Mechanics of Sheet Metal Forming
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Week- 02
Lecture- 04
Sheet Deformation Processes (contd)

So, we will continue our discussion in this module 2. So, in the in this particular lecture. So, in the previous section what we discussed was basically I introduced this yield locus. So, we know how to find the onset of plastic deformation or yielding occurs when you go for any axial type of deformation. Now you have general state of deformation and in that we consider plane stress because of sheet deformation. So, we need something called as yield locus that is what we discussed in the previous section.

So, we are going to continue our discussion you know here ok. So, this yield locus or yield surfaces in 3D ok can be described in the form of some expressions or equations ok. And they follow something called as yield functions or yield criterion as said here ok. And there are several yield functions or yield criteria available for the materials that we are discussing ok.

So, we will discuss some of them in this particular subject. So, the first one is basically called as Tresca yield criterion. We are going to discuss one more yield function after this ok. And after that you know little later in this course we are going to introduce 2, 3 other yield functions ok not in detail ok. So, these two we are going to discuss in detail.

The first one is called as a Tresca yield function or yield criterion. So, the statement as per this Tresca yield criterion ok it goes like this. The statement is yielding occurs ok so or yield point is reached you can say yielding occurs when the greatest maximum shear stress reaches a critical value. Greatest maximum shear stress reaches a critical value. So, in the previous section we have seen τ_1 , τ_2 , τ_3 ok.

So, in that the greatest one ok if it whenever reaches a critical value then we say that yielding occurs as per this criteria ok. So, the greatest is basically you can say for example in general if you want to write it in the form of an equation we can say we had in the previous you know section we had τ_1 , τ_2, τ_3 which will be σ_1 say for example we said here that it is $\frac{\sigma_1-\sigma_2}{2}$, $\frac{\sigma_2-\sigma_3}{2}$ and $\frac{\sigma_3-\sigma_1}{2}$ is not it. So, out of this we are going to pick up one in particular but in general we can say $\frac{\sigma_{max}-\sigma_{min}}{2}$ ok and if it reaches a critical value so what is the critical value ok can be obtained by using uniaxial tensile test type of situation where σ_2 and σ_3 are actually 0 ok. If you put that in any one of that equations let us say σ_2 and σ_3 are 0 so then there will be $\frac{\sigma_1}{2}$ only and that σ_1 we are going to call it as σ_f. So, the $\tau_{crit.}$ can be written as $\frac{\sigma_f}{2}$.

So, I am going to write in general that $\frac{\sigma_{max}-\sigma_{min}}{2} = \tau_{crit.} = \frac{\sigma_f}{2}$ which can be written in this form and with the convention generally if you find all the principle stresses let us say $\sigma_1, \sigma_2, \sigma_3$ ok in the previous section I was telling you one should know how to find this principle stresses given a stress tensor but then if you know how to find out this then it can be arranged in this convention ok. $\sigma_1 \geq \sigma_2 \geq \sigma_3$ this is the convention with which we are going to arrange it. So, if we arrange it like this then out of τ_1, τ_2, τ_3 we are going to pick up only one ok τ that will that can be written as $\sigma_1 - \sigma_3 = \sigma_f$ where this would be the highest value and this will be the lowest value we can say $\sigma_1 - \sigma_3 = \sigma_f$ ok. So, now if you see that we put a condition that if it is uniaxial tensile test for example to test it then $\sigma_3 = 0$ so $\sigma_1 = \sigma_f$, $\sigma_1 = \sigma_f$. So, σ_f is nothing but your yield strength or in general you can say flow stress ok.

Flow stress means the first time it is going to yield let us say that is nothing but your yield strength. So, we are going to use σ_f you know uniformly in all the slides ok. So, now if you can say uniaxial tensile test then σ_1 will become σ_f if that is the case then yielding will start is the thing that we already know that we already discussed in the first section itself whenever reaching the yield strength material is going to enter into plastic deformation right. So, now we are going to have another case like uniaxial you have this you also have yielding in pure shear ok that condition can be written as $\sigma_1 = k$ and $\sigma_3 = -\sigma_1 = -k$ ok. So, $\sigma_3 = -\sigma_1$ is nothing but your pure shear they are equal and opposite in nature they are equal and opposite in nature and that will be equal to $-k$ what is k here k is the shear yield strength of the material this should not be confused with strength coefficient that we have studied in the previous section $\sigma = K\varepsilon^n$ that K is called as strength coefficient that is different this k is called as shear yield strength.

So, during shear deformation there must be some yield strength ok and that is called as a small k here ok. So, when we say $\sigma_1 = k$ and $\sigma_3 = -\sigma_1$ they are opposite in nature and equal that will be equal to $-k$ ok then under pure shear we can write this equation $\sigma_1 - \sigma_3 = 2k$. So, if you can substitute it here you will get $2k$, $\sigma_1 = k$ minus $\sigma_3 = -k$ so plus it will become *2k* it will become *2k* right. So, here one should note that there is no σ_2 part at all in this equation ok in both the equations there is no σ_2 coming into picture because we are referring to σ_1 and σ_3 ok. So, now combined way of writing this ok with respect to your this particular equation ok and combining this equation with this so Tresca yield criterion can be written as $\sigma_1 - \sigma_3 = \sigma_f = 2k$ ok.

This is the appropriate expression important expression with respect to Tresca yield function and you can say that this also gives you relationship between uniaxial yield strength to shear yield strength which is nothing but $\sigma_f = 2k$ ok. So, your uniaxial yield strength will be twice that of your shear yield strength ok or if you want to find k then you can divide this by 2 which will give you k ok. So, shear yield strength of any material if you want to find out if you follow a Tresca yield function then it would be $\frac{\sigma_f}{2}$ ok. So, this Tresca yield function generally can be drawn in a plane of paper in this format ok this is just a schematic of you know the Tresca yield function and is generally written as a hexagon which is not actually regular in nature ok. It is drawn as hexagon which is not regular it is not a regular hexagon ok.

So, we are going to draw between σ_1 and σ_3 in this case ok and you are going to say that so you have this kind of hexagon I am just redrawing here this is the second zone third zone fourth zone then you have fifth zone then you have sixth zone ok. So, zone 1, 2, 3, 4, 5, 6 so in all this zones you can show the state of stress schematically I have drawn here ok. So, one should also know that when you follow one particular α when you follow one particular α , α you know σ_2 by α is nothing, but $\frac{\sigma_2}{\sigma_1}$ that is a proportional path we are going to pick up then once we reach here it means we are entering into plastic deformation. So, on set of plastic deformation would be there. So, our yielding is going to start that is why you are written it as α this α could be any α in this space ok.

So, in this 6 zones you will see the first one you will see that both are pulling type ok this if you pick up an element from a material which will be in this in this zone let us say zone number 1 ok you will see that this element will be pulled in both the directions tensile in nature. If you come to second zone the same element ok will be in pulling only, but different proportion, but they will have different proportion maybe you should pick up one α here they will have different proportion. If you come to zone number 3 ok. So, you will see that one will be pulling another will be compression one will be in tension other one will be in compression is not it. So, your σ_1 is negative is not it.

