Basics of Finite Element Analysis – Part II
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Lecture – 21
Gaussian Quadrature – Part III

Hello. Welcome to Basics of Finite Element Analysis Part II. Today is the third of this particular week and we will continue the topic of discussion which we were having in the last class.

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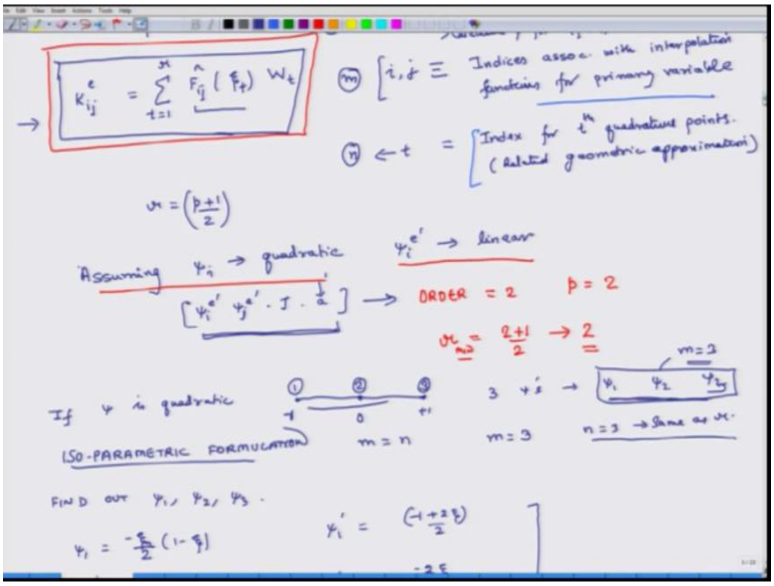

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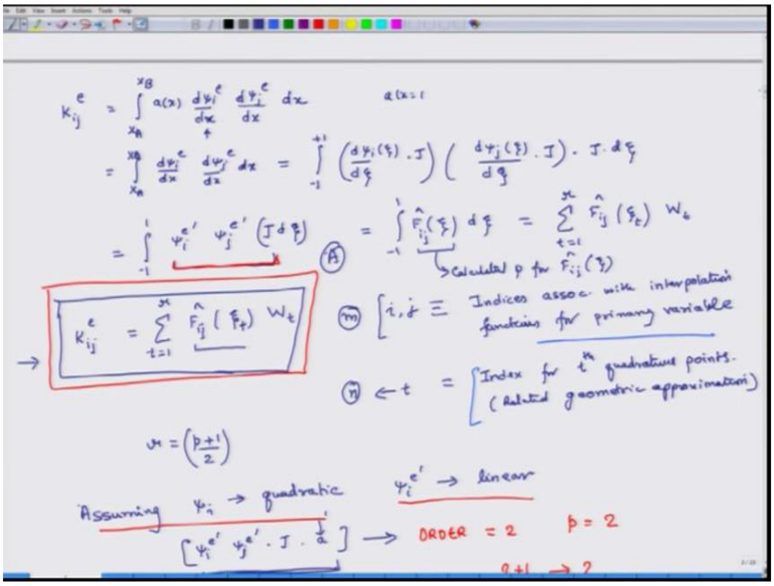

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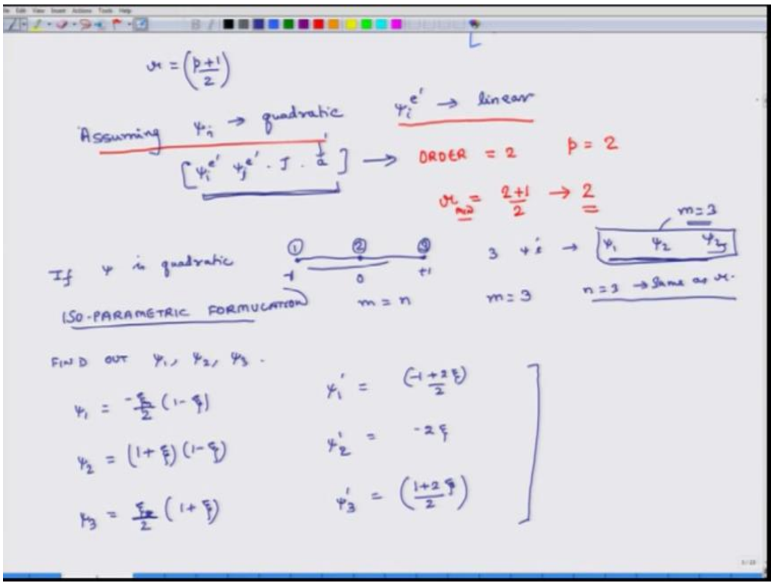

