Basics of Finite Element Analysis – Part II
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Lecture – 28
Interpolation Functions for 2D Finite Element Problems

Hello. Welcome to Basics of Finite Element Analysis Part II. This is the fifth week of this course and today is the fourth day of the week. So, we will continue to develop the finite element formulation for the two dimensional single variable problem and today and may be in the next class, what we will focus upon is the method to develop interpolation functions for this problem formulation.

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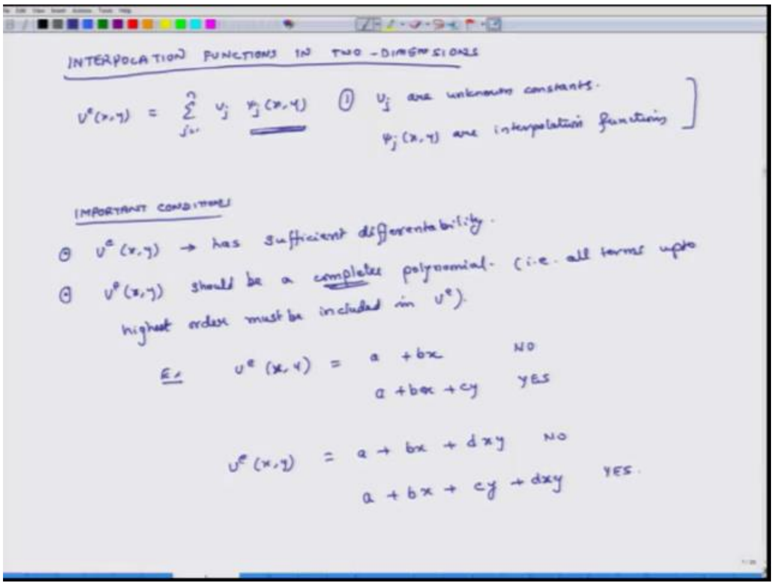

So, we will work on figuring out, how to develop interpolation functions in 2 D.

Now, what does that mean? So, we have said that the displacement or the variable U we can approximate it as U j Psi j x and y here j could be 1 to n and here U j are unknown constants Psi j are interpolation functions for jth node. So, the interpolation function as Psi represents the function for jth node. So, what we are interested in figuring out is what are the nature of this Psi j, because U j is the constant. So, we do not have to worry about it. What is the nature of Psi j? How do we figure that out? As we are excuse me.

So, let us call this equation 1; as we are developing these functions, we have to make sure that some important conditions are met and we had discussed these conditions also when we were doing are 1 D formulation. So, the first thing is so important conditions. So, the first one is, that whenever we pick up Psi j we have to ensure that we pick up Psi j in a such way that U of x y, which is the actual unknown variable has sufficient differentiability and it should have sufficient differentiability.

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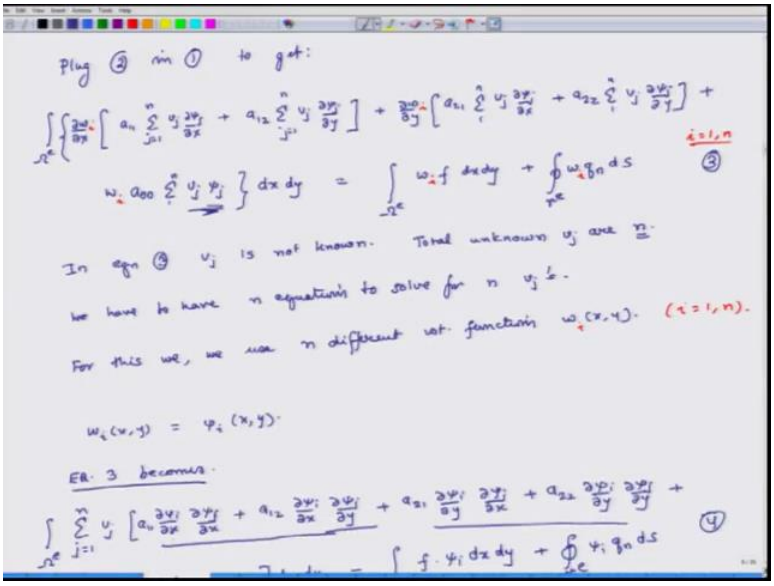

So, what does that mean, because, in this formulation in the weak form differentiability requirement is of the first order. It should be sufficiently differentiable at least once.

