

**Laser Based Manufacturing**  
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**Module # 01**  
**Lecture # 02**  
**Fundamentals of Laser Technology**

Hello everyone. I welcome you all to the second lecture of Week 1 of the NPTEL MOOC course on Laser based Manufacturing. I hope you have enjoyed the lecture 1 which was more on the introduction to these MOOC course, that is, the application of lasers in manufacturing, importance of laser in manufacturing and very basic details about the lasers.

In today's lecture we will go in little more details about the production of lasers in the industry. How exactly lasers works, that is the main objective of this lecture. As we have seen in the previous class, we are using lasers as a beam of high density power and there were many reasons to use it for the manufacturing operation. In the fundamental application of laser in the shop floor in the tool room is machining or engraving or material removal. There we need to have very high density power beam which can remove the material by using vaporization process or by using the melting process. But how to get this high density power by using the lights?

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### Laser technology

Lasers produce intense beams of light:

- **Monochromatic:** the wavelength (color) of laser light is extremely pure ✓
- **Coherent:** have a fixed phase relationship (coherence) with respect to one another
- **Highly collimated:** has very low divergence ✓
- Can be focused to a **very small spot**

Laser has been used in the industry. It provides a beam of the light which is monochromatic. What is the meaning of a monochromatic? Monochromatic means the wavelength of the laser light is extremely pure, the color of the laser beams are extremely pure, the wavelengths are uniform. The beam is having the various laser lights and all these lights are having the same wavelength, that is the meaning of the monochromatic characteristic of the laser beam.

The second characteristics, is the lasers are coherent in nature. The coherence is the very important property that represents or that designates the phase relationship between the laser waves or the light energy waves. The meaning of coherence here is that all the laser beams or

all the energy waves inside the laser beam are having fixed phase relationship. We are in the same phase all the laser light beams are in same phase.

Then the third one is the laser beam is highly collimated - the meaning of collimated is that the lasers beams or the waves in the laser beams are parallel to each other. There is very low diversions, the beams are exactly parallel to each other and that enables to have high power density laser beam which we can apply to a very small spot. As we can apply the laser beam, as we can concentrate the laser beam on a very small spot - we can generate high laser power density and that high laser power density will help us to manufacture the various components: maybe by using machining operation, by using welding operation or by using forming or additive manufacturing operation.

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### Laser technology

- **L**ASER is an acronym for Light Amplification by Stimulated Emission of Radiation.
- **L**IGHT: electromagnetic radiation ranging from 1 nm to 1000 μm in wavelength. wide range
- The **visible spectrum** (what we see) : 400 to 700 nm Amplification of Light Stimulated Emission of Radiation
- From 700 nm to 10 μm : near infrared ✓
- Beyond 10 μm : far infrared
- From 200 to 400 nm is called ultraviolet (UV)
- Below 200 nm : deep ultraviolet

Now let us see what is the meaning of laser? It is an acronym for various group of various words that is 'light amplification by stimulated emission of radiation'. L has come from the light, A has come from the amplification, S is from the stimulated and E from the emission and R is from the radiation. Here we are amplifying the light - amplification of the light. By what procedure? We are following the procedure of stimulated emission, of what? Of radiation. Now we will be learning what is the meaning of the stimulated emission today and why it is essential to have the stimulated emission. Light as we know it is just an electromagnetic radiation which is having the wavelength from about one nanometer to 1,000 micrometer. So, light is a type of radiation which is having a wavelength from 1 nanometer to 1,000 micrometer which is quite wide range.

If we want to look at the classification of the laser light or if you want to look at the classification of light; the visible spectrum of the light is possible when the wavelength is from 400 to 700 nanometer and if the wavelength is more than 700 nanometer that is going to the

range for the near infrared type of light and if it is beyond 10 micron (10 micrometer) that is the far infrared region.

If we are considering the light in between 200 to 400 nanometer, that is called as the ultraviolet light and below 200 nanometer it is the deep ultraviolet light.

