Mechanics of Sheet Metal Forming

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Week- 08
Lecture- 19
Hydroforming

So, this is lecture number 19. So, in this we are going to discuss about hydroforming. This is a more applied process probably maybe we can see it as a recent one of the recent developments in metal forming or in sheet forming or tube forming when compared to other conventional methods. So, until now we have seen your cup deep drawing and of course before that we have seen bending and then stretching right. So, these are the three methods we have discussed and then we applied some simple mechanics and then derived some expressions worked out some numerical problems. And we are at the end of this particular course.

So, in that we are going to discuss about hydroforming. Again here we will try to apply some simple mechanics based discussion and then derive some expressions that is work out to a couple of problems in this. So, as the name suggests hydroforming basically means you are going to form a material maybe in the form of a sheet or in the form of a tube with the help of fluid pressure ok. Like in the previous you know chapter we have seen hydrostatic bulge test ok. This is more an applied version of that ok that is what is nothing but hydroforming. So, hydroforming is a well known and established method which is used to make sheets ok or any tubular components using liquid pressure ok which can also be seen as a soft punch you can say which can be seen as a soft punch one can imagine ok. So, there are lot of you know benefits ok some of them I have listed here. It includes greater drawing ratio suppose if you do deep drawing we have seen limit drawing ratio drawing ratio right. So, it is observed that if you do sheet hydroforming or sheet deep drawing.

So, one can get a greater drawing ratio better surface quality because tool to sheet contact is minimized here. Reduce the spring back ok. So, better dimensional tolerance again you know the contact is pretty smooth. So dimensions and intricate dimensions making of that is comparably better when you go for hydroforming. So, intricate shapes can be manufactured with this ok which can be done easily by pressurizing the fluid and unlike in you know conventional stamping operations where you need a punch ok. A tool which has the particular shape to deform the material. So, here intricate shapes can be made pretty easily when compared to other traditional methods. So, these advantages ok or not the only advantages we have there are several other advantages also which one can you can get it from the existing literature but these are the main things more from industry aspect point of view these are predominantly acceptable ok. And because of this you will see that there are lot of applications the hydroforming is having. So, not only in automotive sector but also in household you know things you know like any items that we see day to day life in our households like tabs ok your you know tubes metallic stainless steel tubes all these things can be made or made basically by hydroforming.

So, some of the applications I have listed are basically in automotive sector real and front axis engine cradles exhaust system parts structural body components like your sheet parts several big sheet parts some of the components in power transmission they are also made by this hydroforming process. So, when we speak about hydroforming so, though we know that the deformation is given by fluid pressure so, which component you are going to make is going to determine the initial raw material required for that application right. So, in that way we can divide this hydroforming into tube hydroforming and sheet hydroforming. So, tube hydroforming and sheet hydroforming the principle remains same ok. Sheet hydroforming you know how is it going to work we have seen that in the previous you know section that your hydrostatic bulge test is a simple example of your sheet hydroforming that is the way it is going to work.

If the raw material is in the form of tube ok then you apply fluid pressure inside the tube and then you allow tube to you know deform ok at a particular location you will see some examples and then you make the component. So, one of the main advantages of any of this hydroforming methods is basically it avoids or it minimizes post forming operations that is one main benefit advantage that practically people see ok. So, post forming operations like for example welding ok many times you know for several applications once you make a component we are going to attach it or connect it with some other member and their welding is required in such operations can be minimized and the entire component can be made using tube or high sheet hydroforming ok without any post with minimized minimal post forming operations. So, other than tube hydroforming sheet hydroforming there are other methods also like for example hydro piercing is also there. So, piercing of sheets ok and you know that what is it and then that can be done instead of a punch instead of tool one can make it with the help of fluid pressure ok.

So, now let us quickly see some schematics of this hydroforming. So, of course sheet hydroforming, shell hydroforming are almost the same except that the shell hydroforming it looks like this the schematic you can see that ok you have initial work piece typically of some structure ok and then you apply this pressure P. This p is actually very important for us our whole discussion mechanics discussion is going to you know move around this P ok fluid pressure. So, if you allow if you apply pressure so naturally this regions will try to move away and then it will try to take the shape of the die and you will get a work piece like this, this is what actually we want. So, people have made a spherical shell ellipsoid shell ok and toroidal shell ok and then applications point of view if you see it is observed that water tanks LPG tanks are made building decorations pressure vessel heads large size elbow joints and single layer double layer materials are actually deformed to make components these all are possible ok one can refer into literature and sheet hydroforming is a similar one ok you can see the schematic this is a simple schematic which says that ok this is your sheet it is written as work piece.

So, this is nothing but your sheet is initial raw material and you will see that this is your die I think you can understand this ok. So, this die is a closed die so naturally there is a shape that you have to give shape could be something as simple as like this a rectangular pan or something like that you can imagine ok and between the work piece and the die there will be a diaphragm ok this diaphragm is basically is a flexible one a flexible diaphragm is frequently applied to the sheet ok. So, if this is the sheet you can people apply you know some sort of you know flexible diaphragm above that of course you have to place it before deformation starts ok which is basically separates your sheet from contacting the fluid ok. So, this diaphragm should not affect the plastic deformation of the sheet as long as that is not going to happen generally ok. So, it is not going to disturb the component fabrication and you will have a nicely formed rectangular pan which is given here you can see that ok.

