Evolutionary Computation for Single and Multi-Objective Optimization
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Module – 04
Lecture – 08
Particle Swarm Optimization

Welcome to the session on Particle Swarm Optimization; it is another EC technique.

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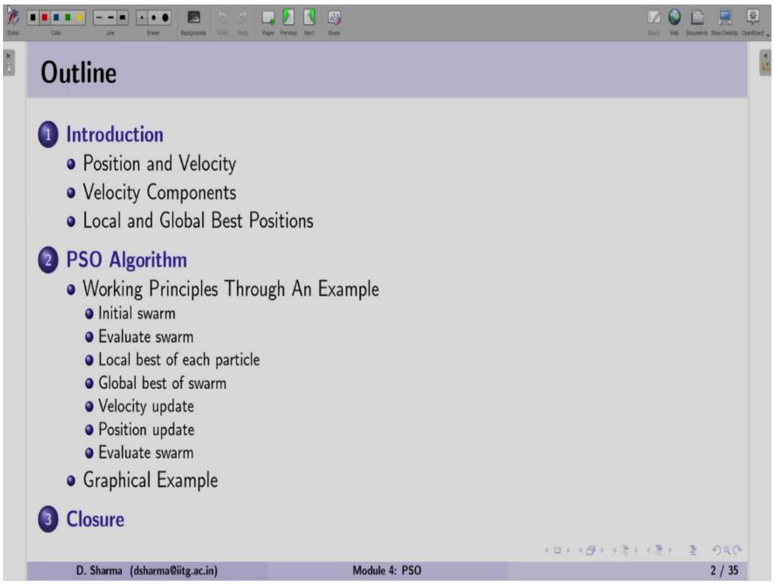

So, in this particular session, we can go through the introduction. Since this PSO is developed using different features of natural evolution so, it has different component such as; velocity update, position update as well as it includes the local best of each particle and the global best of the swarm. So, we will go through the introduction of particle swarm optimization.

Thereafter, we will understand this algorithm through an example. So, working principle of PSO will be understood through Rosenbrock problem and in this problem, we start with initial population followed by the features that are distinct from the other EC techniques, we will discuss them, and we will run for one generation. So, we will perform all the hand calculations for this algorithm.

At the end, we will show you the graphical example of the same Rosenbrock problem and finally, we will conclude this session. So, let us begin with the introduction of particle swarm optimization.

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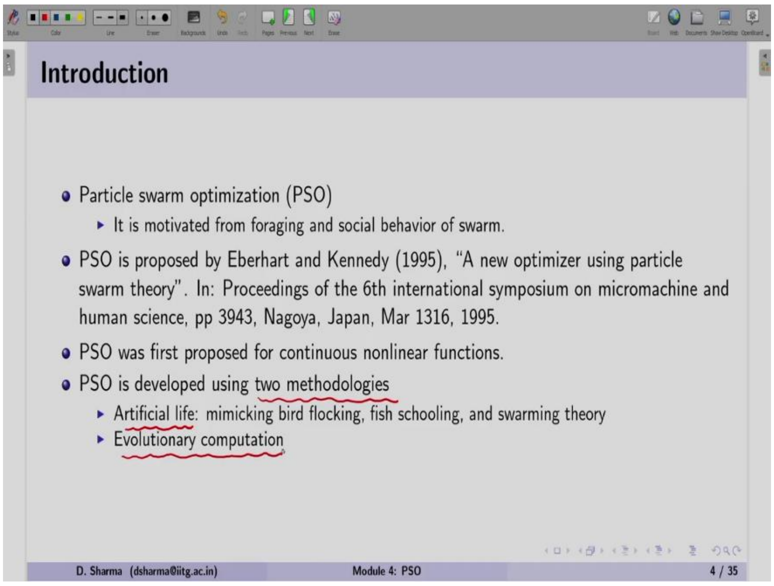

So, as PSO, so, particle swarm optimization as can be seen, it is referred as PSO. It is motivated from foraging and social behavior of swarm. So, as we know that EC techniques are motivated or mimics the natural phenomena or the process. So, in PSO, we mimic the process or the phenomena such as flocking of the bird or a school of the fish.

First, PSO is proposed by Eberhart and Kennedy in 1995. In this particular paper, they proposed PSO as an optimizer for continuous non-linear function and as we know, these methods are direct search method which does not need any gradient information. So, even if a function is discontinuous, we can use PSO and other EC techniques.

In this paper, the authors have mentioned that PSO is developed using two methodologies, first is called artificial life another is called evolutionary computation. So, when we say artificial life meaning that we are mimicking some natural phenomena. So, in this case, we have school of fish, flocking of the bird's etcetera.

Similarly, when they move in a swarm or in a group so, they follow some theory which we call it is a swarm theory. So, by using those concepts, PSO is developed as well as since evolutionary computation idea is used in which the solutions are evaluated, and they are compared, and new solutions are generated in a process to find an optimal solution for the given problem.

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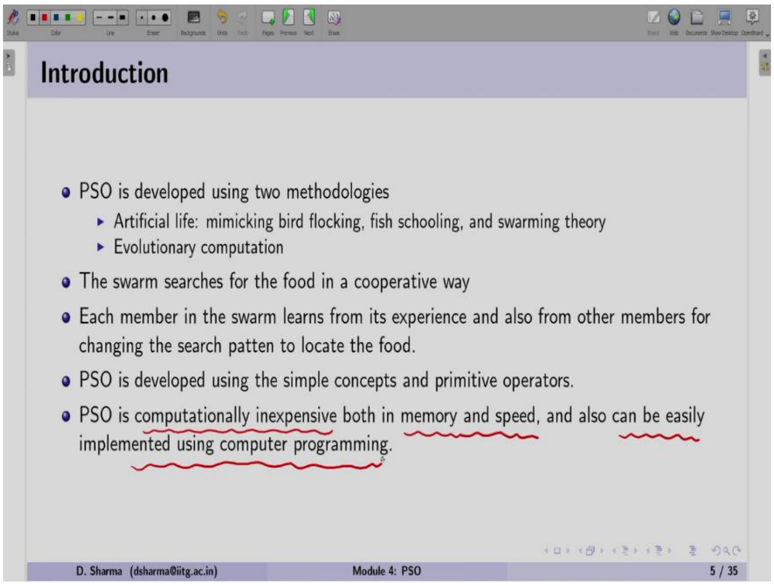

So, as we can see here that since PSO is developed using artificial life. So, in this artificial life, swarm searches for the food in a cooperative way. So, since in a swarm, there are lot of birds or fishes are there so, they work as a in a cooperative manner.

So, we might have observed that there is when lot of birds are flying in an sky and they are moving in a one direction, as soon as any birds sees food, then rest of the bird also follow this particular bird when searching for the food. So, the same concept is borrowed, and particle swarm optimization is developed.

Now, while doing this process, each member in PSO learns from its experience and also from other members for changing the search space pattern to locate the food. Now, it is important why because when we talk about the swarming theory so, what is happening is that when one particle is moving from the current position to the another position so, let us assume that it has started from some position and after say 10 generations, it reach to the another position.

So, in between those 10 generations, teach particle will know what was its best position. So, they keep a track of this position as well as since their task is to search for the food or in an in the parlance of optimization if we say that we are looking for an optimum solution so, these particles also see which particle has the best position so far.

So, that is we refer as a global best. So, they work individually, look for their best position as well as they also see the pattern of other members so that we can find an optimal solution for the given problem.

Now, as it is mentioned in the original paper, PSO is developed using the simple concepts and the primitive operators. Now, when we will be discussing the features or distinct features of PSO, we can see that these are different the concepts of for example, adding the two vector or subtracting the two vectors, those simple concepts are being borrowed and when we will be implementing as an operator say for variation, those implementations are simple because it involves vector operations.

Moreover, PSO is found to be computationally is inexpensive both in terms of memory and speed, and also can be easily implemented using computer programming. So, that is the main advantage of PSO since the operations are so primitive the concepts are simple so, we can make the programming or the code for a PSO in a very easy manner.

Second is since it does not involve any kind of probability distribution so, the calculations are simple. So, we can say that the PSO is computationally inexpensive in terms of memory because we do not store any previous solution as well as we and they are also in terms of speed, they are computationally inexpensive.

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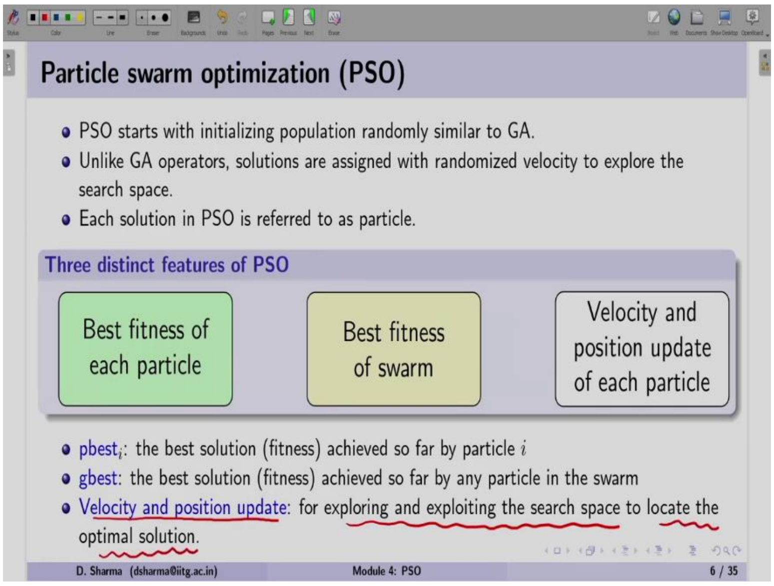

So, let us start with particle swarm optimization, how it works. So, as we can see here, PSO starts with initialising this population randomly. So, we can say that this is very similar to GA and that is the concept of evolutionary computation that we start with set of solutions. But unlike GA, the solutions are assigned with randomized velocity to explore the search space.