So, in the last class what we had covered was we are trying to find the value of K ij and the function F hat is defined here in equation A. So, for this type of function and we are assuming that Psi is a quadratic function. So, using these approach assumptions we developed 3 Psi functions Psi 1, Psi 2, Psi 3 and also developed expression for their derivations. So, our next step is that we have to calculate the value of F hat ok.

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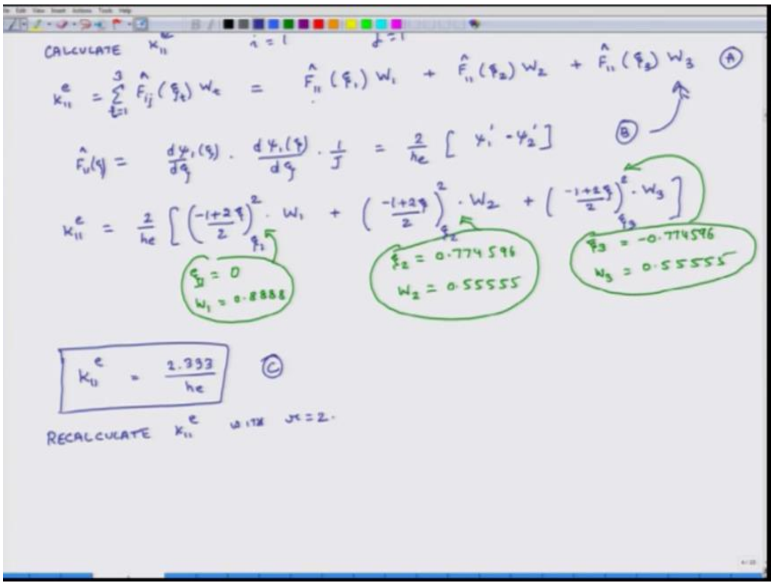

So, actually what we will do is. So, let us say that we want to calculate K 1 1 for the eth element. So, in this case i equal 1 and J equals 1. So, the expression for K 1 1 for the eth element is and its given here Kij e equals F hat i J evaluated at t-th quadrature points multiplied by quadrature weight for the t-th point and then I sum these things from t is equal to 1 2 3. So, first let us write down the expression. So, it is equal to Fij evaluated at zeta t t-th quadrature point and then I am also multiplying it by weight and t changes from 1 to 3.

I am choosing 3 because minimum acceptable value of r was 2, but here we are choosing 3 because we have wanted to do an isoperimetric formulation for isoperimetric situation. So, if I expand this I get. So, i is equal to 1 and J is equal to 1. So, it is F1 1 hat, evaluated at t is equal to 1 times W 1 plus F hat 1 1, evaluated at second quadrature points zeta 2 multiplied by W2 plus F11 hat, evaluated third quadrature point multiplied by accompanying weight. It is not a weight function, it is a weight; it is a number it is not a function.

And here we had assume that A is 1 we had assume that A axis 1. So, read let us write down the expression for F11 hat. So now, So, this is let us call this equation A and then F11 hat equals d Psi 1 over d zeta and Psi 1 is a function of zeta times d Psi 1 zeta over d zeta times J right, this is what we had developed. Psi 1 prime, Psi i prime, times Psi J prime time J that is F hat function and i and J are 1. So, that is why Psi 1 prime Psi i prime is Psi 1 prime Psi J prime is Psi 1 prime and you have J and J is equal to h e over 2 and this i can in brief write it has Psi 1 prime times Psi 2 prime. This is the definition of this F hat right.