So, this is your die and you will see that your sheet has taken the shape of the die you know the die shape ok. So, this approach has the benefit of a simple die structure ok and potential for cost effective production of few pieces. So, the only issue with this is it is not meant for faster production probably by now it might have come. So, it is relatively you know moderately you know may be slow process, but you can make you know cost effective production, but few pieces when compared to conventional forming. This is just one small thing otherwise sheet hydropharming is very well accepted in industries several components are made one can look into the photographs available in the you know any resources.

So, tube hydropharming is of course is what we are going to discuss a lot in this particular chapter ok. This tubular components like you know you can see brackets for bicycle frames or pipe fittings I was telling you are formed using tube hydropharming ok. So, the schematic is the principle is going to remain same ok you can see that there is a die here right and so you will see that you have a you know mandrel type of mandrel type of structure and inside that you have there is a tube which is kept the tube is actually a straight circular tube initially to start with and inside that you are going to apply fluid pressure *P* ok. You are going to apply fluid pressure *P* and the black color one this particular contour is going to tell you one intermediate shape of the tube ok which is deformed using with the help of fluid pressure *P*. So, you will see that this portion is actually can give some cushioning effect ok so that the tube can expand in this direction tube is actually deforming in this direction ok with some restriction ok and the green color portion is going to tell you the second stage ok is going the green color portion is going to tell you the second stage of a tube forming.

So, basically there is a tubular section to start with and this has been converted into this type of tube you can see that it is converted to this type of tubular structure ok. So, this will also give you an idea that only fluid pressure is required or is there any other resistance required to form the tube ok. So, when you are forming this type of shape ok so which is actually shown in this diagram when you form this type of shape let us say. So, you are going to give initially a tube is like this you are going to give pressure the tube in this way right. So, and the material is actually going to move in this direction because of the unavailability of any restriction ok and then you are going to make this type of shape let us say and along with P one can also give F, F is nothing but your axial force ok along with P one can also give axial force to the tube ok.

So that if the compatibility is maintained between P and F you can have a good you know formed tube ok you can have good formed you know well deformed tube actual component can be made. This F is going to actually help the pressure ok to make the component free of defects. If you give large F for a particular pressure if a compatibility is lost between the pressure and your axial force. So, what can happen is you keep on giving this up this particular force ok and the P is not sufficient to push this material in this direction then what can happen is the material will try to settle here more material will try to settle which can create some sort of wrinkle at the this particular corner regions. So one has to be very careful with that but the same time if you do not give F or you minimize F there are other issues that can come into use that we will also see that.

If there is no F what is the thing if you give F what is the advantage we will see that. So in that way if you minimize F, F is not sufficient ok then something else is going to happen we will see that but often this defects can be reduced by having some compatibility between your P and F. So, I just given here that the tube experiences internal pressure in addition to axial force which causes compressive stress in one direction delays thinning and tearing of tube component. So the advantage of giving axial force is actually you are pushing more and more material which can help P to push more material into the high cavity ok. So tube hydroforming has got similar advantages in general like sheet hydroforming one is pot consolidation then weight reduction.

So weight reduction how it can happen is you know some light weight materials which are not formed well using conventional forming can be done in this method. So then in that way basically you can also apply those materials for you know several components which could reduce the weight and of course fuel consumption can also be reduced. So increased part strength and stiffness accurate dimensions so the intricate components can be made with good dimensions, good dimensions less spring back we have already seen lower tooling cost. So initial cost could be higher but then once it is set then since it is going to avoid post forming operations you can reduce the tooling cost ok fewer integrated processes. These are some important advantages of tube hydroforming many of these are valid in general for hydroforming as well.

So as I said because of all these advantages automotive household in aerospace sectors use this particular technology ok. So now in this particular section we are going to see three important topics, three important topics and we are going to put some mechanics to that. First one is called free expansion of a cylinder by internal pressure ok. Free expansion of a cylinder, cylinder means a tube, free expansion of a cylindrical tube you can say by internal pressure ok. Here we are going to consider without axial force, here we are going to consider without axial force and then you apply axial force what is going to happen that is the second case.

Third case is a separate one where you are going to convert a round tube, a cylindrical tube into a square tube some important you know details we are going to discuss in that ok. So the first one what we are going to see number one I am putting here is a free expansion of a cylindrical tube by internal pressure ok and the schematic is shown here for you. So here you will see that I mean this red colour one you can say that this is actually a tube with a wall thickness of let us say t and it has got radius of let us say r ok and internal pressure P is actually applied inside so that it can freely expand ok. It is going to freely expand, it is going to bulge actually, it is going to bulge equally in all the directions. So the shape is going to remain circular only that is way that we expect ok and you want to just expand the tube ok that is why we call it as a free expansion.

So here I have also mentioned as usual our nomenclature I have on the one direction T_θ and two direction I have T_ϕ ok T_θ and T_ϕ and of course we can also understand that there is a σ_θ, there is a σ_ϕ and there is a σ_t or σ_3 we can say. So interestingly you have ε_θ, ε_ϕ, and ε_t or ε_3 any one you can keep it ok. These are already available for us of course these two are going to be tensions T_θ and T_ϕ is going to be your tensions on one direction and two direction. So like in the previous chapters here also we are going to say that the strain in the axial direction would be 0, strain in the axial direction means along this this is your axial direction along this direction your strain is going to be 0. So let us say your ε_t that is $\varepsilon_\phi = 0$ why because there is no constraint ok nothing is actually constrained ok.