So, instead of having those operators which we and which we understood as crossover and a mutation that are based on the probability distribution such kind of crossover and mutations are not needed because the velocity is included into the search and this velocity with the current position of each solution, the solution is perturbed, or the position of the solution is changed.

An important point with PSO is that the solution in PSO is referred to as particle. Now, earlier with genetic algorithm, we refer the solutions, then many point so, sometimes we say points, sometimes we say solutions or individuals, but in PSO, just to make it different from the other algorithm so, the point or the solution is referred as particle.

So, the three distinct features as we talk about earlier that first is each particle is looking for its best position so, best fitness of the each particle is kept. Similarly, the best fitness of the swarm is also we look for it and accordingly, we update the velocity and the position of each particle here. So, when we talk about best fitness of each particle.

So, basically, we target the pbest and the subscript i says that it is a pbest of solution i. So, this pbest is called as the best solution achieved so far by the particle i. How we can know that? When we will be comparing the fitness value of this particle with its previous position so, we keep the column vector of decision variable corresponding to a position which will give me the best fitness.

Similarly, when we talk about the gbest so, gbest is the best solution achieved so far by any particle in the swarm. So, earlier, it says that its best position, but in the gbest, it says that the best solution of any particle in the swarm. So, we are looking this particular swarm in a; in a totality.

And again, if we have to find the gbest, we have to look the fitness of the solution. And finally, these velocities and the position update, for exploring and exploiting the search space, this velocity and position update will help us to locate the optimal solution.

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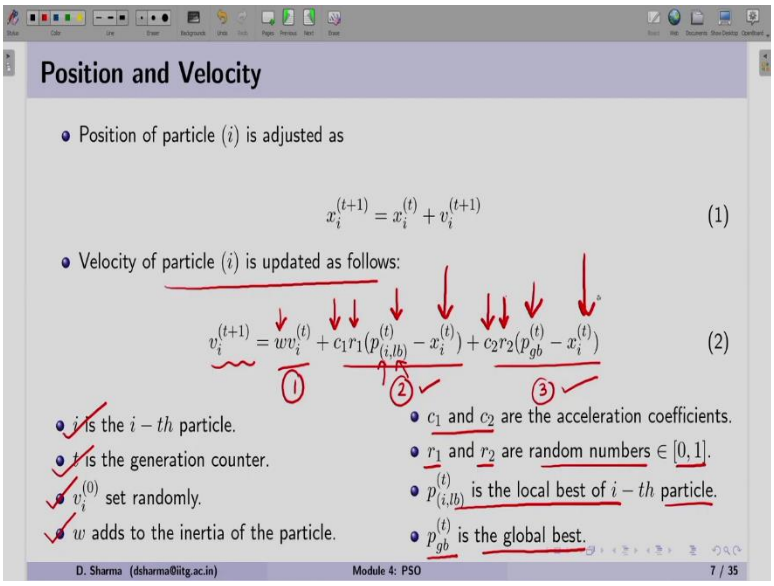

Now, since we know there are three different features with PSO, let us go them one by one. We will start with the position update first because it will behave like a variation operator that will help to change the current position of a particle to its new position.

$$x_i^{(t+1)} = x_i^{(t)} + v_i^{(t+1)} \tag{1}$$

So, here, the position of a particle *i* is adjusted as, as you can see in equation number 1 so, this is a particle *i* in (t +1) generation. So, we are looking for the new position, it depends on the current position and the velocity so, this is the updated velocity $v_i^{(t+1)}$.

Now, the question comes, how we can find this updated velocity? Let us look at the equation number 2. You can see that since each particle is represented as a vector so, we will realise that these 3 component are the addition of three vectors.

$$v_i^{(t+1)} = wv_i^{(t)} + c_1\, r_1\left(p_{(i,lb)}^{(t)} - x_i^{(t)}\right) + \; c_2\, r_2\, \left(p_{gb}^{(t)} - x_i^{(t)}\right) \qquad (2)$$

However, in between so, the component number 2 and component number 3 has some kind of a difference between the two vectors. We will discuss the velocity component in the next slide. In this equation number 2, there are various parameters we have used it. So, since we are saying *i* so, *i* stands for *i* th particle, *t* stands for the *t* th generation counter so, we can say its it is a current generation counter. So, generally $v_i^{(0)}$ so, as you can see 0 stands for the initial velocity which we set randomly.

Now, look at the first term, you can see the w term here, this w adds to the inertia of the particle so, it is a weight, we multiplied with the velocity that is the previous velocity. Then, there are two terms as you can see in equation number 1 and equation number 2, there are c_1 and c_2.

So, these two c_1 and c_2 are called as acceleration coefficients. Along with this, we have r_1 and r_2 into the two components of in the velocity in equation number 2. These r_1 and r_2 are called as random numbers which will be lying between 0 and 1.

Now, come to this representation. So, this representation is called p i so, in the subscript, it is written as i, lb. So, we know the i stands for the i th, particle lb stands for the local best and t is the current generation. So, $p_{(i,lb)}^{(t)}$ is the local best of i th particle. Similarly, if we look at the 3rd component in equation number 2 so, this is $p_{gb}^{(t)}$. So, at the bottom of the slide, we can see that this is called the global best and we know this global best is calculated with respect to the swarm.

Now, apart from that as we know, this these two components $x_i^{(t)}$ in equation number 2, these this is the current position of the particle. So, let us understand the velocity component in the PSO.

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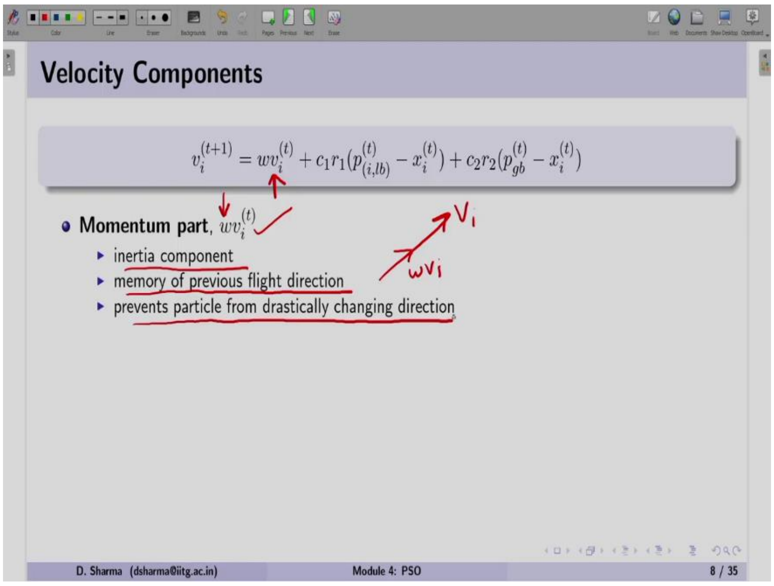

So, since it involves three components, we let us understand all of them one by one. So, first component we call it as momentum part it is because it involves velocity. As you can see so, the velocity is there. So, we have the velocity of a particle in iteration number t when so, that is why, we have this $wv_i^{(t)}$ so, since omega is the weight; to the velocity it is called as the inertia component because we are considering the velocity of the previous iteration *t*.

Now, while we are including this $wv_i^{(t)}$, it says that it has the memory of previous flight direction. So, when so, as we know that suppose we have a vector say $v_i^{(t)}$, in this case, if I am going to multiply say $wv_i^{(t)}$ so, both of them have the same direction. Omega will tell whether the vector is to be shorter or it has to be elongated.

Now, the purpose of the first part is to prevent particle from drastically changing its direction. So, since as it is made in a way that if we are moving in one particular direction or with the velocity, then we should not change drastically, yes, gradually we will change because the other components are added into the velocity.

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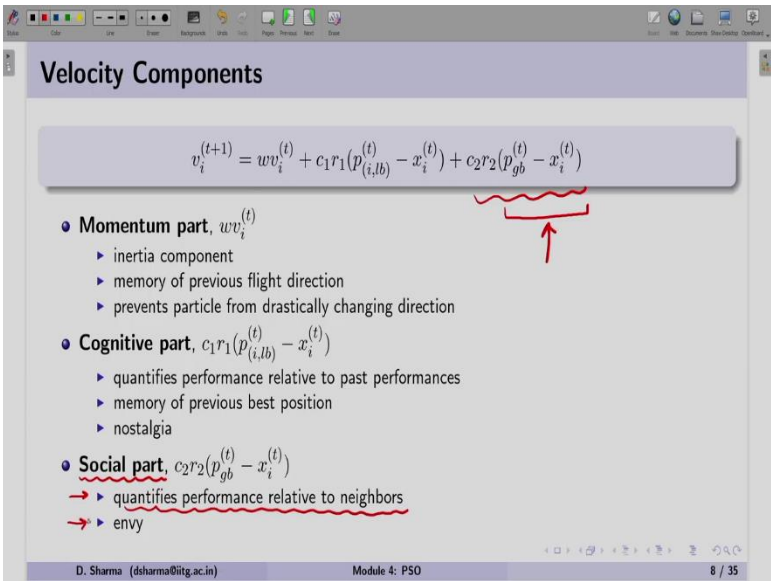

Now, come to the second component. So, this second component is referred as the cognitive part it is because we always have this component which is called the local best of each particle.

What it says that; it quantifies performance relative to the past performance. As we have understood the local best is the position of the particle that is the best position of the particles found so far. So, in this case, we are preserving the best local best position of each particle so that is why in the second point, we have mentioned it is the memory of the best particle.

Now, as you look into the difference part, in this case, you can see that what we are doing is we are subtracting the two vectors. So, in a way, we are saying that the how far this current position from the local best of a; local best of the particle i. And this particular component is also referred as nostalgia because we already know what is the local best of the particle.

Now, coming to the third part, now the third part include the global best. So, here so, this particular part is called social part. So, we remember that when we are using PSO and it is following the swarm theory so, in this case, the particle saves it is the best position so far and it also changes the pattern based on the best fitness of any particle in the swarm so that is why it is called social part here.