So, σ_1 is actually negative. So, σ_3 is anyway positive here. Come to zone 4 ok. So, both will be compression ok both the axis both the elements will be compression in both the sides ok. Same case in 5 also, but 5 will be of different proportion as compared to 4 and when you come to 6, 6 is just opposite to this.

So, you will have pulling on one side, but pushing on the other side compression side, but it is σ_1 and σ_3. So, σ_1 is pulling and σ_3 is actually compression type ok. So, you may see in sheet deformation sheet forming process if you want to make any component you may end up in different state of stress like this at different locations and as per this particular α you are going to reach the yield point in that particular locus that is the meaning of this ok. So, now during strain hardening ok this is the first yield locus. In the previous section we have discussed that the first locus is nothing, but the initial yield locus we say.

So, we can call this as let us say initial yield locus ok. We can call it as initial yield locus. Now, with further deformation because of strain hardening what will happen your σ_f will try to increase ok. So, you can see that $\sigma_3 = 0$ ok $\sigma_1 = \sigma_f$ here. This is what this equation is going to tell you now.

This is what this equation is going to tell you when $\sigma_3 = 0$, $\sigma_1 = \sigma_f$ that is your uniaxial yield strength here ok. If this is initial yield locus ok. So, with strain hardening the σ_f is going to increase ok. So, this is very brief idea about Tresca yield function which is going to tell you when the material is going to start plastically deforming or it will start permanently yielding. Now, a similar one which is also very you know profoundly used in a metal forming is nothing, but Von Mises yield function or Von Mises yield criterion ok.

So, what is the statement here? When the root mean square value of maximum shear stress reaches a critical value yielding is going to start that is the statement. When the root mean square value of the maximum shear stresses ok. So, you have τ_1, τ_2, τ_3 with respect to that you have to get root mean square and if it reaches a critical value then we say yielding is going to start. So, this is what is written here ok. This is a root then it is mean of the squared terms.

So, $\sqrt{\frac{{\tau_1}^2+{\tau_2}^2+{\tau_3}^2}{3}}$. When it reaches a critical value yielding will start. So, what is the critical value we do not know. So, what are we going to do is again we are going to use our own friend that is your uniaxial tensile test which is easy for us to put the condition. So, you know that σ_1 exists in σ_1 is going to be there, there is some value for σ_1 in this $\sigma_2 = \sigma_3 = 0$.

So, what is τ_1? $\tau_1 = \frac{\sigma_1}{2}$, $\tau_2 = 0$, becauseσ_2 and σ_3 will be there, $\tau_3 = -\frac{\sigma_1}{2}$ right. So, I am going to substitute all these things in this equation ok. So, $\sqrt{\frac{\left(\frac{\sigma_1}{2}\right)^2+0+\left(-\frac{\sigma_1}{2}\right)^2}{3}}$. So, there is only one σ here principal stress. So, I am going to use f, there is nothing but my σ_f, σ_1 becomes σ_f here in general σ_f ok because there is only one σ here ok. So, if you calculate this particular function then finally, you can write this 3 way will go off ok you can write $\sqrt{2(\tau_1^2+\tau_2^2+\tau_3^2)}=\sigma_f$. This is the first level of equation for von Mises yield function. Though this is not used you know mostly ok because there is τ_1, τ_2, τ_3 which can be related to $\sigma_1, \sigma_2, \sigma_3$ and that will be the best equation to use for any sheet deformation or any metal deformation process if you want to model it.

So, ok. So, now this since this is one stage of you know equation of course, this is one expression for one masses you can say. Now instead of τ_1, τ_2, τ_3 ok. So, I am going to write this in terms of principal stresses ok. So, again $\sigma_1, \sigma_2, \sigma_3$ I want to rewrite this entire thing τ_1, τ_2, τ_3 now because you know what is τ_1, what is τ_2 and what is τ_3 in terms of $\sigma_1, \sigma_2, \sigma_3$ we have studied that. I have not derived it here, but it is for you to work it out on a small you know paper ok you can easily get it.

So, if you substitute τ_1, τ_2, τ_3 instead of that if you write $\frac{\sigma_1-\sigma_2}{2}$, $\frac{\sigma_2-\sigma_3}{2}$ and $\frac{\sigma_3-\sigma_1}{2}$ and then substitute here and then solve that equation finally, you will get this particular form which is nothing, $\sqrt{\frac{1}{2}\{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\}}=\sigma_f$. This is also written in several resources as $(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2=2\sigma_f{}^2$ ok. You take square in both sides and then bring 2 here it will be $2\sigma_f{}^2$ it will be $2\sigma_f{}^2$. So, that is a one this is one important you know stage of this your Von Mises equation and this is a famous equation with respect to Von Mises yield function. So, now we can also rewrite this equation in terms of deviatoric stresses ok.

So, why because you can always write $\sigma_1, \sigma_2, \sigma_3$ as a function of $\sigma_1{}', \sigma_2{}', \sigma_3{}'$ these are all deviatoric stresses right. So, the $\sigma_1{}', \sigma_2{}', \sigma_3{}'$ can be related to $\sigma_1, \sigma_2, \sigma_3$ in this way ok. $\sigma_1{}'=\frac{2\sigma_1-\sigma_2-\sigma_3}{3}$ this we have studied before this we got it when we discussed about the hydrostatic stress. You remove hydrostatic part from your general state of stress then you will get a $\sigma_1{}', \sigma_2{}', \sigma_3{}'$ that we already discussed. I am putting etcetera here which means like $\sigma_1{}', \sigma_2{}'$, and $\sigma_3{}'$ equation which you have to use ok and rewrite that $\sigma_1{}', \sigma_2{}'$, and $\sigma_3{}'$ ok in terms of or rewrite $\sigma_1, \sigma_2, \sigma_3$ in terms of $\sigma_1{}', \sigma_2{}', \sigma_3{}'$ and then you have to substitute it in this equation and some steps are available then finally you will get yield condition as $\sqrt{\frac{3}{2}\left(\sigma_1{}'^2+\sigma_2{}'^2+\sigma_3{}'^2\right)}=\sigma_f$ ok.

So, this is another form of same equation ok. So, either this form or this form is majorly used well known form or this form can also be seen in the form of to describe your Von Mises yield function ok. Again I am reporting here that like $\sigma_1{}'$ you need to have equation for $\sigma_2{}'$, and $\sigma_3{}'$ and rewrite your $\sigma_1, \sigma_2, \sigma_3$ in terms of $\sigma_1{}', \sigma_2{}', \sigma_3{}'$. So, that you can substitute in this equation and you will get this particular equation after few steps and this also in a way tells you about Von Mises yield criteria ok. So, Tresca and Von Mises yield condition they have different statement ok, but finally will tell you when material is going to yield.

