Basics of Finite Element Analysis – Part II
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Lecture – 11
Convergence
(Part – 1)

Hello. Again welcome to Basics of Finite Element Analysis Part II. Today is the fifth day of the second week. And what we are going to discuss today is continuation of what we are talking about in the last class.

And what we had started discussing is has to why energy convergence in FEA solution happens from above. So, we were developing a mathematical proof for that and that is what we will continue with do in today's class.

So, very briefly I am going to recap what we discussed in the last class. So, we had said that are for m is equal to 1. If m is equal to 1 then our governing differential equation was a1 du over dx excuse me d over dx is equal to f.

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So, this is for the case m is equal to 1 and we had developed it is variation form like this. So, I am in-take rating it from 0 to L so again 0 to L. The integral the variational form, so

this is the governing equation then the variational form is a1 v prime u prime minus vf dx is equal to 0.

And associated with this variation form was it is quadratic functional. And this was 0 to L half a1 u prime whole square dx minus 0 to L.uf dx equals I of u. So, this is the governor this is the integral weighted residual form of the governing equation. V is the variational form of this weighted residual form where I have weekend the differentiability right and excuse me I am sorry here they should not be any integral sign.

So, equation A is the governing differential equation. Equation B is the weighted residual form or variational form of equation A. And equation C is the associated functional which actually also physically represents the energy in the system.

So, this is for m is equal to 1. And I can generalize this by writing the following relations. So, m if m is more than 1 it could be 2 or higher than that. Then the general equation we had discussed and we had shown was minus 1 to the power of Idi over dx i. ai du or di u over dx i is equal to F.

So, this is m is equal to 1 2 whatever number it is on ok. And the associated variation form and just by extending the logic we will just write a directly if you want to find out the mathematical details you can work into that. But we have already developed the mathematical details for the case m equals 1. You can develop for other values FM also.

So, the variational form is integral 0 to L oh sorry here, it is m is equal to 1 2 m and the i equals 1 to M. So, the variational form is half summation I equals 1 to M. So, it is ai di v over dx i di u over dx i minus v f dx is 0.

This is the variational form and the associated quadratic functional is 0 to L half and ration is half here. So, associated quadratic functional is i equals 1 to m, aid i u over dx i. The whole thing square dx minus 0 to L. uf dx is equal to I u ok. So, these are general equations and these are for m equals 1 case and we know that I represent in this case the energy right.

So, let us find out what is the differential energy, so I uh minus I U. So, IU represents the energy associated with the exact solution and IUh represents the energy associated with finite element solution.

Uh represents finite element solution. So, I represent the functional associated with finite element solution. So, this equals 0 to L half and for purposes of gravity I will no longer write a1 I will just write a.

And this is Uh prime square minus u dx plus f of u minus uh 0 to L dx. So, the negative sign has gone this is the negative sign, but this negative sign has gone in the second term in this equation because instead of uh minus u I have done u minus uh that is why.

Now, we know that f equals minus d by dx a du over dx. So, with that understanding I will write this equation again 0 to L half a uh prime square minus u prime square. And then minus and I am going to replace f by this expression. So, it is minus d by dx a u prime and u minus uh and this entire thing is going to be integrated with respect to x.

Now we know that if I have a function g prime. So, if I have a function g and I have a function h, then integration of g prime h between the elements 0 to L is nothing, but integration negative of gh prime dx plus gh 0 to L this we have seen several times in this class.

So, this is like g prime, and this is like h. So, I can 2 integration of this thing by parts. So, I get half a uh prime square minus u prime square and then instead of this entire expression I get plus term because it is minus gh prime and I get au prime and then it is u prime minus uh prime.

And this entire thing is integrated with respect to X and then plus I get gh. So, I get some was other terms. So, what is gh? It is au prime u minus uh 0 to L. Now we had said.

So, this is the bar problem and let say that I am defining this bar problem is such that the value of u at x is equal to 0 and x is equal to L is 0. I want to basically drop these bounded terms ok. So, if it is 0 then this term goes away if the boundary terms are zero this term goes away.

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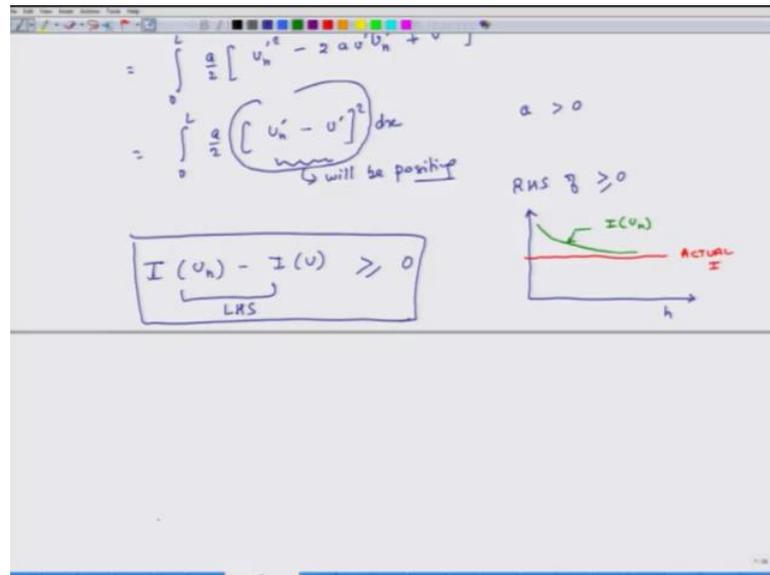
So, what we are left with is have 0 to L a uh prime square minus a write actually I am going to remove this is a by 2. So, minus a u Prime Square plus a u prime square minus a u prime uh prime dx.

So, this term and this term when I add I am left with 0 to L a by two u h prime square minus two a u prime u h prime plus u prime square d x. So, this is equal to zero to l a by two u h minus u prime square d x now a is more than zero we can we assume it because what is a in context of the bar thing it is e time cross sectional area e is positive cross sectional area is positive. So, e is always positive and this will always be this entire thing not the thing inside the bracket this will always you positive because even if the term inside the bracket is negative it is raise to the power of two.

So, it will always be positive. So what that means is that right hand side of this equation is always more than zero or it could be zero also if the term in the bracket is zero then it could be zero if the solution is exact that it will be zero. If the solution is n exact then it will be more than zero it will not be negative that is the point I am trying to make and what is the left side the left side of the equation is I u h minus I u. So, I say that I u h minus I u this will be left side in the right side is always more than zero. So, this is always more than or equal to zero when the solution is exact then it will be zero then the solution is n exact then the finite element solution for energy here we are doing this thing for energy for energy it will always be more than zero.

So, the energy of the system as computed through finite element analysis will always be larger than the actual energy which means that if I am going to plot. Suppose, I am increasing the h size h is represents size of element the actual energy it could be this is the actually energy level and the finite element energy will converge from top this is I u h, it will never converge from the bottom it will never converge from the bottom inside. So, what we have shown in today's lecture is that energy convergence in finite element solutions you always happens from the top.

If you are getting solution at least for simple non-linear for linear conservative systems where energy is coming from the down side then you should thing either a module is not right or something is wrong because theoretically energy should always converge from the top side.

So, with this discussion we will close our today's discussion. And tomorrow we will extend this discussion the little bit more. We will see that how all this relates to different types of solution variables. And how can we make use of this very important observation in context of whenever we are doing finite element analysis. And how can we make use of this important observation. So, thanks a lot and we will continue this discussion tomorrow.

Thank you.