Mechanics of Sheet Metal Forming
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Week- 03
Lecture- 07
Sheet Deformation in Plane Stress(contd)

So, we will continue our discussion where we stopped the previous class. So, we developed stress diagram from β plot. So, we have to convert this diagram to this diagram and so you know that how to convert β to α and finally we get OA, OB, OC, OD and OE in terms of stress diagram, okay. This is nothing but our yield locus and we can mention OA as $\alpha = 1$ and OB as $\alpha = 1/2$ and OC as $\alpha = 0$ and OD as $\alpha = -1$ and this is the last one where $\alpha = -\infty$ we say and then correspondingly there is β values. So, now we will see some details about each one. So, here you can see in the equi-biaxial stretching α is $\alpha = \beta = 1$ that is basically you are if you see the previous one it is OA, this is OA, OA path.

So, if you look into it since $\alpha = \beta = 1$ so we are going to develop some relationships which will be useful for our discussion and you will see that $\bar{\sigma} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$ and this is as per Von Mises yield function and here you will say that if you put $\alpha = 1$ you will see $\bar{\sigma} = \sigma_1 = \sigma_2$, okay. So, effective stress is nothing but the principle stresses and when you go to next one that is your OB, okay this is OB, okay. So, here you will see that $\beta = 0$ and $\alpha = 1/2$, right. $\alpha = 1/2$ here, okay.

So, $\alpha = 1/2$. So, here you will see that if you put $\alpha = 1/2$ here so you will get $\sigma_1 = \frac{2}{\sqrt{3}}\bar{\sigma} = 1.15\,\bar{\sigma}$ and $\sigma_2 = \frac{1}{2}\sigma_1$ because α is $\alpha = 1/2$, right. So, for a given flow stress here you will see σ_1 is greater in this process than any other we will see that, okay. Here σ_1 which is principle stresses is 1.15 $\bar{\sigma}$, 1.15 $\bar{\sigma}$, okay. And if you go to the next one uniaxial tension where $\beta = -\alpha = 0$ if $\alpha = \bar{\sigma} = \sigma_1$, okay and then you will see that your $\sigma_2 = 0$, $\sigma_1 = \bar{\sigma} = \sigma_f$ which is nothing but your flow stress, okay. So, you can see that both are equal here, $\bar{\sigma}$ and σ_1 and σ_f all are equal but here if you see that $\alpha = 1.15\,\bar{\sigma}$ and this occurs in tensile test.

If you go to the next one that is drawing where $\alpha = -1/2$ and $\beta = -1$, if you put $\alpha = -1/2$ here you will get the membrane stresses as $\sigma_1 = 0.58\bar{\sigma}$ which is just half of $\bar{\sigma}$, okay. If you go to the previous one, okay you will see that your $\sigma_1 = 1.15\,\bar{\sigma}$, here $\sigma_1 = \bar{\sigma}$ and here you will say $\sigma_1 = 0.5\bar{\sigma}$ and $\sigma_2 = -0.58\bar{\sigma}$. What does that means? That means if you compare this with the previous one the magnitude of stresses to cause deformation are at a minimum in this process, okay. In drawing, okay when you follow deformation path where $\alpha = -1$, $\beta = -1$ then you will have a magnitude of stresses which is going to be minimum that is in magnitude they are only 58% that means 0.58 times 58 % of stress required to yield a similar element in simple tension. So if you go to uniaxial tension, okay you are deforming the material and you want to yield it so then you need to go to $\sigma_1 = \sigma_f$ but at the same time same material same element you are going to deform it in drawing then it will be only 58 % of that, okay.

So it is going to be minimum in this process whereas you will see that it is going to be greater in this case in plane strain it is going to be greater why it is because 1.15 $\bar{\sigma}$. So here it is 1.15 times here it is 1 time and here it is only half of that 0.58 times, okay.