So, if you pick up this particular equation which is predominantly used ok which is predominantly used you will see that the nomenclatures are $\sigma_1, \sigma_2, \sigma_3$ are again principle stresses and σ_f is nothing, but your yield strength of the material or we can simply say σ_f here. So, now this is I am going to pick up this equation this is a very famous equation we are going to pick up this equation and if I am going to put plane stress if I want to put plane stress condition here that means, $\sigma_3$is going to become 0, σ_3 is going to become 0. So, this fellow will go off and this fellow will go off ok. So, this you have to expand $(a-b)^2$ this is ${\sigma_2}^2$ this is minus ${\sigma_1}^2$.

So, ${\sigma_1}^2$ ok. So, that is what I am going to do it here because we say sheets are going to follow a plane stress type of deformation during any component manufacturing. So, you can rewrite the previous equation ok the form of σ_1, σ_2 when you go for plane stress. So, plane stress meaning σ_3 will be equal to 0 here ok. The previous equation if you substitute it you will get this particular equation ok and this equation can be further simplified as this in this form ok. So, because this is $2{\sigma_1}^2$ then $2{\sigma_2}^2$ is this ok then 2 will come out all will be cancelled.

So, you have $\sqrt{({\sigma_1}^2 + {\sigma_2}^2 - \sigma_1\sigma_2)} = \sigma_f$ and since it is a function of σ_1 and σ_2 I can rewrite this in terms of α . $\alpha = \frac{\sigma_2}{\sigma_1}$ which will give me a simple equation $\left(\sqrt{1-\alpha+\alpha^2}\right)\sigma_1 = \sigma_f$ right. So, what does it mean? If we closely look into look into it so, if I know σ_1 and if I know α I can get σ_2 let us say or if I know σ_1 and α ok then I can substitute in this equation and I can check ok if it is equal to σ_f or not if that is going to happen the material is going to yield ok. So, this equation ok in principle stress space because a function of σ_1 and σ_2 ok can be drawn in this fashion ok σ_1 in Y axis σ_2 in X axis if you see ok it is generally in the form of a fantastic ellipse ok and the same α which I mentioned in Tresca is written here ok you pick up one path if it reaches this particular locus here then you are into P, P means plastic deformation. So, yielding is there and after that you are going to have a plastic deformation that is the meaning right.

So, you can follow any α you want ok. So, this is a von Mises locus in plane stress condition ok von Mises locus in plane stress condition which is an ellipse actually ok von Mises locus in plane stress is going to be an ellipse like this which can be drawn like this ok and this fellow is going to be your σ_f is also uniaxial your σ_f yield strength if it is initial yield locus if it is initial yield locus this is your σ_f. You can also put that condition say for example, if you put $\sigma_1 = 0$ here ok. So, then it will be σ_f would be $\sigma_2 = \sigma_f$. So, you can check it ok σ_1 is let us say $\sigma_2 = 0$ ok. So, this fellow will go this fellow will go square root.

So, $\sigma_1 = \sigma_f$ ok. So, in that way you can evaluate and it can also be shown that the for this is an ellipse right. So, this is a semi major axis it can be written as a $\sqrt{2}\sigma_f$ and semi minor axis can be written as $\sqrt{2/3}\sigma_f$ and I have shown a small derivation here for that. So, same diagram ok. So, Y axis σ_1 and σ_2 here ok σ_1 and σ_2 here and let us pick up one particular stress path ok defined by $\sigma_2 = \sigma_1$. So, $\alpha = 1$, you can say $\sigma_2 = \sigma_1$and I am going to reach this particular point let us assume that this is reaching the yield locus this reached the yield locus ok.

This distance I am going to call it as small a let us say ok. So, because it is reaching yield locus I can say this could be one σ_f in that particular locus. So, this is my first equation which I have derived before right. So, $\sqrt{(\sigma_1{}^2 + \sigma_2{}^2 - \sigma_1\sigma_2)} = \sigma_f$ right. So, this is nothing, but my this equation this particular equation $\sqrt{(\sigma_1{}^2 + \sigma_2{}^2 - \sigma_1\sigma_2)} = \sigma_f$ correct.

So, now, in this particular path I can say $\sigma_1 = \sigma_2$ ok. So, when I am substituting it here you will find out this particular simple expression $\sigma_1 = \sigma_2 = \sigma_f$ which is also mentioned in that diagram ok. So, now, what is a from this figure? $a = \sqrt{\sigma_1{}^2 + \sigma_2{}^2}$ ok. So, I am going to replace it here ok with this condition. So, $\sqrt{\sigma_f{}^2 + \sigma_f{}^2} = \sqrt{2}\sigma_f$.

So, 2 times the $\sqrt{2}\sigma_f$ will become square will become σ_f ok which is what we have shown in that semi major axis right. So, this will $\sqrt{2}\sigma_f$. So, when you go to the other zone when you go to the next zone let us say you are going to pick up this side ok you are going to pick up this side ok this this should be like this ok the if you are going to pick up this particular side ok. So, then what we are going to or in the other side we can say same σ_1, σ_2 is there. So, I am going to pick up this particular point which is let us say touching the yield locus ok and this b strain path can be written as or a stress path can be written as $\sigma_1 = -\sigma_2$ ok $-\sigma_2$ when it reaches yield locus I am going to call this location as σ_f ok.

So, now same equation when $\sigma_1 = -\sigma_2$. So, I will get $\sigma_1 = \sigma_f/\sqrt{3}$ you can check it. So, now this b figure will is going to tell you $\sqrt{\sigma_1{}^2 + \sigma_2{}^2}$ which is nothing but square root of you can substitute these two you know conditions here ok and finally, you will get $\sqrt{2/3}\sigma_f$ ok. In this way one can prove that your a which is a semi major axis and b which is semi minor axis $\sqrt{2}\sigma_f$ and $\sqrt{2/3}\sigma_f$. In many resources this ratio they mention ok the ratio is mentioned ok your major to minor ratio.

So, $\frac{\sqrt{2}\sigma_f}{\sqrt{2/3}\sigma_f}$ this ratio ok you can calculate what is it, it could be $\sqrt{3}:1$,could be $\sqrt{3}:1$. So, that is also very important result for us ok. So, now let us take for example if you take von Mises you know yield locus. So, von Mises yield locus where do you see such situations just example I am telling you ok. So, this is basically I would say this is basically a partially I will say drawn cup ok you can say partially drawn cup right.

So, if you rotate it through $360°$ you will get a full cup and this is actually called as flange region right and this is your cup wall this we already introduced in the first class and this is your cup bottom ok cup bottom region right. So, this is just one quarter of that a part of that is shown in this diagram ok for easy understanding ok and this is your sheet thickness ok this is your sheet thickness instantaneous thickness you can say ok and this was your initial sheet this is your initial situation and it is a partially drawn to this much of height ok. So, now this is a conventional yield locus and an elliptical yield locus is drawn only a part is drawn just for explanation. So, now where are the situations suppose you pick up this particular stress path ok if you particular this particular α let us say it is called as a biaxial it is called as biaxial deformation because your σ_1, σ_2 is going to be equal ok. So, that means an element which is undergoing this type of deformation will have this type of pulling ok it is pulled in both the directions ok.

