Acoustics
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Lecture 1
Module 4
Instantaneous Power Flow

So what we will cover today is couple of concepts, one is instantaneous power flow and the context of utilising this idea is as we had seen in the last class, we have talked about two concepts, kinetic energy density and potential energy density. And so the other concept is instantaneous power flow that at every given point of time if you have an acoustic circuit or an electrical circuit what is the instantaneous power at that point of time.

So we know that volt is real j omega t and I can generalize it as half real then V e j omega t plus V star which is its complex conjugate, e minus j omega t, okay. Similarly current and the reason we are talking about voltage and current is that we will draw these analysis into that posting domain later. So this comes in handy. Is real component of I e j omega t equals half real I e j omega t plus complex conjugate, oh I missed here, complex conjugate e minus j omega t. You are right.

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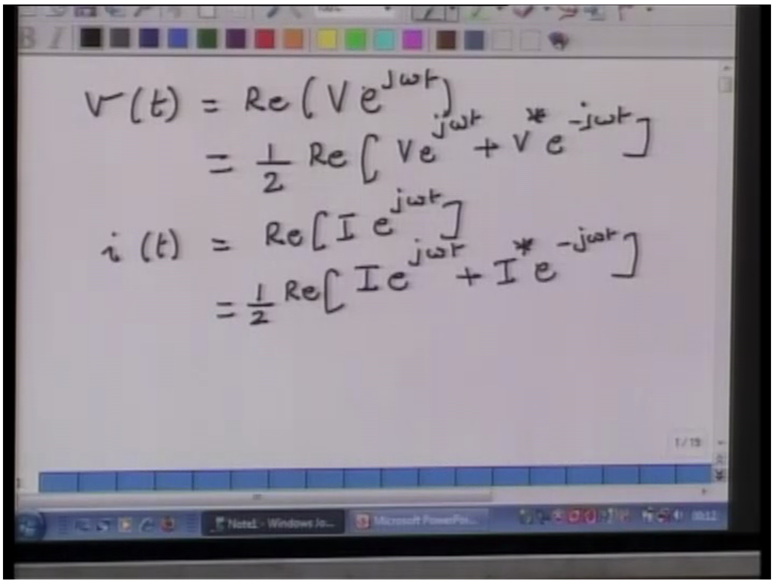

So as you guys rightly pointed out we do not have to explicitly write real because when I add these two V and V star it does give me the real component. So my power equals and then I use this capital letter P t, it is a function of time, is basically a multiple of instantaneous current and instantaneous voltage. So remember we are talking about instantaneous power and that is basically a product of instantaneous voltage and instantaneous current.

If I have to do average power I cannot multiply voltage and current. But when I am talking about instantaneous power it is a valid thing to multiply instantaneous voltage and current. So you have real V e j omega t times half of real I e j omega t, okay.

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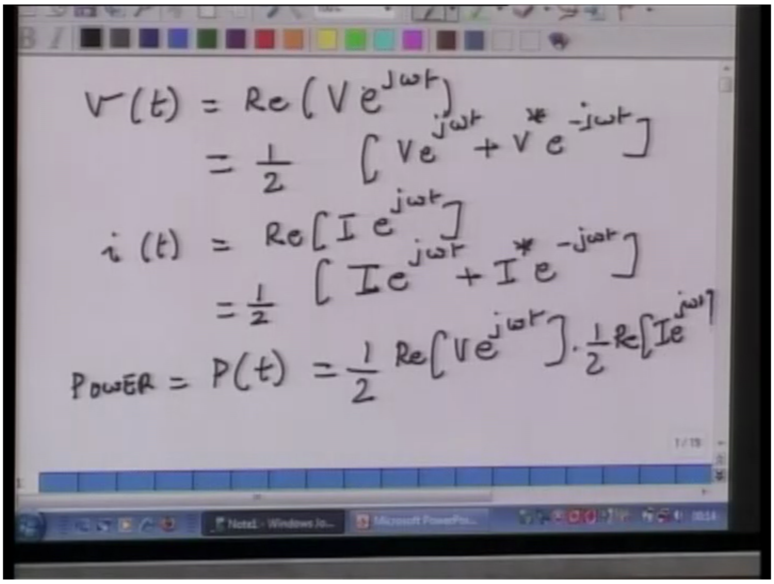

So if I expand that relation I get this long relation. So it is 1 4th V I e 2 j omega t plus V I star plus V star I plus, then I have another component which has both stars, e minus 2 j omega t. So if I expand on V I star and V star I and I add them up, essentially what we will find is that the real portion of V I star is same as real portion of V star I and the imaginary components are negative of each other, so they cancel out. So we know that real of V I star equals real of V star I and imaginary portions V I star equals minus imaginary portion of V star I, okay.

So these two guys I can club them up and I can just sum them up and say that this is V I star plus V star I is twice of real of V I star. Something like that. So I get power as a function of time 1 4th V I e 2 j omega t plus I get V star I star e minus 2 j omega t plus 1 over 2 real V I star, okay.

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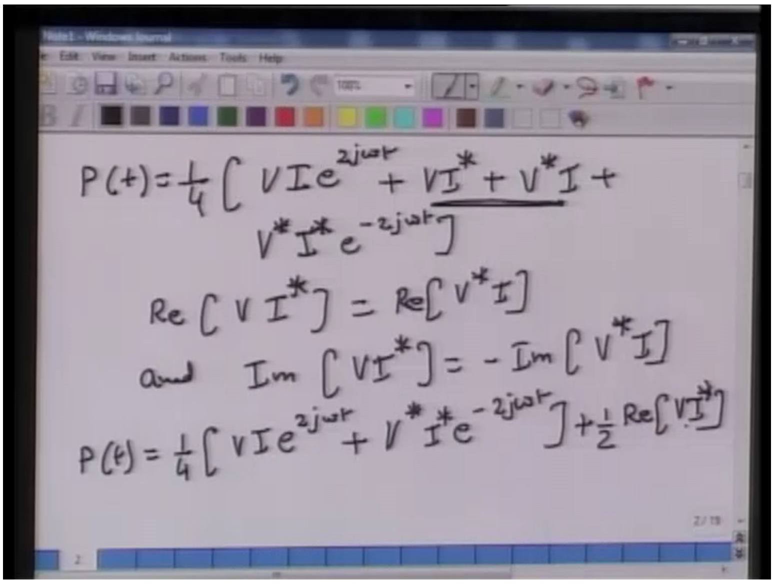

So now what I will try to do is I will try to simplify this term. I simplify this term this has been taken care of. I have to simplify this term. So again if I expand and if I expand V star I star, oh I am sorry, e minus 2 j omega t then I will against see that when I add up the real and imaginary components together, imaginary components you can do this math in your home.

The imaginary components cancel out and only the real portion survives and they are same. So basically what I get is the final expression is power which is a function of time because it is instantaneous power, is half of real portion of V I e 2 j omega t plus half real portion of V I star, okay. This portion its average value is not 0 unless the resistance in the circuit is 0. Then there will not be any power dissipation.

So this portion the average value is not equal to 0. This portion it is a cyclic quantity. So when I average it over a complete cycle which is over a cycle of 2 omega t then it adds up to 0 when I average it over.