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- Light -> electromagnetic radiation that comprises of an electric field that oscillates in both time and space and a corresponding orthogonal magnetic field that oscillates with the same spatial and temporal periodicity (by Maxwell)
- Light is applied -> interacts with the charged particles within the atoms
- Though both magnetic and electric field are absorbed by materials, the interaction of the electric field of light is usually much stronger than that of the magnetic field
- In particular, the term "light" = visible portion of the electromagnetic spectrum
- However, in general (a looser definition) that includes the entire visible and non-visible spectrum

### Electromagnetic Wave

The diagram illustrates an electromagnetic wave propagating to the right. The electric field (E) is represented by a blue sine wave oscillating vertically, and the magnetic field (B) is represented by a red sine wave oscillating horizontally. Both fields are perpendicular to each other and to the direction of propagation. The wavelength (λ) is shown as the distance between two consecutive peaks of the electric field. Handwritten red annotations include 'P1' and 'P2' at the peaks, and a note: 'Wavelength = distance between the peaks / valleys'.

Figure: DECHAMMAKL, CC BY-SA 4.0 <<https://creativecommons.org/licenses/by-sa/4.0/>>, via Wikimedia Commons

Let us see some of the concepts related to light. Light is an electromagnetic radiation. It comprises of an electrical field which is oscillating in both along the time direction and in the space. The light also having the magnetic field component which is orthogonal which is at a 90 degrees to the electric field component and magnetic field is also oscillating with same spatial and temporal periodicity.

If we look at the schematic of the electromagnetic wave as a light, then we can plot the electrical field in this way. This electrical field is oscillating about the axis, oscillating about the direction of its propagation. Here you can see in a blue color the electrical field is oscillating about the direction of propagation and there is magnetic field as well which is also oscillating about the direction of propagation, but travelling in a plane which is orthogonal to the electric field plane.

Here the definition of wavelength can be considered as the distance between two consecutive peaks or two consecutive troughs. When we apply the light on a particular material then the light is interacting with charged particles. Now as we know that the light has both the components that is the magnetic field as well as the electrical field and the interaction of electrical field is found to be much more stronger than the magnetic field.

Therefore in the studies of laser the effect of electrical field has been considered the further computations or further analysis. In particular, the light is considered to be the visible portion of the electromagnetic spectrum. However, in general the light is considered to be both that is the visible spectrum as well as the non-visible spectrum. But to be precise, the light is to be defined as the visible portion of the electromagnetic spectrum only.

The wavelength is defined as the distance between the peaks or the valleys. Here this is peak 1 and this is peak 2 with respect to the electrical field, then the distance between the peak 2 and the peak 1 is considered as the wavelength. In a similar way if we are considering the valleys these two points can be considered and this is the wavelength of the electromagnetic radiation.

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### The Bohr atom

- In 1915, Neils Bohr proposed a model of the atom
- A simple model -> the basis for the field of quantum mechanics
- Not fully accurate by today's understanding, however helpful for demonstrating laser principles
- Electrons orbit the nucleus of an atom
- The Bohr atom has a limited number of fixed orbits that are available to the electrons.
- Under the correct conditions an electron can jump from its ground state (lowest-energy orbit) to a higher (excited) state, or it can decay from a higher state to a lower state, but it cannot remain between these states. The allowed energy states are called "quantum" states.

Now to understand the principle of operation of laser, let us take the theory suggested by the Bohr. In Bohr atom we will see what is the meaning of Bohr atom and by using this Bohr atom model we will try to understand the function of a laser or the production of a laser beam. In 1915, Neils Bohr proposed a model of the atom. It is a very simple model and it is based upon the quantum mechanics. Nowadays, it is considered as not fully accurate and it is not that advance model. Afterwards the Rutherford model came for the explanation of atoms, but for demonstrating the laser principles, the Bohr model can be used.

