

POWER PLANT SYSTEM ENGINEERING

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Module 2

Lec 10: Steam Nozzles (Part II)

Dear learners, greetings from IIT Guwahati. We are in the module 2 for the MOOCs course Power Plant System Engineering. The title of this module is Vapor Power Systems Part 2. In our previous lecture that is lecture number 9, we have initiated the discussions on the Nozzles and its basic theory. So, on this ground we have given the theoretical aspects of nozzle shapes.

So, we will revisit those concepts again here in this lecture. Then moving ahead we have other topics like how to calculate the mass flow rate of the nozzle and there is a concept where the nozzle is to be choked. So, we will find out under what conditions nozzle is going to be choked. Then we will talk about the pressure ratio or what is the importance of pressure difference across the nozzle for desired flow situations.

Then we will discuss about nozzle performance parameters. So, till this point of time nozzles are very generic in nature, which means that nozzles can be designed for gas, which can be perfect gas or liquid or water or any incompressible fluid or it can be steam nozzle. So, steam nozzles falls in the area when we have liquid vapor mixture as well. Now, under what circumstances or what different thermodynamic parameters which are going to be governed, so that we can apply the basic theory of nozzles while particularly handling the steam.

In fact, moving further, steam nozzles normally encounter vapor and liquid simultaneously. So, there is a concept called super saturation, which is a very common phenomena that occurs in steam nozzles. This super saturation is nothing, but an irreversible phenomena. We will discuss its aspects, how the super saturation phenomena takes place in steam nozzles. Now, let us revisit our previous discussions where we discussed about the nozzle shapes. So, in our previous discussions we derived thermodynamic equations about how the flow through nozzles happens.

So, based on that we found out in any given nozzle, for which the area changes in a

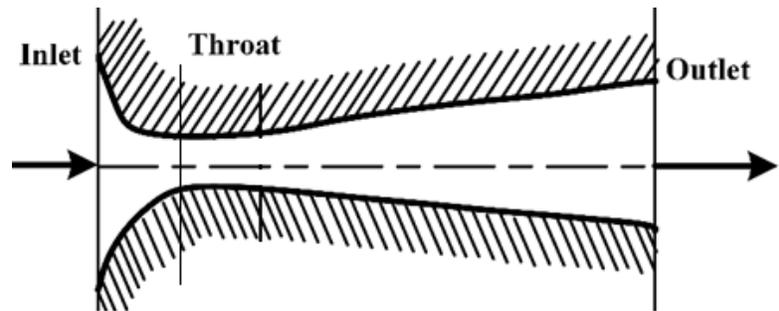
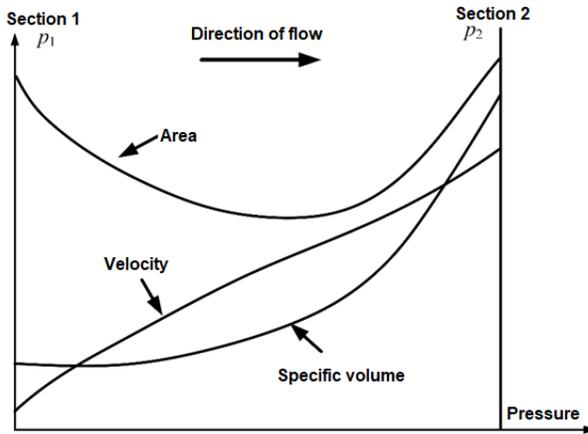
particular direction, we can find out the flow velocity by knowing the enthalpy conditions at the inlet.

$$\text{Flow velocity, } C = \sqrt{2(h_1 - h) + C_1^2}$$

Then we have the fundamental equations for mass flow rate and that is

$$\text{Area per unit mass flow, } \frac{A}{\dot{m}} = \frac{v}{C} \Rightarrow \frac{A}{\dot{m}} = \frac{v}{\sqrt{2(h_1 - h) + C_1^2}}$$

Now, in most of the situations while talking about the nozzles, we normally neglect the inlet velocity ($C_1 \rightarrow 0$) because this inlet condition comes from a reservoir, for which this velocity or flow velocity can be negligible as compared to the velocity at any other cross sections in the nozzles. And this fundamental diagram that comes out that means, if you look at a nozzle shape which is initially convergent then divergence and typically we call this as a convergent-divergent nozzle. And if you try to correlate this particular figure with respect to this graph then what we are going to see is that if you take section 1 and section 2 of the nozzles where the pressure stands p_1 , at section 1 and we will try to see what is the variation of the pressure, how the pressure changes with different shape. So, we will see that if your direction of the flow is from left to right which means that from left to right the area changes, velocity changes and also specific volume changes. One interesting thing is that since flow is going to expand, velocity



will be higher. But if you look at the specific volume graph, initially the change in the specific volume is less. However,

when you go along this length we will find that slope of the line of specific volume is progressively higher as compared to the changes that happens with respect to velocity.

So, this gives some important information that the dominance of specific volume at a later stage is higher as compared to the velocity. And based on this, we will find that the area curve will have a minima, which is the minimum area, and this minimum area, for a particular nozzle is referred as a throat. Throat means this is the minimum dimension what

we come across. So, the nozzle for which the area varies across its length is known as convergent-divergent nozzle and the section of minimum area is called as throat.

