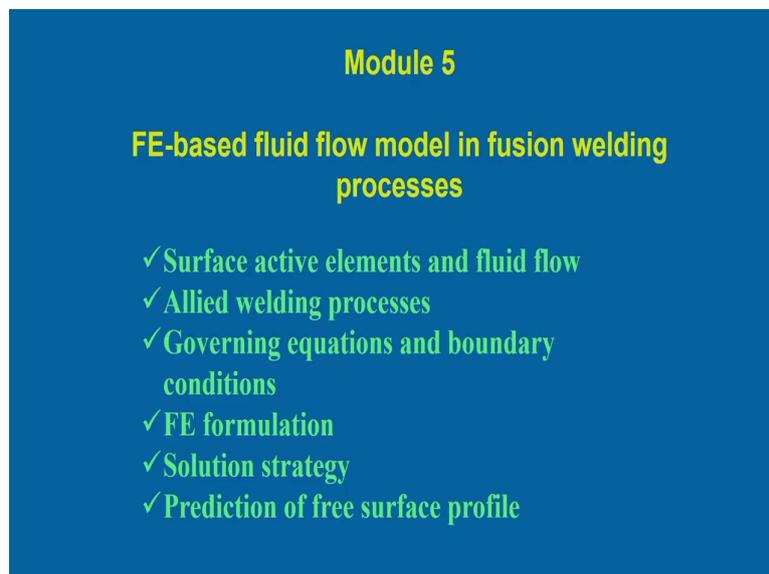


Finite Element modeling of Welding processes
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Module - 05
FE-based fluid flow model in fusion welding processes
Lecture - 25
Fluid flow modeling in welding processes

A good morning to all of you. Today, we will discuss the Finite Element Modeling of in case of heat transparent fluid flow analysis; particular to fusion welding process. So, first in this case, we will try to look into that why this fluid flow is important in case of welding process and second is that what way we can develop some FE model and by solving the governing equation and associated with the boundary conditions.

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Module 5

FE-based fluid flow model in fusion welding processes

- ✓ Surface active elements and fluid flow
- ✓ Allied welding processes
- ✓ Governing equations and boundary conditions
- ✓ FE formulation
- ✓ Solution strategy
- ✓ Prediction of free surface profile

So, in this particular module, this is the module 5. So, FE-based fluid flow model in fusion welding process, we will try to discuss first is the what is the importance of the surface-active elements in the fluid flow or what is the role of the surface-active elements in the fluid flow.

From there, we will get the answer why it is necessary to consider the flow analysis in specific to welding process. Then, in practical aspect of the that even by considering the effect of the surface active elements, what are the different allied welding processes actually developed that we will discuss.

And then, finally, what are the governing equation associated with the material flow and boundary condition also try to understand these things. And then finally, the what are the finite element formulation, then solution strategy or what way we can solve this particular set of the equations that we will see. And finally, we look into over the perspective of that what way we can develop free surface modeling basically in case of the welding process.

So, it is very much obvious that even if you practically observe in any kind of the welding process. So, after welding, we may not get exactly the flat surface, after welding also. So, surface may not be the so. To predict that curvature of the surface, then it is necessary to do some sort of understanding of the free surface modeling and associated with the welding process.

So, we will discuss only on the not in general, but very specific to the welding process in what way we can develop the approaches in specific to the free surface modeling in the welding process.

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Fluid flow

Transport phenomena based heat transfer and fluid flow model

- momentum transport due to
 - surface tension force (material specific)
 - buoyancy force
 - electromagnetic force (current)

solve conservation of mass, momentum and energy equations

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So, to that start with that fluid flow so, fluid flow or we can say the transport phenomena based heat transparent fluid flow analysis. So, basically while talking about the fluid flow analysis, but not only the fluid flow, we are considering; but apart from that material flow, we have to consider the thermal analysis also because temperature distribution is also important at the same time.

So, therefore, in general, if you say the transport phenomena base heat transfer and fluid flow model in welding processes, it is actually there is a energy transfer that means, from the arc and then, what way it induce some amount of the driving force. So, that momentum transport is also there.

So, then the momentum transport may happen within the small weld pool because of the surface tension force and this surface tension force is actually material specific and then, the nature of the surface tension force may change in presence of the surface active elements.