This particular part quantifies the performance relative to the neighbours. Now, if you look at the difference here on the top, this says that what if x since x i is the current position so, the difference of global best minus current position says that how far is the current position of a particle i with respect to the global best of the swarm. So, that is why this part called as envy.

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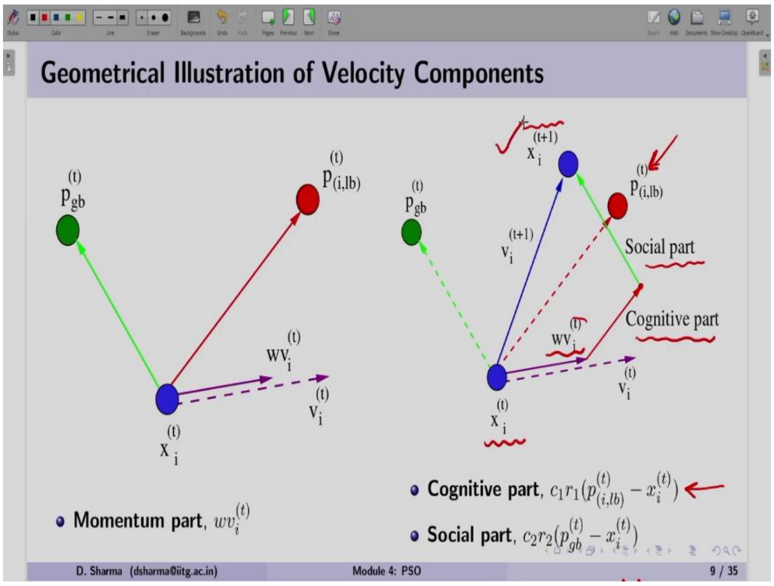

Now, let us understand the velocity components in a graphical way. So, as you can see in the figure, we have x i t which is the current position now, we have v i velocity part, we have local best part and p gb is the global best part. So, here we are showing the three vectors originating from the x i t. Now, the first part is the omega v i. So, you can see this is going to be our first component in the velocity.

Now, let us see the other components. Now, since we know the first part is the velocity part or the momentum part here so, in this momentum part, we add our cognitive part. So, this cognitive part as you can see at the bottom so, this cognitive part is calculated with respect to its local best. Now, looking at those this local best here so, the direction which is parallel to this local best we are adding a small component here so, we will get this.

Thereafter, we add a social part which is represented by the green vector. So, when we are adding these three vectors, we are going to get the new position as x i t plus 1. So, you can see that these three components basically representing the vector addition of those component with the current position of the particle to get the updated position of the particle.

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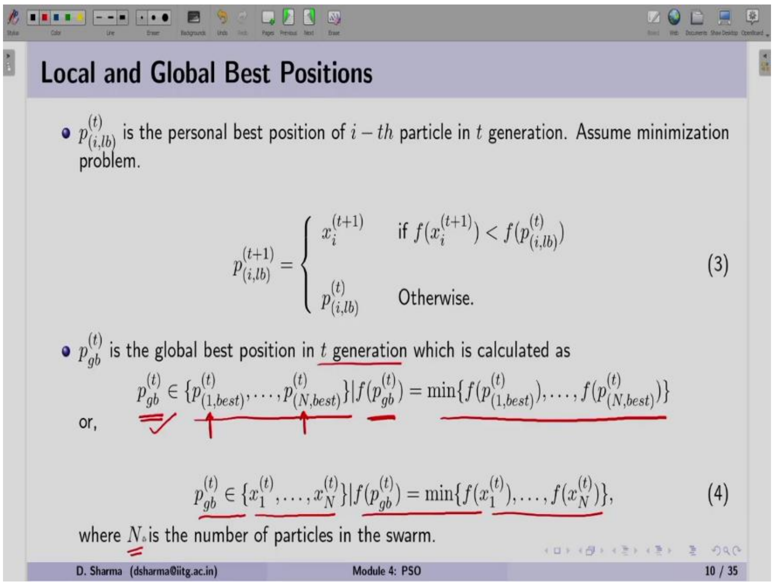

So, we have discussed as of till now, how the position of a particle is updated using the velocity equation similarly, so, the velocity is also updated using three component called the momentum part or the cognitive part or the social part. While doing this particular process, we have two vectors which we have used. One is called the local best of each particle and the global best of the swarm. So, the question comes that how we can find the local best and the global best?

$$p^{(t+1)}_{(i,lb)} = \begin{cases} x_i^{(t+1)} & \text{if } f(x_i^{(t+1)}) < f(p^{(t)}_{(i,lb)}) \\ p^{(t)}_{(i,lb)} & \text{otherwise} \end{cases} \quad (3)$$

Now, looking at the local best, which is called personal best so, sometime it is called personal best position or local best position and in some literature, it is also referred as local leaders or personal leaders. So, in this case, so, the best position of i th particle in the t th generation is calculated.

Here, we are assuming the minimization process so, in problem. So, in this minimization problem, the local best of the particle is updated to the new position of the particle, if the fitness at the new position of the particle i is smaller than the local best so far found for the particle i.

So, since it is a minimization problem so, we are using this smaller than sign. So, if the fitness is like that, we are changing the local best of a particle with the; with the updated position of the particle otherwise, we will keep the same local best of the particle.

$$p_{gb}^{(t)} \in \left\{p_{(1,best)}^{(t)}, \dots, p_{(N,best)}^{(t)}\right\} | \, f\left(p_{gb}^{(t)}\right) \; = \; min \; \left\{f\left(p_{(1,best)}^{(t)}\right), \dots, f\left(p_{(N,best)}^{(t)}\right)\right\}$$

$$p_{gb}^{(t)} \in \left\{x_1^{(t)}, \dots, x_N^{(t)}\right\} | \, f(p_{gb}^{(t)}) \; = \; min \; \left\{f\left(x_1^{(t)}\right), \dots, f\left(x_N^{(t)}\right)\right\}, \qquad (4)$$

Now, comes to the global best. Now, the global best is calculated say in the t th generation. We can calculate as now you can see that the local global best of the particle belongs to one of the local best of the particle. As you can see say suppose this is the particle 1 and this is the last particle N.

So, if we include all of them and we try to find out which one is the local best so, among those local best particle, one of the particle will be the global best. How we can find? So, this we say that global best of the swarm is one of the local best of the particle such that the fitness of the global best of the swarm is the minimum among all the solutions.

So, in this case, what it is suggesting is suppose we have 10 particles, we know they are local best. So, we have 10 local best of each particle. If I am going to compare the fitness of these best position of the particles among them, who so ever is the best will become the global best of the swarm.

So, this is the way we are calculating or in general, at the bottom equation number 4, it is written in terms of vectors since global best of the swarm is one of the column vector, this we find based on the minimum fitness of the particle one particle or the local best of the particle. Now here, since we are using our capital N as a number of a particle so, you can see that all the local best of the particles are included to find the global best of the swarm.

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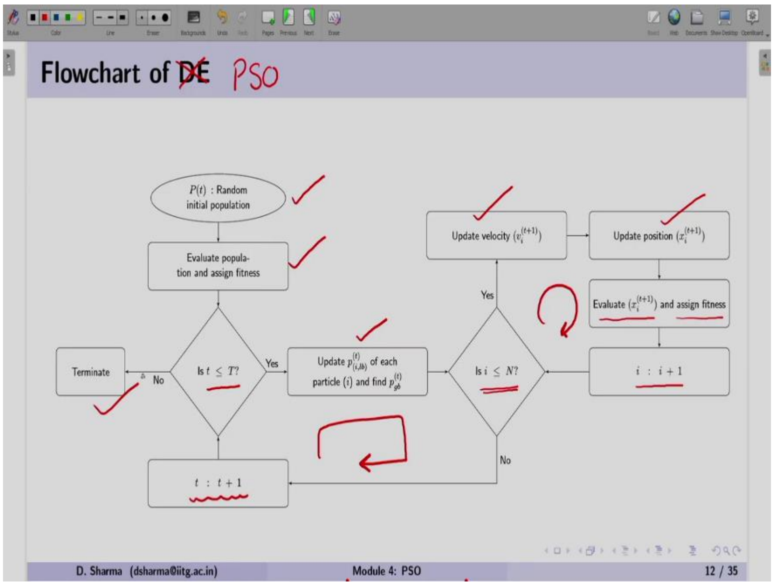

Now, let us understand PSO algorithm. This PSO algorithm first we will understand through the flow chart. In PSO so, this is the flowchart of the PSO in this case, we have a we start with the random initial population and then, we evaluate it. So, these two steps are the same as what we know from the genetic algorithm or we can say the generalized framework.

Then, we enter into the decision box based on the number of generation. Now, assume that t is smaller than capital T so, the first new step which you can see here is we have to update the local best of each particle as well as we have to find the global best of the swarm. Once we know that, then we are in the another loop with respect to the number of particle. So, the operations that are to be done, we do it particle wise.

So, first operation on the particle is we have to find the we have to update the velocity as given in the previous slide, then we update the position here. So, this new position will give me the new column vector. Since, it is new, we have to evaluate this particle and assign a fitness to it.

Thereafter, we increase the counter by 1. This process will be repeated till we have gone through each and every particle. So, in this way, we have calculated the updated velocity as well as the updated position of each particle. Once it is done, we increase the counter of number of generation by 1 and then, we keep on moving in this particular loop till the termination condition gets satisfy. Once it is satisfied, we terminate PSO and we report our result.