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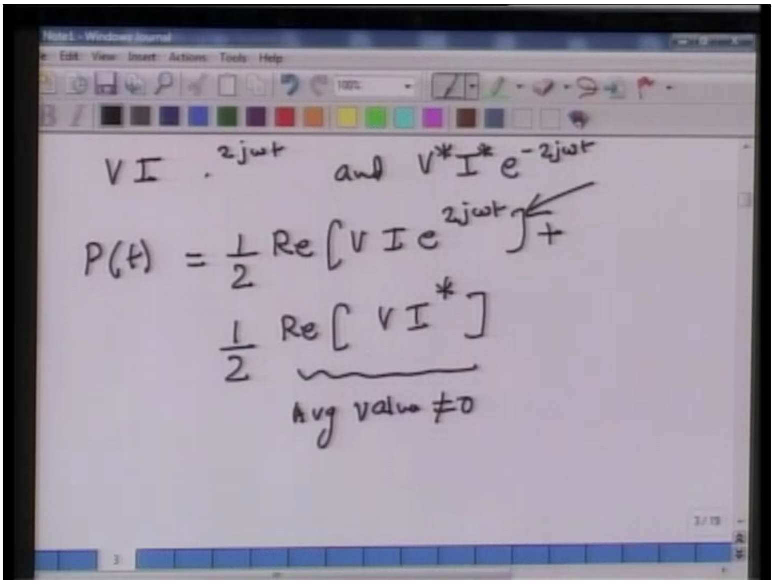

So we will now see an example. So what we will do is we will take two examples. One with a circuit which has pure resistance in it and another circuit which has a resistance load and also an inductor and we will see how instantaneous power changes and also what it tells us a little bit more and how we can relate it to acoustics and also transmission lines in general. So let us look at this circuit. So I have a voltage source and it has an external resistance and I am closing the circuit.

The current is i and my voltage which is a function of time is let us assume 100 cosine omega t and omega is let us say 60 hertz or 60 times 2 pie, 60 is the frequency. And the value of this resistance let us assume it to be 100 ohms. Actually I will change my time and I will make this only 10. So this is 10 cosine omega t.

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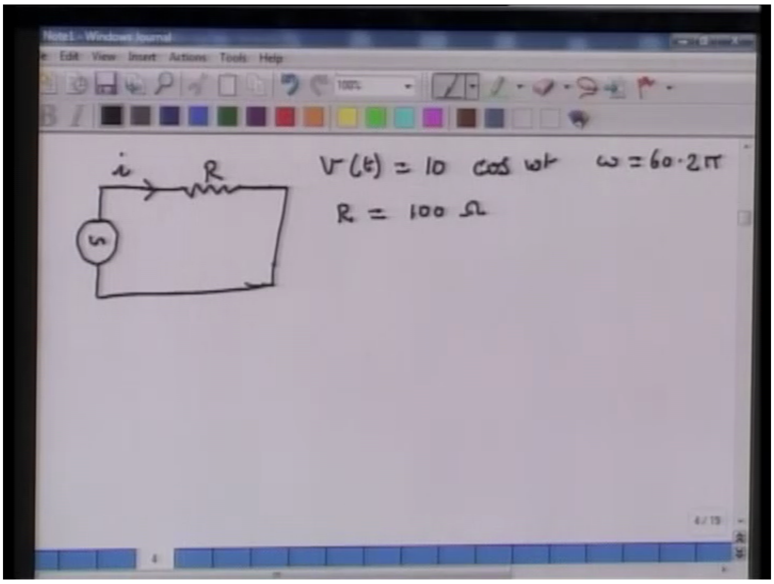

So my instantaneous power is half real V I star. So my V that is the magnitude of this whole thing is 10, right? So that is 10 times I star. So I is V star over Z star. It just happens because this is purely a resistor circuit so V star and V are same and Z star and Z is same as R which is 100 ohms. So 10 over 100 that is I star plus and then I have a cyclic component real V I e 2 j omega t. So V is 10 times I, I is 10 over 100 e 2 j omega t. So if I do all this math I get 0 point 5 plus 0 point 5 cosine of 240 pie t.

So again this is a constant dissipation of power half watts that is being dissipated constantly and this power is a cyclic thing. So in part of the cycle, energy gets stored by the system and another part of the system energy gets dumped back into it. It does not get dissipated it gets dumped back into the system. So it stores and releases the energy going back and forth. The other thing you will notice that the frequency of this is twice the frequency of voltage or current or in acoustic domain whatever is the source signals frequency it will be twice of that.

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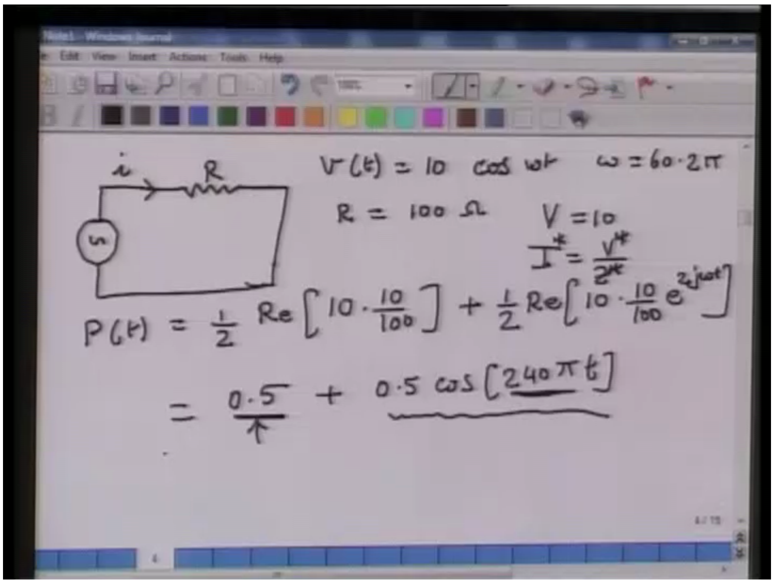

So if I plot this whole thing my plot will look something like this. This is my time axis, this is the total instantaneous power, this is P t. So my axis when I plot it point 5, 1. So it will vary from 1 to 0 to 1 to 0 and so on and so forth. And a complete cycle will be 2 times T which is T is the period of the voltage cycle or the current cycle.

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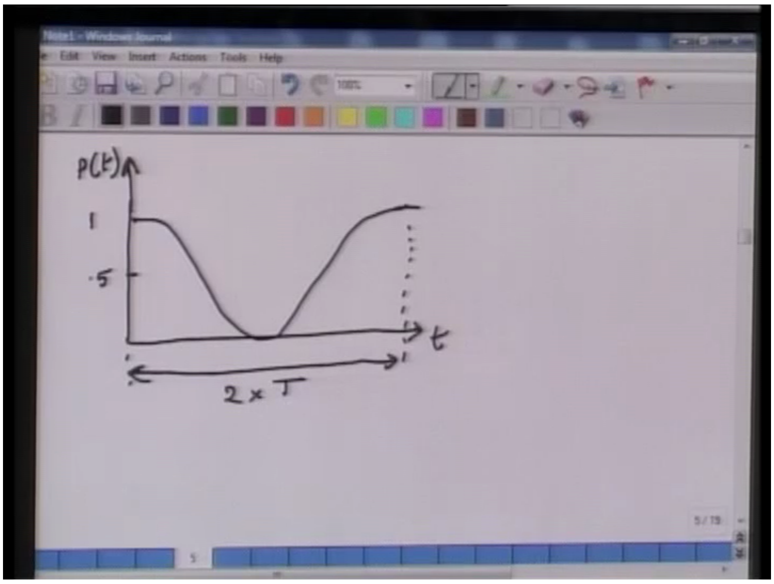

We will do one more example. So here we have the same voltage source. I have an external resistance which is connected to also an external inductor. This is resistance, this is inductance, this is voltage source. Volt equals let us assume 100 cosine omega t. R equals 100 ohms. Oh I made a mistake. This is again 10. Also the value of inductance is 1 over 1 point 2 pie (())(13:09). I am assuming that. So my impedance of the whole circuit is Z. It is basically 1 over R plus 1 over S L and I take the inverse of this.

And I know that S equals j omega equals 2 pie times 60 times j, yes. So my Z becomes, if I do the math basically I get 1 over 100 plus 1 over 100 j and then I have to take inverse of this equals, 1 over 100 I take it out, 1 plus 1 over j. So then I have to do the inversion of this entire thing. So my impedance is 100 over 1 minus j.