We will discuss about this thing, what way we can model the surface active elements effect in welding process and what are the typical surface active elements normally used. Next is the buoyancy force. So, buoyancy force definitely some density differences are there and then based on that, we can get some kind of the buoyancy force even exist within this small weld pool, we can consider as a one of the driving force.

But maybe the influence of the out of the other driving forces, the surface tension force is more significant that is influence more on the momentum transport in a welding process. So, apart from the other driving forces, so even if we consider the arc welding process, so there is a current flow is there.

So, definitely some sort of electromagnetic force, it will generate, that will have that electromagnetic force can be considered as a body force or may be driving force for the material fluids also influence. But once we analyze in case of the laser welding process, then we can neglect the effect of the electromagnetic force, we can consider the driving force of the material flow only on the surface tension force and the buoyancy force.

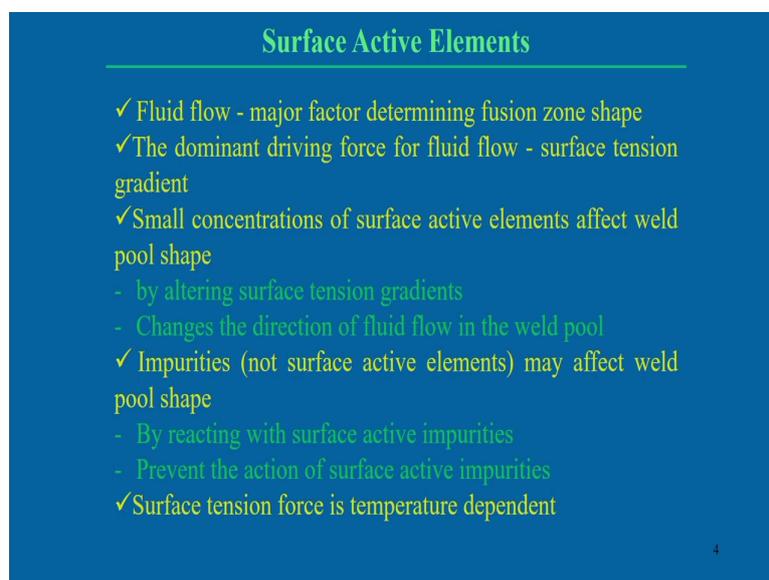
Now, mathematically if we want to introduce or you want to know what is the velocity field within the small weld pool and to do that, we need to solve the conservation of the mass momentum and energy equations. So, definitely, it will be more computationally expensive as compared to the if we consider only the heat conduction within the weld pool or even when you are looking into the when the total solution domain; even you can do get the temperature distribution only by solving the heat conduction equation.

So, definitely, there is a two different several part already discussed about the heat conduction equation. This; that means, what way we from the heat conduction equation, we will get only the temperature distribution in the domain. But if we consider the transport phenomena based

heat transparent fluid flow model in this case, we will be able to get the temperature distribution as well as the flow field also.

Flow field in the terms of the velocity field in that the within the domain. So, that domain that means, the velocity flow field will be able to predict and each and every node point in a finite element based model.

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Surface Active Elements

- ✓ Fluid flow - major factor determining fusion zone shape
- ✓ The dominant driving force for fluid flow - surface tension gradient
- ✓ Small concentrations of surface active elements affect weld pool shape
 - by altering surface tension gradients
 - Changes the direction of fluid flow in the weld pool
- ✓ Impurities (not surface active elements) may affect weld pool shape
 - By reacting with surface active impurities
 - Prevent the action of surface active impurities
- ✓ Surface tension force is temperature dependent

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Now to do that, now coming back to this point the importance of the surface active elements and then, how it is associated with the material flow. So, fluid flow is a major factor determining the fusion zone shape. It is true, because if we consider only the heat conduction analysis, definitely, we will be able to get the geometric shape and size of the weld pool.

But if we consider the material flow also, then it more accurately predict the temperature distribution. In other sense, I can say that probably if we consider the flow field, it actually modify the temperature field and then, this modification modified temperature field maybe more accurate as compared to the only if you consider the heat conduction analysis in a weld pool model.