You will see this type of situation in your cup bottom you can see here in a cup bottom you will see that this particular element is actually pulled in both the directions. Now if you pick up a plane strain why it is plane strain we will see later on because this is σ_1 versus σ_2 and I am going to call this as plane strain that you will see later, but then you can assume this as a plane strain process ok. The same sheet which is deformed along this α which is equivalent to plane strain type of deformation still it is plane stress because σ_3 is not in the diagram ok. So, if that is the case then what is the meaning it means that it is actually pulled in only one direction rather in the other direction the strain is not that much. So, you can keep it as a plane strain that is the meaning you will see that later on.

This type of situation you will see in cup wall region ok. So, if you come to this one this $45°$ is actually called as pure shear ok the same thing which we have discussed in the previous one this one ok. It is pure shear let us say which means that one will be equal and opposite to the other one ok that means it is getting pulled here ok and this is getting compressed here. This type of situation we will see in the flange region ok because here you will see that one is actually pushed the other one is actually pulled ok. So, this type of situations are seen you get to see that center that means cup center and in the wall region and in the flange region ok. Just to give you some example why it is important that is what I was telling you when you deform a sheet different locations may have different this kind of state of stress which can be obtained from the yield locus ok.

So, now I am going to draw a 3D figure of this two yield functions or these two yield criterion and they are going to be called as yield surfaces ok. So, if it is uniaxial type of deformation is just one point called yield strength. If it is in 2D on paper ok then in plane stress type of deformation you have σ_1, σ_2. So, you have a curve that is called yield locus we decided ok and then now if σ_3 is also there ok then this is become a surface ok that is a 3D plot. So, how are they going to you know compare? So, von Mises and Tresca can be compared in this way.

This is a standard diagram you can see in many text books. So, I have referred one of the text books you can say that. So, this can be plotted like this there are lot of features in this first of all you can see that there is $\sigma_1, \sigma_2, \sigma_3$ axis principle stress axis that is what I have written as geometric representation of yield criteria these two in principle stress space in principle stress space. So, what are the features here axis $\sigma_1, \sigma_2, \sigma_3$ are there and then ok you will see that there are two yield surfaces the black one I have written as Von Mises yield locus or is part of yield surface and Tresca yield locus or you can call the entire surface which is going to be generated in 3D form as yield Tresca yield surface ok. Generally this Tresca is inscribed inside Von Mises yield surface that is a way you generally keep it. So, that is why I have given this blue one is actually inside this cylinder ok this hexagonal 3D structure is actually inside the regular cylinder ok that is the way it is drawn ok.

So, and the center of this this axis you can call this is called actually hydrostatic line hydrostatic line means we said hydrostatic stress σ_h right. So, in that case you will see that $\sigma_1 = \sigma_2 = \sigma_3$ in the hydrostatic line ok. So, now ok there is one particular plane called deviatoric plane which is described by $\sigma_1 + \sigma_2 + \sigma_3 = 0$ that is going to cut this hydrostatic line perpendicular in a perpendicular manner that is what it is drawn ok. So, there is a plane you can imagine ok there is a cylinder you can you can you can imagine like a tube like a metallic tube or something like that it is the cylinder given here regular cylinder which is representing one axis and I am going to cut you know perpendicular to its axis. So, I am going to get a plane you know that is nothing, but your deviatoric plane which is described by $\sigma_1 + \sigma_2 + \sigma_3 = 0$.

So, now there is a projection of this locus on the deviatoric plane correct there is your since you are going to cut it ok. So, this shapes two shapes will also be cut right and that is what I have inscribed here. So, this this black line ok is my Von Mises again and this black line this hexagon ok is my Tresca yield locus on the deviatoric plane on the deviatoric plane ok. So, these are the features you have here ok. So, I am going to write this Von Mises yield surface it is radius is nothing, but $\sqrt{\frac{2}{3}}\sigma_y$ ok.

So, we have proved it is one of the you know semi axis ok one of the axis is nothing, but $\sqrt{\frac{2}{3}}\sigma_y$. Now Von Mises surface has got radius $\sqrt{\frac{2}{3}}\sigma_y$ that is one thing you should know. So, I have written here that Von Mises actually right circular cylinder ok and Tresca is going to be your regular hexagonal prism and this regular hexagonal prism is actually put inside your Von Mises in this fashion ok. So, now these are different ways of drawing your surfaces. Tresca we have drawn separately, Von Mises we have drawn separately, in 3D format this is the way you can draw.

So, now this diagram also I am going to use to discuss one important thing which I was discussing with you previously that why hydrostatic part or stress does not affect yielding we removed that is not it and then we said that only deviatoric part is going to play a role right. So, this also can be explained from this particular diagram given here. So, I am going to consider a state of stress OA ok I have given here OA you consider state of stress ok. So, this OA now can be you know resolved into two components OB that is one another vector and OC is another vector ok. So, this OB is along the hydrostatic part there is an equivalent component to that in deviatoric plane that is OC.

So, OB is in hydrostatic part is the hydrostatic part of OA and OC is a deviatoric part of the same OA ok. So, because your state of stress can be divided into two parts hydrostatic plus deviatoric right. So, that is what I have drawn here. So, OB this vector is along the hydrostatic line which is nothing but a hydrostatic part OC is actually on the deviatoric plane and it is a deviatoric part of OA right. So, now what I am going to do is assuming that this A, OA is actually in elastic state A part is not reached the yield surface A part is not reached the yield surface right.

So, when it reaches yield surface we say that material is started yielding plastic deformation is going to start after that. So, now the point here is so any deformation if I want to increase OA without contributing from OC ok. So, without sorry without contribution from OB which is a hydrostatic part ok. So, let us say for example, if I want to increase OA with a contribution only from OB ok. So, only OB is going to increase let us say for example, only hydrostatic part is going to increase.

So, with the same OC ok what will happen this A will never reach the yield surface ok. So, OA I want to increase its length or by increasing the deformation let us say OA I have to deform further right. So, that has got contribution only from OB let us say ok. So, it means that OC is not going to increase let us say then what does it mean? It means this point will never reach the yield surface if you have contribution only from OB ok. So, which means OC has to have some contribution to OA only then this A point will reach the yield locus here ok or yield surface here and then yielding is going to start ok.