What this Bohr model suggest us? The Bohr model is stating that there are electrons in an atom and this electrons are orbiting about the nucleus of that atom. Let us consider there is nucleus which is positive charged and about that nucleus we are having a fixed number of orbits and the electrons are moving in that orbits. Let us consider that there are three orbits. This is an electron which we consider as a negatively charged particle and it is in the energy state 1. The next is energy state 2 and the last one or the outermost is energy state 3.

Now, during the operation or during certain correct conditions, we can consider or we can say that an electron can jump from its ground state that is the lowest energy orbit, here the lowest energy orbit is E1, to a higher state, that is, the excited state. Higher state here maybe E2, so the electron will jump from E1 to E2 or it may even jump to the level 3 or it can decay - the electron decay from the higher energy state. Either it can move from E1 to E2 or E3 that we call jumping in higher energy state or the energy can be decayed from E2 to E1 or from E3 to E2 or E1. That we call decay of the energy, but it cannot remained in between. The electrons should be in one of the orbits around the nucleus and they cannot be in between and these allowed energy states are called as the quantum states. E1, E2, E3 are called as the quantum states and they are designated by using quantum numbers. If I plot the graph here about the energy states, I can draw in a very simple way this is 0, this is ground state. Along the y axis we are having the energy. Energy we are generally representing by using electron volt (eV). Let us consider this is 0 electron volt (eV), this is 5 then 10 and 15. We can call the energy level of about 10 electron volt as the excited energy level E2 and many more energy levels above that. If we plot the energy levels we will simply get the horizontal lines. This is the first excited state and even we can have the more excited states such as this is the second and few more or there may be 'n' number of states.

As we have seen in the previous slide, that electron is jumping from one energy state to the higher energy state or higher quantum state. How it is possible? Automatically it is not possible. We need to have some extra energy, we need to have some outside energy which is applied on that electron, the electron will gain some energy from the outside world and then it will get excited and it will reach to the next quantum state.

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## The Bohr atom

- For an electron **to jump to a higher quantum state** -> the atom **must receive energy from the outside world**.
- This is possible through a variety of mechanisms : inelastic or semi-elastic collisions with **other atoms** and absorption of energy in the form of electromagnetic radiation (e.g., light).
- When an electron moves from a higher state to a lower state, **the atom must release energy** : as kinetic activity (nonradiative transitions) or as electromagnetic radiation (radiative transitions).

*Inelastic collision → there is a loss of kinetic energy →*

*Semielastic collision → some energy lost but objects do not stick together ie less than 100% of KE is conserved.*

How it is possible? There are various mechanisms, by using these, jumping is possible. Here we can consider that the jumping of the electron is due to inelastic or semi elastic collisions with other atoms. What is the meaning of inelastic collision? During the inelastic collision there is loss of kinetic energy during the collision of two atoms. Here we will say that during the collision - during inelastic collision there is loss of kinetic energy or we will say that the energy is not conserved. In semi elastic type of collision some energy is getting lost, but objects do not stick together. That means less than 100% of the kinetic energy is conserved or we can have the absorption of energy in the form of electromagnetic radiation.

If we are applying the light energy on the electrons then they are absorbing the light energy and they are getting excited - the electrons are getting excited. Either you can have the collision of atoms or you can feed the atoms the light energy. By getting the energy - the electrons are getting energized and they are moving to the higher energy level, higher quantum levels. When an electron moves from higher state to lower state. In other way round if there is a electron at a higher quantum level and it is coming down, it is dropping its energy level to the lower one - the atom must release the energy, that is the thing given by the Bohr that when the electrons are at higher energy level, they are moving to the lower energy level, they are releasing certain amount of energy from them. This is considered a kinetic activity, it is a non-radiative transition, or it can be electromagnetic radiation or radiative transition. Either of these phenomena will occur. It can be a kinetic activity or electromagnetic radiation. In laser physics, generally, we are considering that as a radiative transition.

