Mechanics of Fiber Reinforced Polymer Composite Structures
Prof. Debabrata Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology-Guwahati

Lecture - 30

Prediction of Delamination

We have been discussing interlaminar stresses and in our last lecture, we understood the mechanics of formation of interlaminar stresses at the free edges of laminate. Starting from equilibrium equations, we understood how these interlaminar stresses are actually induced at the free edge in a very small length over the free edge. We also discussed the reasons for development of such interlaminar stresses.

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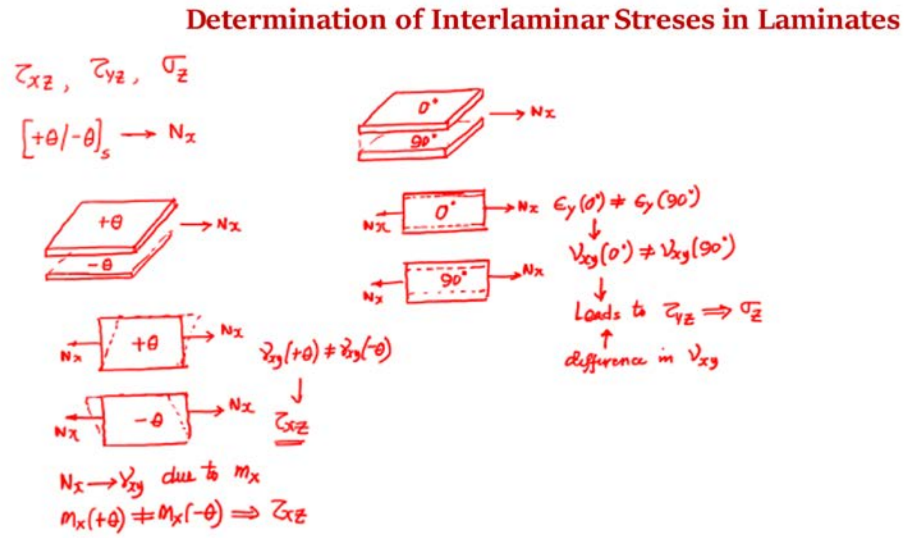

There are three interlaminar stresses viz. two interlaminar shear stresses, τ_{xz}, τ_{yz} and one interlaminar normal stress, σ_z. We understood that if we take a $\pm\theta$ laminate subjected to only N_x (as in Fig.), τ_{xz} is actually induced because there is a gradient of τ_{xy} near the free edge and because τ_{xy} drops down to zero at the free edge. The reason for the same is because of the property mismatch. As shown, $\pm\theta$ laminate is actually subjected to N_x and the layers are perfectly bonded at the interface. Now if the $+\theta$ layer is free and is subjected to N_x then it will actually undergo shear deformation in the x-y plane, okay. Similarly, if the $-\theta$ layer experiences N_x, it will also experience shear strain but in the opposite direction. That means, γ_{xy} for $+\theta$ is not equal to γ_{xy} −\ under N_x. But, because they are perfectly bonded, therefore, they are not allowed to undergo free shear strain and that shear strain is constrained. That leads to the development of interlaminar shear stress τ_{xz}. Thus, N_x causes γ_{xy} due to shear extension couplings, and hence the reason for interlaminar shear stress τ_{xz} is the mismatch in shear extension coupling between the adjacent layers.

Similarly, we also understood that if we take two adjacent layers, one is 0^o another is 90^o, and both of them are actually subjected to N_x. They are perfectly bonded at their interface. Now if the 0˚ layer is free and subjected to N_x, it will have transverse strain along y- direction because of Poisson's effect. Similarly, if the 90˚ layer is subjected to N_x it will also have transverse strain in the y- direction and these two are not equal ie. $\varepsilon_y(0^\circ)$ is not equal to $\varepsilon_y(90^\circ)$. The reason is Poisson's ratio ν_{xy} for 0˚ is not equal to Poisson's ratio ν_{xy} for 90˚. However, they are perfectly bonded and they are not allowed to undergo the transverse strain which they would have experienced had they been free. This is the reason for development of τ_{yz} and τ_{yz} at the interface leads to development of σ_z. Therefore, the reason for development of these two interlaminar stresses is the mismatch of Poisson's ratios of the adjacent layers. Now we need to know how to determine these stresses.