So if you take let us say an axial length of let us say 1 meter tube that will remain almost same as it of 1 meter ok not much change will be there ok. So you can take it as let us say plane strain deformation process. So now having said it is a plane strain deformation process we already developed some equations which are going to use here later on ok in due course we will do that. We are not going to derive it we will simply say these are the equations that will be used. So the tube radius eventually expand while initially staying circular ok that is the whole process here.

So now assuming isotropic material naturally we know that there will be ε_θ right and $\varepsilon_\phi = \beta\varepsilon_\theta, \beta = 0$ why it is a plane strain. Now you will immediately think if $\beta = 0$ what will be my α that is the way we are going to think we will come to that. So $\varepsilon_\phi = \beta\varepsilon_\theta = 0$ so $\varepsilon_\phi = 0$ ok and we know that now $\varepsilon_t = -1(1+\beta)\varepsilon_\theta = -\varepsilon_\theta$ which you have derived long back this fellow is actually 0 so it is going to $-\varepsilon_\theta$ right. So σ_θ will be there and$\sigma_\phi = \alpha\sigma_\theta$ and let us say here $\alpha = 1/2$ ok we have already seen $\alpha = 1/2$ then it is $\sigma_\phi = \frac{1}{2}\sigma_\theta$ and $\sigma_3 = 0$ this is what we have assumed right from the beginning. So I will turn here clearly $\beta = 0,\ \alpha = 1/2$ you can say ok.

So this would be your state of strain and stress ok. So now what I am going to do is as usual our aim is going to get this particular fellow p what is the pressure equation that is the whole idea for us here ok. So if I want to know p, I need to assume some strain hardening law as usual I am keeping $\bar{\sigma} = K\bar{\varepsilon}^n$ strain hardening law it is Holloman power law. I hope you understand that $\bar{\sigma}$ is nothing but effective stress $\bar{\varepsilon}$ effective strain ok your K is strength coefficient and n is strain hardening exponent we all know this and when the material undergoes deformation ok. So since it is a plane strain since it is a plane strain mode of deformation so what I can write is I can write my $\sigma_\theta = \frac{2}{\sqrt{3}}\bar{\sigma}$ which you already developed and of course you have to get it from one my effective stress equation and $\bar{\varepsilon} = \frac{2}{\sqrt{3}}\varepsilon_\theta$ ok so that would be the effective stress with the principle stress you know the relationship we have here.

Now let us go to this pressure equation so the hoop tension t_0 ,T_θ here we have T_θ now this $T_\theta = \sigma_\theta t = pr$ we already discussed about t , pr which is also nothing but pr we also nothing $T_\theta = \sigma_\theta t = pr$ where p is our fellow to be found out and we can also write $T_\phi = \frac{1}{2}T_\theta$ is ok we wrote $\sigma_\phi = \frac{1}{2}\sigma_\theta$. So similarly your $T_\phi = \frac{1}{2}T_\theta$ ok. So now of course whether it will be useful for us we will see later on now this p is a question for us so p I am going to write it as so my $p = \frac{\sigma_\theta t}{r} = \frac{2}{\sqrt{3}}\bar{\sigma}\frac{t}{r}$ which is what is given here this what is given here. So p has been found out now so only thing is you should know the new thickness and the radius ok and of course $\bar{\sigma}$ has to be you know can be obtained from any flow stress model. So now hoop strain and thickness are also very important for us because this thickness is going to be in this equation so I am just going to write $\varepsilon_\theta = ln\frac{r}{r_0}$ yeah so that I know that because your ε_θ is going to be in the circumferential direction which is nothing but connecting the radius so $ln\frac{r}{r_0}$ and $t = t_0 \exp \varepsilon_t = t_0 \exp(-\varepsilon_\theta)$ in this particular case ok.

So now if you put $\varepsilon_\theta = ln\frac{r}{r_0}$ if you put it is going to be, $\exp \ln r$ those things will be taken care and $t = t_0 \exp \varepsilon_t = t_0 \exp(-\varepsilon_\theta) = \frac{t_0 r_0}{r}$ this is also very important for us ok we will use this particular relationship in later on case ok. Let us go back to p now so what do you develop now $p = \frac{2}{\sqrt{3}}\bar{\sigma}\frac{t}{r}$ will be there that is there and then $\bar{\sigma} = K\bar{\varepsilon}^n$ ok and $\bar{\varepsilon}$ would be connected to you know my already developed equation $\bar{\varepsilon} = \frac{2}{\sqrt{3}}\varepsilon_\theta$ this ε_θ is nothing but again connected to $ln\frac{r}{r_0}$ ok so all these things will give me this equation $p = \frac{2}{\sqrt{3}}K\left(\frac{2}{\sqrt{3}}\ln\frac{r}{r_0}\right)^n \frac{t_0 r_0}{r^2}$ ok. So and what else do I have here $\frac{t}{r}$ I have and here I have basically $\frac{t}{r}$ this instead of $\frac{t}{r}$ I am going to write $\frac{t_0 r_0}{r^2}$ this fellow will be substituted here ok so I am going to get $p = \frac{2}{\sqrt{3}}K\left(\frac{2}{\sqrt{3}}\ln\frac{r}{r_0}\right)^n \frac{t_0 r_0}{r^2}$ so everything can be substituted and I will get this particular equation. So using simple mechanics we have got the pressure equation so which is nothing but $p = \frac{2}{\sqrt{3}}K\left(\frac{2}{\sqrt{3}}\ln\frac{r}{r_0}\right)^n \frac{t_0 r_0}{r^2}$ what is K strength coefficient r_0 is the original one r is a new one radius

n is a strain on exponent we know that and t_0 initial thickness if you know t you can use t otherwise $\frac{t_0 r_0}{r^2}$ can be obtained. So this is a simple equation one can get all these characteristics ok.