So, that is why we always say that this hydrostatic part is not going to play any role ok in the onset of plastic deformation or yielding. So, the contribution OB alone if you take then A part will not reach the yield surface you need to have contribution from OC to push this A to the plastic deformation or onset of yielding ok. That is why your hydrostatic part will not play any role in onset of plastic deformation or yielding. But this A point can reach either Von Mises yield locus or Tresca yield locus you know depending on which model you are going to use depending on which model you are going to use and for a given value of α it is said that these two gradient predict less than 15 % difference. So, maximum difference it can have is only 15 % between your Von Mises and Tresca yield function that is generally said.

And you should also note down one important point this Von Mises and Tresca are meant for isotropic materials that is why there is no intimation of plastic strain ratio or any other equivalent term in any of these equations. There is no intimation of R, R is kept as one of isotropic material know right. So, there is no intimation of R or including R in any of this equation ok. So, I hope you understand this particular part why hydrostatic part does not affect yielding is mainly because if OA has to further increase and reach the surface if it has got contribution only from OB then A will not reach the surface it has to have contribution from OC ok. But which yield surface is going to reach first generally it is going to be Tresca ok if that is the case it will yield according to Tresca yield function but they may have maximum difference of the order of 15 % in one particular α ok.

And these two are meant for isotropic yield functions ok. So, with this I am going ahead. So, this two yield locus you know or yield loci ok can be drawn on the deviatoric plane on the deviatoric plane deviatoric plane is this plane right. So, you see from this side ok you see from this side that means you are going to cut the hydrostatic part with the surface and on that how do they look like it will look like this these two will look like this ok. This is the loci of Mises or Von Mises in yield surfaces on the deviatoric plane ok.

One should note that σ_1 , σ_2 axis are not on the deviatoric plane. Let us be careful $\sigma_1, \sigma_2, \sigma_3$ these are not actually on the deviatoric plane ok. So, how do they look like when the state of stress such that it is hydrostatic part is 0 ok when the hydrostatic part is 0 then you have only deviatoric part ok and the geometric plane representation reduces to two you know yield locus on the deviatoric plane. How are they going to look like? One is basically Von Mises criterion which will look like a circle with radius $\sqrt{\frac{2}{3}}\sigma_y$ that is what we have mentioned in the previous 3D plot also which is represented here ok $\sqrt{\frac{2}{3}}\sigma_y$ ok this is the radius ok. Now, Tresca would be a regular hexagon ok which is inside Von Mises circle ok this is a Von Mises circle ok and this is your you know Tresca hexagon is a Tresca hexagon is going to be inside your Von Mises circle which Von Mises circle has got radius $\sqrt{\frac{2}{3}}\sigma_y$ and this diagram also gives you several state of stress you can see here.

These all are basically good denoting Tresca yield functions at different locations ok. So, how do you get these two yield locus? These two yield locus are obtained by generated by intersection of yield surface with the deviatoric plane that is what intersection is this where they are intersection is this it is intersected by deviatoric plane. So, you are going to have these two they are called yield loci on deviatoric plane ok they are called yield loci on the deviatoric plane ok. So, you can imagine that this is plane stress this is plane stress $\sigma_3 = 0$ ok and in principle stress space both are there principle stress space σ_1, σ_2 principle stress space ok and $\sigma_3 = 0$ ok. This one is $\sigma_1, \sigma_2, \sigma_3$ all are shown with a 3D figure and they are going to called as yield surfaces and these two yield locus can be compared on the deviatoric plane in this fashion ok. So, different figures are there one has to be really careful with this and this also is discussed in terms of $\sigma_1, \sigma_2, \sigma_3$ only ok.

So, I am going to move ahead with this. So, these are the tool yield functions which are very important for us and as discussed in the previous slide you should know that these two are meant for isotropic materials for anisotropic sheets anisotropic materials we will see later on ok very briefly what are the important yield functions where R also comes into picture plastic strain ratio. Now when we speak about this yield functions ok there are two important things one is a normality and other one is convexity these two can be explained in this way ok. So, what is this ok? So, it will lead to two important things that one should know ok. Suppose to start with let us pick up a strain diagram ok like this ok let us pick up a strain diagram like this the elastic part is actually zoomed in and little drawn in a bigger way ok and this part is actually transition between elastic to plastic you can say this is nothing, but your yield strength let us say σ_f flow stress yield strength ok. So, σ_A is essentially an elastic part and σ_B I am going to pick up a you know plastic stress and this loop this loop ok is nothing, but a cycle for me.

So, I am going to start from here ok I am going to consider a loop which is starting from σ_A which is an elastic part and I am going to pick up one plastic part and I am going to go along the hardening part ok and I am going to go along the hardening part and I am going to come down and I will reach the initial point and I am going to close that loop that cycle is closed ok cycle is closed. So, it starts with the σ_A goes to σ_B and then the envelope is created by the hardening part of the stress strain graph decreases and it closes the loop the arrow mark is given here for your reference ok. So, this I am going to call it as $d\varepsilon_p$ plastic strain increment ok plastic strain increment ok and this rise in σ I am going to call it as $d\sigma$ this rise in σ is going to call it as $d\sigma$ this $d\sigma$ basically signifies hardening in this loop correct because from let us say this strain let us say this strain the material is hardened this much right. So, I am going to call it as $d\sigma$ ok. So, this brown colour one is area of the rectangle that is going to give me my work done I am just going to call it as W_2 which is nothing but my $(\sigma_B - \sigma_A)d\varepsilon_P$ ok.

So, and this can be seen in the form of a triangle and I am going to call it as W_1 as a work done during the cycle is nothing but $\frac{1}{2}d\varepsilon_P d\sigma$ ok. So, area of triangle is W_1 and this is W_2 it is going to be my work done in this two regions and work done along the closed path is W which is nothing but $W_1 + W_2$ where I am going to write this plus this is a total work done along this particular closed path which has got boundary between σ_A, σ_B and the strain hardening region. So, now if you see in this equation the equation is good because if σ, $d\varepsilon_P$ sorry is 0 , $d\varepsilon_P = 0$ means work done is 0. So, no work done $W = 0$ which means it is a pure elastic response right it means it is a pure elastic response. So, now these are the two parts right $(\sigma_B - \sigma_A)d\varepsilon_P$ and $\frac{1}{2}d\varepsilon_P d\sigma$ I am going to pick up this particular part first and I am written here these two points are very important for us.

I am going to say that the $(\sigma_B - \sigma_A)d\varepsilon_P$ is should be strictly positive it should be strictly positive ok because $d\varepsilon_P$ cannot be 0 ok it is anyway it is going to it is an increment ok plastic strain increment ok. Then this product has to be strictly positive only then plastic deformation is going to happen why because then σ_B is going to be larger than σ_A, σ_B should always be larger than σ_A only then this will remain strictly positive for me ok that is number 1. And number 2 this $\frac{1}{2}d\varepsilon_P d\sigma$ also should be positive that means, your $d\sigma$ should also be positive for me it is also signifies that strain hardening is going to happen strain hardening is going to happen right. If this is not going to be positive negative then it means that there is $d\sigma$ which is going to be negative which is actually due to the downfall of your two stress or load like that which is not describing your strain hardening behavior ok. So, my this product $(\sigma_B - \sigma_A)d\varepsilon_P$ should be strictly positive $\frac{1}{2}d\varepsilon_P d\sigma$ should also be strictly positive ok which means my W_1 this fellow and W_2 both should be greater than 0 for a stable plastic response.