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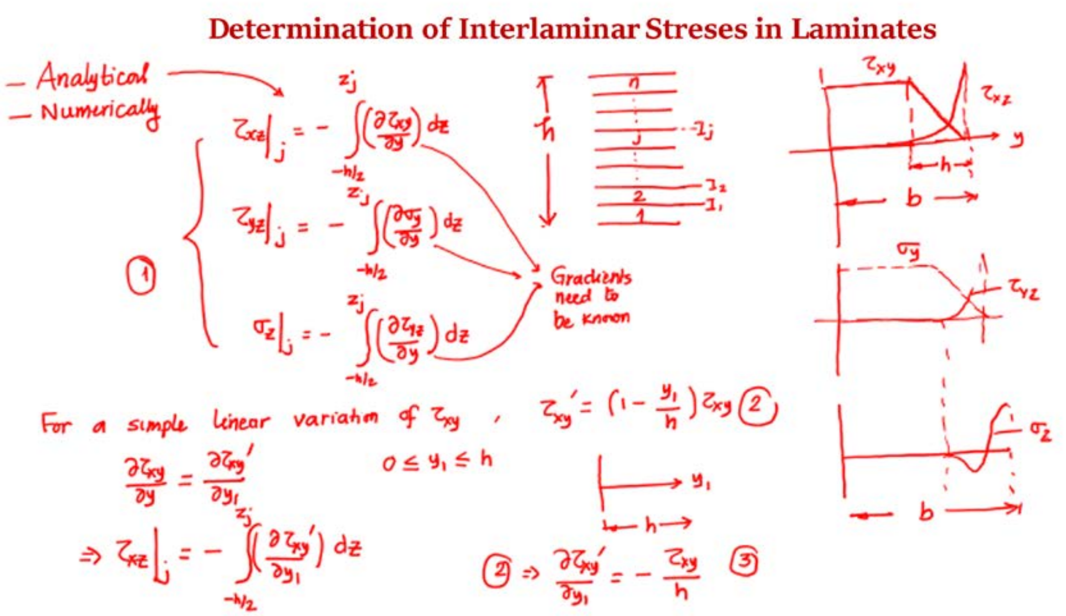

For an N layer laminate having N-1 interfaces like interface 1, interface 2, … interface j, … interface N-1 as shown in Fig.

So for an N layer laminate τ_{xz} at an interface j was written as

$$\tau_{xz}\Big|_j = -\int_{-h/2}^{z_j}\left(\frac{\partial \tau_{xy}}{\partial y}\right)dz$$

Similarly,

$$\tau_{yz}\Big|_j = -\int_{-h/2}^{z_j}\left(\frac{\partial \sigma_{y}}{\partial y}\right)dz \quad \text{and} \quad \sigma_{z}\Big|_j = -\int_{-h/2}^{z_j}\left(\frac{\partial \tau_{yz}}{\partial y}\right)dz$$

So, at any interface, if we need to know the interlaminar shear stress or normal stress we must know how these stresses are varying near the free edge.

Referring to Fig. for example, for a simple linear variation of τ_{xy}, we can write the variation of τ_{xy} in the boundary layer as

$$\tau'_{xy} = \left(1-\frac{y_1}{h}\right)\tau_{xy}$$

That means, we define a coordinate in the boundary layer which is y_1 so that at y_1=h, τ'_{xy} =0 and at y_1=0, τ'_{xy} is nothing but τ_{xy}. Therefore, we can replace $\frac{\partial \tau_{xy}}{\partial y}$ by $\frac{\partial \tau'_{xy}}{\partial y_1}$ as there is no variation of τ_{xy} other than the boundary layer region.