In most of the applications there are two types of nozzles that we normally come across one is mainly convergent. So, when you say convergent nozzle, it is mostly used for incompressible fluids or mainly liquids. Now, when you deal with the gas, it is normally preferred to have a convergent-divergent nozzle. Now, the basic difference between these two is that when you deal with the convergent nozzles, we will see that your outlet area is always less than the inlet area which means that minimum area is always encountered at the outlet itself. Whereas, in case of a convergent-divergent nozzle, to make this C_1 to be 0 this initial area is made larger and suddenly it is brought to this minimum area, so that the assumption of taking C_1 close to 0 holds good. That means, the flow at the initial state is almost stagnant or velocity is very negligible as compared to the flow velocity at any other cross sections.

Now, considering this fact, all convergent-divergent nozzles are designed and for which we have the shape of this geometry from larger area to the minimum area and rest of the expansion takes place in the diverging portion which normally is regulated by the pressure ratio between the throat and the outlet condition. We will come back to this point later. In fact, in our previous discussions we also defined parameters like critical velocity, critical pressure, critical temperature. And these conditions for a perfect gas are stated below.

$$\text{Critical pressure ratio, } \frac{p_c}{p_1} = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}}$$

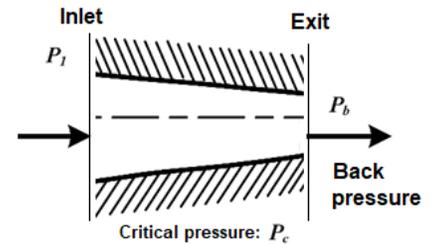
$$\text{Critical temperature ratio, } \frac{T_c}{T_1} = \left(\frac{2}{\gamma + 1} \right)$$

$$\text{Critical velocity at throat: } C_c = \sqrt{\gamma R T_c}$$

Critical pressure which normally occurs at the minimum area for a convergent nozzles or at the throat conditions. It has also been shown that the critical velocity of the throat is nothing, but the sonic velocity.

So, these are the some of the important aspects that depicts how the nozzle shapes are decided. Then we will move on to next important topic that is-for a given nozzle how do you find out the mass flow rate. And based on this mass flow rate, we will say that whether the flow is choked or not. Choking is a phenomena which says that what is the maximum mass flow rate that a nozzle can handle or in other words, what is the maximum velocity that can be achieved.

So, let us try to understand the first case, when you are looking at a convergent nozzles what happens is that we have some inlet state and we have the exit state. So, initial pressure is P_1 that means, nozzle is operating between two thermodynamic states one is P_1 , other is the pressure what we call as back pressure P_b .



Back pressure is the word that we normally use when the flow expands to a particular space, the pressure of that space is known as the back pressure. Of course, that back pressure can be regulated the way we want. So, by controlling this back pressure we can actually define what velocity we can achieve in this flow or what will be the exit velocity at the minimum area in a convergent nozzle.

In the beginning when the flow does not occur which means that $P_1 = P_b$. Now, let us see under what conditions the flows will initiate in the nozzle. So, for that reason, we have to reduce this P_b , so that some flow will occur within this converging space. So, if you keep on reducing this P_b , we will arrive at a particular condition where $P_b = P_c$, that means the velocity at the exit of the nozzle will be sonic. Further reduction of back pressure will not change the mass flow rate and for that exit condition or exit velocity we know the exit area. So, that way we can find out what is the mass flow rate at the exit. And that state when the flow is sonic, the flow mass flow rate will be maximum.

But further reduction of the back pressure means the mass flow rate will not change but the gas will expand very violently at the exit of the nozzle. So, whatever expansion that nozzle is going to give it has given, but the rest of the things that the subsequent expansion from P_c to P_b happens at the exit of the nozzle. Now, this maximum mass flow rate in the nozzle happens when you have this critical pressure ratio. Critical pressure ratio is nothing, but P_c/P_1 , P_1 is the inlet condition of the nozzle. And for that we have expressions like critical pressure ratio, critical temperature ratio, and critical velocity at the throat that we derived in our previous lecture. And now under these conditions once you have this critical velocity at the throat then we can find out the mass flow rate based on the information of critical velocity as minimum area of this throat.

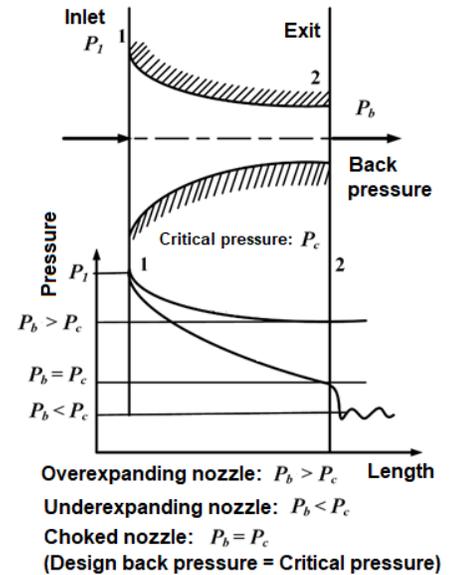
Now, let us understand more details of the practical utility, how this nozzles are normally operated. In fact, when you design this nozzle we have two conditions. One is the through the area information as the minimum area will give you the choke conditions. Now the other parameter that regulates the flow is the pressure ratio or pressure difference across this nozzle. Side by side the true simultaneous effect of area change and the pressure difference, we will talk about what velocity we are going to achieve and what is the physical behavior or nature of the flow in the nozzle. So, based on that we will discuss about two important concepts one is called as under expansion in the nozzles other is the over expansion in the nozzles.