So now suppose for a strain hardening material which is what is generally seen $n > 0$ we know that the pressure will tend to increase as the material deforms right. So once you expand the tube pressure is going to increase. So what will happen during that time the tube wall will thin right so tube wall is expected to thin ok and radius will grow the tubular part is allowed to expand freely both effects will work to reduce pressure ok. So on one hand your tube wall will thin at the same time ok so your radius also is going to change which together is going to affect your change in pressure. So that one has to keep it in mind and at some stage like in other process your maximum pressure is also reached right.

So maximum pressure is also reached ok. Now if you differentiate this particular equation and put $dp = 0$ let us say ok for example maximum pressure ok you want to get p_{max} ok. So one way is you can take all some values for this and you can graphically plot this you will get p_{max} the other way is we can put $dp = 0$ at maximum pressure then that maximum pressure will happen when $\varepsilon_\theta = n/2$. So similar relationship were obtained for previous other process also for example $\varepsilon_u = n$ we said $\varepsilon_1^* = \frac{n}{1+\beta}$ a general equation all these things were derived. Similarly here also ok the maximum pressure condition you want to put then that can be obtained if ε_θ what is ε_θ for us $\varepsilon_\theta = ln\frac{r}{r_0} = ln\frac{t_0}{t}$ ok. So that if you put you will get a value $n/2$ that means what you have to keep on measuring let us say ε_θ when it becomes half of the strain bonding exponent then you can imagine that your $dp = 0$ maximum pressure is reached ok.

After that what will happen is the tube will swell locally ok there will be very localized deformation ok in the tube to create a new shape ok. Next stage would be splitting occurs in the locally swelled region ok splitting occurs in locally swelled region and because this is a plane strain forming ok splitting occurs when the hoop strain the same fellow your ε_θ ok circumferential strain as a value approximately equal to n ok. So that means when you plot forming limit curve let us say this is ε_θ for example this is ε_θ ok and this is ε_ϕ you are going along this particular path to reach plane strain mode of deformation and here your forming limit is reached in this particular free expansion of tube ok. So if blue color line is a forming limit curve then you are actually following a plane strain deformation path to reach forming limit curve ok and you will see that this would be your n this would be your n because I think we already discussed about that basically $\varepsilon_1^* = \frac{n}{1+\beta}$ if you put $\beta = 0$ so n will come. So that is another way to maybe you can assume that way also one will get that type of result.

So there are two stages one is $dp = 0$ maximum pressure $n/2$ is going to be there and when it reaches a forming limit curve you will see it is going to be n ok. So now the limiting case I am going to get is nothing but $\varepsilon_\theta = ln\frac{r^*}{r_0}$ where r^* belongs to $r^*$is nothing but the radius at that critical moment when maximum pressure is reached when maximum pressure is reached. That means what only when r_0 becomes r^*that is a maximum ε_θ you can otherwise eta you have to stop it you cannot go beyond that ok. So r should can become r^* that is all ok when r^* is reached you have to stop like for example when $\varepsilon_\theta = n/2$ or $\varepsilon_\theta = n$ ok there are some instabilities know reason conditions for instability we say similarly you can deform the material here up to $\varepsilon_\theta = ln\frac{r^*}{r_0}$. And when r becomes r^* then you can get p^* also from this equation it is $p^* = \frac{2}{\sqrt{3}}K\left(\frac{2}{\sqrt{3}}\mathrm{n}\right)^n\frac{t_0 r_0}{r^{*2}}$ ok.

This r^* we are replacing this r by r^* , $ln\frac{r}{r_0} = n$ only and this fellow is going to become p^* is nothing but where r^* and p^* are values at tube splitting. When tube splits r becomes r^* and p becomes p^* where p^* is nothing but your maximum value you have to be careful with that ok. And you will see the splitting occurs in that particular region where there is local swelling and one stage before that is this particular stage ok. So, now we have seen that we are moving along plane strain strain path ok to reach this particular forming limit curve which is actually a conservative window. So, conservative window means your formability is limited in that particular direction.

So, you want to have a better forming behavior ok. So, then you need to deform this tube in a non-plane strain condition you should not deform it in plane strain condition ok. The problem is you can deform it but you need a higher n value ok. Suppose if n value is let us say 0.2 ok then as per this discussion the material will split at when ε_θ becomes 0.2 ok. But you cannot make a full tube from that may be successful forming is not done which means you have to increase the n value of the materials. You need another material n is equal to let us say 0.27 ok to get a good form tube ok. So, that is the only disadvantage here. So, in the plane strain expansion the material fail approximately when the hoop strain reaches n.