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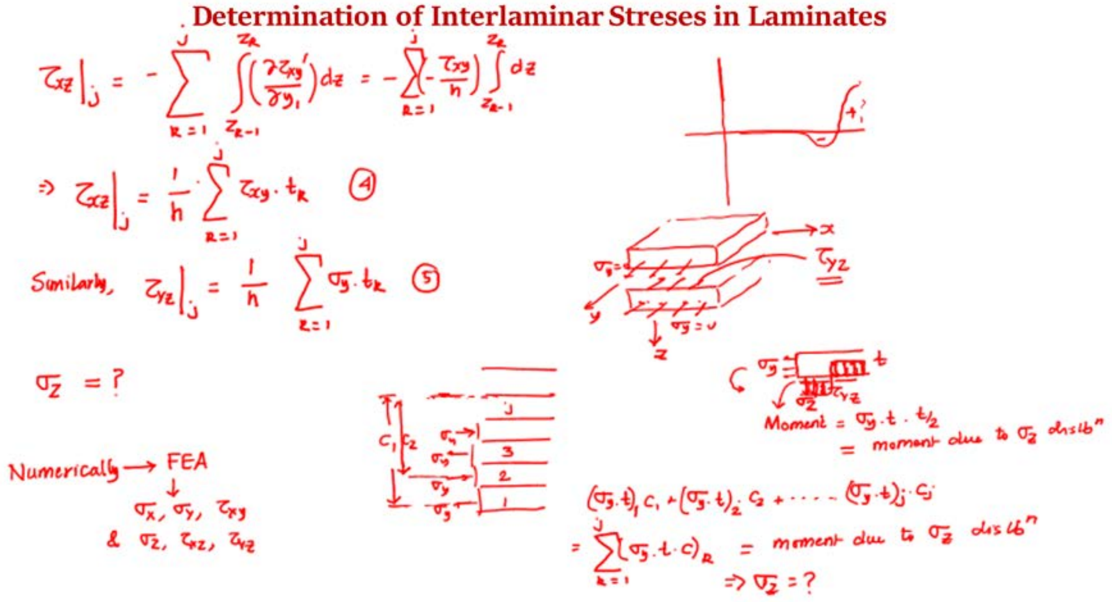

Therefore, we can write that τ_{xz} at interface j as

$$\tau_{xz}\Big|_j = -\sum_{k=1}^{j}\int_{z_{k-1}}^{z_k}\left(\frac{\partial \tau'_{xy}}{\partial y_1}\right)dz = -\sum_{k=1}^{j}\left(-\frac{\tau_{xy}}{h}\right)\int_{z_{k-1}}^{z_k} dz$$

Now we can replace this continuous integration by sum over the layers from layer 1 to layer 'j' as

$$\tau_{xz}\Big|_j = \frac{1}{h}\sum_{k=1}^{j}\tau_{xy}t_k$$

So this is how we could obtain τ_{xy} at any interface 'j' for a simple linear variation. For other variations also we could obtain τ_{xy} at any interface 'j', but then we need to know the variation of τ_{xy} in the boundary layer. Similarly τ_{yz} at any interface 'j' considering linear variation of σ_y could be written as

$$\tau_{yz}\Big|_j = \frac{1}{h}\sum_{k=1}^{j}\sigma_y t_k$$

Then how to determine σ_z? For that we need to understand the distribution of σ_z. Even though the force due to τ_{yz} and force due to σ_y balances each other, but then there is a net moment because of the σ_y (as the lines of actions are not same). As shown in Fig. the net moment at the j-th interface will be $(\sigma_y \times t)_1 \times c_1$ for the first layer, $(\sigma_y \times t)_2 \times c_2$ for the second layer and so on to obtain $(\sigma_y \cdot t)_1 c_1 + (\sigma_y \cdot t)_2 \cdot c_2 + \cdots - (\sigma_y \cdot t)_j \cdot c_j$ .at the jth interface. Therefore, the net moment is

$$\sum_{k=1}^{j}\left(\sigma_y.t.c\right)_k$$

So this net moment must be equal to moment due to σ_z distribution, i.e.

$$\sum_{k=1}^{j}\left(\sigma_y.t.c\right)_k = \text{moment due to } \sigma_z \text{ distribution.}$$

For a given distribution of σ_z, we could thus find out the value of σ_z. Therefore, we can analytically determine the interlaminar stresses but we need to know the distribution of τ_{xy}, σ_y and the σ_z. It is important to notice here that depending upon the sign of σ_y, the σ_z distribution will be different. Therefore, if the σ_z distribution is such that near the free edge it is positive then it will lead to delamination. If the near the free edge if it is negative, then the effect is not as severe as in the case of positive σ_z. Numerically also we can determine using finite element method.

We can use finite element analysis to determine all the stress components, the in-plane stresses σ_x, σ_y, τ_{xy} and the out-of-plane stresses σ_z τ_{xz} τ_{yz}. However, there are issues of proper mesh refinement and choice of proper element to accurately determine the magnitude and distribution of stresses in the free edge.