Stable plastic response means the way we understand plastic response right start with this deformation across the yield point ok elastic point that part is covered then you have to go for plastic deformation strain hardening is going to happen ok. If that has to happen then this work done W_1 should be greater than 0 and W_2 should also be greater than 0 that is a very important condition for that ok. So, now convexity ok let us pick up one thing convexity this convexity is going to tell you why yield locus is convex at each and every point in that locus ok. Why yield locus is a convex let us say σ_2 versus σ_1 we have let us pick up first quadrant you take any point and put a tangent here ok. So, we say that the material has to have maintain this point should be the yield locus should be convex at each and every point in that particular yield locus why is it so ok.

So, for that we are going to consider a case just opposite to that this is actually the yield locus this is actually your yield locus let us say and there is a small region where there is a small dip it is going to come down it is going to increase like that ok. So, I am going to map the σ_A the same σ_A here σ_B I am going to map it like this. So, σ this diagram you will see σ_A is elastic. So, I am going to keep it below the yield locus correct σ_B is on the yield locus because it is in plastic deformation ok because it is in plastic part, but a is still in the elastic part let us say it is inside the yield locus ok. So, now you will see that the main requirement is $(\sigma_B - \sigma_A) d\varepsilon_P > 0$.

So, I am going to pick up this particular part I am going to pick up this particular part this particular part ok my W_2 part ok that is the main requirement why otherwise you will not have stable plastic response ok. This means this means that $\sigma_B > \sigma_A$ otherwise plastic deformation will not happen. In this figure you will see that σ_A looks larger than σ_B which is actually not acceptable which means that the $\sigma_B - \sigma_A$ has negative projection on $d\varepsilon$ which also means that your W_1 is actually less than 0, but for stable plastic response we said that your W_1 should be greater than 0 ok. So, this is not accepted that is happening why why because $\sigma_A > \sigma_B$ which is forbidden from plasticity point of view ok.

So, we can also say that no elastic states can be available outside the tangent line to the ill locus. So, you draw a tangent line to the ill locus ok at any point here, here, here ok. So, let us pick up this particular point you are drawing a tangent let us say for example no elastic states can be available outside the tangent line. So, that is here σ_A which is actually in elastic part ok σ_A is an elastic part which is actually beyond the tangent line to the yield locus ok. So, always $\sigma_B > \sigma_A$this is possible only when the yield locus is convex at every point, this is possible only when you have yield locus which is a convex at every point this is an important condition.

So, this should be greater than 0 only then your work done is greater than 0 that is point number 1. Next one is there is something called normality condition which will tell you the direction of $d\varepsilon$ which will tell you the direction of $d\varepsilon$ for that I have drawn a simple schematic here let us say this is your yield locus ok this is just your yield locus ok. So, now we are going to pick up the next one another part in that work done which is $\frac{1}{2} d\varepsilon_P d\sigma > 0$ ok for strain hardening to happen for strain hardening to happen that is where this fellow comes into picture right. So, now in this you can say $d\sigma$ and $d\varepsilon$ can be seen as in general as vectors. So, we can write this as $d\sigma . d\varepsilon > 0$ this means $d\sigma$ and $d\varepsilon$ have a positive projection on one another what does that mean? That means, it is a dot product know positive projection on that means, angle between them is less than 90°. So, maximum angle it can have is 90° between them for any choice of $d\sigma$ that produces plastic deformation for any choice of $d\sigma$ that produces plastic deformation ok.

What does it mean? That means, suppose this is a point I am going to pick up the material is let us say following one particular α and reaches this particular point ok. So, then I am going to draw a tangent I am going to draw a tangent to this and I am going to pick up any choice of $d\sigma$ 1, 2, 3 ok, 4, 5, 6 maybe here also any choice of $d\sigma$ I can have the simplest choice of selecting the direction of $d\varepsilon$ because it can have a maximum angle of 90° between them is actually perpendicular tangent drawn here this particular point. So, I am going to draw a line perpendicular arrow perpendicular to the tangent and that I am going to represent as $d\varepsilon$ that will be my direction of $d\varepsilon$. So, I written that the simplest choice of direction of $d\varepsilon$ is normal to the yield surface f let us say this is your yield locus or surface f ok. This choice of direction when you are choosing of $d\varepsilon$ is called as a normality condition or normality rule ok.

So, the maximum angle it can have is 90° ok it can have is 90°. So, the best choice is to pick up $d\varepsilon$ direction perpendicular tangent drawn at any point in the yield locus. If you pick up this point then if you draw this tangent then perpendicular to that is a direction of $d\varepsilon$ that is what is told by this normality condition. So, if you pick up a work done in a closed path in a closed loop ok W_1, W_2 and they can explain why the yield locus has to be convex at each and every point in the yield locus ok. So, it has to be convex why because only then your W_1will be greater than 0 only then plastic deformation will happen the way we understand it ok. And the other part why or the direction of $d\varepsilon$ how should it be it should be perpendicular to the tangent drawn at any point in the yield locus and that is mainly because your the angle between any choice of $d\sigma$ you pick up the angle between $d\sigma$ and $d\varepsilon$ should be less than 90° or maximum it can have is 90° ok.

So, this normality condition can be written in a mathematical way in this way of course this can be derived but we have not done it here one should remember this $d\varepsilon_{ij} = d\lambda\left(\frac{\partial f}{\partial \sigma_{ij}}\right)$ of course there is one small change here you can see this ε_{ij} and $d\sigma_{ij}$ are I think integral notation ok. So, which we are not discussed but I think you can go back and refer it the σ_{ij} is nothing but is going to describe stress tensor and different elements in the stress tensor and this is going to give you different elements in the strain tensor. But the strain increment $d\varepsilon_{ij} = d\lambda\left(\frac{\partial f}{\partial \sigma_{ij}}\right)$ where f is your nothing but your yield function that you have going to derive or you already derived it could be Tresca yield function or von Mises yield function ok. And $d\lambda$ is actually an arbitrary constant we will also see in due course $d\lambda$ can be replaced during any derivation ok. So, this equation significance of equation is basically you can use this equation to find the strain increment suppose if you want to find $d\varepsilon_1, d\varepsilon_2, d\varepsilon_3$ ok you can partially differentiate the yield function which is a function of

$\sigma_1, \sigma_2, \sigma_3$ you can differentiate with respect to $\sigma_1, \sigma_2, \sigma_3$ and then you will get $d\varepsilon_1, d\varepsilon_2, d\varepsilon_3$ that is the significance of this equation maybe the assignments or in the you know in the example problem we can show one or two examples how to get $d\varepsilon_1, d\varepsilon_2, d\varepsilon_3$ ok.