So, if that is the case normally high strain hardening material is preferred. Why? Because you are deforming in a conservative forming window. So, now instead of that so instead of this $\beta = 0$ this particular path as I mentioned here from 0 to here you reach instead of that ok I can take it to $\beta = -1$ that is a constant thickness process ok that is called constant thickness forming or constant thickness process we know that $\beta = -1$ what will happen we already discussed about it which is nothing but an arrow mark indicating this. You are going to deform the material along this path in which case it may not meet the forming limit curve or it may meet very late. So, even you can use a lower n value to get a good tube forming.

So, that is why I reduce the height of this forming limit curve here you can see this is about let us say 0.25 here it is let us say only 0.1 ok it is only 0.1, n value of 0.1 can be used ok. So, larger strains are possible by taking lower n value material and using a constant thickness strain path that is $\beta = -1$. So, if material is the restriction you have then better you deform the material in $\beta = -1$ ok by some way we will discuss it now. So, that you do not reach the forming limit before the component is fabricated. If you have higher n value material then no problem ok you can deform in plane strain mode itself and then you can make the component. So, that is the advantage of deforming in constant thickness forming.

When $\beta = -1$ ok why it is constant thickness you can get $\beta = -1$ means $\varepsilon_t = -1(1+\beta)\varepsilon_\theta$. . So, ε_θ I am going to put here and $\beta = -1$ you put $\varepsilon_t = 0$ ok. If you put $\beta = -1$ we already discussed it ok it means your $\varepsilon_t = 0$ ok which means thickness is going to remain same and it follows this particular path. So, let us put some mechanics to it and in the constant thickness process ok the hoop and axial tensions and stresses would be equal and opposite correct.

So, when you say $\beta = -1$ your $\alpha = -1$ which means your $\frac{\sigma_2}{\sigma_1}$ or in terms of this if you say $\frac{T_\theta}{T_\phi}$ they are also equal and opposite right. So, $\alpha = -1$ says $\sigma_\theta = -T_\phi = T_\theta$ right. And the main difference how you convert a free expansion the case we have seen before in plane strain mode of deformation to constant thickness forming is by providing this axial force *F*. This is what I was telling you before everything remains same in the schematic except that to this tube you are going to provide some axial force on both the sides. Everything remains same *p* is going to be there *r* will be there you have thickness and T_θ , T_ϕ everything will remain same for us.

So, along with fluid pressure *p* you are going to add *F* to it. So, *F* is also going to actually help *p* in deforming the material. So, as usual now what we are going to do hoop tension is given by $T_\theta = pr$ and $T_\phi = \alpha T_\theta$ which is nothing but $\alpha = -1$. So, $T_\phi = -pr$. So, now is axial force *F* is nothing but it is compressive in nature.

So, $F = -2\pi r T_\varphi$ ok. So, it is basically $F = -2\pi r T_\varphi = 2\pi r p r = 2\pi r^2 p$ will come that will be your force. So, this much amount of force you have to give ok to have a successful forming. So, that is why I said that this *F* and *p* are dependent ok. And as usual we can for constant thickness forming also we can get this relationship $\bar{\varepsilon} = \frac{2}{\sqrt{3}}\varepsilon_\theta$, $\varepsilon_\theta = ln\frac{r}{r_0}$ and $\sigma_\theta = \frac{1}{\sqrt{3}}\bar{\sigma}$ you can get all these things from these two relationships we already have here right. So, $\alpha = -1$ if you put -1, $\sqrt{(1+1+1)}\,\sigma_\theta = \bar{\sigma}$ correct. So, this gives a $\bar{\sigma} = \sqrt{3}\,\sigma_1$. So, $\sigma_1 = \frac{1}{\sqrt{3}}\bar{\sigma}$. So, σ_1 for us is σ_θ here. So, you can get and this relationship $\bar{\varepsilon}$ relationship you can get from Von Mises effective strain equation.

So, now as usual we are as going to assume your Holloman you know power law equation and pressure how do you get this *p*? $p = \frac{2}{\sqrt{3}}K\left(\frac{2}{\sqrt{3}}\ln\frac{r}{r_0}\right)^n\frac{t_0}{\mathrm{r}}$ we already know this and $\sigma_\theta = \frac{1}{\sqrt{3}}\bar{\sigma}, \bar{\sigma} = K\bar{\varepsilon}^n$. So, *K* is there $\left(\frac{2}{\sqrt{3}}\ln\frac{r}{r_0}\right)^n$ is there and $\frac{t_0}{r_0}$ of $\frac{\mathrm{t}}{\mathrm{r}}$ we are writing $\frac{t_0}{\mathrm{r}}$ why because *t* and t_0 are going to be same constant thickness should be noted here that is the only difference we have. So, let us be careful here of course you can put *t* also here with understanding that t_0 and t_r one and the same here ok. So, there are two pressure equations one for free expansion without axial force which is in plane strain mode of deformation a conservative window and forming limit. The other case is if you give axial force so you are going to do the same operation free expansion of tube with the help of axial force and pressure the advantage here is you are going to be in the constant thickness forming mode and two pressures are developed here and these two are compared here.