So, previously we have shown that at the minimum area, you will get the sonic conditions and that is nothing, but the critical conditions. At that critical conditions the flow is choked as long as that critical pressure ratio is maintained. But not necessarily we will always have choking in the nozzles, because back pressure is the main culprit as that normally talks about whether the flow will be choked or it will have some other effect.

So, let us understand what happens in a convergent nozzle. That means, how you talk about the design pressure ratio. So, design pressure ratio is normally encountered by the pressure ratio at the inlet conditions to the minimum area conditions. Now, this minimum area for the convergent nozzle will be interpreted as the throat. Of course, there is no diverging portions, but since it is a minimum area it will be throat.

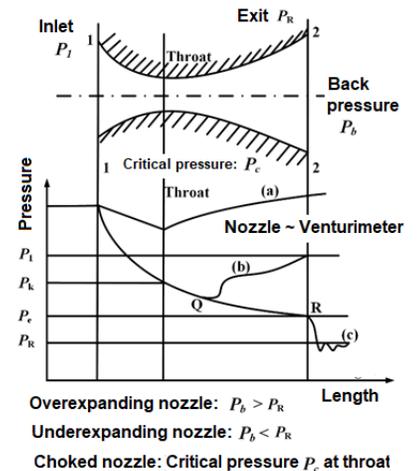
Now, if the pressure differences are adequate then flow will be choked for that pressure difference. So, there are two types of nozzles, one is under expanded nozzles & the other is over expanded nozzles. Then what does this mean? So, let us see that when we have a choke conditions that means, there are two pressures, one is the inlet pressure, other is the back pressure & third one I am interpreting as exit pressure. So, here let us say it is at P_c that means, we will achieve this sonic conditions at the exit.

So, when we say $P_b = P_c$, that means, the flow expands. So, the pressure drops till the exit point. So, at this point we can say it is P_b , since the back pressure is maintained at the exit condition of the nozzle that means, that is the design condition of the nozzle then we say that flow is choked and it will follow this graph that means, pressure will drop along this length of the nozzles till it reaches to P_c . Now, let us see that if your exit condition is not at P_b either $P_b > P_c$ or $P_b < P_c$. When $P_b < P_c$, we will call this as a under expanded nozzle which means that flow has already expanded to the point 2, but beyond that, the pressure is below the exit pressure. Which means when the flow goes out, it further expands outside the nozzle and this expansion occurs in an irreversible manner. Although 1-2 process is reversible, but beyond this it is an irreversible one. After coming out it expands violently outside the nozzles till it reaches the back pressure. And it happens in an irreversible manner till the flow achieves the back pressure conditions.



But in case, the back pressure of the nozzle is above the design value, P_c then it is said to be an over expanded nozzle. So, here the exit pressure is greater than the critical pressures. When it happens, its net effect is to reduce the mass flow rate through the nozzles. And this particular over expansion is more relevant when you go for the convergent-divergent nozzle.

So, let us see what happens in a convergent-divergent nozzle. So, in a convergent- divergent nozzle, we have already said that the shape of the nozzle is designed in such a way that, the flow initially starts from inlet condition P_1 , suddenly the area changes to the minimum area & then the divergent portion happens.



So, expansion mostly takes in this diverging portion only. Here since the minimum area occurs at the throat, your critical condition P_c, T_c, C_c is achieved at the throat. The exit conditions is now governed through this throat condition that means, we can get information at the exit condition 2. So, that is what we interpret this P_R as the exit condition right now for a convergent-divergent nozzle. Now, once you know the information of throat we can get back the information of the exit conditions.

So, we have one more information that is exit pressure P_R which is normally called as design pressure and side by side we have also this back pressure. So, here the possibilities

could be like this- $P_b > P_R$ or $P_b < P_R$. So, same concept whether the flow is an over expanding nozzle or under expanding nozzles is applicable. And other possibility could be that the mass flow rate is very small, so that the pressure at the throat of the nozzle is well above the critical pressure. That means, at the throat we will not achieve critical pressure because mass flow rate is very small and then this nozzle behaves in a manner, which we commonly use in the incompressible flow that is known as Venturimeter. But Venturimeter is not used although the shape is similar to a convergent divergent nozzle, as it does not operate for expansion purposes.

So, the concept of Venturimeter is not in this part of our course. What we look at right now is that when the pressure difference is more which means that P_b back pressure is further lower. So, by controlling the back pressure, we can design the shape of this graph in such a way that, if you have critical pressure at the throat then we will achieve the design pressure ratio condition R at the exit. Now beyond this pressure, it is will be an under expanded nozzle or over expanded nozzles. So, when we have over expanding nozzle, $P_b > P_R$ and when we have under expanding nozzle $P_b < P_R$. And in between at some intermediate pressures, if the throat condition is achieved, then the flow will try to expand in the diverging portion, but again there will be recompression because adequate back pressure is not maintained.