So, with respect to Von Mises and Tresca yield function this is what we discussed and then normality and convexity is two for any yield function which can be simply explained in this way ok. So, now let us go to Levy-Mises flow rule ok there are small small things now this is very important for us ok why you will know now. So, this Levy-Mises flow rule ok is going to tell you something which we already discussed briefly we said that deviatoric stress components that is you are in principle format you can say $\sigma_1', \sigma_2', \sigma_3'$ together with hydrostatic components make up actual stress state correct that we know. As a hydrostatic stress is unlikely to influence deformation in a solid that deforms at constant volume that is during plastic deformation why we have seen that before in the 3D yield surface why hydrostatic part of the stress or hydrostatic is unlikely to influence the deformation we have seen during plastic deformation. It may be said that it is the deviatoric components ok that will be the ones associated with the shape change right what you do in plastic deformation ok this is the hypothesis of Levy-Mises flow rule this is the hypothesis of Levy-Mises flow rule.

So, using this it can also be stated that the ratio of strain increments will be same as a ratio of deviatoric stresses. The ratio of strain increments will be same as that of the ratio of deviatoric stresses strain increments $d\varepsilon_1, d\varepsilon_2, d\varepsilon_3$ ok 1 along length 2 along width 3 along thickness. So, how are they related to length width and thickness we already seen and given a particular case let us say for example, uniaxial or something like that you can relate all these 3 using constant volume equation correct that you already discussed. Ratio of deviatoric stresses what are deviatoric stresses $\sigma_1', \sigma_2', \sigma_3'$ ok which can be written in a mathematical way in this way like this $\frac{d\varepsilon_1}{\sigma_1'} = \frac{d\varepsilon_2}{\sigma_2'} = \frac{d\varepsilon_3}{\sigma_3'} = d\lambda$ ok. The same thing written little differently the ratio of strain increments will be same as a ratio of deviatoric stresses $\frac{d\varepsilon_1}{d\varepsilon_2} = \frac{\sigma_1'}{\sigma_2'}$. In other way you can write $\frac{d\varepsilon_1}{\sigma_1'} = \frac{d\varepsilon_2}{\sigma_2'} = \frac{d\varepsilon_3}{\sigma_3'}$ let us be careful here it is $\varepsilon, \varepsilon, \varepsilon$ here it is σ, σ, σ strain increment here deviatoric stresses that will be equal to $d\lambda$ ok.

So, this is the expression for Levy-Mises flow rule and with this also you can get the strain increments ok. You can say $d\varepsilon_1 = \sigma_1' \times d\lambda$ correct σ_1' is already known to you it is a function of $\sigma_1, \sigma_2, \sigma_3$ that we already derived right it is a function of $\sigma_1, \sigma_2, \sigma_3$ ok. So, in that way you can find , $d\varepsilon_1$. Similarly, $d\varepsilon_2$ can be found , $d\varepsilon_3$ can also be found out ok can also be found out if you know $\sigma_1, \sigma_2, \sigma_3$ ok. So, now I think this also we derived in the previous class previous discussion is not it. Your $\sigma_1', \sigma_2', \sigma_3'$ as a function of α, $\sigma_1', \sigma_2', \sigma_3'$ in the function of α I think we already discussed which means that σ_3 is not there because you are having only σ, α now ${}^{\sigma_2}/_{\sigma_1}$ which means σ_3 is not there we already discussed this derived this ok.

So, now what we can do we can substitute the $\sigma_1', \sigma_2', \sigma_3'$ in this equation and we can write $d\varepsilon_1$ divided by your, by you know your σ_1 ,3, σ_1, 3, σ_1, 3 are common it will it will get removed ok. So, then you can write $\frac{d\varepsilon_1}{2-\alpha} = \frac{d\varepsilon_2}{2\alpha-1} = \frac{d\varepsilon_3}{-(1+\alpha)}$ ok. So, this equation can be modified to this form using this relationship which you already discussed fine. So, if a material is deforming in plane stress proportional process ok plane stress meaning here $\sigma_3 = 0$ proportional process means α remains same ok. The above equation can be integrated we have already seen three conditions right the above equation can be integrated in terms of true strains and you can modify the previous equation like this ok.

So, what is the equation $\frac{d\varepsilon_1}{2-\alpha} = \frac{d\varepsilon_2}{2\alpha-1} = \frac{d\varepsilon_3}{-(1+\alpha)}$ that is already there here $d\varepsilon_1$ is integrated. So, we get ε_1 it will become ε_2 it will become ε_3 these three parts are already known to us ok for a plane stress proportional process you can write this. Other than these two there I am going to add two more which will help which will be helpful for us now what is this I am going to this is completed this also complete this completed ε_2 is nothing, but $\beta\varepsilon_1$ right $\beta = \frac{\varepsilon_2}{\varepsilon_1}$. So, $\varepsilon_2 = \beta\varepsilon_1$, I am going to put here this will remain same ok $\varepsilon_3 = -(1+\beta)\varepsilon_1$ this also we derived before which I am going to substitute here divided by $-(1+\alpha)$ it can be rewritten in this way $\frac{\varepsilon_1}{2-\alpha} = \frac{\varepsilon_2}{2\alpha-1} = \frac{\beta\varepsilon_1}{2\alpha-1} = \frac{\varepsilon_3}{-(1+\alpha)} = \frac{-(1+\beta)\varepsilon_1}{-(1+\alpha)}$ will give me this ok. So, now there is one important point that using this equation we can find a relationship between α and β relationship between α and β for an isotropic material how are we going to find it is very simple here ok.

So, now what I am going to do is I am going to compare this fellow and this fellow I am going to compare these two I will pick up let us say this ok. So, I am going to compare this with this this part ok. So, ε_1 and ε_1 will be cancelled. So, I can directly write $\beta = \frac{2\alpha-1}{2-\alpha}$ right $\beta = \frac{2\alpha-1}{2-\alpha}$ right. So, which is what I have given here which is what $\beta = \frac{2\alpha-1}{2-\alpha}$ then of course, you can also get this relationship from this ok $\alpha = \frac{2\beta+1}{2+\beta}$ ok.

You can get this relationship from this relationship you can rewrite this and find out you will be able to get $\alpha = \frac{2\beta+1}{2+\beta}$t ok. So, relationship between stress ratio and strain ratio β can be obtained from the Levy-Mises flow rule in this way ok. So, just to get a quick picture what is α for uniaxial α for uniaxial ${}^{\sigma_2}/{}_{\sigma_1}$. So, $\alpha = 0$ uniaxial tensile test 0 if you put 0 here what will happen $-\frac{1}{2}$.
So, $\beta = -\frac{1}{2}$ which is what we have seen in the previous class ok. I hope you remember

that which class we have seen that it is this particular class this particular section somewhere we have seen yes. $\beta = -\frac{1}{2}$, $\alpha = 0$ for uniaxial tension test right right. So, this we have discussed without knowing the relationship now we are proving that if you put α is $\alpha = 0$, $\beta = -\frac{1}{2}$ uniaxial tension test that we have already seen right that is what we are getting in this equation also right. So, just to summarize there are important points that you should know ok. So, it may be seen that while the flow rule ok Levy-Mises flow rule gives relationship between stress and strain ratios α and β ok it does not indicate the magnitude of strains.