So, tube expansion without axial force plane strain tube expansion with axial force constant thickness these are the two expressions we have derived and you will see that the pressure is reduced for constant thickness expansion compared with the plane strain you can find out ok. So, you can also derive some ratio between these two ok what ratio of pressure you have in constant thickness forming with respect to your plane strain you can find out but it is you can also put some numbers here and then find out that pressure the constant thickness expansion must be smaller than the plane strain one. But you have to have the you know set up for axial force so along with the pressure internal pressure set up in the actual you know the hydraulic press you have along with the internal pressure you can also have an axial force that is given ok that set up also you need to have ok. So, axial force actually helps pressure to form the component so these are the second one.

The first one is that this is the second one for us. So, now let us go to the third one ok so this third one is nothing but converting a cylindrical tube into a square section which is what is given here the schematic ok. So, this is a square die right and there is a circular tube here you know of a particular thickness you can imagine and you are giving some internal pressure to this internal pressure is given like this and what do you do basically here the material is actually going to freely expand but of course after sometime it will locally deform which is what is shown here. So, you can see that the tube is almost becoming a square section and with further you know push you will see that the corner region will be filled fully to make a almost a square tube ok. So, you can imagine by conventional forming how you make it, it is not going to be so easy but here one can do this. So, now this corner is region you want to analyze mechanics part of it, corner region is actually zoomed here and some dimensions are provided here so this is a situation the corner of the tube I have given.

So, only just you know one corner is shown situation remains same in all other corners. So, what are the things given here you can see that r_0 is the initial radius and t_0 is the thickness in the contact region. The tube is already contacting here and there is an unsupported region here right that is the situation now. So, the contact region you will see that either this region ok or this region you can say ok is nothing but you have t_0 and the contact length is l let us say ok that is the initial status and initial status also you have r is the corner radius. So, now with if you compare this with the next stage, next stage is given by the dotted lines is given by dotted lines your r becomes $r + dr$ ok that means your dr is actually a negative quantity you are going to become a small corner ok this corner ok is going to become a smaller corner small corner ok.

So, your corner radius basically decreases this dr is actually a negative quantity ok. Anyway, so in the next stage of deformation that is the dotted lines you can see that your r becomes

$r + dr$ where dr is a negative quantity and during that time you will see that this l becomes $l + dl$ this l becomes $l + dl$ and your t ok here t becomes $t + dt$, t_0, t, dt three are there. So, t_0 is in the contact region which is already established contact and let us say your t is in the corner region and $t + dt$ would be the thickness in the corner region in the next stage like that you can imagine ok. So, a portion of the tube has been enlarged so that wall touches the die at point a. So, this point is now contacted ok this point is actually in touch this is all are a points a points are actually reached now.

Assuming that the thickness is less than the radius and the contact length l is yes ok the current contact length your l. So, this $l = r_0 - r$ correct. So, l is nothing but your $r_0 - r$ that is your l. So, the corner radius ok so decreases to $r + dr$ that is r is a negative value during an increment in the process while the contact length grows to $l + dl$. So, when the contact length l becomes $l + dl$, r becomes $r + dr$ that means it is becoming a small corner radius decreases ok. So, from this equation we can write $dl = -dr$ or $dr = -dl$ ok. $dr = -dl$. So, the change in you know radius is nothing but change in length only but they are opposite fine. So, now there is one important thing here. So, internal pressure what we are giving will force the tube against the die wall at the contact zone.

So, now the contact zone is established this l is a contact zone ok this has been established now ok and there is an unsupported corner region and there is an unsupported corner region ok. When the material slides along the die so you can see this a figure you can see that so this is your die ok and as I already pointed out so this region is already contact is established here in this portion of the may now your this one and here you will see that there is an unsupported region ok and internal pressure p is applied and this is your s distance let us say. So, the material slides along the die friction opposes the motion ok and tension varies throughout the tube wall. So, basically it is said that so you if you see tension ok T_θ which is applied here it will change with respect to the S with respect to S ok. Now what will happen is the strain will eventually become insufficient to stretch the wall resulting in sticking zone.

So, while the tube is deforming what will happen is in the contacted portion already you will have a sticking zone. Sticking zone means the material is already stuck to the die and there will be a sliding zone in which further deformation can happen that is what is given here a sliding and sticking ok. So, the strain is actually insufficient to stretch the wall region and a sticking zone is getting established as shown is figure a this is your tube. So now the same situation I am going to put an equilibrium equation here ok you will see that the equilibrium equation for an element so I am going to pick up an element in the sliding zone ok I am going to pick up a small region in the sliding zone ok and I have zoomed it in figure b and I am going to apply some equilibrium to it to get some idea what is the change in tension. So, I am going to say that if this is my element this is my element and since the material is deforming in this direction ok.

So, μp is going to act in my right hand side ok and since this is the die wall and let us say the element size is ds ok and this is a general S distance ok so ds is along S right. So, now I have

T_θ acting in this direction so there will be some change in T_θ , $T_\theta + dT_\theta$ will be there so I can write $T_\theta + dT_\theta = T_\theta + \mu pds1$ is in the same direction $\mu Pds1$ so which will give me my $\frac{dT_\theta}{ds} = \mu p$. So, I am picking up a small element in the sliding zone and I am going to get this particular equation this equation says that how my T_θ is going to vary with respect to S that is along the tube wall and that is going to be a function of μp and that is going to be a function of μ which is nothing but μp. So, this is nothing but slope so if you draw a graph between T_θ and S the slope is going to be μp ok.