Now, next we will move to the nozzle performance indicators. We have come across this terms previously while dealing with the steam turbines, but the for the sake of continuity let me repeat these parameters again. So, three important parameters are considered for the nozzle design. One is nozzle efficiency, which is defined as the ratio of actual enthalpy drop to the isentropic enthalpy drop between same pressures. Second parameter is velocity coefficient, which is defined as the ratio of actual exit velocity to the exit velocity when the flow is isentropic between same pressures.

And third one is coefficient of discharge, which is defined as the ratio of actual mass flow rate of the nozzle to the mass flow rate which would be passed if the flow is isentropic.

So, based on our definitions, mathematical expressions are given here.

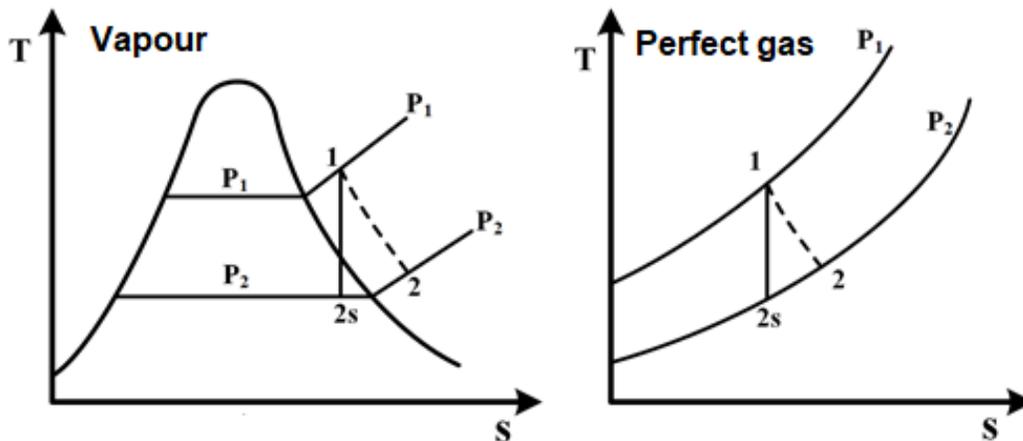
$$\text{Nozzle efficiency, } \eta_n = \frac{h_1 - h_2}{h_1 - h_{2s}}; \text{ For a perfect gas, } h = c_p T \& \eta_n = \frac{T_1 - T_2}{T_1 - T_{2s}}$$

$$\text{Recall steady flow energy equations: } h_1 + \frac{C_1^2}{2} = h_{2s} + \frac{C_{2s}^2}{2} \& h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

$$\Rightarrow \eta_n = \frac{C_2^2 - C_1^2}{C_{2s}^2 - C_1^2} = \frac{C_2^2}{C_{2s}^2} \text{ (} C_1 \text{ is negligible small at nozzle inlet)}$$

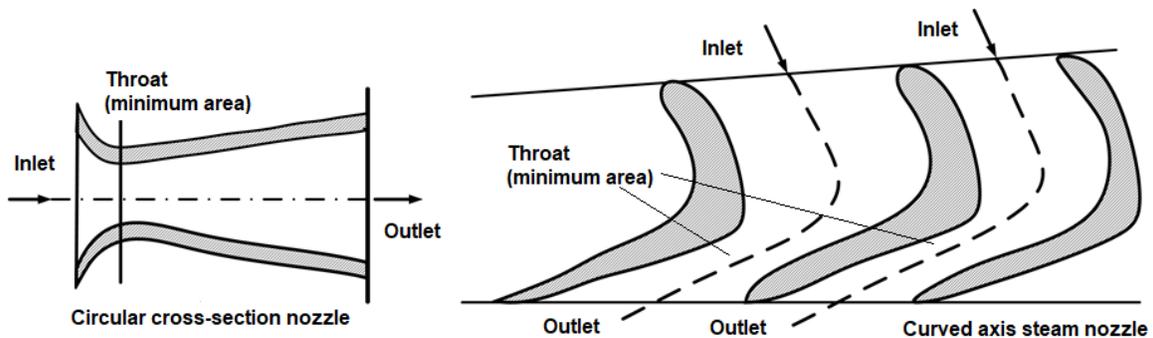
Velocity Coefficient, $k_v = \frac{C_2}{C_{2s}}$ & Coefficient of discharge, $C_d = \frac{\dot{m}}{\dot{m}_s}$

Referring to this particular figure, there are two cases. One is when you are dealing with vapor, which means that particularly in the steam nozzles when the flow expands initially, it is at superheated vapor and after expansion it falls to the dome of liquid vapor mixtures. So, the isentropic process can take place from 1-2s or 1-2. But when you deal with the perfect gas there is no dome. Rather the expansion takes place between two pressure differences, in isentropic manner from 1-2s and in irreversible manner from 1-2.



So, once we know all these information, we can recall our definitions to calculate nozzle efficiency, η_n then velocity coefficient, k_v and coefficient of discharge, C_d .