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## Photons and Energy

- Light is made up of particles : “photons” -> exhibit both particle-like and wave-like properties
- The intrinsic energy of a photon can be determined by using the equation

$$E = h \nu \quad \text{--- ①}$$

where  $\nu$  is the frequency of the light ;  $h$  is the Planck's constant.

$$\lambda \cdot \nu = c \quad \text{--- ②}$$

$\lambda$  : wave length of wave       $c$  : speed of light in vacuum

$$E = \frac{h \cdot c}{\lambda}$$

- ✓ longer the wavelength of the light -> lower the energy of the photon
- ✓ ultraviolet light is much more energetic than infrared light

Now, let us look at the light constitutes. What exactly the light comprised of? Light is made up of particles and that particles are called as photon and these photons are exhibiting the

properties of both. They are behaving like a particle or they are behaving like a wave. What is the energy that is of a photon and how to compute that energy? let us find out.

The intrinsic energy of a photon can be determined by using an equation and this equation is:

$$E = h \cdot \nu \text{ --- (1)}$$

where  $\nu$  ( $nu$ ) is the frequency of the light and  $h$  is the Planck's constant. Now we got the frequency of the light here. Now every light wave is having  $\lambda$  (lambda), that is, its wavelength. What is the correlation of the wavelength with the frequency of the light with respect to the speed of the light? That we know very well that:

$$\lambda \cdot \nu = c \text{ --- (2)}$$

Here  $\lambda$  is the wave length of the light wave,  $\nu$  is the frequency of the light and  $c$  is the speed of the light in vacuum. We got this is Equation (1) and this is Equation (2). Let us use Equation (2) in Equation (1) to get the energy of a photon. So, I can write here:

$$E = (h \cdot c) / \lambda$$

If you look at this equation you will find that if the wavelength is higher - if we increase the wavelength then the energy is getting reduced - meaning is that the higher wavelength light beams will have lower energy photons. From this particular equation what we can understand that longer the wavelength of the light, lower would be the energy of photon. Therefore, the ultraviolet light is much more energetic it is having low wavelength, a small wavelength than the infrared light.

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## Photons and Energy

- For an atom to absorb light : for the light energy to cause an electron to move from a lower energy state  $E_n$  to a higher energy state  $E_m$ ) -> the energy of a single photon must equal (almost exactly) the energy difference between the two states.
- Too much energy or too little energy and the photon will not be absorbed.
- Therefore, the wavelength of that photon must be,

$$\lambda = \frac{hc}{\Delta E}$$

$h = 6.626 \times 10^{-34} \text{ joule}\cdot\text{s}$   
 $c = 2.998 \times 10^8 \text{ m/s}$   
 $\Delta E = E_m - E_n$   
 $\lambda = \frac{1.99 \times 10^{-25}}{\Delta E} \text{ joule}\cdot\text{m}$

Now regarding the absorption of the light, for the light energy to cause an electron to move from lower energy state to the higher energy state, the energy of single photon must equal the energy difference between the two states. What we have seen that we have to pump certain amount of energy to the electron to move it from one energy state to the higher energy state. How much energy has to be put in to excite the electron to move to the next state? That is the exactly the difference of the energy levels at higher energy and the lower energy. If we apply too much energy that will also not work and if too little energy also that we apply that will also not help to get the exact absorption or exact operation.

Therefore, the wavelength of that photon can be computed by using this particular correlation:

$$\lambda = (h \cdot c) / \Delta E$$

where,  $\Delta E$  = the higher energy state ( $E_m$ ) – the energy of the lower energy state ( $E_n$ ).

Now let us put the values of the Planck's constant and the speed of light here. We are having the Planck's constant as  $h = 6.626 \times 10^{-34} \text{ J.s}$  and we are having the speed of the light as  $c = 2.998 \times 10^8 \text{ m/s}$ .