So in uniaxial compression you will see that since σ_2, σ_1 is 0 if you substitute here you will get σ_2 will be equal to on the negative side which will be your $-\sigma_f$ and this is going to create wrinkling on the flange of the sheet and we have seen this example uniaxial compression at the edge of the flange region. So you will see that you are going to have wrinkling because of this, okay. So this 5 parts OA, OB, OC, okay and then you have OD and then you have OE, okay all are coming from this particular one OA, OB, OC, OD, OE and you will see that if you are deforming in this particular you know stress path where $\alpha = -1$ you can deform the material to reach yielding, okay pretty easily and it is about 58 % of that what is required in uniaxial tension that is what we have shown in this particular comparison in this one and this one, okay. So just to complete the last section of this particular then we will go for one couple of problems you will see that principle tractions or tension, principle traction or tension, okay. So tension is nothing but force per unit length transmitted in the sheet, okay or you can also say it is traction, traction means just pulling, okay.

So $T_1 = \sigma_1 t$ and $T_2 = \sigma_2 t$. So $\frac{\sigma_2}{\sigma_1} \times \sigma_1 \times t = T_2$ and you will see that this is going to be your T_1, okay. So and this is nothing but your α. So $T_2 = \alpha T_1$. So $T_2 = \sigma_2 t = \alpha T_1$.

So $\frac{\sigma_2}{\sigma_1}, \frac{\sigma_2}{\sigma_1} = \alpha$. Similarly we can say $\frac{T_2}{T_1} = \alpha$. Both are one and the same. So now you can see that like σ_1 versus σ_2 you have a yield locus, okay. We can also have tension locus T_1 versus T_2 , okay.

So this can be obtained by from σ_1 and this can be obtained from σ_2, okay. And you will see that this there is an yield locus, okay. Similarly there will be a tension locus which is given here, okay. There will be a tension locus which is given. This is similar to what we have seen in yield locus, okay.

So there is something called $\bar{\sigma}$ we discussed, okay. And $\bar{\sigma}$ will lead to, $\bar{\sigma}$ is nothing but effective stress, right. So effective stress will lead to effective tension which is given by $\bar{T} = \bar{\sigma} t$. So you can directly write that as $\left(\sqrt{1 - \alpha + \alpha^2}\right)\mathrm{T}_2$, okay. So $\bar{\sigma} = \left(\sqrt{1 - \alpha + \alpha^2}\right)\sigma_1$, right.

So this will be T_1, okay. This has to be T_1 I think, okay. What is this one? Into σ_1 now. So this will be T_1, okay. So if you know this $\bar{T}$ then you can get this yield locus, right.

So we are not going to use it. Rather we are going to use only the yield locus. This will be helpful for us when you evaluate the tensions. And now we are going to introduce something which will be useful for our next chapter, okay. So what is that? Suppose in this equation $T_1 = \sigma_1 t$, okay.

$T_1 = \sigma_1 t$. In this equation, okay, if you use the material law $\bar{\sigma} = \mathrm{K}\,\bar{\varepsilon}^n$ then what will happen? You can say that this $\sigma_1 = \frac{\bar{\sigma}}{(\sqrt{1-\alpha+\alpha^2})}$, isn't it? So $\bar{\sigma} = \mathrm{K}\,\bar{\varepsilon}^n$. So this you can combine as 1 and $T = t_0 \exp[-(1+\beta)\varepsilon_1]$. So if you combine this you will get this particular equation. So $T_1 = \sigma_1 t$ where $\sigma_1 = \frac{\bar{\sigma}}{(\sqrt{1-\alpha+\alpha^2})}$ and $T = t_0 \exp[-(1+\beta)\varepsilon_1]$.

This we have developed in the previous section itself, okay. So now you will see that this equation is a function of some material properties and your β and α and your ε_1 strain, right. K and n you know strength coefficient and strain hardening exponent. t_0 is initial thickness. β is you know your strain ratio and α is the stress ratio and finally only variable is ε_1 and for ε_1 you can get $\bar{\varepsilon}$ also because by knowing β you can get ε_2, ε_1 and ε_2 can be obtained.