So, K 1 1 e equals h e over 2. So, here I should have 1 by J which means this should be 2 over h e. So, this gives me 2 over h e. Now so I am going to put B in equation A, I am going to put equation B in equation a. So, I get the entire expression as. So, F1 1 hat evaluated at zeta 1 and what is F1 1 hat? So, Psi 1 prime is minus 1 plus 2 zeta divided by 2 and this gets multiplied by itself for K 1 1. So, I get minus 1 plus 2 zeta divided by 2 whole thing is square and this function is going to be evaluated at zeta 1 and it has to be multiplied by W1 plus F11 hat evaluated at zeta 2. So, in the bracket the expression does not change minus 1 plus 2 zeta divided by 2 whole thing square evaluated zeta 2 multiplied by W2 plus minus 1 plus 2 zeta by 2 square times W3 and this function is evaluated third quadrature point. So, this is my expression for K 11.

Now, what I do is here zeta 1 equals if I look at the table which I have given earlier for 3 quadrature points zeta 1 equals 0 and the associated weight function is 0.8888, for the second part zeta 2 equals 0.774596 and the associated weight is 0.55555 and for the third one zeta 3 equals minus 0.774596 and W3 equals 0.55555 and so and so forth.

So, if I plug this in this term, this in this term and these values in these terms then I can calculate the value of K 11. So my K 11 it comes out to be if I do all the calculations what I get 1 get is K 11 e equals 2.333 divided by h e understood. So, this is equation C. So, everyone understands how we will calculate values of K. Now we will do 2 more examples, see 3 more examples. So now, what we will do is we will recalculate K 11 e with r equals 2 do we expect that we will get correct answers with r is 2 or not we should get that answers are correct because we have calculated the that the value of minimum value of r should be 2.

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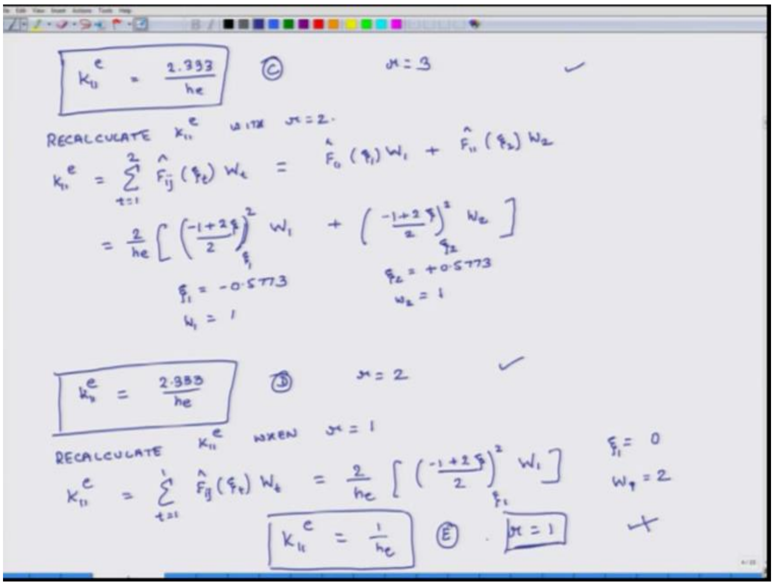

So, let us do the calculations and see whether we get it the same answer or not. So K 11 e if I use r is equal to 2, then the expression is Fij evaluated at t-th zeta point multiplied by t-th weight and this t varies from t is equal to 1 to 2. So, here r is equal to 2 and I am just doing it twice.

So, this is equal to or this is F hat. So, this equals F1 1 hat evaluated first quadrature point times, first weight plus F1 1 hat evaluated second quadrature point times second weight. So, from here I get 2 by h e in to minus plus 2 zeta by 2 whole squares. What does this mean? F11 is what? F1 1 is this entire expression does not depend on r. So, this does not change. So, that is why I am not see this entire expression F1 1 hat it only depends on induces i and J are does not come in to this picture right. So, this expression does not change. So, this will be evaluated at first quadrature point times first weight and then again minus 1 plus 2 zeta divided by 2 whole squares, W2 evaluated at second quadrature point.