Now moving further, we have already emphasized that when you actually deal with the nozzle, the type of fluid that we are going to handle can be a gas or perfect gas or air. In other category, it can be vapor. Particularly when water vapors or superheated steam expands in the turbine and the shape of this passage of the turbines are nothing but of a particular fashions for which we say that they are called as curved axis steam nozzle. So, what it means to us for the nozzle calculations? Basically they are nothing, but a convergent-divergent nozzle. So, here I have shown two particular pictures, one is a simple one dimensional convergent-divergent nozzle, typically we call this as a quasi-one dimensional flow which means that only area changes across X axis.



So, for that in the initial part, as I mentioned, it starts from a big area and suddenly it converges to the minimum area which is the throat and then we design this diverging portion based on the area which is available at the throat. Now, normally when you use this nozzle for perfect gas the divergence angle, referred as θ , is always kept below 20° . That is because if it is more, then there will be a breakaway of the fluid from the duct wall and it increases the friction losses. The other part of the nozzle is the diverging portion in which the velocity increases or flow accelerates. And whatever losses that is supposed to happen, will happen in the diverging portions only. So, this we say is the circular cross section of a nozzle which is used mainly for perfect gas and that to generate the thrust.

But you use this particular things in a steam nozzle because our main purpose is to expand the flow not to give thrust, but rather to give power. So, for that reasons, we call it as a curved axis steam nozzle. So, normally in our turbine viewpoint, we say that the curved blades are designed in such a way that, flow is aligned in a particular fashion and the shape of the nozzle is such that the axis is a curved axis & that typically gives the shape of this flow passage. So, steam nozzles are generally curved and when the nozzle is used for perfect gases it can be either circular or rectangular. And I have already said the main other difference, that is the main intention of convergent-divergent nozzles is to generate the thrust and this curved axis nozzle's main intention is to generate power.

Now, we will move on to next topic that is steam nozzles. Till this point of time we have been discussing about the theoretical aspects of nozzles which mainly deal with the gases. Now, when you talk about steam, same thermodynamic concepts can be used here with a reasonable approximations. For examples when you use the isentropic flow for gases, we use $pv^\gamma = \text{constant}$, but when you use this for steam, γ is replaced by a parameter k , which is nothing, but the polytropic index for the steam.

Then rest of the theory remains as it is. So, by replacing γ with k , we can have good estimates for critical pressure ratio, critical velocity at the throat and all other cases. Now, when you deal with this steam nozzles, the initial state of the steam can be at superheated

state or it can be at saturated state. So, there are two cases one is dry saturated state other is superheated state. For that a suitable approximation has been made for which, $k= 1.135$ for a dry saturated state of steam in the nozzle and $k= 1.3$ for superheated state of the steam entering in the nozzles. So, based on that we can simplify all these expressions, we can also find out the critical velocity of the throat. Of course, here another difference we see is that for enthalpies we have to use steam tables rather than $C_p T$ because it will not give good approximations. And based on that we can recall our tedious equations then we can find out what is the enthalpy difference between two states.

Isentropic law for steam: $pv^k = \text{constant}$ or $vp^{\frac{1}{k}} = \text{constant}$

$$\text{Critical pressure ratio, } \frac{p_c}{p_1} = \left(\frac{2}{k+1} \right)^{\frac{k}{k-1}}$$

Steam entering nozzle at dry-saturated state: $k = 1.135$ & $\frac{p_c}{p_1} = 0.577$

Steam entering nozzle at superheated state: $k = 1.3$ & $\frac{p_c}{p_1} = 0.546$

Critical temperature and enthalpy (T_c & h_c): use steam tables at p_c & $s_c = s_1$

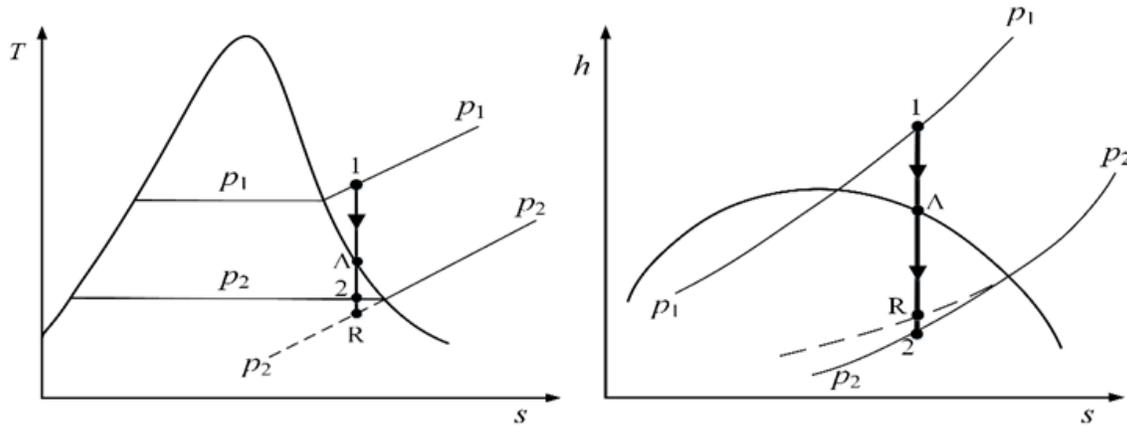
Critical velocity at throat: $C_c = \sqrt{2(h_1 - h_c)}$

Recall T-ds equation: $Tds = dh - vdp$; Isentropic flow: $dh = vdp$

$$\text{Integrating between two states, } h_1 - h_2 = \frac{k}{k-1} (p_1 v_1 - p_2 v_2) = \frac{C_2^2 - C_1^2}{2}$$

So, these are the some theory which we can apply to find the thermodynamic states, while dealing with the steam nozzles.