So ε_3 can be obtained. From this you can get $\bar{\varepsilon}$. Finally you will see that T_1 as a function of T_1 as a function of ε_1 and T_2 as a function of ε_1 can be drawn and here it is given, okay. So from this we can get T_1 and T_2 where T_1 and T_2 are related by $T_2 = \alpha T_1$, okay. So from this equation you can fetch any values for this and you can get T_1 versus T_2 here, T_1, okay or T_2 versus , ε_1 you will get. So and you will see that for those cases when you have $\beta > -1$, $\beta > -1$ means that means this fellows, $\beta > -1$ means this side, okay.

So for those cases where you have thinning, for those cases where you have thinning you will get maximum load, okay. You will get maximum load and after that you will see that maximum load is going to decrease or tension is going to decrease, okay. So T_1 is obtained and T_2 can be obtained by knowing α, right. So if you want to find this maximum tension, okay, so then you can get it by taking $\frac{dT_1}{d\varepsilon_1} = 0$ and if you solve it you will get $\varepsilon_1{}^* = \frac{n}{1+\beta}$ and this value will be this one from 0 to this, okay. So you can solve it and find out, okay or you can do it graphically also where you can choose K, n, t_0, β, ε, α you can choose and if you change ε_1 as a variable you will get T_1 and T_2 in this way and only for $\beta > -1$ you will get such situation, this kind of situation.

If it is < -1 then it will never come down, okay because β is not going to be, you know, > -1then thinning will not happen, okay then you will not see this maximum tension, okay. So only if you have $\beta > -1$ you will get this particular situation and the maximum tension can be obtained when you have $\varepsilon_1{}^*$. We are going to call this as star because it is going to tell the maximum value $\varepsilon_1{}^* = \frac{n}{1+\beta}$ where n is your strain warning exponent and β is your strain ratio, β is your strain ratio, okay. So if you know $\varepsilon_1{}^*$ you can get $\varepsilon_2{}^*$ star as $\varepsilon_2{}^*$, sorry $\varepsilon_2{}^* = \beta\varepsilon_1{}^*$considering proportional process the β is not going to change let us say. So now for uniaxial tension suppose $\beta = -1/2$, okay, $\beta = -1/2$ if you put $\varepsilon_1{}^* = 2n$ and for plane strain if $\beta = 0$ you will get $\varepsilon_1{}^* = n$.

What does that mean? That means if you deform a material in uniaxial tension, okay, so the material is going to fail at a strain of 2 times of strain warning exponent at the same time if you deform the material, same material when you have plane strain process $\varepsilon_1{}^* = n$, okay. So what does that mean? That means the material can extend to a larger value of you know ε_1 when you deform the material in uniaxial tension as compared to plane strain. So in plane strain the material can fail or material can reach this maximum value which is a indication of something like instability, okay, then that will be reached early in plane strain process, okay, at $\varepsilon_1{}^* = n$. So if $n = 0.22$, so $\varepsilon_1{}^* = 2 \times 0.22 = 0.44$ in case of uniaxial but the same time it is plane strain means it is 0.22 only, okay. So in that case the material is going to reach failure or instability in plane strain much early as compared to uniaxial, okay. This is going to lead to a good next chapter some theories we are going to develop for instability this will be a basis for this. So where once this star indicates that material will be in some sort of instability or a maximum load is reached, maximum tension is reached, okay.

So we will stop this theory part with this. We will solve two problems in this now, okay, which are going to be useful for us, okay. So first problem is, okay, a small circle of 5 mm diameter is printed on the surface of undeformed low carbon steel sheet with thickness of 0.8 mm. So, t_0 is nothing but 0.8 mm. So we will take t_0 as 0.8 mm and initial diameter of the circle let us say d_0 as 5 mm. Let us pick up this way and it is a plane stress proportional process and it is a plane stress proportional process. So after deformation the major dimensions are given, minor dimensions are given. What are they basically? It is let us say d_{major} as you can call it as 6.1 mm and d_{minor} as 4.8 mm, okay, and effective strain relationship is a standard one. So we need to get α here and then T_1 and T_2 and then you can get effective strain.