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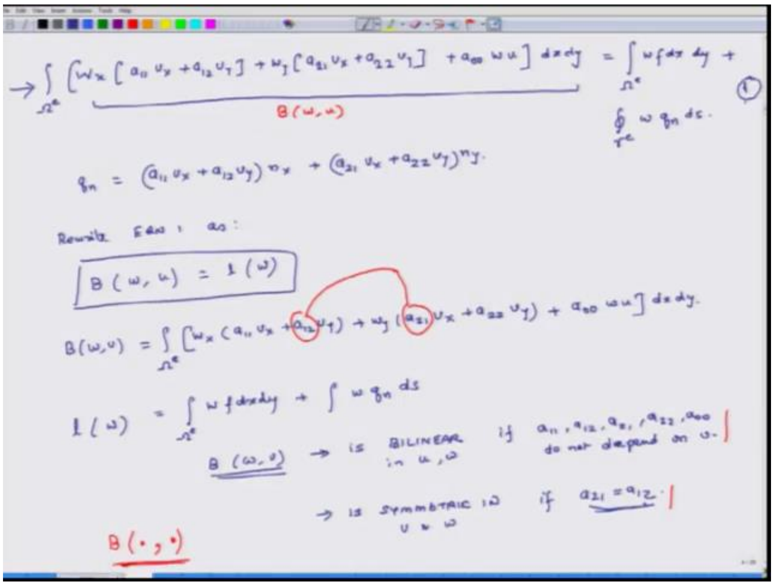

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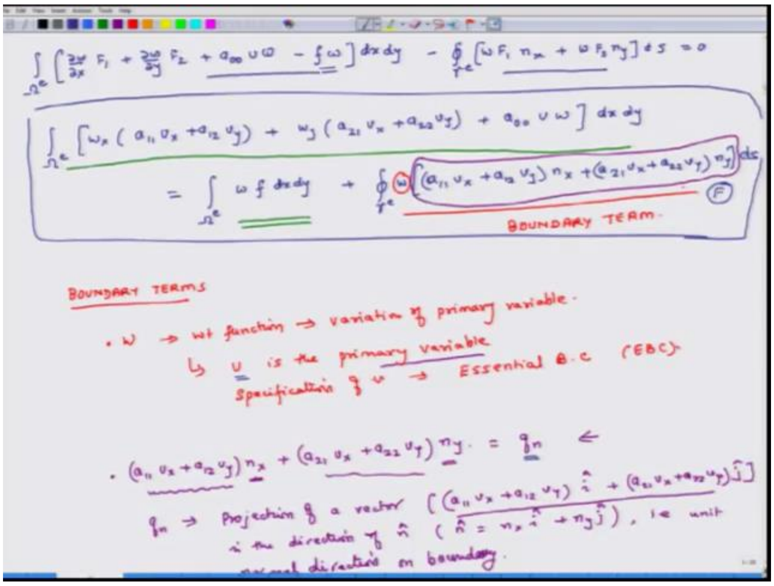

If we had used the strong form, which should be this form, then the differentiability would be of second order. So, we have to take Psi j in such a way that U is sufficiently differentiable this is important. The second thing is that the Psi j there will be polynomial function right. So, we have to pick up Psi j in such a way that U e of x y should be a complete polynomial. That is all terms up to highest order must be included in U e.

So, what does that mean? So, U e would be a polynomial function; example, U e x y is equal to a plus b x, is it a complete? So, this is a function of x and y. is it a complete function? No. Why? Because I have not included the y x terminate the y terminate. So, a plus b x plus C y this is a complete polynomial. Another example U e x y equals a plus b x plus d x y, is it a complete polynomial? No because I am missing the y term. So, a complete polynomial would be a plus b x plus C y plus d x y.