So, this equation is going to give me my this particular diagram. So, tension in the unsupported corner will increase as the radius decreases that we already know that and as per the equation for T_θ the tension reduces linearly towards the center owing to friction right this particular dotted line if you see so this is the same diagram at the bottom ok. So, this is your sticking zone and this is your sliding part and your T_θ sorry T_θ is acting here ok and with respect to this S this is your S and you can see that X axis is S, Y axis is T_θ and you will see that your $\frac{dT_\theta}{ds} = \mu p$ right. So, now what will happen now here is the tension in the tube wall is less than that necessary for yielding to right of the point where sliding stops and there is no additional deformation in the sticking zone. So, what basically is going to tell is when you move from sliding to sticking region because of the sticking region ok the deformation in the cup wall in the sticking region is going to stop and what can happen is that can happen only in this localized region which is unsupported region and you will see that there is a transition between sliding and sticking and at that place there is a thickness t_S ok. So, now if you want to get this t_S ok if you want to get this t_S so what are we going to do is the main thing now for us and if you pick up $\sigma = K\varepsilon^n$ ok since this deformation is also plane strain this deformation is also in plane strain mode of deformation that means you will see that perpendicular to the plane ok perpendicular plane the deformation is not going to be there.

So, you can say that is a plane strain process for that this is already known to us $\sigma_\theta = \frac{2}{\sqrt{3}} K \left(\frac{2}{\sqrt{3}} \varepsilon_\theta\right)^n$ this is a plane strain forming ok. So, the tension at the critical point that is your t_S now this particular point if you want to get then it can be obtained in a similar way $T_{\theta s} = \sigma_\theta t_s = \frac{2}{\sqrt{3}} K \left(\frac{2}{\sqrt{3}} \ln \frac{t_0}{t_s}\right)^n t_s$. So, this T_θ at that particular location can be obtained from this particular equation provided you know what is t_S provided you know what is t_S ok and when you compare this sticking zone and sliding zone in the sticking zone there is no sliding naturally and the slope of the tension curve ok would be less than μp that is obvious that we know and if you want to evaluate pressure in the unsupported corner then it can be obtained from the simple equation which we already derived ok. So, when you convert your circular tube into square tube what we have seen is basically how tension changes when you move from sticking to sliding. Sticking zone is that zone basically where the tube is already in contact with the established contact with the die wall and your sliding exist in the unsupported closer to unsupported corner and if you keep on monitoring tension it will follow your μp ok it will follow μp and you will see that there is one particular transition thickness t_S that is a critical one ok which can be evaluated ok or tension at that location can be evaluated using the simple equation provided you know what is t_S fine.

So with this we will stop this particular chapter discussion. So, we have seen three important one is you know topics one is a free expansion of tube without axial force which is nothing but plane strain mode of deformation next one is to keep it more advantageous constant thickness forming can be done by giving axial force that is the second one third one is converting around tube into square tube ok which is all about filling a corner mainly and that pressure can be obtained from this equation but other than that we have seen what is happening at that transition of sliding to sticking.

So, let us work out two important problems which are actually theoretical in nature only ok let us do it quickly. So, a circular tube with a radius R and thickness t_0 ok which is what you have seen before is deformed into a die with a square cross section through hydroforming. So, you need to find a relationship between r and internal pressure p when there is no friction in the frictionless case. So, r has to be related to p sorry p has to be related to r ok you need to get p which is a function of r when there is no friction at the die metal interface frictionless case ok.

And the schematic is shown here you can see that red color one is the initial one R and it is got t_0 thickness and in the next stage it is becoming a square tube with a small corner radius of r and it is become t let us say the contact length is established l here and here also it is l ok fine. So, now what we do is we need to get p actually we need to get p this fellow we need to get so I am just going to equate volumes in these two stages ok one is the initial one which is nothing but my $\frac{\pi}{2}Rt_0$ times and which is equal to my new stage which has got actually two regions one is a corner region the other one is a already contacted region ok. So, $\frac{\pi}{2}r\mathrm{t}$ would be my corner one plus I am writing $t(R-r)$ ok two times for the two walls you can say $t(R-r)$ these two I am going to add ok to equate these two from this I can get $t, t = \frac{t_0}{\frac{4}{\pi}-\left(\frac{4-\pi}{\pi}\right)\frac{r}{R}}$ where r is your corner radius and capital R is your initial tube radius. So now again let us assume plane strain these details are already known to you we already discussed about it σ_θ how is related to $\bar{\sigma}$, $\bar{\varepsilon}$ how is related to ε_θ this will give me my σ_θ ok as there is only one change now so basically we are relating t to t_0 that is all otherwise the pressure equation is going to be remain same ok that will come back to this in the next slide. So $\sigma_\theta = \frac{2}{\sqrt{3}}\bar{\sigma}, \bar{\sigma} = K(\bar{\varepsilon})^n$ right so instead of this there will be let us say a small change here let us say we put $\bar{\sigma} = K(\varepsilon_0 + \bar{\varepsilon})^n$ for example let us say so $\sigma_\theta = \frac{2}{\sqrt{3}}K(\varepsilon_0 + \bar{\varepsilon})^n$ is $\sigma_\theta = \frac{2}{\sqrt{3}}K\left[\varepsilon_0 + \frac{2}{\sqrt{3}}\ln\left(\frac{4}{\pi} - \left(\frac{4-\pi}{\pi}\right)\frac{r}{R}\right) + \bar{\varepsilon}\right]^n$ this is my σ_θ so now we know what to do we are going to pressure equation so $p = \frac{\sigma_\theta t}{r} = \frac{2t_0K}{\sqrt{3}r\left[\frac{4}{\pi}-\left(\frac{4-\pi}{\pi}\right)\frac{r}{R}\right]}\left[\varepsilon_0 + \frac{2}{\sqrt{3}}\ln\left(\frac{4}{\pi} - \left(\frac{4-\pi}{\pi}\right)\frac{r}{R}\right)\right]^n$ so you will get this particular equation.