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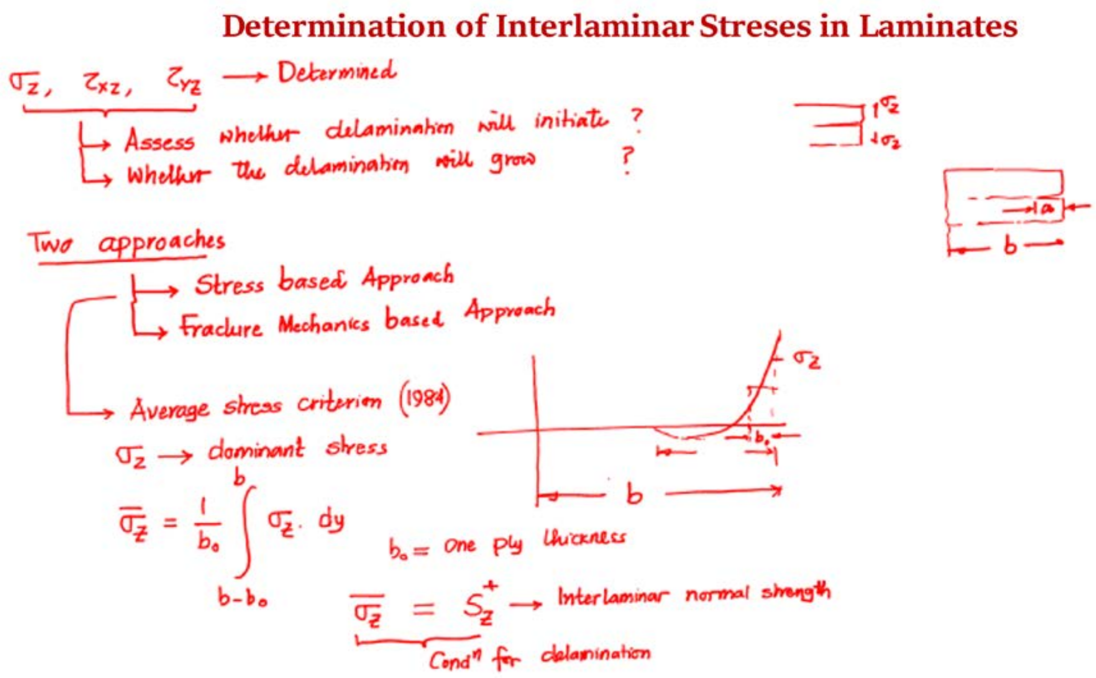

Once we determine these interlaminar stresses using appropriate numerical or analytical methods, we need to assess whether delamination will initiate. Because, due to the existence of interlaminar stresses, whether delamination will initiate or not is decided by the corresponding interlaminar strengths in a laminate. Say for example, if at the free edge there is positive σ_z, it will try to separate the adjacent layers (delaminate). But then if the interlaminar normal strength is sufficiently high compared to σ_z then even though there is a σ_z, for a small value of σ_z it may not be able to cause delamination. Similarly for the other stresses also. Therefore, we need to understand whether delamination will initiate or not.

Again, once the delamination initiates it is important to assess whether the delamination will grow or not. These two important questions need to be answered. Therefore, it is important to have some criteria for delamination initiation as well as once a delamination initiates, whether it propagates. There are clearly two approaches for that viz. the stress based approach or mechanics of material approach and the fracture mechanics based approach. In stress based approach interlaminar stresses are evaluated and compared with the corresponding strengths to assess whether delamination will initiate or not. Among different criteria, a more general criterion involving all the stress components is

$$\left(\frac{\bar{\tau}_{xz}}{S_{xz}}\right)^2+\left(\frac{\bar{\tau}_{yz}}{S_{yz}}\right)^2+\left(\frac{\bar{\sigma}_z^t}{S_z^+}\right)^2+\left(\frac{\bar{\sigma}_z^c}{S_z^-}\right)^2=1$$

This is the condition for delamination initiation.

$\bar{\tau}_{xz},\bar{\tau}_{yz},\bar{\sigma}_z^t,\bar{\sigma}_z^c$ are average interlaminar shear and normal stresses.

$S_{xz},S_{yz},S_z^+,S_z^-$ are the corresponding strengths.

t and c, + and - for tensile and compressive.