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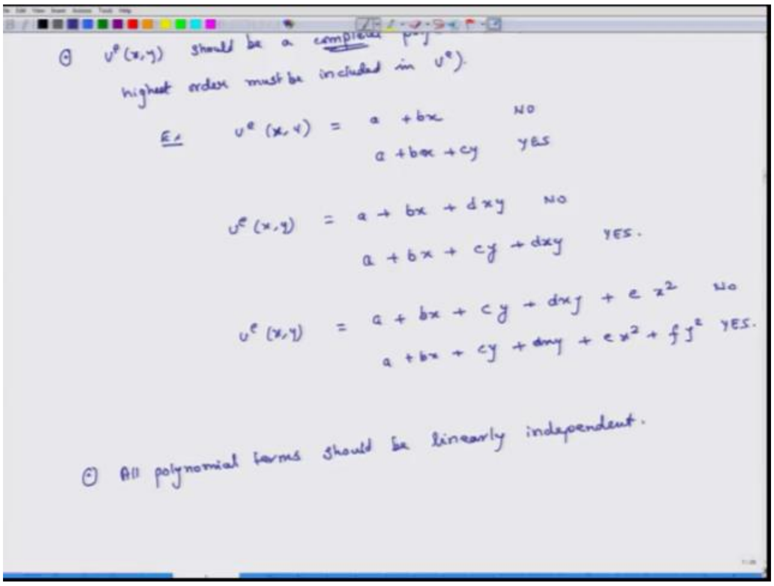

This is a complete polynomial. One-third example could be, U e x y is a plus b x plus C x oh sorry C y plus d x y plus e x square. This is not a complete polynomial, but I can make it complete polynomial. This is a complete polynomial.

So, it has to be complete polynomial otherwise, our results will be inaccurate because they will be skewed in one particular dimension strongly right. So, that is there, the third thing is that all polynomial terms should be linearly independent. So, they should be linearly independent. There should not be linear combinations of two things. So, I cannot have a plus b x plus C x because b x plus C x they are all not linearly independent. So, that is important thing. So, with this background let us look at some functions.

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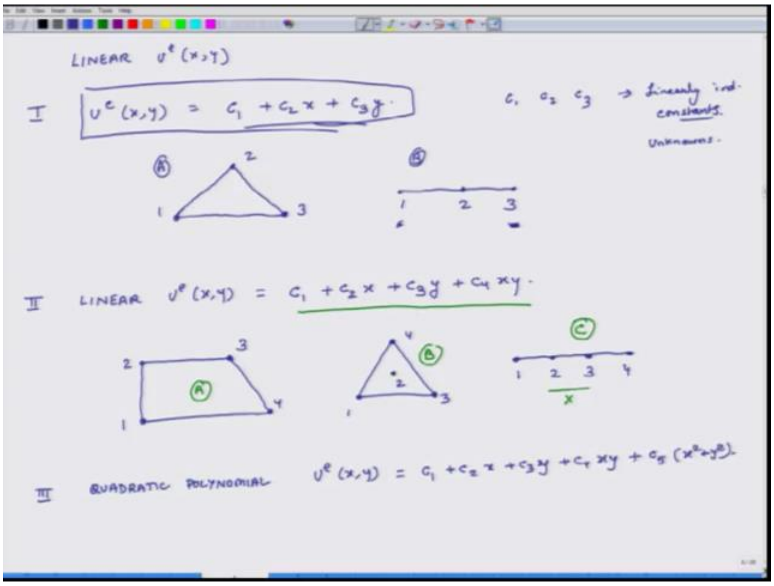

So, let us look at linear U e of x and y. So, we are still not talking about interpolation functions. We are right now on U e not on size, but from U we will go to size later. So, if U e is linear then the simplest expression would be constant C 1 plus C 2 x plus C 3 y C 1 plus C 2 x plus C 3 y.

So, here C 1 C 2 C 3 linearly independent constants and this is linear in x and y and C 1s and they are unknowns. So, we have three unknowns and we have three unknowns then we have to know find them out, we have to know conditions at three different points right. So, we have to know conditions at three different points, which means this type of an expression, could correspond to this type of an element. So, one can also say that to find these three unknowns. So, this is node 1 2 and 3.

So, this is option A, this is option B. Here I have 1 2 and 3, here also there are three points. If I know three conditions at three locations and in option A, I have conditions in three locations, but I then I can find out C 1, C 2, C 3 for case A, but I may not able to find it for case B why? Because these three conditions at 1 2 and 3, they are not linearly mutually independent because if I know condition at point, if I know condition at point 2, condition at point 3, condition at point 2 is somewhere you know an interpolation of that right.