So, we can say that the wavelength of the photon must be:

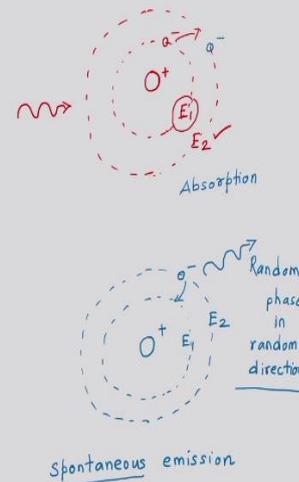
$$\lambda = (1.99 \times 10^{-25}) / \Delta E$$

If we get a product of these two, multiplication of these two we are getting the value  $1.99 \times 10^{-25} \text{ J.m}$ . To decide the wavelength we must first understand, we must first compute how much amount of energy difference is there between the two quantum states between which the jumping is to be expected or desired.

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## Laser technology: spontaneous and stimulated emission ✓

- When an electron is in an excited energy state, it gets decayed to a lower level -> releasing a photon of radiation. This is called spontaneous emission.
- The photon is emitted in a random direction and a random phase.
- The average time it takes for the electron to decay is called the time constant for spontaneous emission, and is represented by  $\tau$ .



Now this emission of photon from an electron during its change in state from higher to lower energy level is called as the emission - it is emitting the photon. Now how it is possible, what are the various modes to have this emission of photon. The first mode of emission is the spontaneous emission and the second mode or second type is the stimulated emission. When an electron is in an excited energy state and it will be vibrating, it is not stable and from that unstable state it is coming to the lower state and during this change in state, it is releasing certain energy in the form of photon. There is a decay of the energy of that electron and the release of the photon by using radiation it is called as the spontaneous emission. There is no external source that has been applied over here. The electron is at the higher energy state at higher quantum state and it is coming to the lower energy state spontaneously by releasing a photon that is the spontaneous emission. These photons are getting emitted in a random direction, they are moving in a random way, their wavelengths are different, their phases are also random. The emission in a random manner will occur. The average time which is getting consumed for the electron decay is called as the time constant. So, we have to find out or we have to note down how much time the decaying operation is taking place during the spontaneous emission and that is designated by using the later tau.

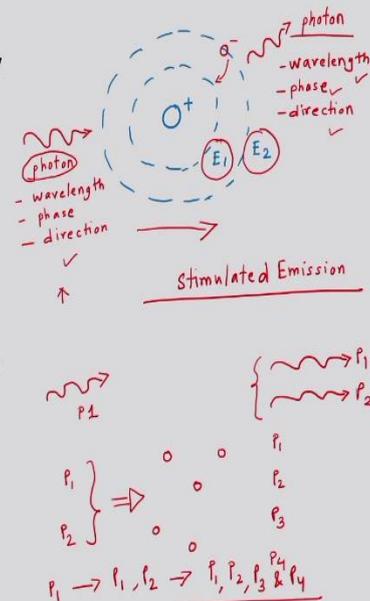
In our previous slides we have seen the process of absorption. Let us consider we are having a positive nucleus and there are two orbits around the nucleus. This is energy state 1 and energy state 2 and let us consider there is an electron negatively charged particle. When we apply the energy - so a photon has been applied here. This negatively charged particle which is at the lower energy state is getting energized and it will jump to the next energy state, that is  $E_2$ . This would be move to the next energy state over here. This process is called as absorption. So, it is absorbing the energy and utilizing that absorbed energy to move to the next state. The spontaneous emission is exactly the reverse procedure. Here we are having a positive nucleus and let us consider that again there are two orbits around the positive nucleus and there is an electron which is at the higher energy state. When this higher energy state electron will come to the lower energy state it is releasing certain amount of energy. This is called as the process of spontaneous emission.

However, the problem with this spontaneous emission is that the photon which is getting emitted during this process is having a random phase and it is moving in random direction. But our entire objective in the laser is to have three basic characteristics that is the wave should have same phase, they should have same wavelength and they should be collimated - they are in parallel to each other. How to achieve that? For that purpose we have to apply an external energy to get the required characteristics of the photons so that we can utilize them as a high power density beam for our intended purpose.