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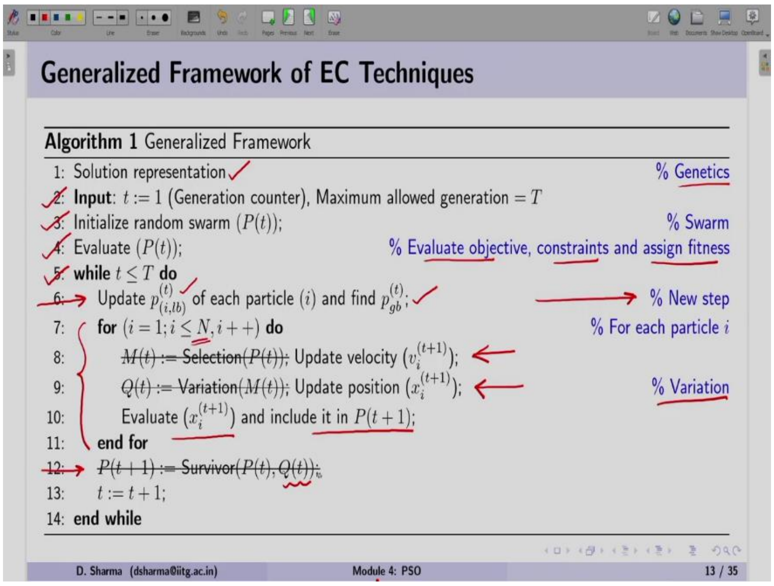

Let us fit our particle swarm optimization into the generalized framework of EC techniques. So, as you can see here, we start with the solution representation. So, since we have we are using PSO for continuous search space so, we have to store those values so, it called genetics as we know.

In a step number 2, we give the input, required input to start our PSO, then we initialise our swarm. So, this swarm we generate randomly and then, each particle is evaluated. So, basically calculating the objective function, constraints and then, we assign the fitness. So, till step number 4, all the steps which we have done similar to GA are the same.

Thereafter, in step 5, we are in the standard loop of number of generation. Now, step 6 is something new; why because we have to find the local best of the particle as well as we have to find the global best of the particle and that is why it is referred as a new step. Now, there is a small change in this generalized framework that we have one extra for loop here. This for loop is basically to update each particle one by one.

Therefore, what we do here is inside first of all, we have to find the updated velocity in step number 8, once we calculate it; calculated the velocity; we have to update the position.

Now, the position update means we are changing the column vector of decision variable. So, we can say that this is the variation in the current swarm by updating the position.

Now, since the position has been updated, we have to calculate the particle, the new position of the particle and we can include in P t plus1 because in the particle swarm optimization, we keep on changing the position of the particle and we do not keep another population such as offspring population with respect to GA.

There is no step 12 in as for as the survival because we do not have any offspring. Every time we update the position of the particle by keeping its local best and finding the velocity with respect to the local best, global best and the previous velocity. So, here, these small changes if we can make into our generalized framework, we can fit PSO into this EC this framework of EC techniques.

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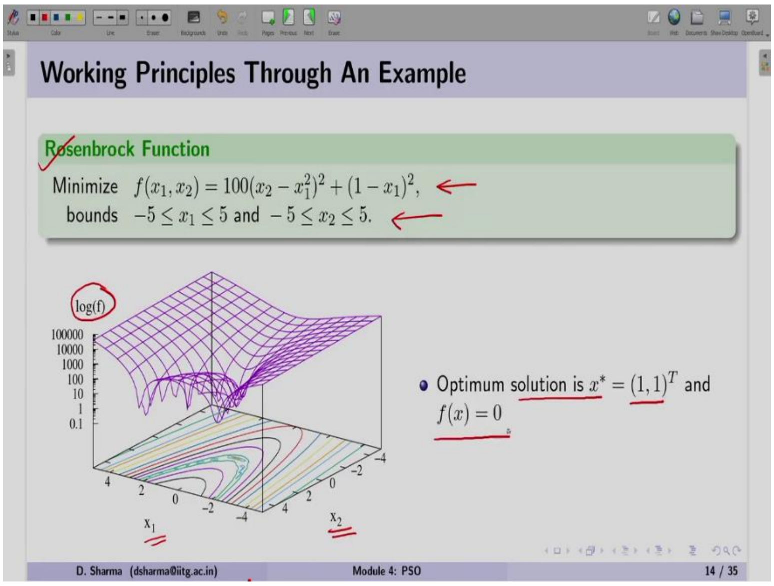

So, let us understand the working principle of a PSO with an example. As you can see that we have taken the Rosenbrock example here and which is currently we have taken for two variable, the bounds on two variables are from minus 5 to plus 5.

$$Minimize\ \ f(x_1, \ldots, x_n) = \sum_{i=1}^{n} \left(100(x_{i+1} - x_i^2)^2 + (1 - x_i)^2 \right)$$

$$bounds\ \ -5 \le x_i \le 5\ \ and\ \ i = 1, \ldots, n$$

On the right-hand side, on the third axis, we have taken logarithmic of the function so that we can see the nature of the surface and at the bottom, which is x_1 and x_2 plane, we can find the contours of this particular function. So, since we know this function has many local optimum, but there; there is one; one global optimum which is at 1 in a 1 and the function value at this solution is 0.

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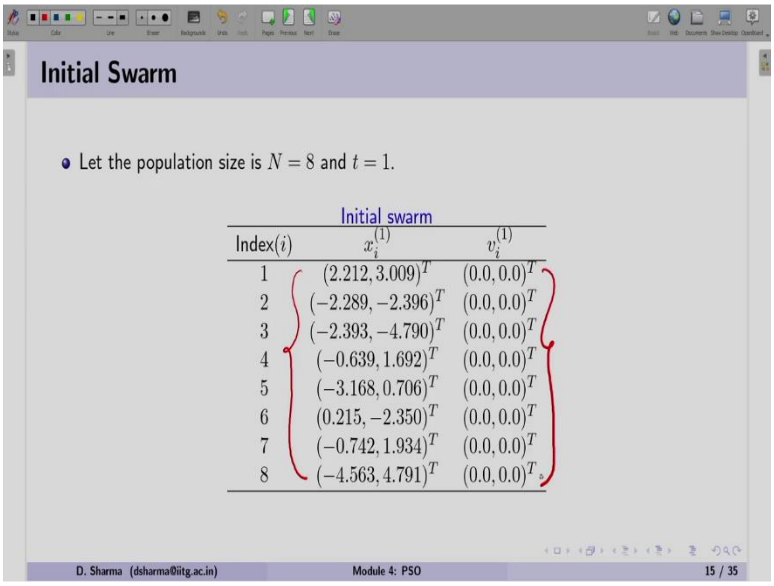

Let us begin with the initial swarm in which we are considering that there are 8 particles and since, it is the first generation so, we are saying that t is equals to 1. So, let us generate this initial swarm. Since it is generated randomly so, you can find it out that for all 8 particles, x 1 and x 2 values are generated for Rosenbrock problem. Similarly, if you we look at the third column of this table, the velocity is currently assigned as 0. So, since we can take any value. So, we are starting with the velocity 0.

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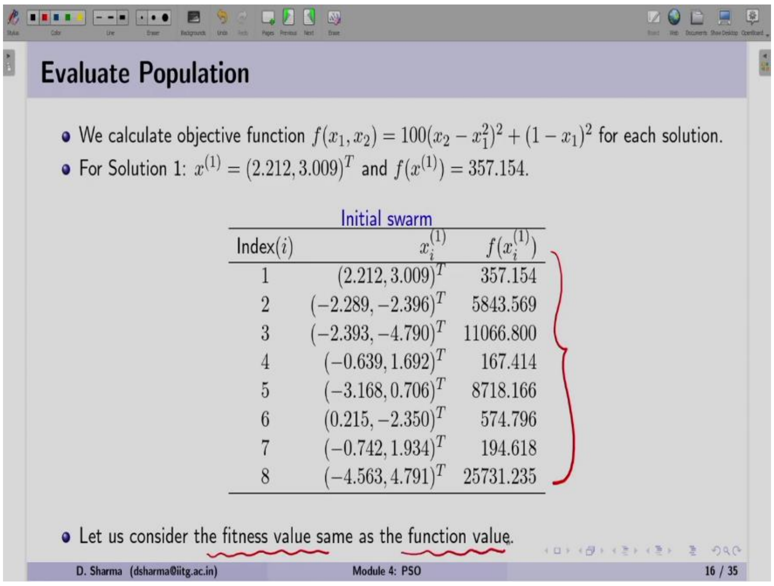

Now, thereafter we have to evaluate this swarm. So, we know that our objective function has given on the top; we have to include the value of x 1 and x 2 from the previous table to find out what is the function value.

So, for solution 1, we have x 1 and x 2 value. So, we are representing in the column vector of decision variables, putting into the objective function, we get the function value as 357.154. This process we can repeat for each and every particle so that, we can find the function value. So, you can look at the third column that the function value of each particle is calculated and reported here.

Now, here since we have to assign a fitness to each particle as of now, we are considering the fitness is the same as the function value. We are at the t equals to 1 generation so; we are going into the loop. Inside the loop, the first task which we have to do is find the local best of the particle.

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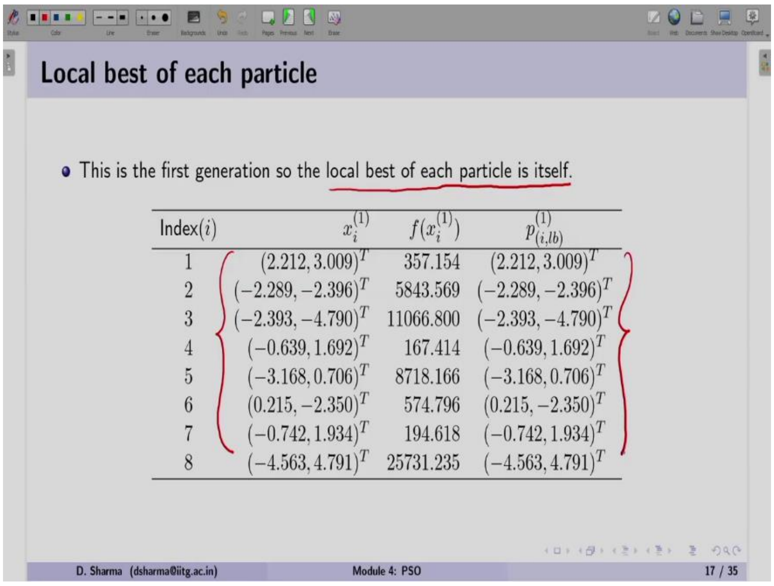

So, the local best since it is the first generation, we did not know the previous positions. So, what we do here is that the local best of each particle is itself because it is a first generation. So, you can look the second number column and the fourth number of the column, all of them are the same it is only because we are assigning its current position as the local best for each particle in the iteration number 1.