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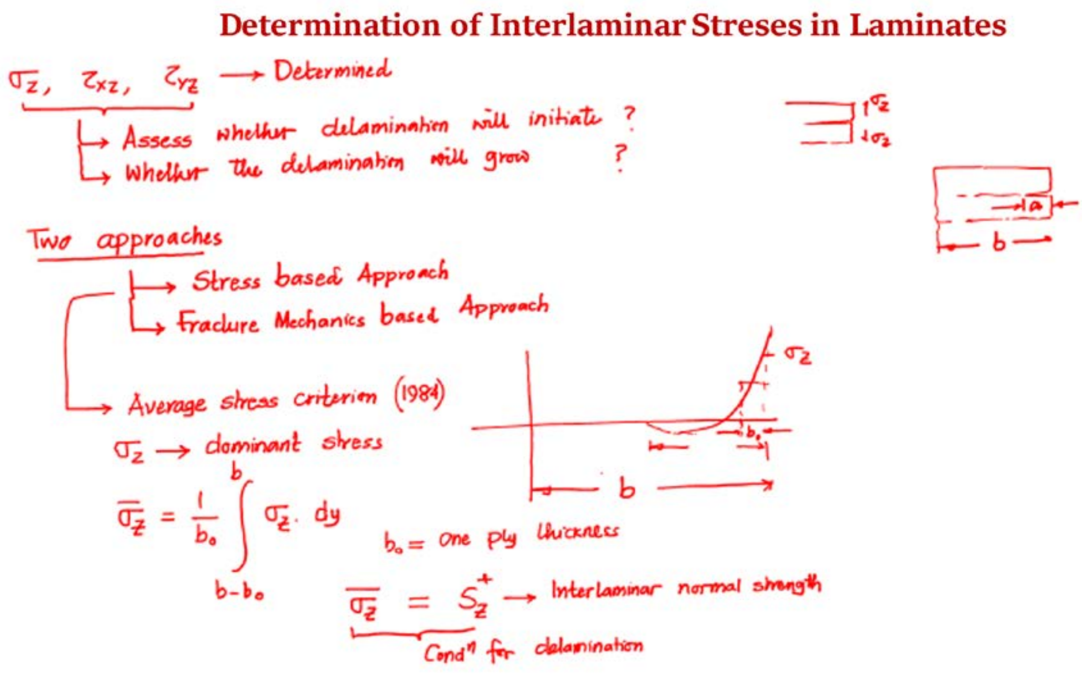

where b_o is one ply thickness. So these are the stress based criterion for delamination initiation. We can actually use numerical techniques like finite element analysis to determine the stresses and once we have those stresses we could actually use one of these criteria to actually assess whether delamination will grow or not. Now one thing about the delamination is that it leads to catastrophic failure, but when a delamination propagates, it also leads to stiffness degradation which characterizes the delamination growth. For example, suppose we have a symmetric laminate and it is intact, there is no delamination. We have seen discussed in analysis of laminates that we can have an effective Young's modulus in extension for a laminate as $1/hA_{11}^*$ where A_{11}^* is the first element of the $[A]^{-1}$ matrix. And h is the thickness of the laminate. Now suppose this laminate is completely delaminated at a particular interface ie., at a particular interface the bonding is completely lost. Therefore, it actually divides this laminate to two sub laminates. In that case the Young's modulus of the delaminated laminate will be

$$E_d = \frac{\sum_{i=1}^{n} E_{x_i} \cdot t_i}{h}$$

That means, if the laminate is actually subdivided into n number of sub laminates, then we can determine the Young's modulus of this laminate as the weighted sum of each of these sub laminates. So this is for the completely delaminated laminate. Now E_x is the Young's modulus in extension for the intact laminate and E_d is when it is completely delaminated. Now for a partially delaminated, suppose it is actually delaminated not over the entire width but say over a length, a. So naturally at a = 0, it is E_x. At $a = b$ it is E_d and in between for partial delaminate the value of E will be between E_x and E_d. Therefore, if we consider that E actually varies as a linear function of a, we can write the expressions of E as

$$E = \left(E_d - E_x\right)\frac{a}{b} + E_x \, .$$

So this is for a partially delaminated laminate with a delamination width a the Young's modulus of that particular laminate. So the stiffness degrades.

Now once there is a delamination whether that particular delamination will grow or not could be actually addressed using fracture mechanics approach.