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# Laser technology: spontaneous and stimulated emission

- Whereas, if an electron is in energy state  $E_2$ , and its decay path is to  $E_1$
- However, before the electron has a chance to spontaneously decay, a photon is passing near to the electron. The energy of the photon is approximately  $(E_2 - E_1)$ .
- Then there is a probability that the passing photon will make the electron to decay in a way that a photon will be emitted at exactly
  - same wavelength ✓
  - same direction ✓
  - same phase ✓
 As of passing photon
- This process is called "stimulated emission."



Now, let us take the same atom which is having a positive nucleus and there are two orbits, that is orbit  $E_1$  and  $E_2$ . Previously we have seen that if the electron which is at higher energy state  $E_2$  is getting dropped to  $E_1$  spontaneously - that we can consider as the spontaneous emission.

But now let us take one example. We are having an electron in its higher energy state. Consider there is a photon which is applied which is just passing near to this particular atom, this is a photon - the photon has a certain wavelength, it has phase and it has direction as well. When this photon is just passing near the atom during the process of decaying here we are considering from  $E_2$  to  $E_1$  and during this process let us consider a photon is just passing over - it is passing near to that particular atom and that energy of the photon is exactly  $E_2 - E_1$ . So, if the photon energy is  $E_2 - E_1$ , then there is the high probability, there is a probability that the passing photon will make the electron decay. Naturally, there will be generation of photon during this process, but what would be the wavelength phase and direction of this generated photon? Without this passing photon there is a generation of photon as well, but if the photon is passing near the atom it will influence, it will generate, the passing photon the passing photon will influence or it will generate a photon of similar wavelength as of the photon which is passing similar phase and similar direction. In this case we are getting two number of photons which are having same wavelength, same phase and same direction. That is called as the stimulated emission - a photon which is passing over the atom is generating a similar wavelength phase and direction of that photon which is getting generated during the decaying operation from  $E_2$  to  $E_1$ . And that is called as the process of stimulated emission. Ultimately a single photon is coming over the atom and we are getting two photons at the end of the operations, that is,  $P_1$  which is passing over and  $P_2$  which is getting generated during the decaying operation. However, both of them are similar.

Now let us consider this P1 and P2 photons are again moving ahead and they are doing the similar operation with the other atoms which are there. As the P1, P2 are doing the similar operation that has been done in this phase the same operation would be carried out. Now we are getting the P1, P2 which are already there and P3 and P4 which are getting generated during this emission, that is, the stimulated emission. P1 is generating P1 + P2 of same wavelength, same direction and phase that is again moving ahead and we are getting P1 P2, P3 and P4 of the same wavelength, same direction and same phase. We are calling this as the amplification of light energy. We are calling the amplification of the waves by adding the energy to that to make it more streamline, to make it monochromatic beam of light which can be applied to the work particle or the materials to get the intended operations.

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### Laser technology: spontaneous and stimulated emission ✓

- Consider:
  - ✓ A group of atoms : exactly the same excited state, and most are effectively within the stimulation range of a passing photon. ✓
  - ✓  $\tau$  is very long ✓
  - ✓ Probability for stimulated emission is 100 %. ✓
- The incoming (stimulating) photon interacts with the first atom, causing stimulated emission of a coherent photon
- These two photons then interact with the next two atoms in line, -> four coherent photons -> continues..... ->
- Many photons with identical phases and all traveling in the same direction.
- That means the initial photon has been (amplified) by a certain factor. ✓
- Essential to note that the energy to put these atoms in excited states is provided externally by some energy source which is usually referred to as the (pump) source.