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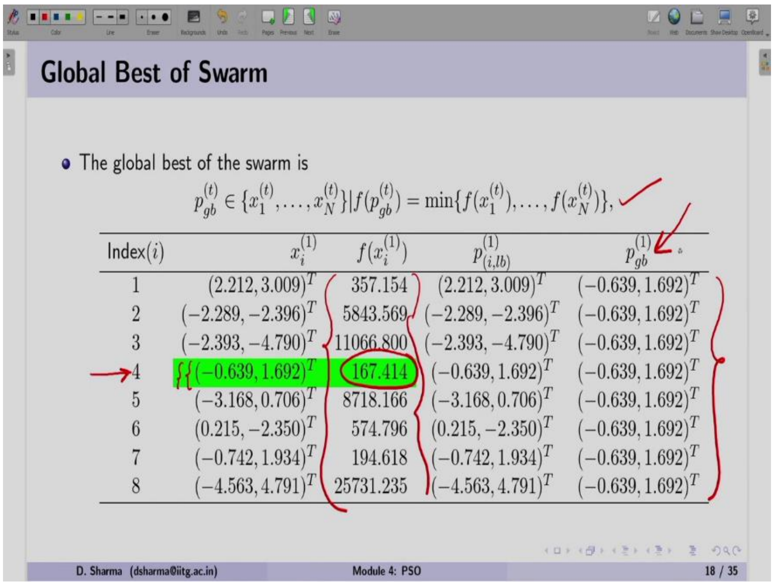

Now, once we have calculated the local best of each particle now, we have to find what is the global best of the swarm. So, as we know the formula given here, we have to what we have to do is we know the each particle so, the fitness for each of the particles so, I should say these are the fitness of the local best among those fitness, what this method say; what this global best says that which particular local best is having the minimum fitness value.

So, as we can see that 167 is the minimum value so, the global best in the generation number t corresponding to the particle number 4. Now, looking at this particular column vector here, the same column vector is copied at the last column. So, this last column is corresponding to the global best of the swarm in iteration or generation number 1.

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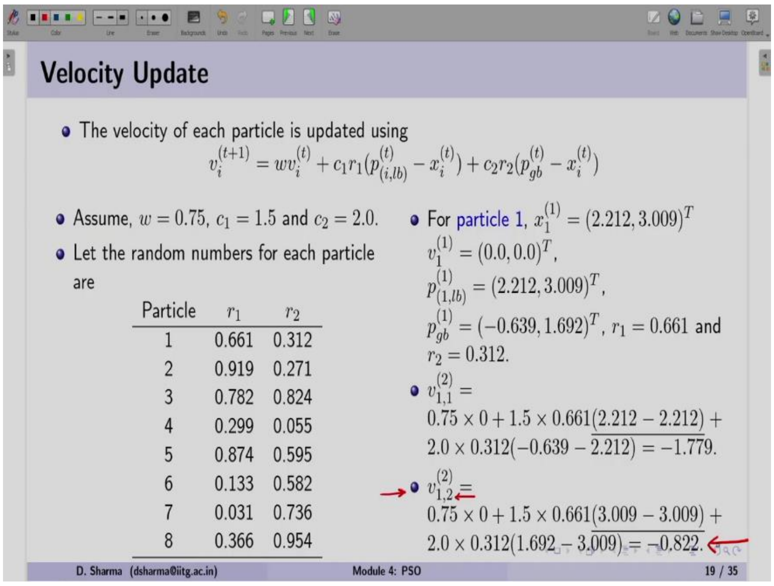

So, once we calculated the local best and the global best now, we have to update the velocity. In this particular velocity as we know, it includes three component as we have understood earlier. Now, let us see how we can calculate this velocity. As of now, we are assuming that omega is 0.75, constant c 1 is 1.5 and c 2 is 2. So, these are the three inputs we have to give to PSO to start with.

Now, since in this equation it involves r 1 as you can see here and r 2, both are the random numbers. So, these random numbers we can generate using our computer. So, set of random numbers are given in the table on the left-hand side where r 1 component will be used for the cognitive part and r 2 will be used for the social part of the velocity. For each particle, distinct random numbers are given. So, let us use them to update the velocity.

Consider the particle number 1 on the right-hand side. So, let us write for our simplicity, we are writing the column vector of the current position which is x 1 1. So, the velocity is 0 as of now. The global best of the swarm as you can see sorry local best of the swarm as of now is the same as the particle because it is the generation number 1.

The global best we have calculated in the previous slide, we have copied here and the two random numbers r 1 and r 2, we have taken from the table which is on the left-hand side. So, let us find the component of velocities variable wise.

So, first component as you can see here, we are representing a term and the last component is v 1 1 so that says that this is the first component of the velocity. So, now, we are putting everything into the equation as given on the top so, we are going with variable wise.

Now, this particular part as you can see, we have underlined its only because to see that at it at iteration or the generation number 1, the personal best has, or the cognitive part has no role because the position of the particle itself is the local best of the particle.

And then, we have the social part at the last. Putting altogether, we get the 1st component of the velocity of particle 1 is minus 1.779. Similarly, including the velocity and for the 2nd component, we can calculate, and we get the second component of the velocity for particle1 is minus 0.822.

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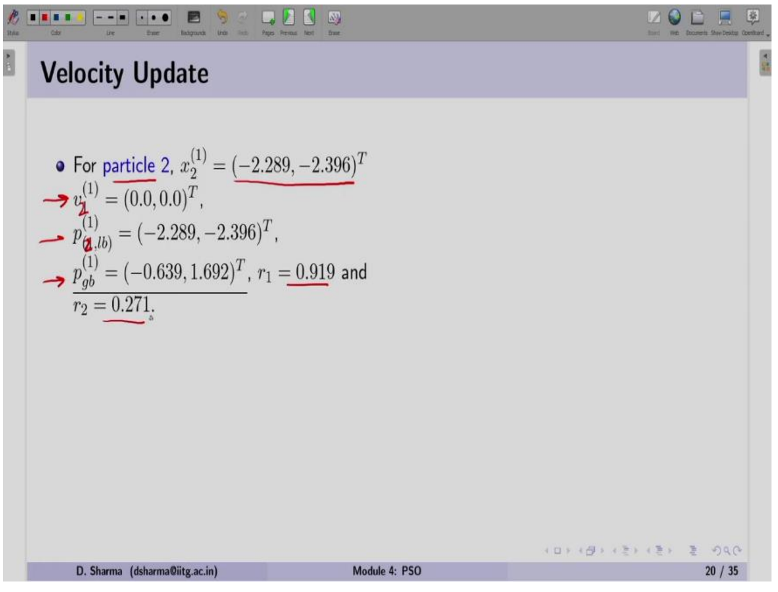

Let us perform the calculation one more time for a particle number 2, here we have mentioned the column vector of the current position of particle 2, velocity will be the same will be the 0 because we started with the 0 velocity and we have the local best of the particle and similarly, we have the global best of the particle.

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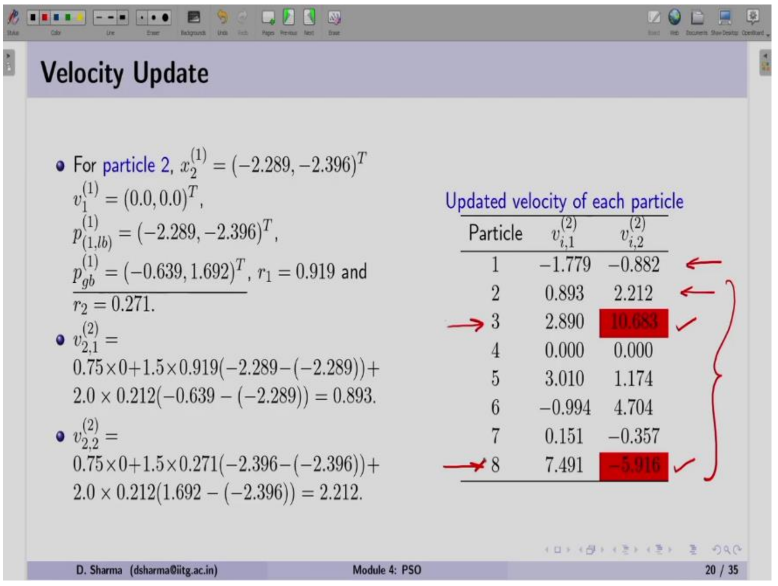

Now, by putting all of them together and taking r 1 and r 2 from the table in the previous slide, we can calculate the velocity the 1st component of the velocity here coming out to be 0.893 for particle 2 and again including into the formula, we will get the 2nd component of the velocity for a particle 2 is 2.212.

Now, if we perform the same kind of calculation basically using the formula, putting the value of x i vector that is the current position, the local best of the particle and the global best and the velocity, we will calculate all the velocities, all the component of the velocity for each particle. So, the table on the right-hand side shows you the component of the velocities for each particle.

Now, there are two columns which we have put it in a red colour it is because you remember that the variable bound for a given problem is lying between minus 5 to plus five, but this particular component are already a large value which is more than plus 5 in case of velocity for solution for a particle 3 and it is less than minus 5 in case of particle 8.

So, these two components we are showing that when we are performing the row operations to calculate the velocity of the particle, one or both of the component can go beyond the limit of the variables. Since, we have calculated the velocity of the particle; now let us calculate the position. So, this is the updated position of the particle.