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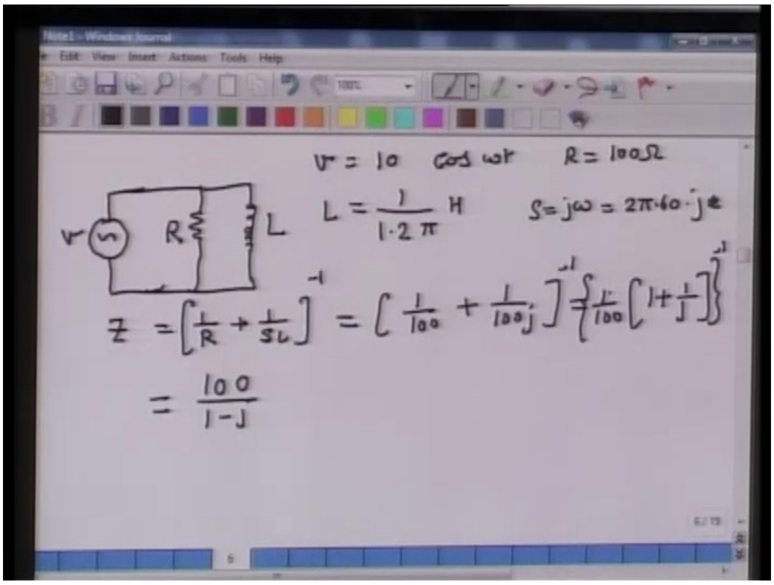

Now if I look at the denominator it is basically there is a constant real portion to it and then imaginary portion. And I know that if I use the complex principle I can express 1 minus j as square root of 2 times e minus pie over 4 j. So you have your real component this is 1 and then you have imaginary component minus j. So the magnitude is root 2 and this is the angle. So e minus j pie over 4, okay.

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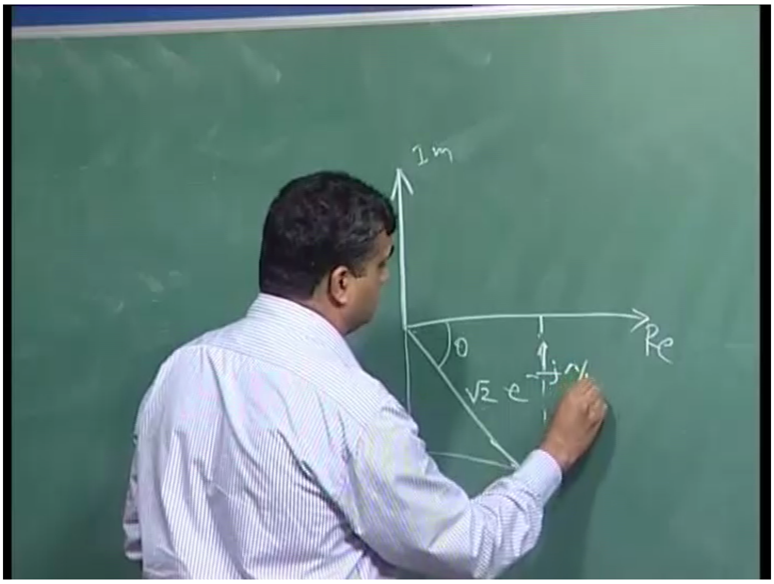

So my Z is 100 times root 2 e minus j pie over 4. Everything is in the denominator. Or it is 100 over root 2 e j pie over 4, everything up in the numerator. So my Z star is complex conjugate of Z which is 100 over root 2 e minus j pie over 4, okay. So now I can use Z star to evaluate what is I star? So I star equals V star over Z star, root 2 over 10 e j pie over 4. That is I star.

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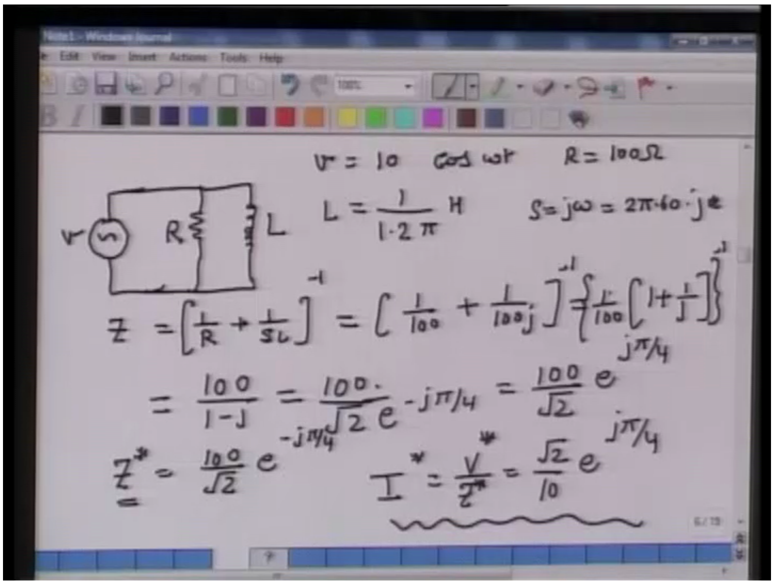

So moving on I go back and start calculating power dissipated. So instantaneous power equals half of real component of V I star plus half of real component of V I e 2 j omega t. So this I get as half real component of voltages. V is 10 times root 2 over 10 e j pie over 4 plus half real component of 10 times root 2 over 10 e minus j pie over 4 e 2 j omega t, where omega is 125.

So if I take the real component of this I get 1 over root 2 times real component of exponent of j pie over 4 plus again I get 1 over square root of 2 real component of exponent of j omega t minus pie over 4. Going back, e j pie over 4 is cosine pie over 4 plus j sin pie over 4. The real component is cosine of pie over 4 which is 1 over root 2. So that becomes half and then the oscillating component of the power is again 1 over square root of 2 cosine omega t minus pie over 4.

Yes, 2 omega t. It is minus pie over 4. So again this is my average power and this component of the power is fluctuating and there is a difference of pie, the pie over 4 here.

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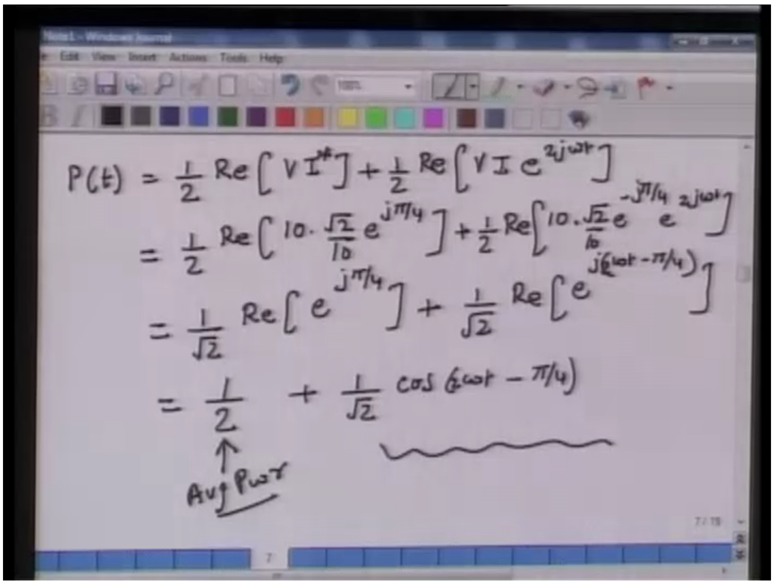

So now if you plot this entire expression you will see that instantaneous power not the average power averaged over a cosine, instantaneous power can become positive and also it can become negative. But the fact that it is becoming negative is not that it is getting absorbed permanently somewhere. Getting saved into system and then gets dumped back into the system back and forth.