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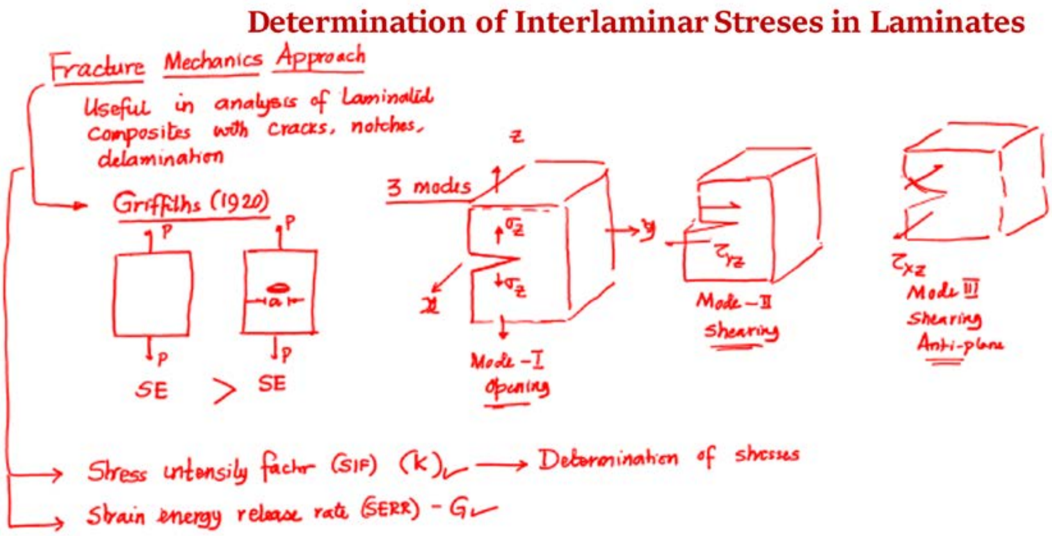

Since many of you may not be conversant with the fracture mechanics approach, the basics of fracture mechanics will be briefly discussed here because the objective here is to understand how fracture mechanics approach could be used for analysis of delamination. Fracture mechanics approach is useful in analysis of laminated composites with cracks notches and for analysis of delamination.

We will try to understand how delamination could be analyzed using fracture mechanics approach. Strain energy of the uncracked component is more than the strain energy of the cracked component. Or when a crack grows, strain energy is released and that strain energy is actually utilized for generation of new surfaces. The surface energy associated with this generation of new surfaces is actually obtained from the strain energy. So if the strain energy that is released is sufficient for the generation of two new surfaces then crack grows. That is what the reasoning has been, okay. In linear elastic fracture mechanics there are three modes of fracture, three distinct modes.

Subjected to load the delaminated surfaces may slide either along x or tear along y lead to shearing and tearing mode and the stresses responsible are τ_{xz} and. τ_{yz} (refer to Fig.). So considering a delamination as a crack here, it may open in the mode I or it may try to slide over in the mode II or it may grow in the mode III. Though this fracture mechanics is actually based on homogeneous and isotropic material, but these are also used for heterogeneous and anisotropic material with proper modification.

So in fracture mechanics, there are again two parameters which are extensively used, viz. stress intensity factor (SIF) denoted by *k*, and strain energy release rate (SERR) denoted by G. In mechanics of material approach we compare stresses to corresponding strengths, maybe independently or following some interactive criterion. In fracture mechanics however, the approach is that there is an existing flaw or crack and these parameters like stress intensity factor and strain energy release rates are actually used to assess whether that particular crack will grow under loading or not.

We will restrict our discussions here to strain energy release rate with reference to how it could be applied to delamination growth. So let us understand what actually strain energy release rate is. As we have discussed already that if a crack grows strain energy is released.

Now if the strain energy released is at least equal to the energy associated with the new surfaces due to the crack growth, then the crack will grow. To make it independent of the dimension of the crack the strain energy release rate ie. the energy released per unit area of the crack is used.