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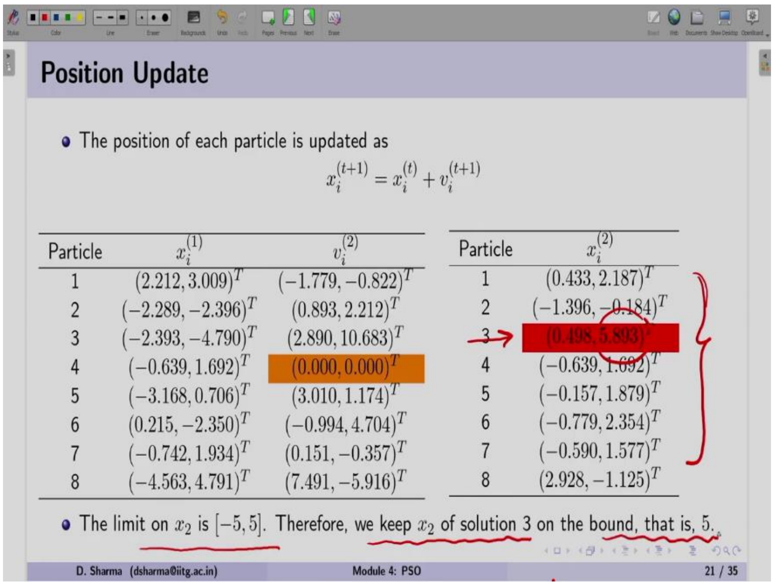

Since the formula is so straight-forward that we have to just add these two component so, let us see these two things. Now, in this table, the column number two represents the position of the particle, this represents the velocity, the updated velocity and here, one particular row of solution 4 is made in a orange colour; in an orange colour it is because the velocity component is 0.

Why it is so? Since it is the generation number 1 in which the local best of the particle is same as the current position so, the cognitive part will become 0. Moreover, when we calculated the global best of the particle, we know that the particle 4 was the global best. So, meaning that when we will be doing the difference between the global best and its [vocalize-noise] and the current position of the particle, the social part is also 0 for solution 4.

Now, we are left with velocity component. Since we are starting with the 0 velocity for all particle so, it means say it says that the updated velocity for particle 4 is 0 because it is the local best, it is the global best as well the initial velocity is 0. Now, since it is a simple addition; addition of the vectors, you can see in the table on the right-hand side that we have the updated position for each particle.

Now, the column; the row number 3, we made it in a red colour because the second component of this particle has the value more than 5 and since the limit on the variable is from minus 5 to plus 5 so, these this particular component as you can see which is the second component 5.893 which is more than that, then we have to keep this particular variable on the bound. So, as you can see at the bottom so, limit on x 2 is minus 5 to plus 5. Therefore, we keep x 2 of solution 3 on the bound that is 5.

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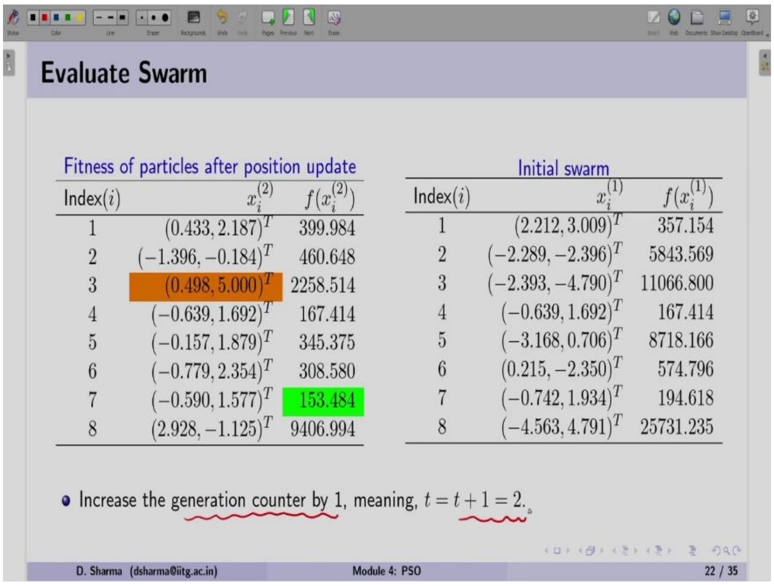

So, since we have calculated the position, updated position so, we have to evaluate it. Now, here the component of the particle 3 is again shown and just to show you in an orange colour, this particular component so, the second component of the particle 3 was out of the bound so, we kept it at 5 and accordingly, the fitness is calculated.

Now, the fitness of the position after fitness of the particle after position update all of the quantities are given in this particular table on the left-hand side. Just to see, when we started this initial swarm, the best particle was 4 and it has a fitness of 167.414 and after applying the velocity update and the position update to get a new position of the particle, what we found that the solution 7 is updated and this is the position or this is the new position which has the better fitness than the initial swarm.

And that is what we expected from every evolutionary computing technique that generation by generation, using the operators, the solution should improve, and all these solutions or particles will move towards the optimum solution. Now, at the end, we increase the generation counter by 1 so, currently we are at the generation number 2.

So, generally for the algorithm, we go for 1 generation, but since in PSO, when we started, the velocity was kept 0 as well as the local best of the particle was the same as the current position of the particle.

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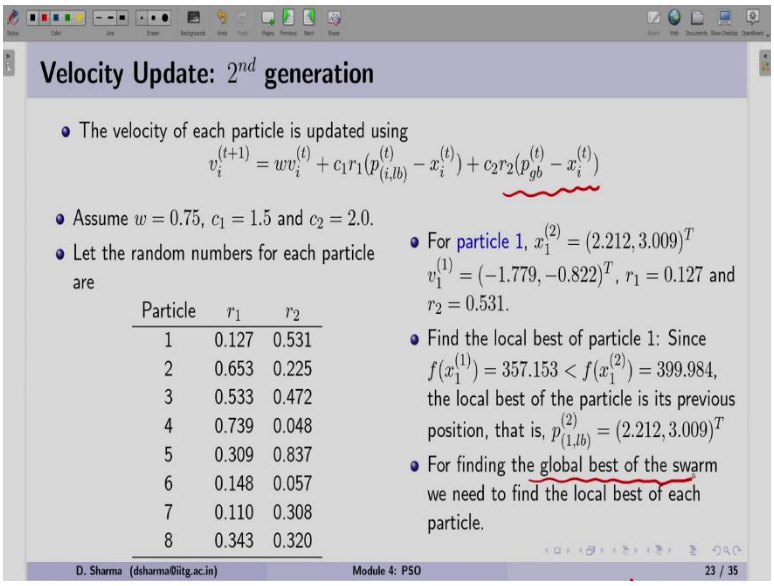

To understand more on this particular phenomena, we will show you the hand calculation for one more generation. So, let us start with that. So, first component is we have to update the velocity. So, the given according to the given formula, we are using the same concept. We are keeping the same value of omega c 1 and c 2. We generated another set of random number r 1 and r 2 because those are needed in the cognitive part and the social part.

Thereafter, we calculate the 1st component of the velocity for particle 1 as you can see in the right-hand side, we have the updated position; we have the updated velocity r 1 and r 2. To include this particular; this particular calculation into the velocity, first we have to find what is the local best of this. How I can find it? We have to compare the fitness as you can see the fitness in the previous position and this is the fitness at the current position.

So, what we can see that the previous position was better than the current position. So, the local best of the particle is its previous position that is the p i, lb 2 remains the same as the previous; previous one.

Now, we have calculated the local best, but to calculate the social part as you can see on the top, we have to find the global best of the swarm. So, in this case, we have to first find the global best and then, we can use the formula. So, this process goes like that, we have to find the local best of each particle first, then find the global best of the swarm and then, we can update the velocity. So, let us see how we have updated the local best.

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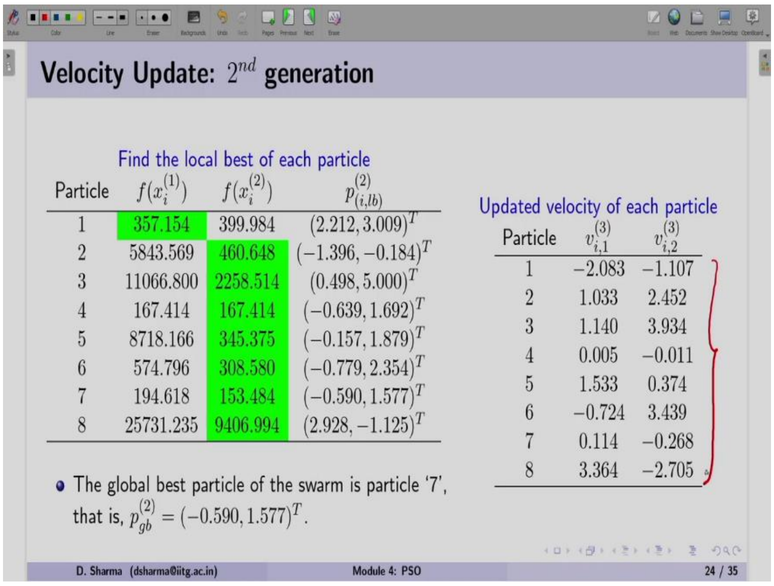

Now, looking at the table on the left-hand side, particle 1, this we discussed in the previous slide, we compared the fitness, and the green colours shows that the previous position was better so, the local best remains the same. For the particle number 2, the green portion shows that the updated position has a better fitness so; therefore, the local best of the particle is updated here.

Similarly, if we compare all of the particles position, the previous position and the current position so, in this case, what we will find that the new position has updated the local best of the particles. Now, since we have calculated the local best of the particle, we have to now update the global best of the particle.

How we can find? We will be looking; we I am just looking into the green part here. So, if I compare the fitness of the local best of the particles which are in the green colour among them the best is corresponding to solution number 7 meaning that this solution 7 will become the global best of the swarm and p g 2 will become the global best of the swarm for generation number 2.

Now, using the formula, we can update the position. Since we already know the formula and calculations so, while putting all of them together, you can see on the right-hand side, the velocities, the component of the velocity of each particle is updated. Now, this component will be used to update the position of the particle.