So if I plot it my graph will look something like this. So this is my time axis and this is my vertical axis is let us say watts. So power and so the maximum value will be half and then cosine 2 omega t minus pie over 4. At the maximum it will become 1. So half plus 1 over square root of 2 that is the maximum value. So that is the maximum value which is half plus 1 over square root of 2.

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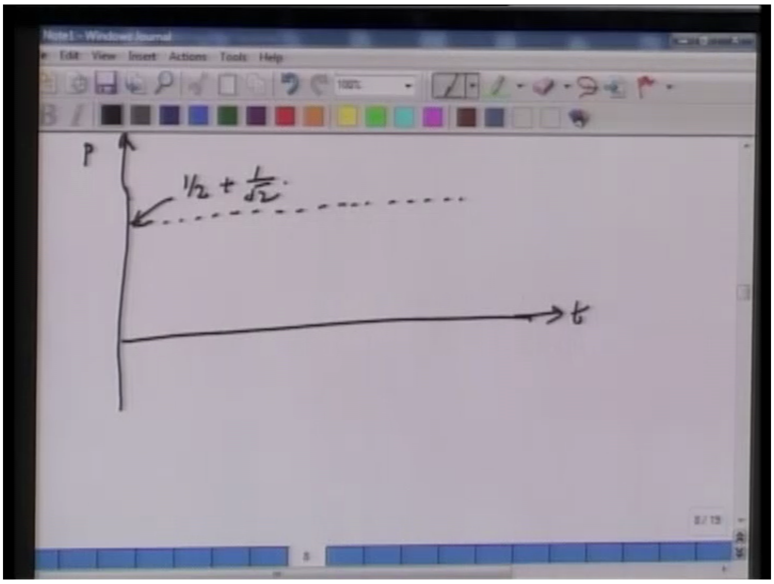

Now at t equals 0 the value will be half plus 1 over square root of 2 cosine pie over 4, right? So it will be half plus half that is 1. So at time t equals 0 this is there. And then my power cycle will look something like this and so on and so forth. So in certain parts of the cycle energy is getting stored into the system. In some other parts energy gets dumped back into the system but the average power dissipated does not change from a (resi) purely resistant circuit which is half watts. Even the peak power does shift and it does exceed 1 watt because of this number.

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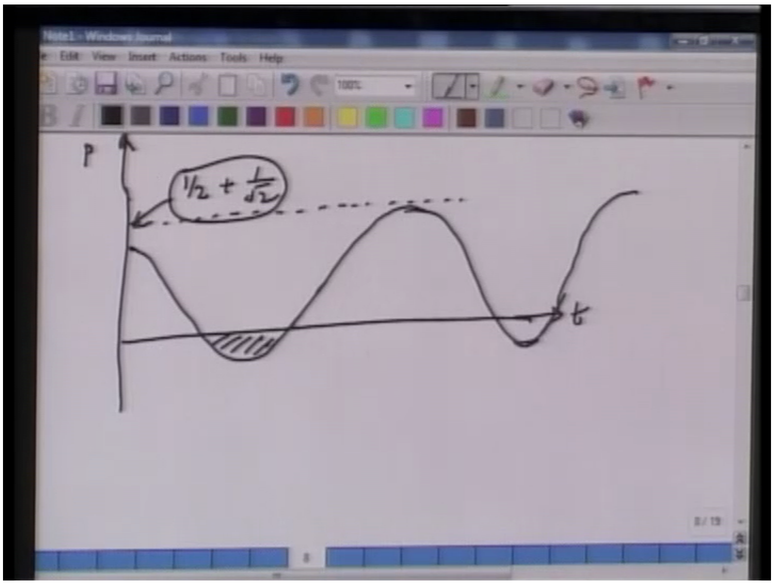

So this is something we have to understand and also because they will be mapping some of these concepts into the acoustic domain as we move on which you will see either in the next class or in the class after that one. So, again average power remains same if the resistive load does not change. The other thing is that peak value can shift because of presence of inductors and capacitors or because of presence of elements in the circuit similar to inductors and capacitors.

And the third thing is that it has a strong bearing especially in the electrical domain is that this term V over Z. So V is voltage, Z is the total impedance that basically influences the total current, right? Now Z in a fairly resistive circuit we saw was 100 ohms. But in case of the other circuit were we had an inductor the magnitude of Z was 100 over square root of 2. So it was not 100 ohms, it went less than 100 ohms by a factor of 1 point 414. What that means is that in general the circuit is drawing more current into the system.

Now in transmission lines and this does not have a strong bearing on acoustics itself but it is worthwhile to know. In transmission lines we have a power generator unit and that current gets transported over let us say 1000 kilometres and it come to your home. If your home has a purely resistive circuit then it will consume some amount of watt. If your home has resistors and also let us say inductors, it will still eat the same amount of heat but it will draw more current.

What that means is that the heat dissipated along the transmission line goes up significantly. And most of the loads in your houses what do they have? The refrigerator has a motor means it has an inductor. Vacuum cleaner, they have inductors. Mixie, it has an inductor. Most of the circuits in household appliances, television it is an electronic device but it also has inductors. Most of the circuits which have motors they have inductive loads. So what that means is that it draws more current.

So what that means is that the heat dissipated in the transmission line goes up. So basically the (sys) overall efficiency of the system goes down. So what people do is they try to rectify by putting some capacitive load also in parallel to the inductor and that kills the influence of inductive load so that the current comes back to the original number. So that we will very quickly capture and see what happens.

So if I have a resistor, an inductor and a capacitor and let us say my R is 100 ohms, L again is 1 over 1 point 2 pie and capacitance is 1 over 12000 pie, then my total impedance will be 1 over 100 plus 12 pie over j times 120 pie. So this is the inductive part plus 120 pie j over

12000 pie. And then this whole thing I have to invert it. And if I do the math correctly it again brings the overall system impedance to a purely resistant number which is 100 ohms.

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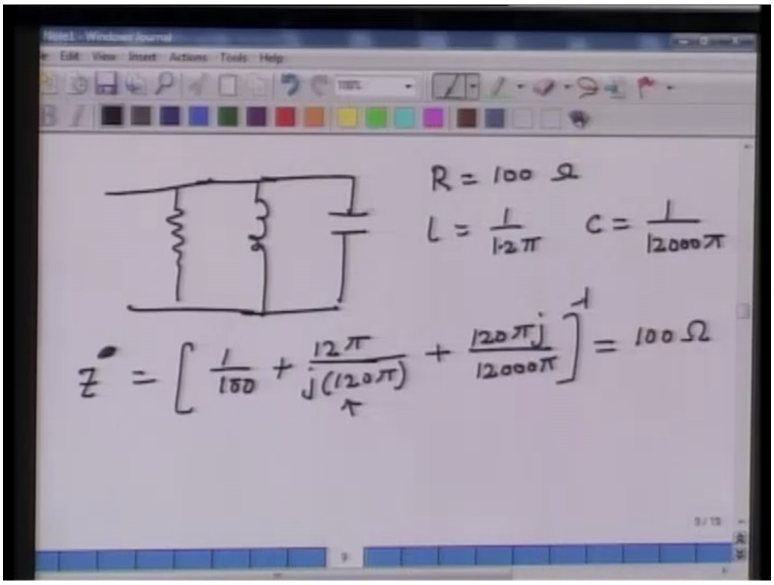

So by adding a capacitor you can change the phase back to 0 degrees. Similarly if you have a predominantly capacitive circuit and if you want to reduce the power consumption in an overall sense then you put some appropriate value of inductors and you can bring it back. So in this context there is another term and it is called power factor correction and this is also used in acoustic industry.

