Basics of Finite Element Analysis – Part II
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Lecture – 29
Interpolation Functions for Linear Triangular Elements (Part – I)

Hello. Welcome to Basics of Finite Element Analysis Part II. Today is the fifth day of this particular week which is the fifth week and what we are going to discuss today is the method of developing interpolation functions for a linear triangular element.

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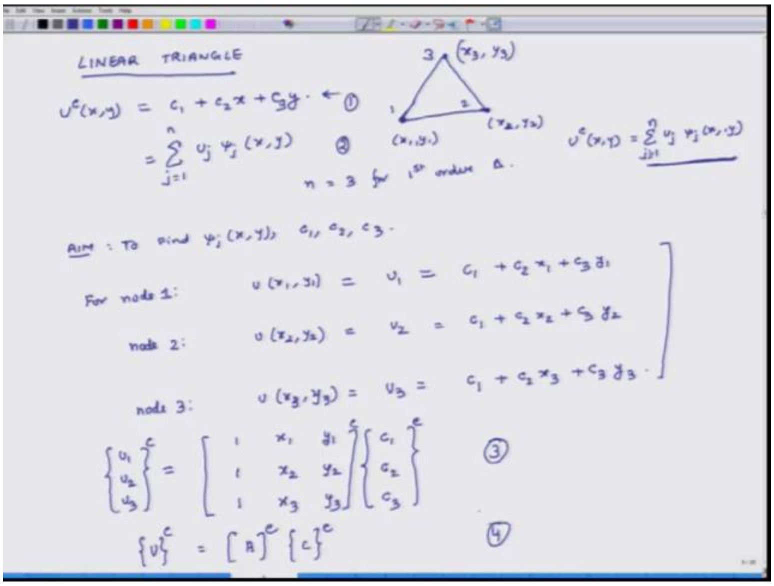

So, that is what we will do linear triangle. So, a linear triangle we are discussed. It will have 3 nodes; node 1, 2 and 3. The coordinates are x 1, y 1; x 2 y 2. So, the locations of these nodes are known. So, that is from actual geometry x 3, y 3 and what we are interested in is figuring out how does U e, which is the function of x and y it varies over this element and if it is a linear element and triangular element then it will have 3 unknown constants.

So, U e will equal C 1 plus C 2 x plus C 3 y. Now earlier in our FEA formulation we had said that, we had expressed U e x y as summation of U j times Psi j right and j is equal to 1 to n. In this case n is, what is the value of n in this case for a linear triangle? 3 for a linear triangle n is 3, n is the number of nodes. So, this is how U varies over the area of the element, but alternatively we had also expressed U in this form. So, an alternate expression for the same equation is U j Psi j x and y and j is equal to 1 to 3, I will still right it as n and n is equal to 3 for first order triangle.

So what our aim is to find Psi j. These are the functions we have to find. I can write U e as C 1 plus C 2 x plus C 3 y. I can alternatively write it as U j times Psi j sum of that and so on so forth. But our aim is to find Psi js and also C 1, C 2, C 3. Now what is the value? So, for node 1 U of x 1 y 1 and node 1 what will be the value of U? It will be U at x 1 y 1 because the coordinates are and that equals. So, I will call this relation 1, I will call this relation 2. If I use relation 2 what will be the value of U at node 1?

What will be the value of Psi j at node 1, Psi 1 at node 1; Psi 2 at node 1 0, Psi 3 at node 1 0. So, the value of U at node 1 will be U 1 and if I use equation 1, then what will be the value of U at node 1; C 1 plus C 2 x 1 plus C 3 y 1; similarly, for node 2, U equal x 2 y 2. So, once again for node 2, the co ordinates of node 2 are x 2 y 2. If I use equation 2, what is the value of U at node 2? U 2, because the value of Psi 1 is 0 at node 2, Psi 2 is 1 and Psi 3 is 0.