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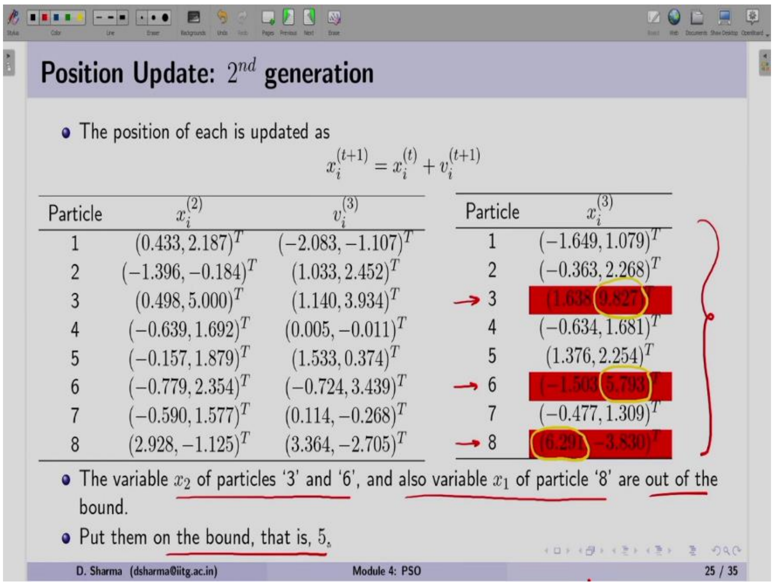

Since, we have calculated the velocity, the updated velocity in the previous slide, now we have to calculate the updated; updated position of the each particle. It is done using the simple formula as given on the top. So, the table on the left-hand side column one represents the current position of the particle and the third column represents the updated velocity of each particle. As the formula is so simple, we have to just add those components to vectors basically to get the updated position of each particle.

Now, here in this table on the right-hand side, what you can see that the particle 3 and 6 and 8, they are made in red colour it is because you look into the; you look into the component here, the basically the this particular component so, this component is greater than 5 now, look at the particle 6, this component is also greater than 5 and look at the solution number 8 so, the x 1 so, the first component of the particle is more than 5.

So, what we have to do here is we have to keep them on the bound. So, as you can see at the bottom, variable x 2 of particles 3 and 6 and variable x 1 of particle 8 are out of the bound so, we have to put them on the bound that is 5 as per the question given.

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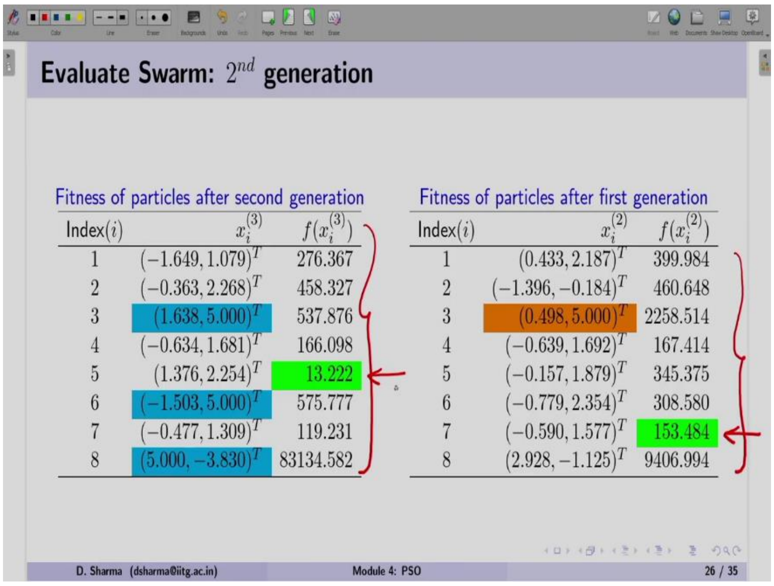

Now, in this case, the blue column; blue rows corresponding to the particles 3, 6 and 8 they are changed as you can see, we have written 5 because they were out of the bound. Now, just looking at the third column so, this third column says that when we have updated the; when we have updated the particle position and when we find the fitness of them so, the fitness has been improved to 13.22.

If we compare the fitness with respect to the previous generation, in the previous generation on the left; right-hand side, you can see that the best fitness we get is 153, but in generation number 2, we get the best fitness as 13.222 and that is the whole motive here that the velocity update and the position update which behave like a variation to the for the particle which updates their position.

They should be changed in a way that the particle should move towards the optimum solution. In a way, every generation, the fitness of the particle should improve as we can see from the left-hand and right-hand side tables.

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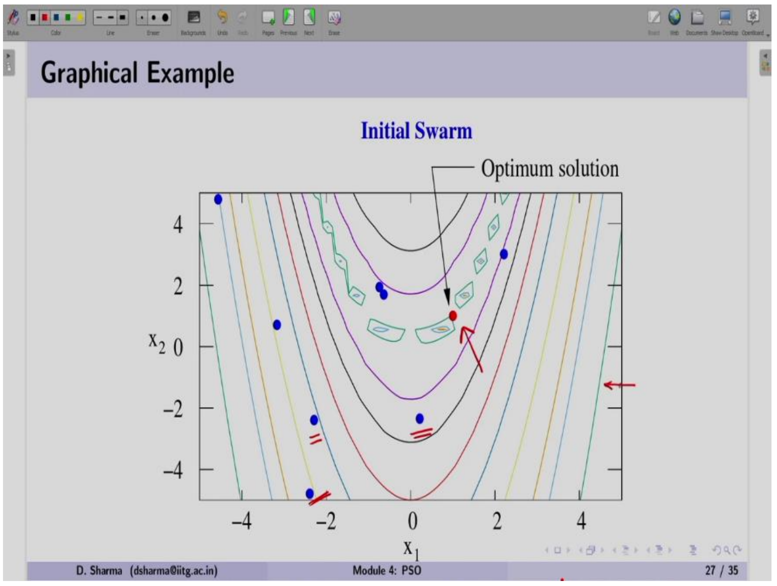

Now, come to the graphical example here. So, the same iterations or the generations which we have gone through, we will show how the particles are moving towards the optimum solution. So, this is was the initial swarm as you can see and the red dot as you can find it out is our optimum solutions and the solutions which are shown in the blue colour, they are distributed randomly in a plane of x 1 and x 2 and the contours of the functions are also shown here.

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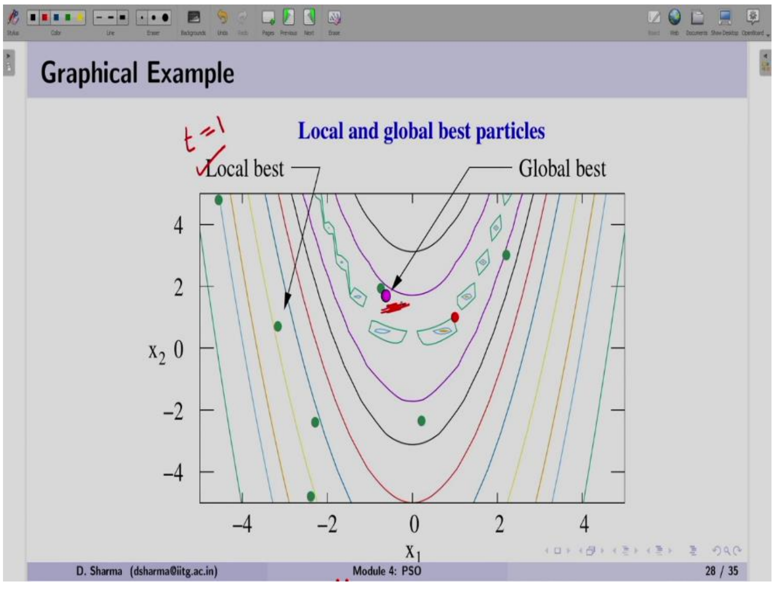

Now, once we have started with the random swarm, we have to find the local and global best. As we know, this is the iteration or the generation number 1, the local best is the same as the as a; local best is the same as the current position of the particle so that is why you cannot see any change in the local best.

However, to find out the global best, we have to find which is the; which particle or which local best of the particle has the least fitness so, accordingly this pink colour point suggest that this is going to be the global best of the swarm in iteration or generation number 1.

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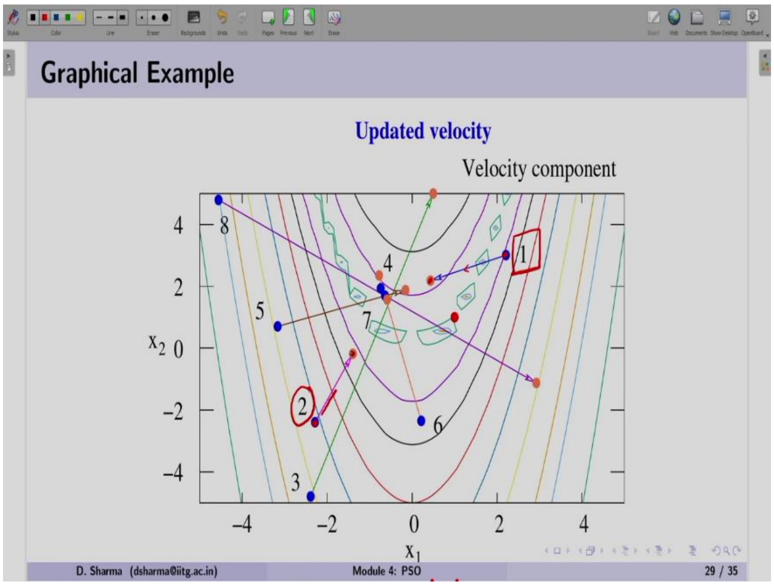

Thereafter, we update the velocity. So, here the vectors are drawn to understand how the particles are moving. So, let us see the particle number 1 here and this was its current position and when we are adding all the three components, the particle is updated to this. So, it is not a particle, it is a velocity. So, the velocity is updated. So, the particle will be updated according to this velocity. So, this particular component will behave will be the addition of the vector component.