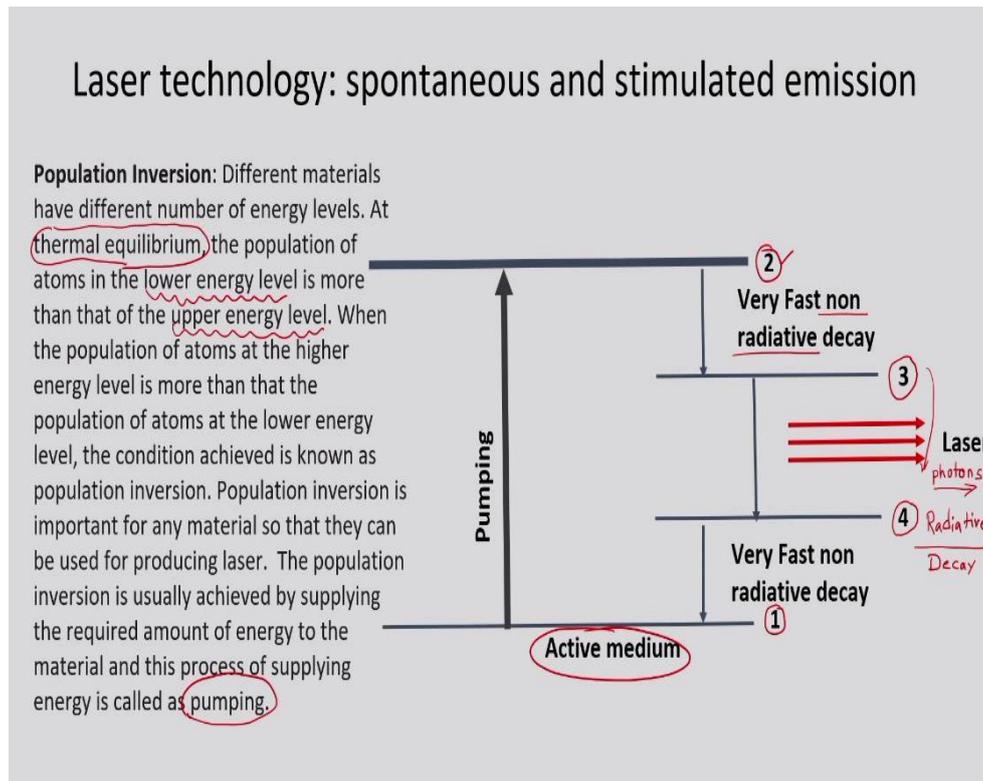
Now, let us summarize the group of activities which are being carried out during this stimulated emission. We are having a group of atoms and they are exactly the same excited state and most are effectively within the stimulation range of passing photon. These group of atoms are in the range of the passing photon, stimulation range of passing photon.

And let us assume that the time constant or the time is varying very long and the probability of stimulation is 100%. These three assumptions we are considering during this particular theory of generation of photons during the stimulated emissions. The incoming photon interacts with first atom and causing stimulated emission of coherent photon. As we have seen in our previous slide we are getting incoming photon which is interacting with first atom and we are getting a stimulated emission of a coherent photon.

These two photons then interact with next two atoms in line and we are getting four coherent photons and this is getting continued further. Many such photons with identical phases and are travelling in a same direction that means we are amplifying by certain factor - it maybe 2, 4,

10 or that certain factor may be  $n$ . Then and then only we can have a number of photons of the same phase, same length, in the same direction by using this stimulated emission process. So, more number of electrons should be at the higher energy level and less number at the lower energy level so that they decay and when the passing photons are stimulating them to be in the same direction, same wavelength, same phase we can achieve our objective to have a monochromatic coherent and highly collimated beam.