So as we discussed in the previous chapter, so now the point here is once the dimensions are given we have to get principle strains, right. So ε_1 is you can directly get $ln\left(\frac{6.1}{5}\right)$, okay, that will be about 0.199 and then ε_2 can be obtained $ln\left(\frac{4.8}{5}\right) = -0.041$ which is going to be negative value. If you know these two you can get $\varepsilon_3 = -(\varepsilon_1 + \varepsilon_2)$ as which will be this, okay. So now we need to get α, so for that we need to get β because strains are known, from β you can get α .

So what is β? $\frac{\varepsilon_2}{\varepsilon_2}$, so $\frac{-0.041}{0.199} = -0.21$ and from here you can get α as 0.324, okay. So α has been found out. So now α has been found out, it is fine, then we need to get the effective strain, so that is also another question. So it is understood that it is Von Mises equation. So $\bar{\varepsilon} = \sqrt{\frac{4}{3}[1 + \beta + \beta^2]}\varepsilon_1$, so you substitute β, this particular β value here you will get $\bar{\varepsilon} = 0.21$. And the tensions T_1 and T_2 can be obtained, we just now derived it, okay.

T_1 and T_2 we just now derived, isn't it? So T_1 is this. If you know T_1, $T_2 = \alpha T_1$. So $T_1 = \sigma_1 t = \frac{K \cdot \bar{\varepsilon}^n}{\sqrt{1-\alpha+\alpha^2}} t_0 \exp(-(1+\beta)\varepsilon_1)$. So all values are known to us, so this is your K, right and this is your n, so all are given.

So $\frac{600 \cdot 0.21^{0.22}}{\sqrt{1-0.324+0.324^2}} \times 0.8 \times \exp(-(1-0.21) \times 0.199)$. If you calculate T_1, it is going to be 329.3, about 329 kN/m, okay. So you have to check the units, please check the units, it has to be consistently used, please check it. And T_2 you can obtain by αT_1.

What is α? It is already known to us $0.324 \times T_1$, it will be just one third of this, so this will be your 106.7 kN/m, okay. So this is one easy problem, okay. So these all are known to us from the previous chapter, ε to get ε_1, ε_2, ε_3, β and then α, $\bar{\varepsilon}$ we already done.

Only difference is now we calculated, we derived an equation for T_1 and we derived an equation for T_2 and T_1 and T_2 are nothing but major tension, minor tension, okay. So that is the only extension in this chapter and you know how to get it. Similar problem, okay. So suppose you are doing deep drawing, okay, so this deep drawing, cup deep drawing you know. The strains in the center of the base A, so this point is let us say A, halfway up the cup wall, this is your cup wall B, okay and in the middle of the flange that is in C, okay.

These three values are given as 0.015, 0.015, 0.050, 0.15, −0.1. These all values are given, okay. This is nothing but strains that is ε_1 versus ε_2, okay. ε_1 and ε_2 are given for these three locations A, B and C.

This A is here, B is somewhere in the middle and C is somewhere in the middle here, okay. The strain hardening of the material is negligible so that effective stress is constant, okay. So here they are saying $\bar{\sigma}$ as a constant value. Constant value means what? Okay, there is no hardening in the material. So which means that your flow stress is going to remain constant at 300 MPa which is easy for us to calculate.

Initial thickness is given as 0.5, so t_0 is given, okay. So what do you need to get is basically at each point, this A, B, C you have to get the new thickness and major tension that is T_1, new thickness t and major tension T_1. That is all you need to get. So now how do we go ahead? So like now strains are given, so directly we start getting one by one what was the requirement we have. So you need to get t, that is the new thickness, right. So new thickness depends on the original thickness by $t = t_0 \exp(\varepsilon_3)$, correct.