So, the magnitude of strains one should get from the original definition it should just give you the ratio only. If the element deforms on the given stress state let us α is known the ratio of strains can be found from the above equations ok that is what we got just now ok. Just to give you a quick picture of this entire thing in one diagram ok some of this you have not discussed much but I am just going to summarize that ok this is one diagram ok you have got both strain increment as well as principal stresses σ_1 in Y axis σ_2 in X axis $d\varepsilon_1$ in Y axis $d\varepsilon_2$ in X axis both are drawn here which means α and β can be represented in one diagram right. So, 5 different in α and β are given 1 2 this is 3 this is 4 and this fellow is going to be 5, 5 different I was telling you when we discussing about α and β that only uniaxial that is the case right. So, there could be many other α and β values prominent of them are this 5 of course, you can have in between also in between also you can have, but these are all prominent ones.

We will let us quickly discuss about it let us pick up this particular line let us pick up this particular line ok. So, here you will see that this is both are 1 ok let us say for example, you have a $\sigma_1 = \sigma_2$. So, $\alpha = 1$. So, $\alpha = 1$ here $\alpha = 1$ if you substitute in the previous equation $\beta = \frac{2\alpha-1}{2-\alpha}$, $\frac{2-1}{2-1}$. So, $\beta = 1$ which means $d\varepsilon_1 = d\varepsilon_2$ fine.

So, now, let us pick up this particular stress path where it is $\frac{\sigma_2}{\sigma_1} = \frac{1}{2}$ ok. So, let us put $\alpha = \frac{1}{2}$ here what will happen $\alpha = \frac{1}{2}$. So, $1 - 1$ so gone. So, $\beta = 0$ that means, $d\varepsilon_2 = 0$ right this is a known thing for us Y axis along Y axis this particular stress path is known thing for us right why because here your $\sigma_2 = 0$ that means, it is uniaxial. So, which means $\alpha = 0$ if $\alpha = 0$ we already seen that $\beta = -\frac{1}{2}$ which is what I have written $d\varepsilon_2 = -d\varepsilon_1/2$.

So, $d\varepsilon_2/d\varepsilon_1 = -1/2$ this is on the other side ok is on the other side. So, what is α here? So, this is going to be my α, $\alpha = {}^{\sigma_2}/_{\sigma_1}$ which is $\alpha = -1$ ok if it is $\alpha = -1$ what will happen? So, this is $\beta = \frac{2\alpha-1}{2-\alpha}$, $\frac{-2-1}{2-(-1)}$ $\frac{-3}{3}$ which will be $\beta = -1$ which is -1. So, $\beta = -1$, $\alpha = -1$, $\beta = -1$ like that ok and this is actually just opposite in this direction in this direction. So, one can work it out and find out that this is nothing, but $\beta = -2$, $\beta = -2$. So, you will see that your β value is going to move from $\beta = -1$ here ok it is going to cross $\beta = 0$ and then $\beta = -\frac{1}{2}$ and then $\beta = -1$ and then you are getting $\beta = -2$. So, it starts from 1 it goes up to -2 that is what is shown in this diagram and we have already seen in this diagram the deep drawing one part of the component I have shown here know this part.

So, you can coincide this diagram with this diagram which I have shown and you can see what type of things are going to come in which part of different deforming when you deform the cup right. So, you can coincide these two because there is also σ_1, σ_2 here also σ_1, σ_2. So, you can find out where it is going to come ok. So, a brief explanation for this with this we are we will stop it here ok.

So, we are going to say that a simple example problem I have just shown here the current flow strength of a material element is 300 MPa ok. The current flow strength let us say you are we do not current flow strength means we do not know what is it.

So, we write $\sigma_f = 300MPa$. In a deformation process the principle strain increments are 0.012 and 0.007 right. So, the larger value you can always take it as 1 ok. So, I am writing $d\varepsilon_1$ as 0.012 ok and my $d\varepsilon_2$ as 0.007 in 1 and 2 directions it is already given. Determine the principle stresses associated with this in a plane stress process the question is given. So, we can directly say $\sigma_3 = 0$. So, it is a $\sigma_3 = 0$ process ok. What do you want to find? You need to find principle stresses. So, you need to find σ_1, σ_2 only ok σ_3 is known.

So, how do you proceed? Whenever you have got a strain increment directly you can find out β. This is the way you have to think about ok. So, whenever you have strain increment one thing directly you can get it is your β .

So, I written here that $\beta = \frac{d\varepsilon_2}{d\varepsilon_1}$ is nothing, but $0.007/0.012 = 0.583$. So, $\beta = 0.583$ right $\beta = 0.583$ means it is somewhere in between these two is not it.

This is $\beta = 1$ this is $\beta = 0$, 0.583 means it is somewhere in between the somewhere in between 0.5 and 0.6 you can keep ok. So, it is somewhere in between that is what is just a simple reference to that. So, β is found out α can be found out right $\frac{2\beta+1}{2+\beta}$ which you already derived.

So, you substitute β value in this equation you will get 0.839 that is α that is all. So, what do we need we need σ_1, σ_2. So, α is known which means σ_1 can be found out first ok. How do you find out σ_1? $\sigma_1 = \sigma_f/\sqrt{(1-\alpha+\alpha^2)}$, how do you get this? This you are getting from Von Mises yield function unless otherwise said we are going to follow Von Mises for all the problems. We just derived this we just now derived is not it this equation where is it this equation comes here yes this equation know this fellow this equation. So, $\sigma_1 = \sigma_f/\sqrt{(1-\alpha+\alpha^2)}$ ok and σ_f is already given current the current strength or flow strength is already given 300 MPA.

So, this would be your will $300/\sqrt{(1-0.839+0.839^2)}$ give you can calculate it should be about 323 MPA that is all. If α is known then $\alpha.\sigma_1$ will be 271 MPA. This is the route whenever new dimension and old dimensions are given or whenever strain increments are given directly ok you can use this ok. If strains are not given and if it is original new dimensions are given then you have to calculate $\sigma, d\varepsilon_1, d\varepsilon_1$to get β then you can get α by knowing α and by knowing current flow strength you can get the principle stress ok 1 first principle stress by knowing α you can get a σ_1 which is nothing, but σ next principle stress 323, 271, 0 ok. Together is going to cause this particular flow strength as 300 MPA, but whether this is less than the yield strength of the material or greater than we do not know.

So, only this much of information we can get from this particular problem ok. So, we are stopping here and we will continue our discussion in the next part. Thank you.