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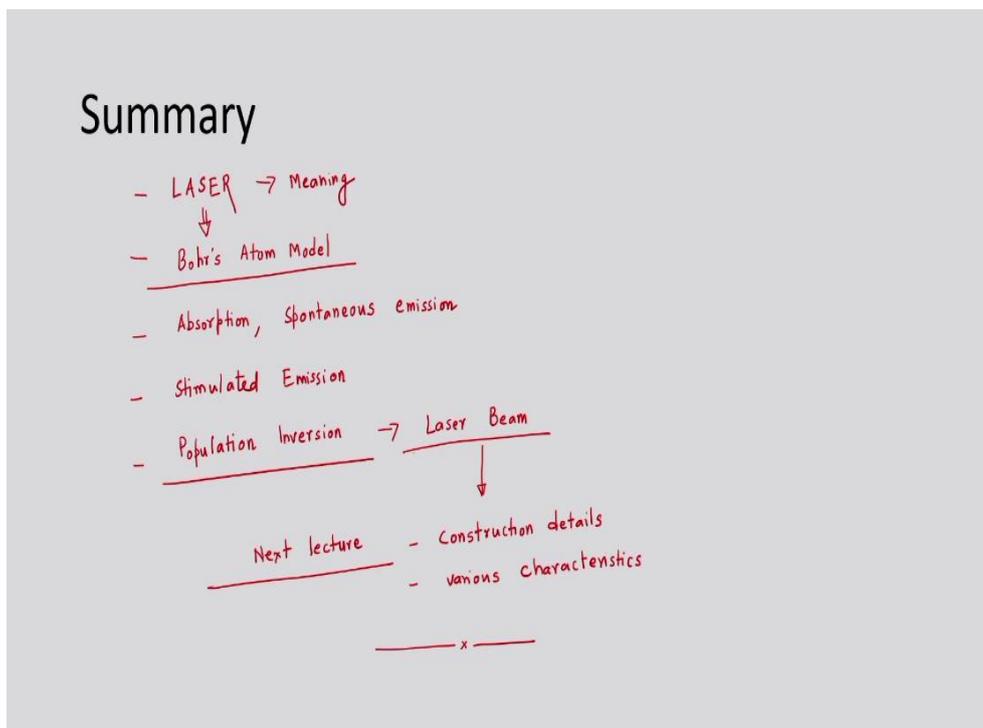
But in general, in thermal equilibrium of a material the population of atoms in the lower energy level is more. That is the characteristics of number of atoms, the number of electrons in the lower energy level is more than the upper energy level. But for having the laser action we should have more electrons at the higher energy level, but naturally more electrons are at the lower energy levels we have to pump - we have to apply energy so that this lower energy level electrons will move to the upper energy level and from there they will get de-energized by releasing the photons. To carry out this operation we should have a active medium and when we pump the energy to this active medium it will go to the next energy level or the higher energy level and from there it will get decayed. These decaying operations would be in a step-by-step manner.

Now let us consider we are having the ground state as 1 and the highest energy state as 2. From highest energy state the decaying is done to the energy state 3, but there is no radiation from 2 to 3. The time duration is very less of the decay. There is no radiative decay will happen - over here the electrons will just jump from state 2 to state 3. But from state 3 to further lower state that is the state 4, the duration is little longer and during this state transition from 3 to 4 there

is generation of the photons which are having the same phase, the same direction and same wavelength as the passing photons. Here we consider that the transition from 3 to 4 is generating radiative decay - it is providing the radiative decay phase and it is generating photons that will lead to the laser beam.

So, the process of having more number of electrons at higher energy state is called as the population inversion operation and that is possible by using pumping through the external energy in the laser operation and we will see how this pumping operation is being carried out, what are the various modes of pumping operation in the next lecture.

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Now, let us summarize today's class. We have seen the laser term, its acronym or its meaning that we have seen today and we have studied how the laser beam is getting generated. For that we have used the Bohr's atom model. After that we have seen the process of absorption, spontaneous emission and stimulated emission and at the end of the lecture we have seen the concept of population inversion which is of great importance to generate the laser beam of desired quality and strength.

Now, in the next lecture we will try to find out how to generate a laser beam, its constructional details and various characteristics.

Fine, till then good bye, see you in the next lecture. Thank you very much for watching and listening this lecture 2 bye.