It is good to know that. So it is power factor correction. So we saw that power is half real component of V times I star plus half of real component of V I e 2 j omega t. So this is half real component of V V star over Z star plus this oscillating component.

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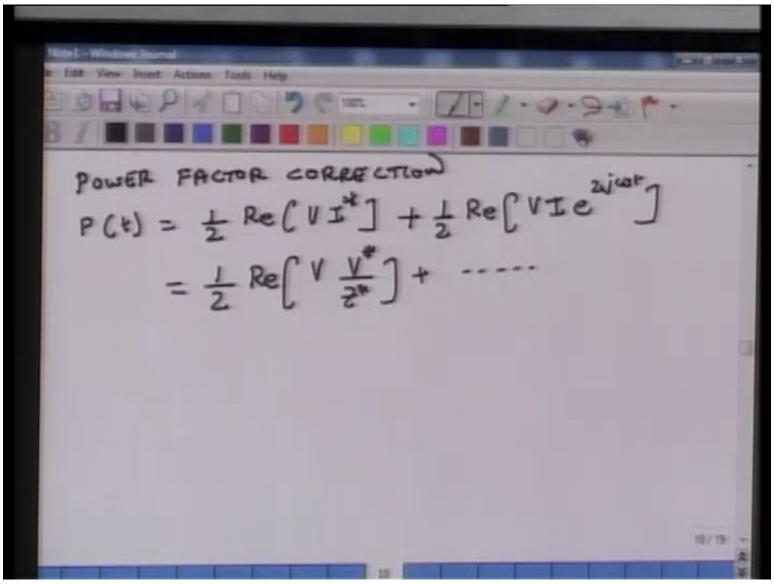

And I know that V V star is square of the magnitude of V. So I can bring it out. Now let us say that Z is magnitude of Z times e j psi. So if I plug this relation in here I get magnitude of V square over 2 and then I get real component of, so I will still have and then 1 over e minus j psi. And then the oscillating components, right? And so it becomes V square over 2 into 1 over magnitude of Z times cosine psi plus other things. This number cosine psi is power correction factor.

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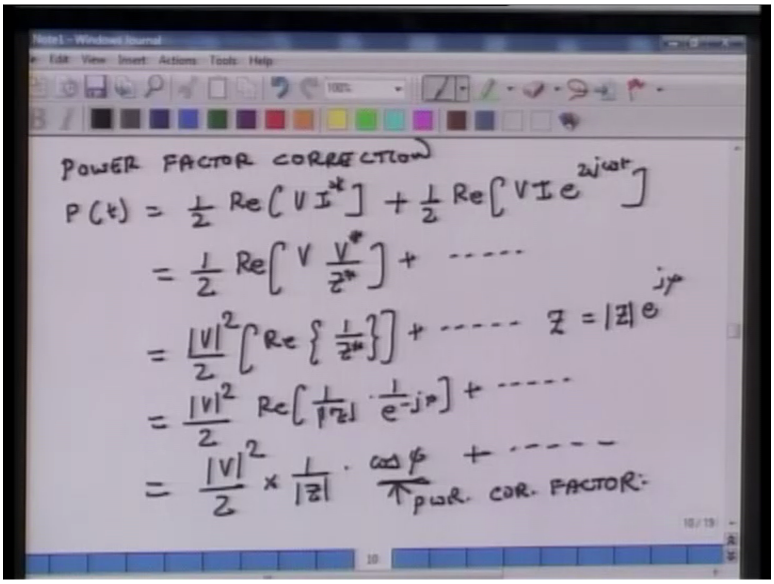

If psi is 1 then my transmission losses in the circuit are minimised. Not psi, cosine of psi is 1 then my transmission losses get minimised. If it is more than 1, not more than 1, if psi is more than 0 degrees then I start seeing losses in the circuit especially in the transmission. So there are countries especially in Europe which have legal requirements for all product manufacturers that if you have to buy a refrigerator it has to have this correction factor built into its system so that transmission losses in the country do not become large.

In India we lose about 5 percent of total electricity just because of this power correction factor. So we have this nuclear deal and all, these nuclear plants will be coming up. So total energy generated by these plants will be something equivalent to, if we just fix one small power correction factor in a product. That much energy can be conserved by one change. So, wanted to share this.

So we have seen electrical power at an instant of time. So now we map the same idea into acoustic domain. Now in acoustic domain it happens that we use pressure and velocity. So pressure times velocity is essentially not power but power per square area if you do the dimensional analysis. So a lot of times people use the term acoustic power but what they are in reality imply is instantaneous power per unit area.

So you have to be careful that when you are doing the math that scaling happens accurately. So, acoustic power in common language again is essentially basically instantaneous power per unit area. So let us call that W, acoustic power. So it is basically a dot product of pressure at a given point and time times velocity.

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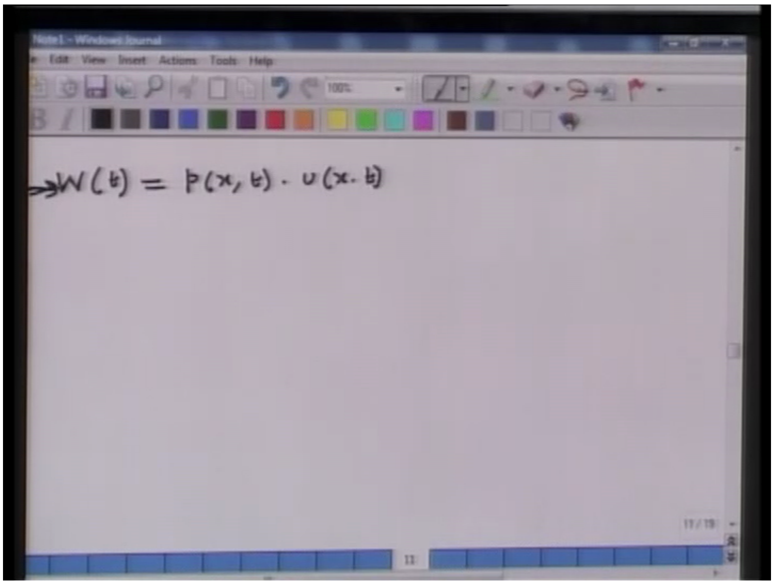

So that is my instantaneous acoustic power per unit area. And again the velocity and pressure variables they have to be normal to each other so if pressure is acting on this plane the velocity has to be normal in this direction. So I expand this actually. So I know that P x t is real component of P x omega e j omega t.

Similarly U x t equals real component of U x omega e j omega t. So I substitute these pressure and velocity functions back into the equation for W and what I get is W x t equals half real P dot U star plus. So remember I am using exactly the same analogy which I had in electrical engineering. P dot U e 2 j omega t, okay.

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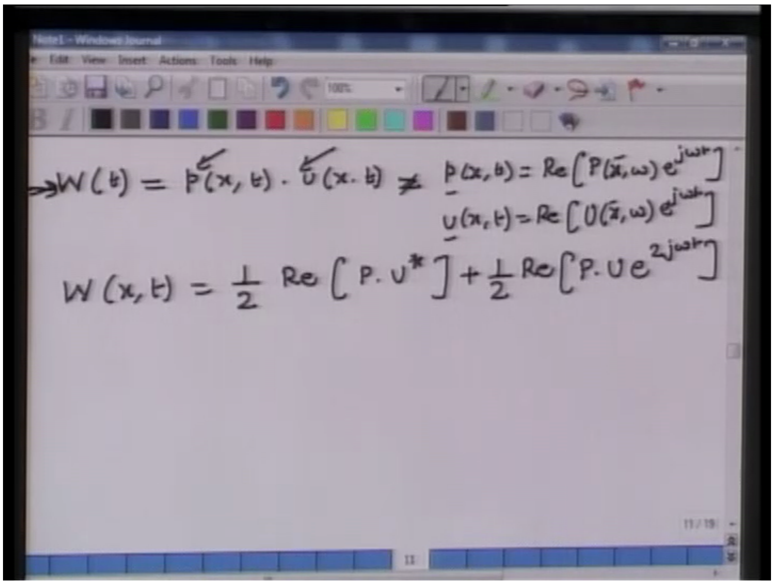

So now what we will do is we will evaluate what is the power dissipated in a closed tube and also in an open tube of infinite length and see how it works out. So we will go to a closed tube. So I have a tube closed at this end. So it is a rigid perfect closure so everything is getting reflected such that (reflec) reflective coefficient is 1 and nothing is getting excited by a transducer here like this. This is my x equals 0. Here x equals minus L. My input excitation is that at x equals minus L t the velocity is U S. It is a pure number, cosine omega t.