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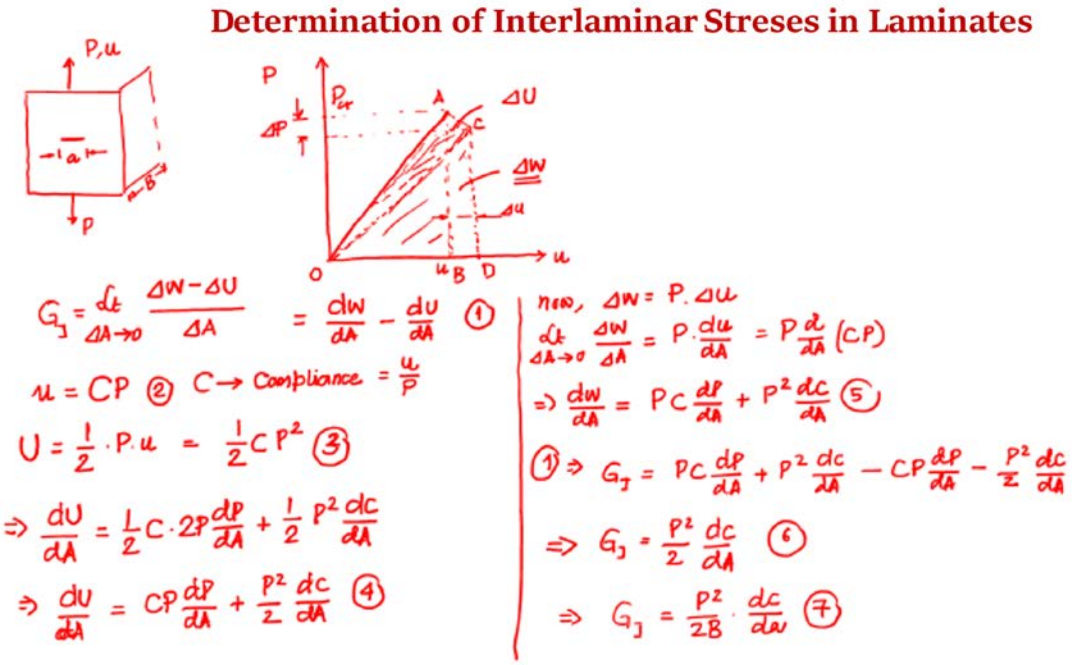

Referring to Fig., suppose we have a component which is loaded in pure mode I ie. the load is perpendicular to the plane of the crack. Suppose the load is P and the corresponding displacement in the direction of P is u. So as P increases the corresponding displacement also increases, So after some time as P keeps on increasing at some point ($P=P_{critical}$) of time the crack grows. As soon as there is a crack growth then there is a load drop and there is a corresponding increase in u. say, Δu.

Referring to the Fig., the strain energy release rate $G\ (\Delta A \rightarrow 0)$.

$$G_I = \frac{dw}{dA} - \frac{dU}{dA} \quad (1)$$

$$u = CP \ (2)\ C \rightarrow \text{ compliance} = \frac{u}{P}$$

$$U = \frac{1}{2}\cdot P\cdot u = \frac{1}{2}CP^2 \quad (3)$$

$$\Rightarrow \frac{dU}{dA} = \frac{1}{2}C\cdot 2P\frac{dP}{dA} + \frac{1}{2}P^2\frac{dC}{dA}$$

$$\Rightarrow \frac{dU}{dA} = CP\frac{dP}{dA} + \frac{P^2}{2}\frac{dC}{dA} \quad (4)$$

$$\text{now, } \Delta w = P \cdot \Delta U$$

$$dt = \frac{\Delta W}{\Delta A} = P \cdot \frac{du}{dA} = P\frac{d}{dA}(CP)$$

$$\Rightarrow \frac{dw}{dA} = PC\frac{dP}{dA} + P^2\frac{dC}{dA} \quad (5)$$

$$(1) \Rightarrow G_I = PC\frac{dP}{dA} + P^2\frac{dc}{dA} - CP\frac{dP}{dA} - \frac{P^2}{2}\frac{dc}{dA}$$

$$\Rightarrow G_I = \frac{P^2}{2}\frac{dc}{dA} \quad (6)$$

$$\Rightarrow G_I = \frac{P^2}{2B} \cdot \frac{dC}{dw} \quad (7)$$

We understand that as the crack area increases, the compliance changes. Therefore, we need to find out what is $\frac{dC}{da}$. Therefore, we can write this as G_I as

$$G_I = \frac{P^2}{2}\frac{dc}{dA} \quad (6)$$

where A is the crack length and B is the width of the thickness of the component.