So, I get only U 2 and if I use equation 1 then I get relation C 1 plus C 2 x 2 plus C 3 y 2 and finally, for node 3 U at third node equals U 3 excuse me, this equals U 3 using equation 2 and if I use equation 1 I get C 1 plus C 2 x 3 plus C 3 or I can express these 3 equations in matrix form, U 1, U 2, U 3 equals 1 x 1 y 1, 1 x 2 y 2, 1 x 3 y 3, C 1, C 2 C 3. So, let us call this equation 3 or I can also in brief, I can write it as U e. So, this is for the eth element is equal to some matrix a for eth element.

So, this 3 by 3 matrix I call it matrix A times a C vector for eth element. Now matrix A for eth element is, is it known or unknown? It is known because it is a triangle we have discreatized the domain and we have broken it up into lot of triangles. So, for each triangle I know the coordinates of all 3 nodes. So, we know matrix A. What we are interested in knowing is the values of C because Cs are unknown. So, for we said that we are interested in finding the values of Cs and we also want figure out the expression of Psi j. So, first we will worry about finding Cs from Cs then we will develop expression for Psi j.

So, we want to develop an expression for Cs, in terms of U and A that is the first step. When you look at equations 3 or 4, the solution for Cs will exist; if all these 3 equations in equation 3 are having 3 equations, if these 3 equations are mutually independent.

What that means, is that the rows, the first row, the second row, the third row they should be different, they should not be linear combinations of each other. If they are linear combinations when you go out your matrix knowledge, if they are linear combinations of each other then you cannot invert this A matrix.

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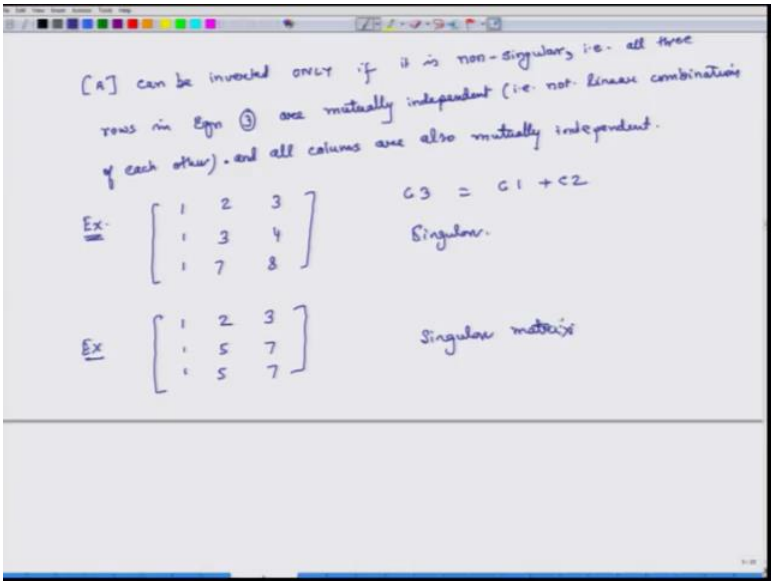

Similarly, the columns the first columns, the second column, the third column they should also not be linear combinations of each other. If you can if they are linear combination then you cannot invert it correctly.

So, matrix A only can be inverted, if it is non singular; that is all 3 rows in equation C are mutually independent, that is not linear combinations of each other and all columns are also mutually independent. So, rows should be mutually independent and columns should be also mutually independent. What does that mean? Example, suppose matrix A is 1 1 1 2 3 7, equation 3. So, this is one example 1 2 3 1 3 4 1 7 8 are these linearly independent equations? So here column 3 equals. So, this is a singular matrix column 1 plus column 2. If it is singular matrix when I try to invert it is determine will be 0. So, I will not get a good valid solution or I will not get a solution.

Another example 1 1 1 2 3 5 7 5 7 here 2 rows are mutually dependent they are not. So, this is again a singular matrix. So, we should not have this condition. If we have this condition then we will have problem inverting the matrix.

So, this concludes our discussion for today. We will continue this discussion in the next class, and we will then cover and actually develop the interpolation functions for this system.

Thanks, and look forward to seeing you tomorrow.