So in complex notation I can write that as real component of U S e j omega t, okay. So I calculate instantaneous acoustic power per unit area. So that equals half U Z, that is my pressure, right, times U star plus, oh I missed real, real component of U Z which is the pressure function times velocity e 2 j omega t.

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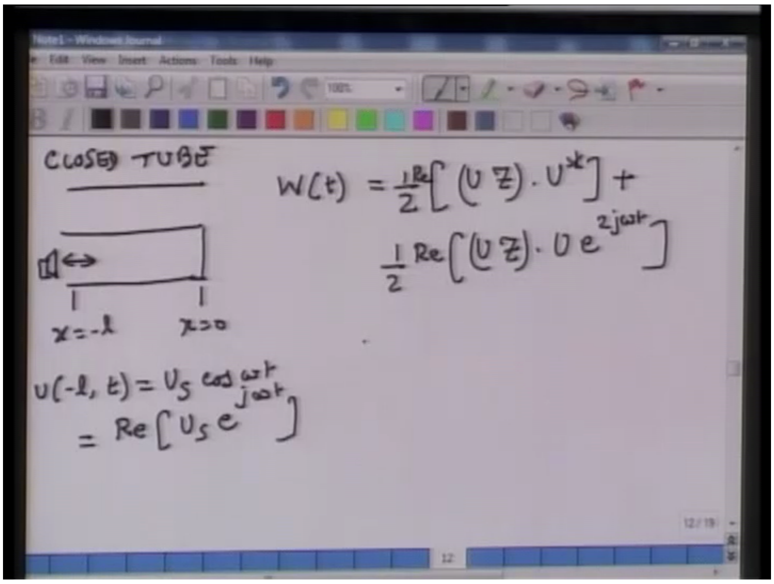

Now we know that impedance in a closed tube we had seen in earlier classes at minus L omega equals minus j Z 0 cotangent of omega L over C. So my power equals 0 plus U S square over 2 real component of a very big term j Z 0 cotangent omega L over C times e 2 j omega t.

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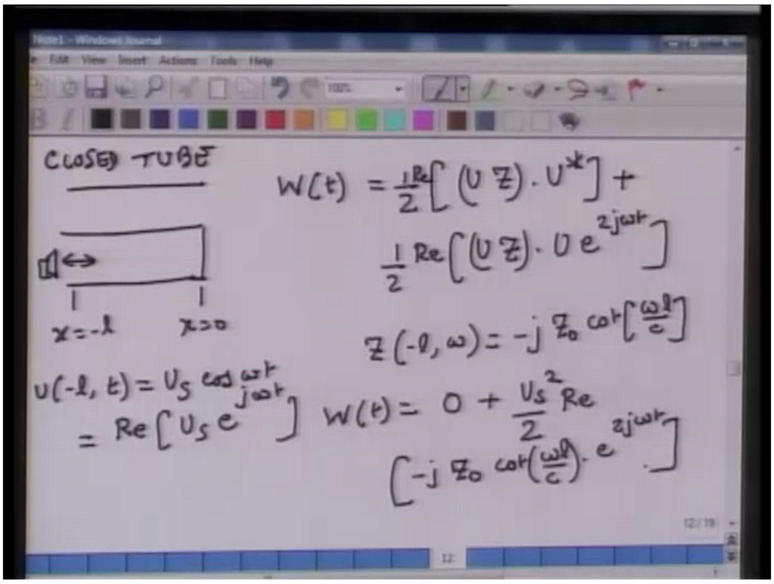

So my acoustic power is 0 plus U S square over 2 cotangent of omega L over C sin 2 omega t. So in a closed tube the total acoustic power being dissipated if there are no dissipative losses in our wave equation we assume that they no dissipative losses happening because of the medium, is 0.

Average power being consumed by the system is 0. However energy (doe) transfer does happen between potential and kinetic states. So my plot for the power would be something like this. Where this is time so this distance will correspond to twice the frequency of the excitation.

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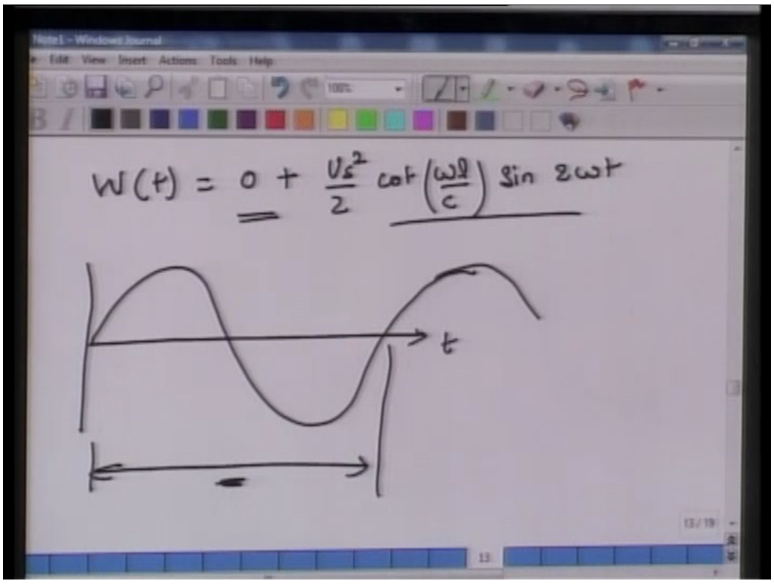

So we will one more example and that is of an infinite tube and again I am exciting it at this location through a velocity source, x equals 0 at this location so also I know that at x equals 0 velocity equals U S cosine omega t. And in complex notation I can write it as a real component of U S e j omega t. So again instantaneous power per unit area is half real U times Z times U star plus half real component of U Z times U e j omega t, closing the bracket. Yes?

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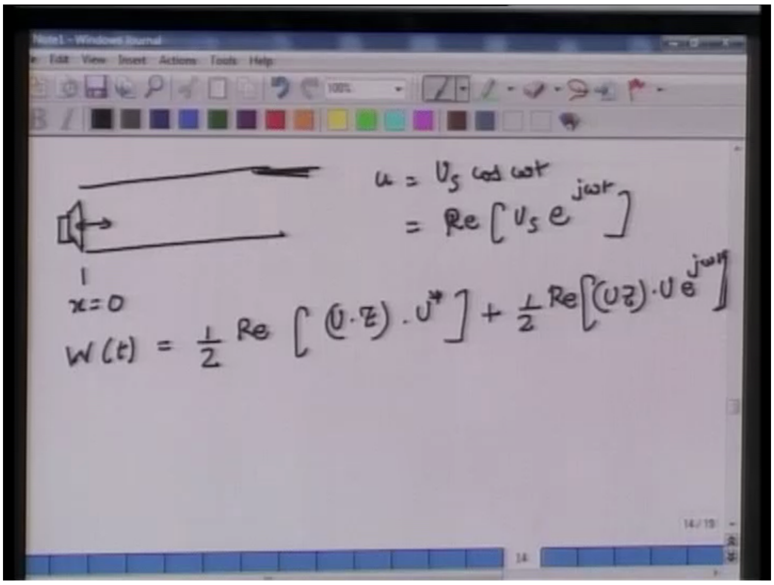

Student 1: In the previous expression sir at omega pie C, there was a term containing omega pie over C and this omega becomes something like, it becomes an integral multiple of pie. There something called infinite (())(38:39).