So now here you need to get ε_3 because t_0 is known to you, t_0 is already given, okay. ε_3 has to be found out. So $\varepsilon_3 = -(1+\beta)\varepsilon_1$, right. So for this β should be known, for this ε_1 also should be known which is already given for each points.

So how do you get β now? β is nothing but $\frac{0.015}{0.015} = 1$, then $\frac{0}{0.05} = 0$, $\frac{-0.1}{0.15} = -0.667$. If you put β is equal to 1, 0, -0.667, in this you can get α. You know that for $\beta = 1$, $\alpha = 1$. For 0 it is 1/2 , this is known, okay.

This is balanced by axial stretching and this is plane strain, okay. So and here this is going to be a variable, okay. It is 0.251, okay. So it is plane strain.

Plane strain is half, isn't it? So plane strain is 1/2. We say plane strain is 0, $\beta = 0$ and $\alpha = 1/2$. β =0. $\beta = 0$, $\alpha = 1/2$.

That has been obtained. So now everything is known, okay. So all the values are known here. ε_1 is already given in the value 0.015. So $-1+\beta$ is 1. You substitute everything here. You will get this value, this value and this value for the different parts, A location, B location and C location, right. So now our question is there is only one variable that is ε_3 here that you can substitute in this.

You will get new thickness 0.485, 0.456, 0.456, right, 476. So there is less decrement in balance by axial. You will see more change in thickness in case of these two. Now you want to get T_1 . What you need to do? You know $T_1 = \sigma_1 t$.

So t has been found out already. You need to get σ_1. $\sigma_1 = \frac{\bar{\sigma}}{(\sqrt{1-\alpha+\alpha^2})}$. That can be found out, right. So $\bar{\sigma}$ is nothing but already given, 300 mega Pascal. You have to just substitute as it is because it is not going to be variable. Divided by $\sqrt{1-\alpha+\alpha^2}$, you can substitute α value and you will get 300, 346 and 262, right.

So you can see that $\bar{\sigma}$ and σ_1 are equal, isn't it, for balanced by axial stretching, you know. So that is what we have seen in the previous one. You know, you can see that $\sigma_1, \bar{\sigma}$ are equal, isn't it.

So that is what we got here in this problem as well. So which means it is good. So then if σ_1 is known to you, $\sigma_1 t$ will give you T_1, that is major tension. t is already known, okay.

This is about half of each, 300 into 0.485, 340 into 0.476, 262 into 0.476, you will get these three values. So now by knowing, you know, T_1 and T_2, you can get not only the tensions corresponding to the principal stresses but also at which stage of deformation you will get maximum tension which is equivalent to some instability that is going to start. That can be obtained when you put $\varepsilon_1{}^* = \frac{n}{1+\beta}$, right. So basically in this chapter, we were discussing about how to find your strains at different locations of the sheet and then some concept called strain signature we introduced, that is through strain diagram. Then we introduced how to interpret strain diagram, some important features we understood, then how to convert that into stress diagram and then some important features of this five different α s and β s we understood and at the end we introduced major tension, minor tension, how to get T_1 and T_2 from σ_1, σ_2 has been found out and some problems we solved just now, two problems, two examples, okay.

So this, a small circle means it is given in one location, right. Similarly you can imagine a sheet of 200 mm by 200 mm dimension. In all that location you have so many such circles, okay and in each circle you will have so many, you know, variables inside. Each location will have $\varepsilon_1, \varepsilon_2, \varepsilon_3$ will be varying, each location will have its own β α, then $\bar{\varepsilon}$ will come and tension $T_1$and T_2 will change accordingly, okay. And this example is the best one to represent that. So different locations A, B and C, you will see that A is deforming in this fashion, B is deforming in this fashion and C is deforming in this fashion, okay and your α, β, thickness strain, σ_1, σ_2, everything is going to change as per the location and strains that we get. So we stop here. So in the next class. Thank you.