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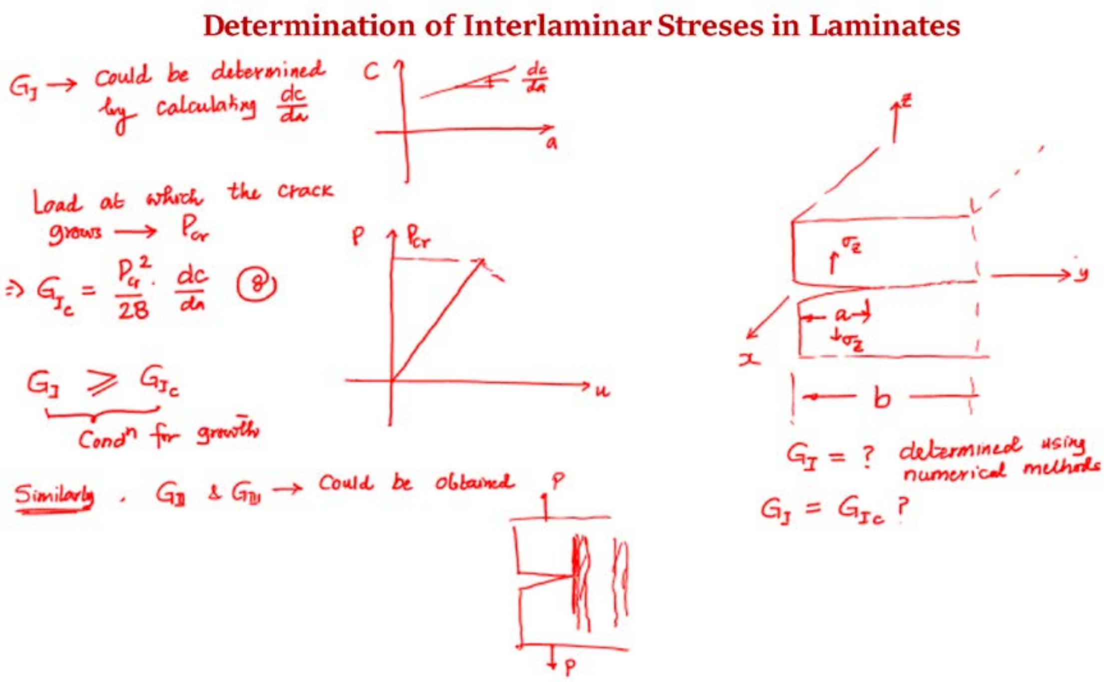

The energy release rate G_I could be determined by calculating dC/da. If we plot A versus C, we can find out what is dC/da. Knowing $\frac{dC}{da}$ we can find out what is G_I. Now the load at which the crack grows say it is P_{cr}.

From the load deflection curve, when there is a crack growth the load drops and we can find out what is P_{cr}. Therefore, corresponding G_I is called the G_{I_C}, critical strain energy release rate as

$$G_{I_c} = \frac{{P_{cr}}^2}{2}\frac{dc}{dA} \quad (7)$$

Therefore if G_I is equal to G_{I_C}, it is the condition for crack growth. There are other issues also like stable crack growth, unstable crack growth and we are not going into these details here. But we can summarize that for a given crack subjected to load, we can find out the strain energy release rate and for that particular material if we know the critical strain energy release rate, we can compare the strain energy release rate to the critical strain energy release rate to assess whether that particular crack will grow or not . Here we have considered only for mode I and similar expressions for G_{II} and G_{III} could be obtained.

And then we can also determine the critical strain energy corresponding to mode II and mode III. Considering an edge delamination, we could apply this concept of fracture mechanics. We need to determine G_I, G_{II} and G_{III} and comparing those with the corresponding critical values like G_{I_C}, G_{II_C} *and* G_{III_C} we can assess whether this delamination will grow not. That means, given a component suppose there is a delamination already there, we can take a decision under the given load whether this delamination will grow further or it will not grow. Referring to the Fig. suppose, we have a laminate where there is a transverse crack perpendicular to the fibres subjected to load. Now what happens when the crack grows and it encounters a fiber, it cannot penetrate the fiber as the strength of the fiber in the longitudinal direction is far higher compared to that of the matrix and therefore this crack actually grows along the length of the fiber between the fiber and the matrix. Therefore, it is not that easy to assess crack growth in such cases. However, in the case of delamination, because it grows along the interface, this fracture mechanics approach could be actually applied using strain energy release rate to take a decision whether the delamination grows or not.