So again it will transfer between kinetic and potential energy.

Student 1: Sir, but the term is infinite.

Theoretically because you are assuming that damping is not happening. So even in purely spring mass systems where you excite at a resonance theoretically, I am again giving you a parallel analogy, when you excite a system which is purely having a mass and a spring at resonance that is placement goes to infinity, right? But that does not mean that the energy required to induce that kind of thing has to go to infinity because it gets compensated , I mean it fluctuate between kinetic and potential energy.

So all this (hap) what is happening is energy transfer from potential to kinetic back and forth. So going back that is my acoustic power per unit area is half real and then I get U S square times Z plus half real U S square times Z e j omega t. I missed the 2 here. So again I have U S square over 2 real component of Z plus U S square over 2 real component of Z e 2 j omega t.

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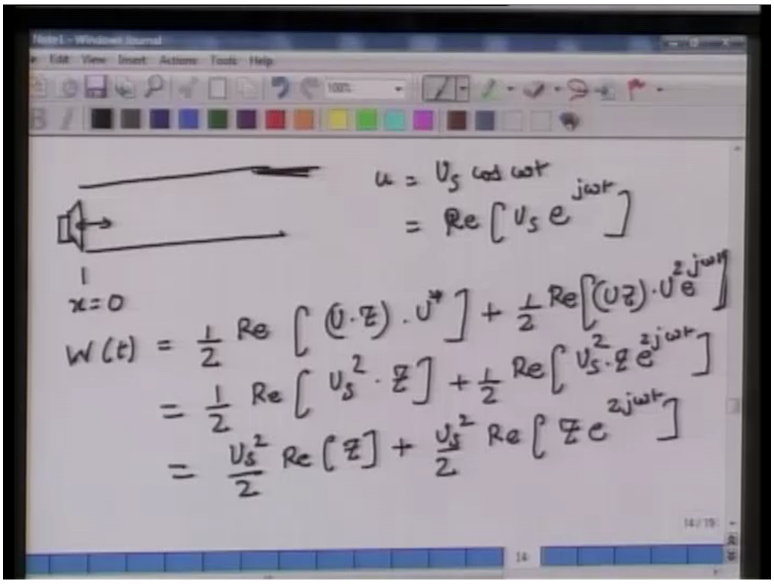

Now we know that for an infinite tube Z equals Z 0 equals rho 0 C. We have seen this, right? And it stays constant throughout the length of the tube. So if I make this replacement then instantaneous power equals U S square over 2 Z 0 plus U S square over 2 Z 0 cosine omega t. So here average power is non-zero and this is the oscillating component of the power.

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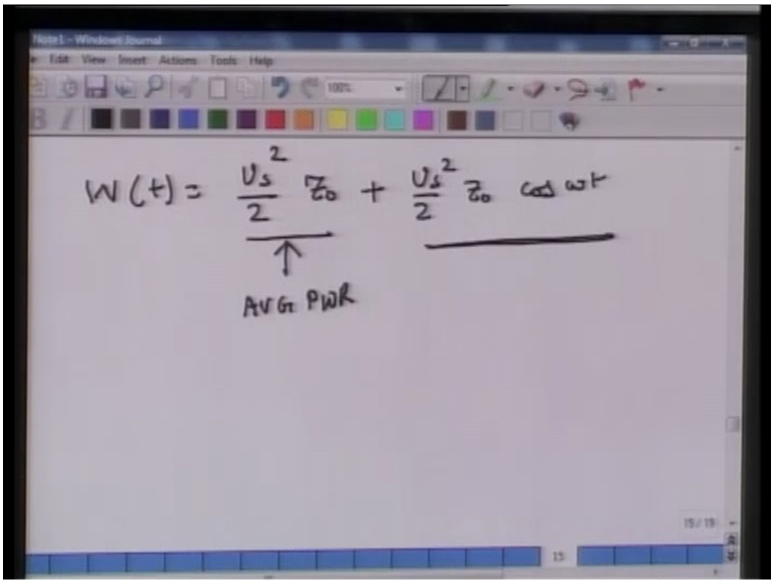

So in a closed tube we saw that because all the energy gets reflected by the closed end surface, the consumption of power by the x at the source point is 0. But here because we have infinite tube and if I am exciting a membrane back and forth that energy keeps on travelling and never comes back. So my average power consumption is non zero. So physically also it makes sense even though in both the situations we do not have any damping happening.

Student 2: (())(41:46) in a closed tube and an open tube.

So if you have an infinitely long tube, you have a membrane in an infinitely long tube and it is exciting air. So that membrane is initially exciting air at this location, then after sometime it excites air at this location. So it has to induce kinetic energy and potential energy in medium which is infinitely long. So it will keep on consuming power. That energy will not come back to it at all because the tube is infinitely long. If you have a tube which is closed at the end then that energy comes back, it gets reflected because this t minus is non zero.

So the energy required to excite is not non zero, it is 0 in that case in a closed tube even though in both the systems you have no damping. So this is what I wanted to capture today in terms of instantaneous power and also about how it relates to acoustics, how it helps us understand total power consumed in an open tube, in a closed tube.

The other thing I wanted to share with you is a brief introduction to how acoustic transducers are made so that you become familiar with the structure of it. So the notion of transduction and then how do we start establishing equal answers between electrical and mechanical components.

So we will just for few minutes go over this then in the next class we will go deeper in this. So what you have here is let us say you have a metallic structure what I call frame. This is made of metal on which the whole transducer of the loud speaker is mounted.

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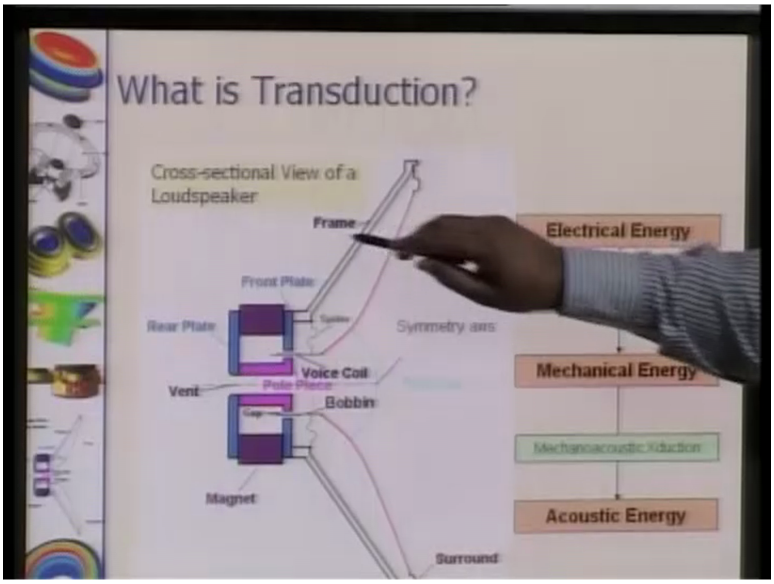

So this is a metallic structure and this is I have made a cross section, so you can rotate it by 360 degrees and you will get a better (())(43:59). Then at the back side you have what we call a back plate or a rear plate, this blue thing. In some of these speakers there is also a hole because heat gets generated this hole helps dissipate the heat. So this is what we call vent. Then between back plate and there is another plate called front plate, there is a magnet.