But the dominant driving force for the fluid flow already explain the surface tension gradient or surface tension force is the main driving force in the fluid flow model also. But this small-small, very small concentration of the surface active elements actually influence the surface tension force or I can say the influence of the surface stress value and then finally, it will influence the material flow pattern and that pattern decides the shape of the weld pool.

So, that concentration there is a two way the surface active elements influence on the material flow; one is the by altering the surface tension gradient and changes the direction of the fluid flow or material flow field in the within the weld pool; so that, we will discuss that how it influence the weld pool also.

Then, other part is the impurities. So, impurities may also exist within the material when during the welding process also, that but impurities is not the surface active elements. But it may affect the weld pool shape. But in a different way, the impurities may affect to the weld pool shape; one is that by reacting with the surface active impurities.

Maybe it can react with the surface active impurity; in that sense, it may influence the weld pool shape and second, in other way also it can prevent the action of the surface active impurities. So, both way, it can react or both way it can work. So, until and unless, we do not have some kind of model surface tension model, the effect of the impurities we do not analyze until and then, we may not predict the what is the exact result. I am talking about purely from mathematical basis.

So, maybe this surface active, effect of the surface active elements, it is well-established some equations are already developed and simply, we can using this equation, we can implement in

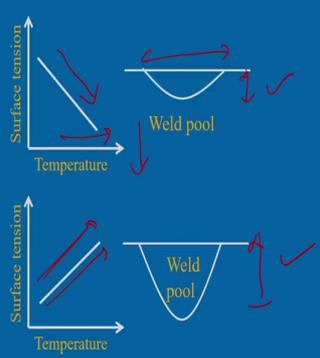
the finite element based model and we can get the more accuracy. But this effect of the impurities is not well-established things; maybe in that cases, to establish this effect of the impurities, there is a need of the lots of experiments to establish some kind of the mathematical model of this particular aspect.

So, definitely, once talking about surface active elements or when you try to model the basically once we affect the surface active elements, its necessary to model the surface tension force. So, once we model the surface tension force, it is definitely, it should the surface tension should vary as a function of temperature as well. We will see what way we can consider the surface active elements in a fluid flow analysis part.

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Marangoni Convection Mode

- ✓ Magnitude and direction of surface tension gradients - Marangoni convection
- ✓ Surface tension decreases with increase in temperature - negative slope
- ✓ Small addition of surface active element - change the surface tension temperature coefficient to a positive value
- ✓ Overall, affect the direction of the liquid material flow
- ✓ Surface tension of most liquid metals is substantially altered by the presence of small amounts of oxygen and Sulphur



Marangoni convection mode in weld pool

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So, here, we can see the magnitude and direction of the surface tension gradients that is sometimes called the Marangoni convection or Marangoni shear stress is acting the associated stress; we can say the Marangoni shear stress is acting on the surface.

That is mostly associated with the effect of the surface tension flow in the material flow and thus, definitely, the surface effect of the surface tension force must be there because the interaction may happen between the molten material.

This is one medium liquid medium. And when it interacting with the shielding gas also, through different medium acting so in between these two surface, there must some amount of the surface tension will be there. But sometimes these surface tension force is more active in the sense that in the presence of the surface active elements or maybe more influential to the results of the weld pool shape, when in presence of the surface active elements.

So, from that point, it is necessary to consider that effects also, during the analysis or when we wants to develop some kind of the material flow model associated with a fusion welding process. So, then Marangoni convection, it is definitely depends on the magnitude and direction of the surface tension gradients or we can say the surface tension force also. So, surface tension decreases with increase in the temperature.

So, we represent thus effect of the surface tension in pertinent to the fusion welding process; like that in the first case, we can see that surface tension force and the temperature as a function. So, in these cases surface tension force actually gradually decreasing; if you see this figure, gradually decreasing surface tension force with increasing temperature.

So, then if this is the situation or nature of the surface tension force in a weld pool, then we can expect the weld pool behavior something like that that the wide the depth is low and the width is may be more. And it entirely decided by the nature of the material flow pattern.

Now, this small addition of the surface active elements and change the surface tension temperature coefficients to a positive value. For examples, with a small addition of the

surface tension, surface active elements then it may drastically change the nature of the surface tension force.