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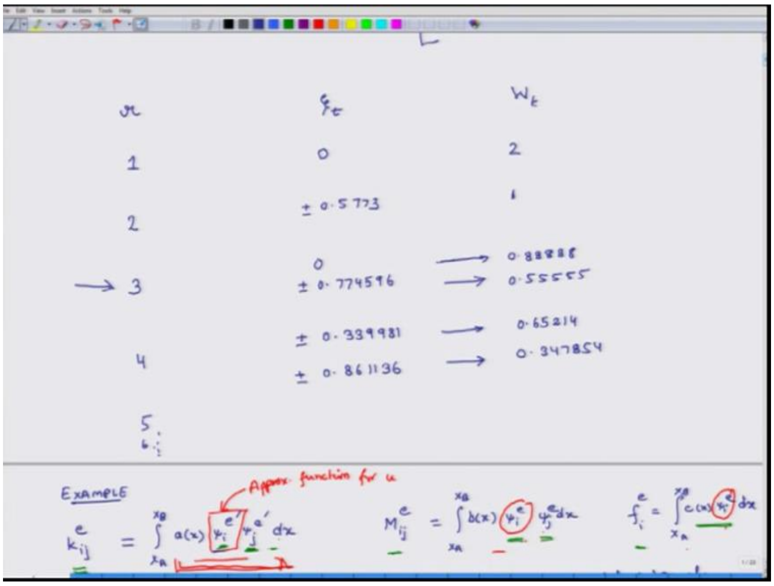

Now what is the value is zeta 1 zeta 2? What is the value of r? In this case r is 2. So, the table which we had developed we will not use this one, rather we will use this line. So, the values are zeta 1 and zeta 2 is plus minus 0.5773 and the associated weights is 1. So, zeta 1 equals minus 0.5773, W1 is 1, here zeta 2 is equal to plus 0.5773 weights is 1 and if you do this math you will find. So, all you have to do is now just substitute these things straight forward. So, the answer you get is still 2.333 divided by h e.

So, in case d c r was 3, in case d r was 2. Now we will do 1 more calculation recalculate K 11 e when r is equal to 1. So, we do this recalculation. So, here K 11 e equals t is equal to 1 to 1, Fij hat evaluated t-th point times t-th weight and this will have just 1 single term. So, the expression for this is 2 over h e, 1 plus 2 zeta, minus 1 plus 2 zeta divided by 2 whole square times weight 1 and here and this is evaluated zeta 1 and zeta 1 equals if you look at your table for r is equal to 1 zeta 1 is 0 and weight is 2.

So, the answer from here what you get is K 11 e is equal to 1 by h e. So, this is e equation e, when r is equal to 1. So, you see that for r equals 3 we get correct results for r equals 2 we get correct results, but we do not get correct results for r equals e 1, if I use r is equal to 4, I will still get the same results. So, these are the correct results, but I am getting a different number when I am using r equals 1 because the assumption that r should be 1 is incorrect because does not meet the rule that r should be equal to p plus 1 divided by 2 or the next higher integer. So, this is important to understand.

So, we will do 1 more case. So, here we had calculated K 11. So, both the first index i and J they were having the same value. So, we will do 1 more calculation to make things there are in any other lingering doubts.

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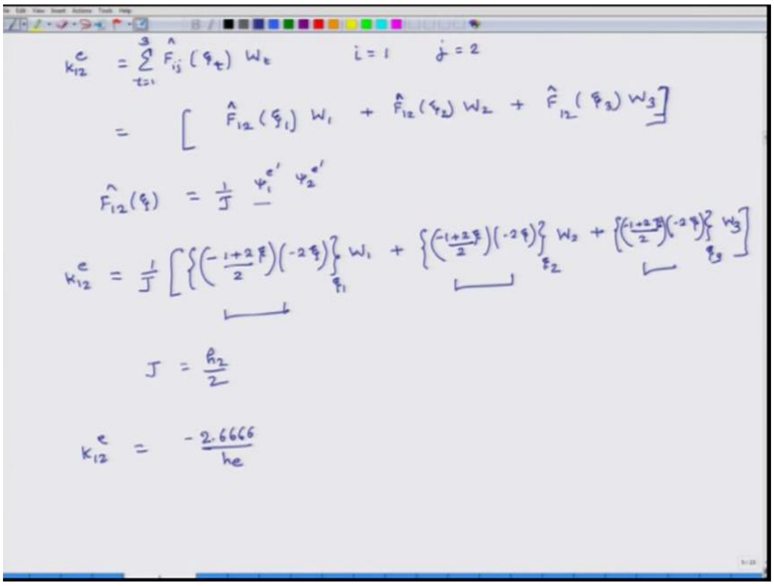