Similarly, looking at the solution number 2, this is the current position of the particle, this particular arrow which is drawn in a pink colour is the velocity component added to it and finally, we get the new position. So, every particle from 1 to 8, the arrows are shown, those arrows represent the velocity component of each particle.

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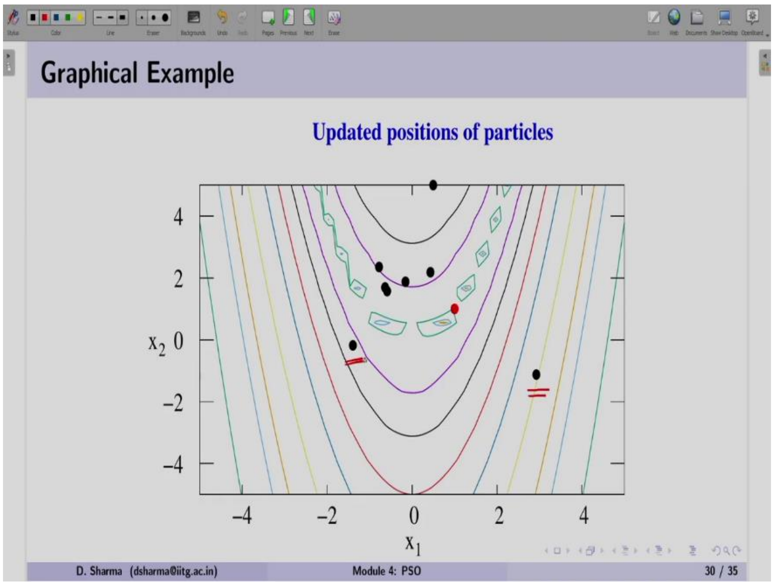

Thereafter, we have to update it. So, the current position plus the velocity will give us the updated position of each particle. So, those are shown in those black circles and you can see that the solutions have started moving towards the optima.

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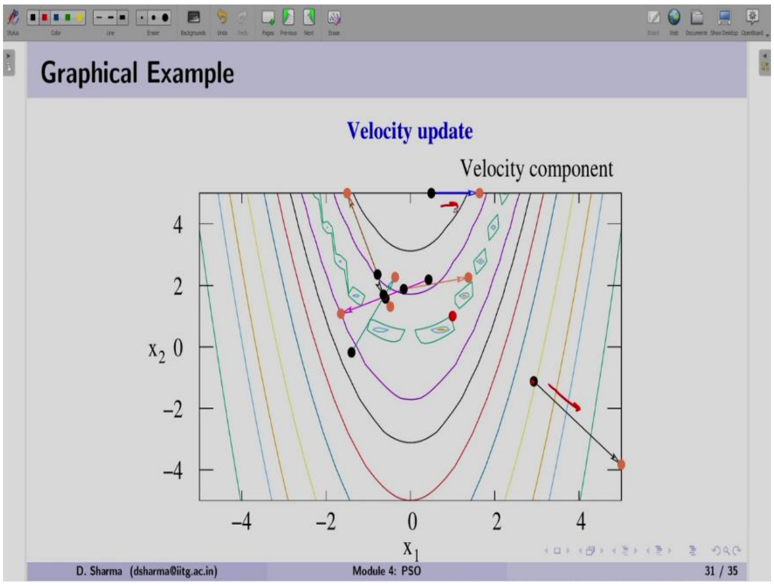

We have done one more generation to understand the local best and a global best. So, in this 2nd number of a generation so, velocities are updated. So, in this case, the solutions are shown, and the velocity components are also shown as you can see for every particle.

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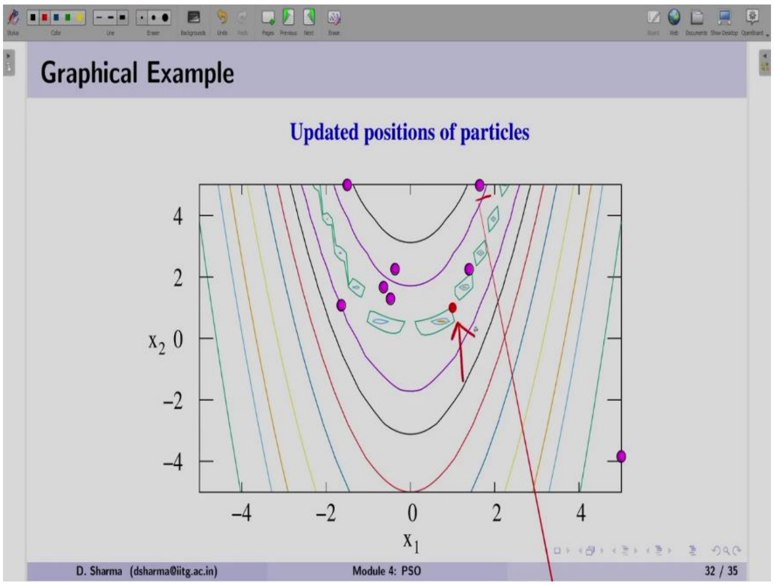

Once we know the current position and the velocity update, we will get the updated position of each particle in the generation number 2. So, these pink dots all are the updated positions here and we can see in the generation number 2, solutions are again started moving towards the optima which you can see in the red dot.

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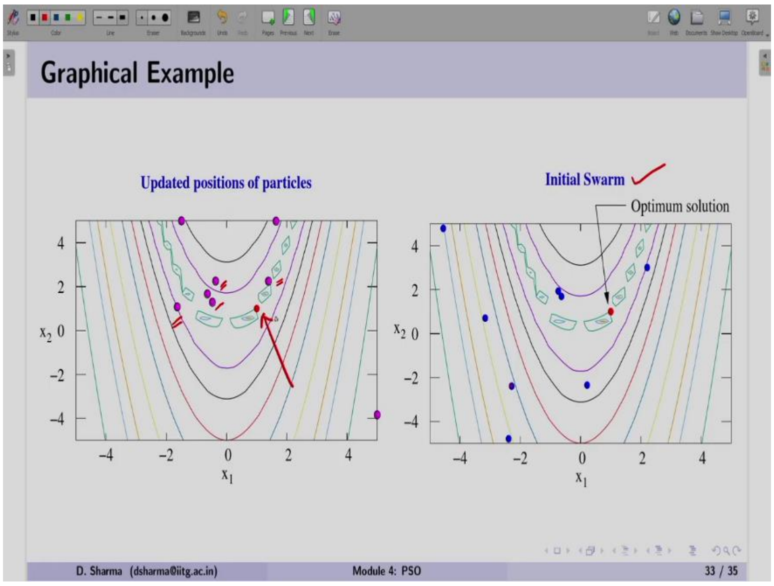

Let us compare how the solutions have moved so far. So, this was the initial swarm where the solutions were randomly generated in x 1 and x 2 plane. After two number of generations, you can see how these pink points are moving towards the optimum solution and this process when we will be continuing for further generations, we will find that the solutions or the particles will move to the optimum solution or the particles will locate the optimal solution for the Rosenbrock problem.

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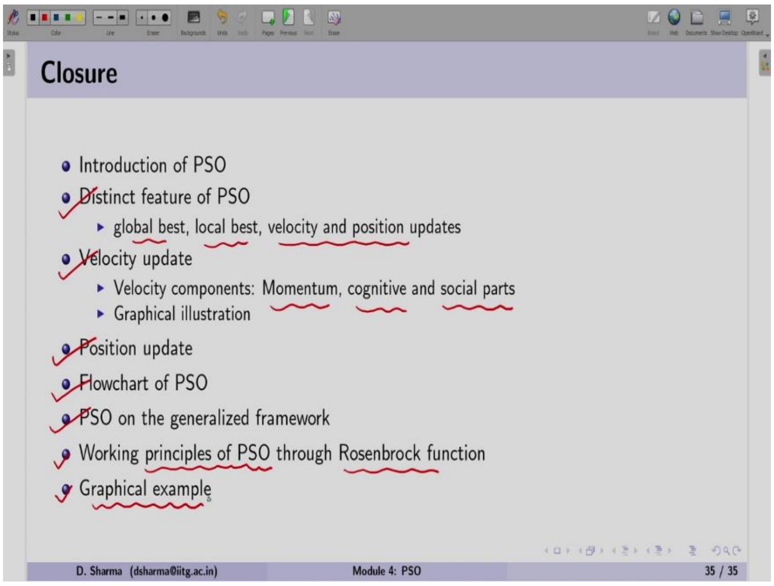

So, with this particular introduction and the working principle of PSO, let us come to the closure of this session. So, in this session, what we have gone through is the introduction of the particle swarm optimization. Since it works differently than the genetic algorithm so, we have gone through the PSO.

Its development based on the flocking of the birds and thereafter, once we understand the physical or the biological phenomena or the process of the swarm, there are three distinct features that we understood. So, those three distinct feature we understood as a global best, local best and the velocity and a position update of each particle.

Once we did it, we have certain formulas to update the velocity of the particle. It constitutes of momentum part, cognitive part and the social part and we have also gone through the graphical illustration. So, what we understood that the velocity is velocity of each particle is the summation of three vectors whereas, the social and the cognitive parts are the vector operation that is the difference between the two vectors.

Thereafter, we have gone through the position update. So, it was a simple formula to use it and we understand the PSO with the help of a flow chart. So, in that flow chart, a new edition was done in which the position update and the velocity update was done for each particle one by one.

We also fitted this PSO on the generalized framework so that we can see what are the small changes we have to do it and the same generalized framework, we can use it for particle swarm optimization. We understood this particle swarm optimization through the working; through the example of a Rosenbrock function. So, we understood each and every component such as local best, global best, velocity update and a position update and we perform two hand calculations for two generations.

And the same example, we have shown graphically to understand how these particles have been they started with the random position in the initial swarm and after performing two generation, the fitness has improved a lot and if we keep on continuing, the particles will finally, move to the optimum solution. So, with this, I conclude this session on introduction on particle swarm optimization.

Thank you very much.