So this t can also be replaced with this particular equation again so you will get finally this is the pressure right. So instead of deriving a full equation for pressure in this particular section we are working out as a problem that is the only difference here but here we are restricting our case to frictionless case restricting our case to simpler one which is nothing but a frictionless case we could have derived it in the previous derivation itself but we are keeping it as a separate problem so it is very simple the only thing is you need to get this t as a function of t_0 that is the only difference otherwise is a regular root with which we evaluate it. So finally it is addressed so p is related to your corner radius r in this way you can create a pressure versus r you know a graph.

Let us go to next one next one is also an application of what we discussed in the Q1 number 1. So there is a steel tube of 180 mm diameter and thickness of let us say 4 mm ok that has to be expanded by internal pressure to a square section ok. So a circular one should be converted into a square tube but there is one constraint that the maximum pressure available with the person is 64 MPa ok or the machine has got a limit of 64 MPa ok that is a maximum pressure you can have within that you have to make this particular tube. So the material has got a strain behaviour with a $700\bar{\varepsilon}^{0.2}$ n is given K is also given. So now what you need to do is you need to get the minimum corner radius, minimum corner radius basically you need to get r that can be achieved under frictionless case which is easy for us to understand.

So determine the minimum corner radius ok. So now we need to understand one important thing this minimum corner radius that is r^* we said no something like that you can imagine minimum corner radius means that is a critical limit we have ok. Let us say this is give me your r^* which you have seen in the previous derivation right. So this r^* has to be reached means this can be reached either because the tube is net or maximum pressure is reached ok. So either it has to reach making either it has to reach a forming limit curve ok or it has to reach $dp = 0$ let us say maximum fluid pressure is reached ok. So now depending on our availability of data we will do something but before that let us list out what all the things we know this equation is known to us just now we derived it for a frictionless case and we also know this is not it for a plane strain process $\varepsilon_\theta = -\varepsilon_t = ln\frac{t_0}{t} = ln\frac{r}{r_0}$ fine let us keep it like this.

So now what we are going to say this for local necking of tube wall ok I am going to say that is $\varepsilon_\theta = n = 0.2$ correct that is why you know we got one previous equation ok you can go back and check this particular equation free expansion you can see that know this p^* is obtained by that condition only $\varepsilon_\theta = n$. So now what we do is if you put this $\varepsilon_\theta = 0.2$, $0.2 = ln\frac{4}{t}$ let us say ok and from this you can get t of course $t < 4$ ok.

So $t < 4\ mm$ which I got it as 3.27 mm I got it as 3.27 mm so this is the thickness limit we have 3.27 mm ok. So now we are going to somehow bring in the 64 MPa right you have to bring in somehow 65 MPa so pressure equation also should be used this pressure equation was derived just in the previous slide only I have just written here for our convenience. Now here what we are going to do is we are going to do some iterations and we have to do that calculation and get the answer. So now here what we are going to do is as we know that r is going to change with p will also change you can draw a graph between p and r right something like that you can imagine so I am going this equation I am going to change r values from let us say 30, 40 to 30 mm ok 40 to 30 mm and I am going to find t, σ_θ and p using all known equations ok my t can be obtained using this equation ok r also can be obtained ok sorry t can be obtained and σ_θ can be obtained my previous derivation which already said the equations are there and p is this equation ok this also here we can obtain it.

If we can obtain p directly that was also fine with the help of t. So now you may take maybe 10 different values 40, 39, 38, 37, 36, 35, 34, you know 32, 31, 30 like that if you can pick up 10 different values after this 10 iterations ok you will see that when $r = 31$ you reach a pressure of 63.5 MPa as per this particular formula as per this particular formula you can just work it out ok one after another you can put ok. So r then you can put t then σ_θ and p so here you can say 10 different values you can see find t, σ_θ and p you will see that when r is equal to 31 mm you will see p is going to be about 65.5 MPa, 65 MPa which is actually the limit of the machine which is set as 64 MPa ok then one can say that hence a minimum corner radius is 31 mm ok and is limited by maximum pressure and is limited by the maximum pressure which is nothing but 64 MPa, 64 or 65, 64 only, 64 MPa so 31 is a minimum corner radius.

So the question is what is the minimum corner radius that can be achieved under frictionless case ok because it is a frictionless case this formula we have chosen if it is with friction then we do not know your model could be equation could be different ok. If this is the frictionless case then this is the formula you have to choose ok and you can get different values of r values you can substitute and you can get maximum pressure and where it is going to cross 64 you need to see at 31 mm it will be closer to 64 so you can say 31 mm as a minimum corner radius right. So let us stop here we will discuss the further parts in the next session. Thank you.