So, it can change something like that in the direction or surface tension coefficient may change in this particular from negative to positive. So, once the surface tension force acting in this way or surface temperature coefficients of the surface tension is something like that when you plot it in between surface tension and temperature, then this weld pool can be completely different.

So, here you can say the here you can expect the more penetration and the width can be less as compared to the first case. So, that is why it influence the nature of the material flow pattern and indirectly to the shape of the weld pool. So, there is a some influence of the surface tension force.

Now, overall affect the direction of the liquid material flow. So, we are getting this kind of profile also and we are getting this kind of different kind of the profile. The just change in the in these two cases the difference is the presence of surface active elements or dot. If there is a presence of surface active element, it simply indirectly affect the or directly affect the material flow pattern.

So, that material flow pattern decides the shape of the weld pool. So, that sense, so overall effect of the direction of the material flow pattern in presence of the surface active elements and surface tension of most of the liquid metal is substantially altered by the presence of small amounts of the oxygen and all and sulphur.

So, these are the two small quantity of the addition of the liquid metal, the oxygen or small quantity of the sulphur is acts as a surface active elements and change the flow pattern or its completely change the surface tension mode or surface tension explicit of the surface tension as a function of these surface active elements.

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Surface active elements in welding

- Presence of surface-active agent in the liquid metal in significant amount, $\partial\sigma/\partial T$ can be changed from negative to positive
- Marangoni convection influence the weld pool
- Presence of Sulphur and oxygen in Stainless Steel acts as surface active elements
- Example: 180 – 600 ppm oxygen in SS304 produce maximum weld penetration

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So, therefore, presence of surface active elements agent in the liquid metal in significant amount, then change surface tension force. So, change of the surface tension force or with respect to temperature, then means we can say the gradient or coefficients of the surface tension or temperature coefficients of the surface tension, it can change from negative to positive.

That we can say negative is the normal condition, we can assume that negative gradient in the previous slide, we have shown that indicates that there is no surface active elements within the material. But once it change to a positive, then we are assuming that there must amount of the surface active elements.

So, therefore, you can change these temperature coefficients surface tension from negative to positive value and definitely, the Marangoni convection influence on this thing the weld pool.

So, because of the surface tension force, the Marangoni convection equation Marangoni convection actually changes the weld pool shape and size.

Now, in quantity if you quantify the or you have to try to model the surface effect of the surface tension force, then most in practical the presence of sulphur and oxygen and that a also experimental experimented the these two cases, their presence of sulphur or presence of oxygen in stainless steel acts as a surface active elements and change the model of the surface tension force.

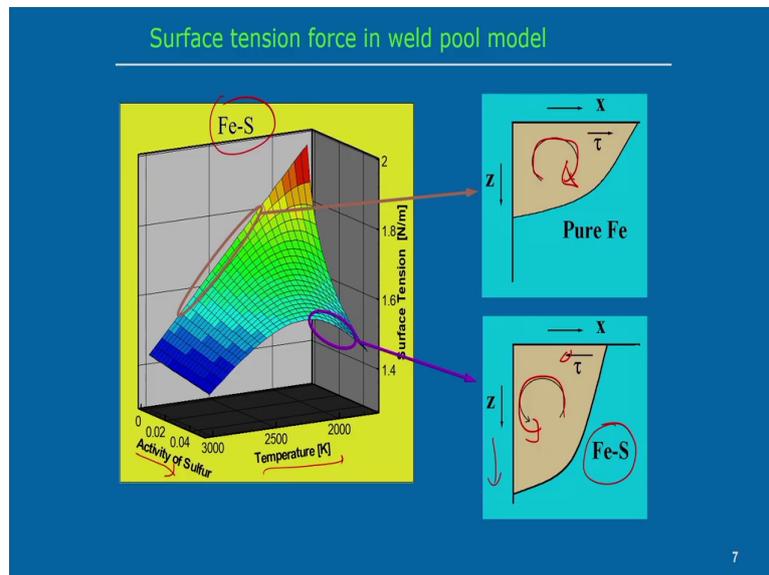
So, one example we can take here that is for example, the we are talking about small quantity; but 180 to 600 ppm oxygen in SS304 and can produce the maximum weld penetration. It means that this presence of the 180 to 600 ppm oxygen very small quantity oxygen in SS304 and this stainless steel, that produce the maximum weld penetration.