Now, moving further for steam nozzles, when you think about the expansion in the nozzles another practical phenomena that drops in, which we call as super saturations. Similar term super saturation, we also have encountered, when we were dealing with the steam turbine as well. Of course, for the sake of continuity, we will move little bit more, what is this state. This super saturation state is an irreversible phenomena and normally it is called as metastable state. So, now to understand what this means, you recall our Mollier diagram, h-s diagrams and T-s diagrams here.



We see that the steam in its superheated state that is state 1, expands from the pressure P_1 to P_2 and this expansion process is in isentropic manner. So, 1-2 is the expansion process which occurs in an isentropic manner. Now, when it expands, since it is a liquid vapor mixture, we can think of a dome construction. And while expanding from 1 the steam crosses this dome at point A, which means that we can divide this entire expansion process 1-2 as 1-A and A-2. So, here also in the Mollier diagram, we draw the process in this manner. When it reaches the point A that means, when it touches the saturation point A, the steam is said to be at metastable condition.

Why? Because it has been seen that, although it is a superheated state, yet it expands further down the line. That means, at point A, the condensation is supposed to occur. But unfortunately what happens is that, since the dynamics of this change is so fast that instantly the fluid does not condense. So, it has to take some finite time and after that finite time the condensation starts. So, instantly the condensation does not happen. So, the condition of the steam, at that particular state, is called as a metastable state and the expansion is known as super saturated expansion and this can occur within the nozzle or outside the nozzle.

Now if you look at the same figure here again, then what we can see practically is that, same process 1-2 and point A is defined at the saturation point or where the steam crosses the saturation point. And we will see that, from 2-R the state at which the steam is said to be super saturated which means that, if you can draw a particular line at that point R, we will have actual pressure is P_R and temperature will be T_R So, P_R & t_R are known to us. We will see that $P_R < P_2$ & $t_R < t_2$. So, this gives a practical information that steam has undergone the super saturation and the process from 2-R is a metastable state or irreversible phenomena and the actual value of 2 is represented as R.

Which means that we can define a temperature that will give information about super saturation temperature and degree of super cooling that is Δt .

Degree of super cooling, $\Delta t = t_2 - t_R$.

Other way of interpreting the super saturation phenomena is that we can find a parameter χ . "Degree of super saturation", $\chi = \frac{p_2}{p_R}$

We also 2 velocities, one is exit velocity, C_2 if the steam goes from 1 to 2, other is exit velocity at super saturation, C_R when it goes from 1 to R.

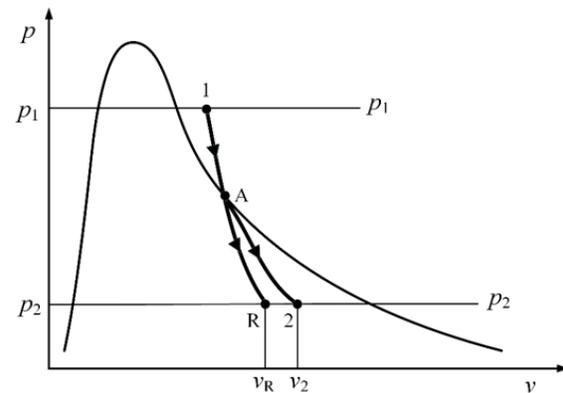
$$\text{Exit velocity, } C_2 = \sqrt{2(h_1 - h_2)} \text{ \& } C_R = \sqrt{2(h_1 - h_R)}$$

$$(h_1 - h_R) < (h_1 - h_2) \Rightarrow C_R < C_2$$

So, from this information we can say that $C_R < C_2$.

Now, moving further we can also get the information of enthalpy drop and the degree of super saturation.

Then going back to the final curve that is in the $P-v$ pressure and specific volume plot, it will give the information about the mass flow rate. Because for mass flow rate, we require the specific volume information. Now, same phenomena is being plotted that when the expansion takes place from 1 - 2 and we have 2 constant pressure lines P_1 and P_2 . While coming from 1 - 2, it intersects the dome at point A that means, at this point the steam interacts with the saturation point. At this point the steam tries to condense or vapor tries to condense, but instantly it does not condense at a time or suddenly. So, it takes some finite time to continue this condensation process. And during this process what happens is that, if the expansion takes place from 1-2, we will have a specific volume v_2 , and if the expansion has to take place from 1- R we will have specific volume v_R . So, as you can see your $v_R < v_2$. So, based on that we can apply the isentropic law for the steam that is from process 1- A that is,



$$\text{Isentropic law: } pv^{1.3} = \text{constant}; \text{Expansion}(1 - A)$$

$$\text{Isentropic law: } pv^{1.135} = \text{constant}; \text{Expansion}(A - R)$$