We will also calculate K 1 2 e where i and J do not have the same index. So, if that is the case then using this formula first we write down the expression, this is equal to Fij evaluated zeta t-th point multiplied by weight, t is equal to 1 to and here let us assume that r is equal to 3, because that 3 is working and we want to do met isoperimetric formulation now here i equal 1 and J equals 2. So, this means this is equal to 1 over J times F, excuse me F1 2 evaluated at zeta 1 point times W1 plus f 1 2 hat evaluated second quadrature point times second weight plus F 13 evaluated zeta 3 times third quadrature point, I am sorry here should be 1 2.

And F 1 2 hat for zeta is equal to 1 over J Psi 1 e prime, Psi 2 e prime. So, K 1 2 e equals 1 over J. So, Psi 1 e prime the derivative of first function we had calculated is this, derivative of second function is this one. So, it will be minus 1 plus 2 zeta divided by 2, this is Psi 1 e prime multiplied by minus 2 zeta and this entire expression is evaluated at zeta 1 times W1 plus minus 1 plus 2 zeta by 2 minus 2 zeta, entire thing evaluated at second quadrature point times W2 plus minus 1 plus 2 zeta by 2 times minus 2 zeta evaluated at zeta 3 W3.

So, once again i r zetas are for r equals 3 these are zeta 1 zeta 2 and zeta 3 and these are the weight functions, W1, W2 3. So, I will just write down the same things. So, these individual numbers we calculate at them up and J is equal to h e by 2. So, we put it in and what we get is K 1 2 e equals minus 2.6666 divided by h e. So, this is the overall methodology for doing numerical integration using Gaussian quadrature.

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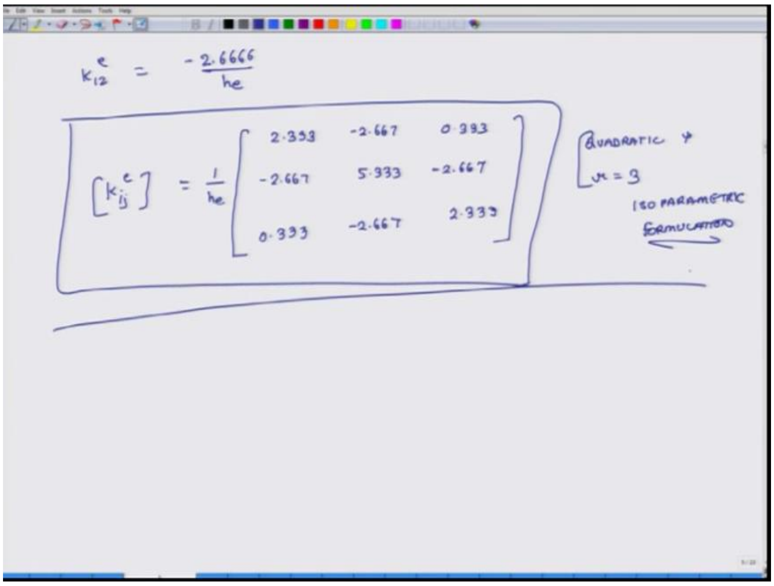

So finally, if I have to calculate my matrix for the eth element, then it looks like something like this 2.333. So, this is K 1 one K 1 2 is minus 2.667, K 1 3 is 0.333 and this is a symmetric matrix, K 2 2 is 5.333 and this number is minus 2.667, 0.333, minus 2.667, 2.333. So, this is my K matrix. When for quadratic functions and r is equal to 3. So, we have done isoperimetric formulation.

Likewise, we can also develop mass matrices F vectors or whatever using Gaussian quadrature. So, this concludes our discussion for today in the next class, it we will do a recap of this entire Gaussian quadrature scheme and also we will talk about briefly about another quadrature scheme known as Newton course quadrature.

With that discussion we will close the discussion and coverage of numerical integration and move on to 2 dimensional problems. So, that is the conclusion.

Thanks a lot. We will meet tomorrow, bye.