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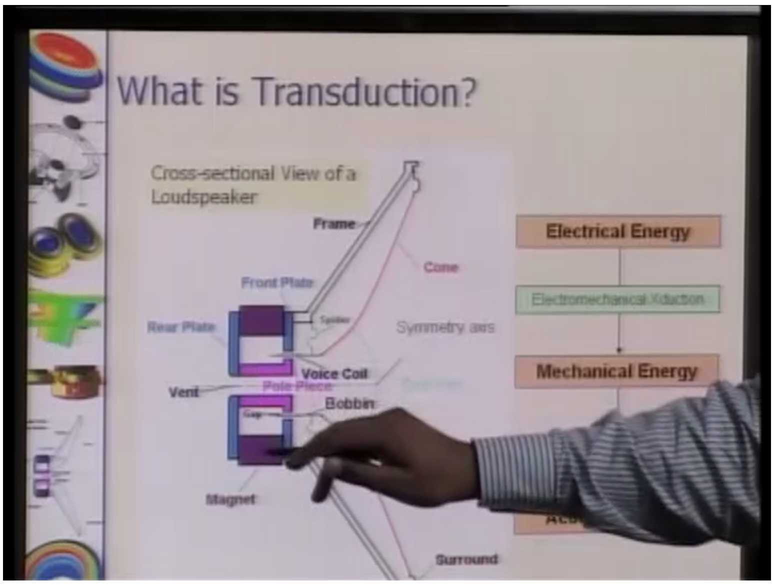

And then there is also a metallic piece which is this magenta in colour and between the magenta piece and the front plate you have this thin film like thing and it is called a bobbin which we talked about in the last class. And on bobbing is bound the voice coil. So as (electrici) electricity comes into it, AC current comes into it, it goes through and spins around the bobbin and then gets out and because of electromagnetic induction the voice coil moves back and forth.

As the voice moves back and forth its motion is opposed by spring force which is generated by this green thin film called the spider and it is also forced by this brown piece of material called the surround. And then you have a rigid diaphragm called the cone. And finally we have the blue membrane called the (())(45:38).

So what is happening is that you have electrical energy, it comes into the system, it gets transformed into mechanical energy and the transformation is for the electro mechanical transduction. Then the mechanical energy gets transformed into sound through mechano acoustic transduction.

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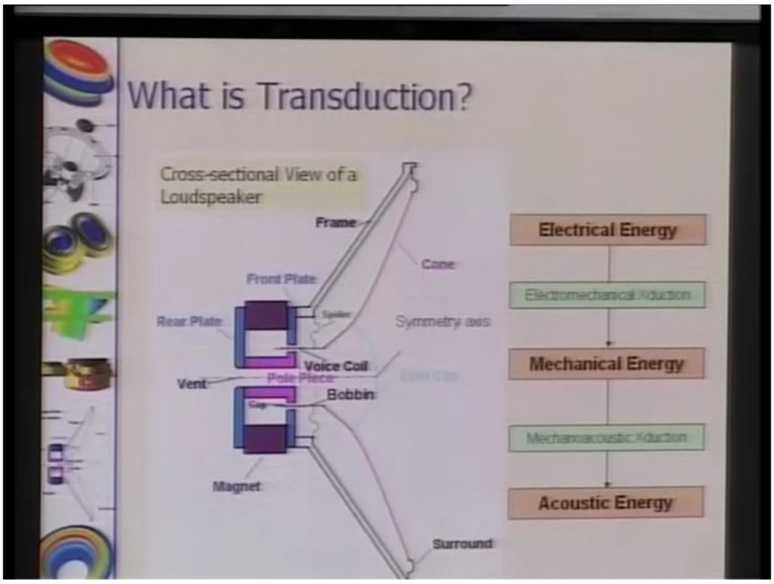

Student 2: What is the exact definition of a transducer?

Transducer is any device which transforms one type of energy to another type of energy. So you have piezoelectric transducers where you have electricity being transformed into motion or vice versa. So this is a three dimensional view of the same object so you get a better understanding. That is your spider, this is your surround. It is typically made of foam. This is the diaphragm which moves in and out.

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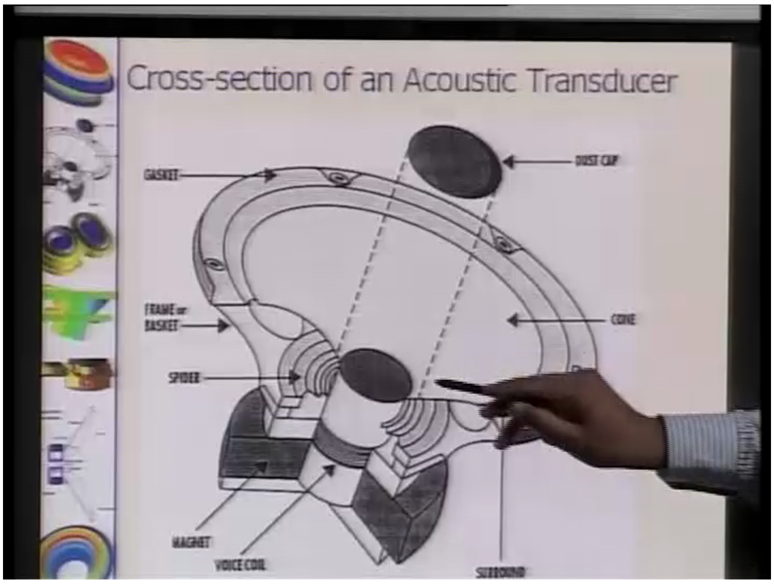

To close the band hole you have this dust cap. You have a voice coil here, magnet is here. Magnet is sandwiched between the front plate and the back plate.

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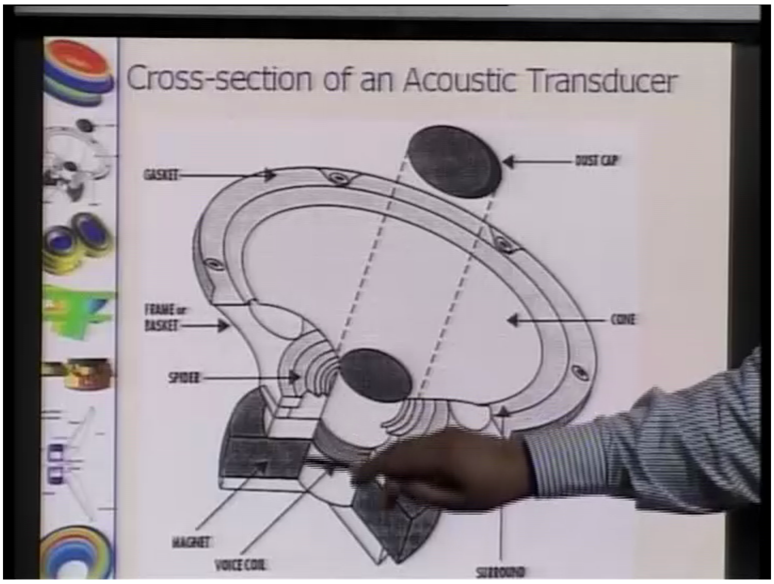

So I just wanted to give you a brief understanding of how a transducer is constructed. Because in next class what we will try to develop is that as (electri) electricity is coming in and it is a signal, it is not a DC current. It has all sorts of frequencies in it. It goes through that winding which has an inductance in it. There may also be an external capacitor mounted on the speaker. So how do you bridge the electrical element of the (sig) circuit to the mechanical element, to the acoustic element.

How do you make that bridge between these three domains, electrical, mechanical and acoustics? Because at the end of the day what you want to know is that so much of current is coming into the system at such frequency, what is the pressure I am going to hear? That is what is needed. That is what we want to know. So how do we bridge that gap and what kind of a theory we should use to understand that fundamental question? So that is what I wanted to share today. And with that I think we will close.