At given exit area(A_2), mass flow rate:

So, from these 2 information, we can find out 2 mass flow rates one is the mass flow rate when the fluid expands from 1- 2, other is mass flow rate when the fluid or steam expands from 1- R.

$$\dot{m} = \frac{A_2 C_2}{v_2} \text{ \& } \dot{m}_R = \frac{A_2 C_R}{v_R}$$

Since, $C_2 \sim C_R$ & $v_R < v_2 \Rightarrow \dot{m}_R > \dot{m}$

So, from this we can find out the mass flow rate in a saturated super saturation phenomena is higher than the mass flow rate for an equilibrium flow. So, basically why we are discussing this phenomena because of the fact that the concept was not known. But in reality when the mass flow rate was measured, then people tried to understand, why this mass flow rate is supposed to be higher than the mass flow rate in the conventional case or equilibrium flow case. Then so many theories were developed and the concept of super saturation was framed.

So, what it says is that, the experimental evidence from this mass flow rate leads to the discovery of the phenomena of super saturation in the steam nozzles. So, that means, people tried to understand the steam nozzles in a conventional way, but unfortunately when they found that the mass flow rate typically becomes higher than the equilibrium flow then they tried to find out what is the theory behind it. So, with this, we say that the super saturation phenomena is a very critical aspect for the steam nozzles as well as for the steam turbines. So, with this we conclude the nozzles. Now we will try to solve some numerical problems based on our discussions today.

Q1. A fluid enters the converging nozzle (7 bar & 100°C) and expands isentropically to a space maintained at 3.5 bar. Calculate the mass flow rate per square meter of exit area if the fluid is, (a) helium; (b) ethane.

So, the first problem talks about a situation where a fluid enters in a converging nozzle and it starts with an initial condition of 7 bar and 100°C and expands isentropically to a space which is maintained at 3.5 bar. So, we do not know the fluid condition. So, we have to find out what is the mass flow rate per square meter of the exit area of the fluid, if the fluid is first case helium and other case is ethane.

So, let us understand the physical concept of this. So, we have a converging nozzle to initial condition

$$p_1 = 7 \text{ bar}, T_1 = 100^\circ\text{C} = 373\text{K}$$

And condition 2 that means, at space. So, this space is maintained at $p_b = 3.5$ bar.

- a) So, let us start this problem for the case 1 when the fluid is helium. When the fluid is helium we also need to have some information about the helium gas.

$$C_p = 5.19 \text{ kJ/kg.K}$$

As helium is a mono atomic gas, $\gamma = 1.667$,
Characteristics gas content for Helium, $R=2079 \text{ J/kg.K}$.

So, now recall the expression for Critical pressure.

$$\text{Critical pressure ratio, } \frac{p_c}{p_1} = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2}{2.667} \right)^{\frac{1.667}{1.667-1}} = 0.487$$
$$\Rightarrow p_c = 7 \text{ bar} \times 0.487 = 3.4 \text{ bar}$$

This $p_c = 3.4 \text{ bar}$ is whereas, your $p_b = 3.5 \text{ bar}$.

Which means $p_b > p_c$, that means nozzle is not choked or in other words we say that back pressure does not allow the nozzle to be choked. As nozzle is not choked, we do not have to recall the expression for choking condition of the nozzle, because once the nozzle is choked, all other flow conditions were governed by the choked parameters. But here the nozzle is not choked, so we have to revisit our isentropic relation to get the value of velocity at the exit.

Isentropic relation,

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{7}{3.5} \right)^{\frac{1.667-1}{1.667}} = 1.32$$

$$\Rightarrow T_2 = T_1 / 1.32 = 373 / 1.32 = 285.6 \text{ K}$$

$$C_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2C_p(T_1 - T_2)} = \sqrt{2 \times 5.19(373 - 285.6)} = 952.5 \text{ m/s}$$

$$v_2 = \frac{RT_2}{p_2} = \frac{2079 \times 285.6}{3 \times 10^5} = 1.69 \text{ m}^3/\text{kg}$$

$$\text{Mass flow rate, } \dot{m} = \frac{A_2 C_2}{v_2}$$

The question is asked mass flow rate per square meter of area exit area.

So we need to find,

$$\frac{\dot{m}}{A_2} = \frac{C_2}{v_2} = \frac{952.5}{1.69} = 563.6 \left(\frac{\text{kg}}{\text{s}} \right) / \text{m}^2$$

So, this is what we get when the gas is helium. So, we understood that for helium gas, the nozzle flow condition was not choked.

b) Now, let us see what happens when it is ethane.

$$C_p = 1.88 \text{ kJ/kg.K}, \quad \gamma = 1.172, \quad R = 277.1 \text{ J/kg.K.}$$

So, first we need to find out what is critical pressure.

$$\text{Critical pressure ratio, } \frac{p_c}{p_1} = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} = \left(\frac{2}{1.172 + 1} \right)^{\frac{1.172}{1.172 - 1}} = 0.57$$

$$\Rightarrow p_c = 7 \text{ bar} \times 0.57 = 3.99 \text{ bar}$$

This $p_c = 3.99 \text{ bar}$ is whereas, your $p_b = 3.5 \text{ bar}$.