So, it can be a combination of 3 and 1, but in case of case A condition at 2 is not a linear combination of 1 and 3. So, if I have this type of a function such that the displacement over the element is varying in a linear way then that corresponds to a triangular element with 3 nodes. Another example, so this is first case. Now let us look at second case. Linear U e and x and y and this is equal to C 1 plus C 2 x plus C 3 y plus C 4 x y. So, here I have 4 constants, 4 unknowns C 1, C 2, C 3, and C 4. To find out C 1, C 2, C 3, C 4 I should know conditions at least 4 points.

So, what could those points be? So, these 4 points could be vertices of a quadrilateral or they could be for a triangle or they could be for a line. So, once again I have case A, case B and case C. Now if we consider case C, the values of U at 2 and 3, if it is a linear variation then it is not independent of, it is not linearly independent of 1 and 4 right. So, this is not possible a valid option. The same thing is true for case B also where, 0.2 is located at the centroid of the triangle.

So, the 4th node is located at the centroid of the triangle, but here also the same thing is the logic which we used in the case C, it also applies for case B. If you do the mathematics you will find it out. So, if I have a function U e which is equal to C 1 plus C 2 x plus C 3 y plus C 4 x y, this type of variation of U can exists on a four nodded quadrilateral. Third case we consider a quadratic polynomial U e.

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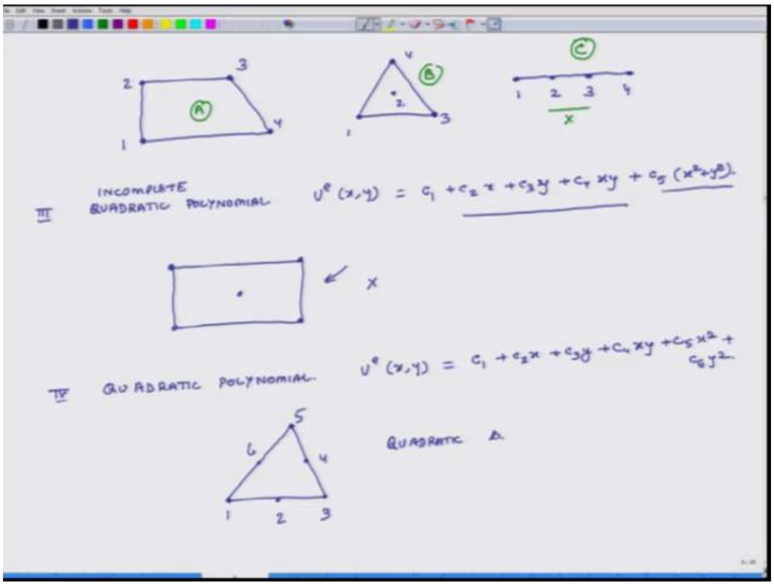

So, this is a quadratic polynomial, but we have seen where the weight on x square and y square is same. So, we have violated our own condition, that this weight should be independent of each other right. Earlier we had said all polynomial term should be linearly independent, here the weight on x square and y square is not same. So, we have violated our own condition. This type of function you would think it could exists on a square or a quadrilateral with a mid node, but when you apply this on this thing, you will again see the compatibility issues. So, this is not a valid option.

So, this is an incomplete quadratic polynomial. So, this is not a valid option. Fourth case could be a complete quadratic polynomial. So, here U e of x y equals C 1 plus C 2 x plus C 3 y plus C 4 x y plus C 5 x square plus C 6 y square. So, here there are 6 unknowns to know these 6 unknowns, we have to conditions at 6 different points right and this is they are square terms in the expression. So, one possible topology or actually the only possible topology would be this, here the variation of U from 1 2 3 is not linear, it is quadratic.

So, I need conditions one at additional points same thing is true for 3 to 5. The variation of U from 3 to 5 is not linear, but it is quadratic. So, for a quadratic expression I need, one additional data at one additional point same thing on the third edge. So, this is a quadratic triangle. The first triangle which we had discussed was a linear triangle. So, like this we can develop quadratic quadrilateral and higher order elements in this way. So, this completes our discussion for this particular lecture.

In the next lecture what we will do is? We will develop the interpolation functions for a linear triangular element and what we have discussed today are different types of elements. So, we have a linear triangular element then we have a first order linear quadrilateral, then we saw that this incomplete quadratic polynomial does not give us a valid element.

Then finally, if we use a quadratic polynomial for U, then that will work if we use a quadratic or a second order triangular element. So, this completes our discussion for today and we will continue the discussion on interpolation functions tomorrow as well.

Thank you.