substituted u by t j psi j, some of t j psi j, so this is the error. And I am multiplying this error by your weight functions psi i for the eth element and this is also a function of r. And this entire thing is getting integrated over the equating it to 0, and what are the limits of integration, so this this equation is for eth element.

So, for eth element the first, starting of eth element is, h 1 e and the ending of eth element is h 2 e. These are the coordinates, so for each the eth element this is there, now we said that i is equal to 1 to n and n is the number of interpolation functions. So if I have so just look at this thing here, n is the number of interpolation functions right, how many unknowns we have integer in in this functions. So I all s I js are known or n known there known, t js are n known, how many t js are there n. So total number of unknowns is total number of unknowns is n. Because t js are n in number so total number of n known is n, now in this equation i can assume how many values i is equal to 1 to n. So first I multiply the error with psi 1, I get one equation. Then I multiply this error with psi 2 an integrated I get second equation.

So, total number of equations I will get at element level will be n because i is equal to 1 to n and total number of t js which are unknown a n. So the total number of t js is n and total number of equations which we generate at element level is n. So this is not one equation, this represents n equations, because i equal 1 to n. This is not one equation this represents n equations, because i equal 1 to n. Not because j is going from 1 to n, because j is being summed. So this term does not change, it is the psi I it changes as I changes. So this represents n equations and there are n variables at element level. So at least at the element level we have the number of equation and number of unknowns is same.

So, this is step three, so what we will do is we have done these 3 steps, what we will do is next is we the way we have explained is that step 4, is we assemble all these equations. And in the step following that is, that once we have done the assembly. Then we apply the boundary conditions, and once we have applied the boundary condition, then we solve for these unknowns which are t js and that is how we get the solution.

So this completes our lecture for today and in the next lecture will develop this discussion even further. And we will also review this whole notion of week formulation in that context. So with this we will close for today and will continue this discussion tomorrow.

Thank you.