Which means $p_b < p_c$, that means nozzle is choked or in other words we say that back pressure does not allow the nozzle to be choked. So, once the nozzle is choked, we have to use the choking relation.

Choking relation,

$$\frac{T_c}{T_1} = \left(\frac{2}{\gamma + 1} \right) = 0.92$$

$$\Rightarrow T_c = 0.92 \times T_1 = 0.92 \times 373 = 343.5 \text{ K}$$

$$C_c = \sqrt{\gamma R T_c} = \sqrt{1.172 \times 277.1 \times 343.5} = 334 \text{ m/s}$$

$$v_c = \frac{R T_c}{p_c} = \frac{277.1 \times 343.5}{3.99 \times 10^5} = 0.2385 \text{ m}^3/\text{kg}$$

$$\text{Mass flow rate, } \dot{m} = \frac{A_2 C_2}{v_2}$$

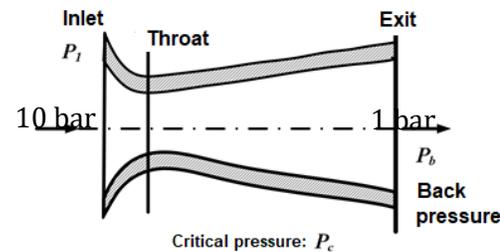
$$\Rightarrow \frac{\dot{m}}{A_2} = \frac{C_2}{v_2} = \frac{334}{0.2385} = 1413 \left(\frac{\text{kg}}{\text{s}} \right) / \text{m}^2$$

So, when the nozzle is choked for ethane gas, we find the mass flow rate per square meter of the exit area is $1413 \frac{\text{kg}}{\text{s}} / \text{m}^2$.

Q2. Calculate the critical pressure and throat area per unit mass flow rate of a convergent-divergent nozzle expanding steam from 10 bar (dry saturated) down to atmospheric (1 bar).

So, the last problem is a steam nozzle problem which says that in a steam nozzle, the flow expands in a convergent-divergent nozzle from a condition which is 10 bar dry saturated and it expands to a location, where it is atmospheric which means 1 bar. So, we need to find out the critical pressure and throat area.

So, this is your throat area A_c and try to plot it in a temperature entropy diagram. So, initially the steam is at superheated state, point 1 is can be located at the dome that is 10 bar dry saturated and it expands to a condition C and this condition is nothing, but your critical condition.



So, for that we have to recall the expression

$$\text{Critical pressure ratio, } \frac{p_c}{p_1} = \left(\frac{2}{k+1} \right)^{\frac{k}{k-1}}$$

Since it is dry saturated, $k = 1.135$.

$$\text{So, } \frac{p_c}{p_1} = \left(\frac{2}{k+1} \right)^{\frac{k}{k-1}} = \left(\frac{2}{2.135} \right)^{\frac{1.135}{0.135}} = 0.577$$

Initial condition, $p_1 = 10 \text{ bar}$, $p_c = 0.577 \times p_1 = 5.77$

So, basically steam expands from 10 bar to a critical pressure at throat. And once we know this critical pressure, then we can find out the other conditions. Other condition we require is the throat area & to calculate this throat area, we also require mass flow rate.

So, from this we have to say that for state 1, we have to refer steam table.

State 1: At $p_1 = 10 \text{ bar}$ (dry sat)

$$\Rightarrow s_1 = 6.5863 \frac{\text{kJ}}{\text{kg.K}}, h_1 = 2778.1 \frac{\text{kJ}}{\text{kg}}$$

State C: Critical condition: At $p_c = 5.77 \text{ bar}$ (liquid vapor)

$$\text{Dryness Factor, } x_c = 0.962, h_c = 2675 \frac{\text{kJ}}{\text{kg.K}}, v_c = 0.316 \frac{\text{m}^3}{\text{kg}}$$

I am omitting some of the steps to calculate this, because it is a constant entropy process. Maintaining same entropy at h_c , we can find out what is x_c and we can find out enthalpy at c and of course, we can find out specific volume. So, once you have this information, you can find out C_c .

$$C_c = \sqrt{2(h_1 - h_c)} = \sqrt{2 \times 1000(2778.1 - 2675)} = 454 \frac{\text{m}}{\text{s}}$$

$$\text{Now, } \dot{m} = \frac{A_2 C_2}{v_2}$$

In the question it is asked to find out the throat area per unit mass flow rate that is

$$\frac{A_2}{\dot{m}} = \frac{v_2}{C_2} = \frac{v_c}{C_c} = \frac{0.316 \text{ m}^3}{454 \text{ kg/s}} = \frac{0.316}{454} \times 10^6 \frac{\text{mm}^3}{\text{kg/s}} \approx 696 \frac{\text{mm}^3}{\text{kg/s}}$$

So, we got this critical pressure, $p_c = 5.77$ bar, we also find out the throat area per unit mass flow rate $696 \frac{\text{mm}^3}{\text{kg/s}}$. So, with this I conclude. Thank you for your attention.