It means that it can act as a surface active elements and basically, it is very important when this oxygen or sulphur will act as a surface active elements and that there is some optimum quantity and that optimum quantity is the system dependent, basically depends on these things. In general, see we can see that it in between 180 to 600 ppm.

So, if it is too much of surface active elements, then it may not affect the material flow pattern or may not achieve the very good penetration what we can expect in this in presence of the optimum quantity of the surface active elements. So, therefore, this is the one task to decide what is the or to develop the surface tension model as a function of this surface active elements.

It means that if very low amount of the surface active elements may not influence or may not act as a surface active elements that actually influence the flow pattern. At the same time, if surface active elements are too high also, that may not act as a surface active elements to influence the flow pattern. So, some optimum quantity; so, therefore, that optimum quantity can be decided based on the experiments also.

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Here, we can see that what is the effect of the surface active elements. So, this is the graphically, we can see the activity of sulfur. So, small quantity, the activity of sulfur, if you see this one axis; other axis represent the temperature and other axis represent the z axis basically represents the surface tension and Fe-S system. So, system is the Fe-S.

Now, this Fe-S system. So, once Fe-S system means it is a binary alloy system, we can say something like that. Presence of the this way we assume the pure iron, but within this pure iron, these there is a small quantity of presence of the small quantity of the sulfur. Then, how surface tension behave on this particular situation? See, if this part this zone is indicating that there is a presence of the surface active elements is very small; so, 0 basically.

So, in that case, we can see it is a pure iron system. So, once it is the pure FE that in that particular material system say this is a typical pattern of the weld pool shape and this is the

flow pattern is something it is like that the shear stress is vertically Marangoni shear stress is acting on the top surface; but it is acting in this outward direction, from centre to outward periphery.

So, that influence the material flow pattern is something like that. So, it is we can say, it is a kind of clock wise direction, the material flow will there. So, once this kind of material flow pattern, definitely it will try to enhance the width of the weld pool and decrease the depth of penetration or we can say it is a normal mode, if there is in absence of the any kind of the surface active element, this is the typical flow pattern.

Now, if you look into the other condition. So, you can see there is a some very high quantity of the surface active element or we can say the very small optimum quantity of surface active elements presents in this particular FE system. So, in this Fe-S, in this particular material system, you can see since there is a presence of the surface active elements and if we assume that, that particular quantity of surface active elements with the influence maximum.

So, such that it will simply reverse the direction of the Marangoni shear stress. So, Marangoni shear stress will be acting in the from the outer periphery to the centre; so, towards the centre, in that particular direction. So, then once it is acting this particular direction, then flow pattern will change like this.

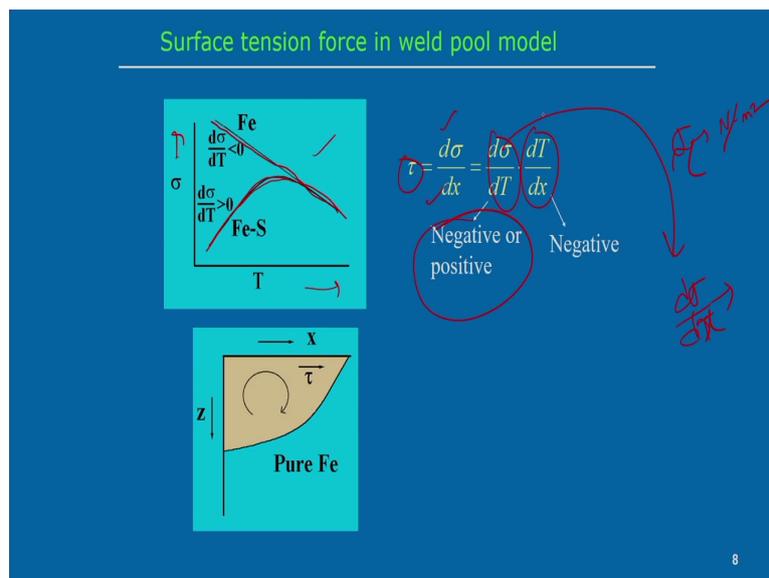
It is a it is it will be acting in the anticlockwise direction. So, once the flow pattern is followed in this particular situation the direction, then we can expect that it will enhance the weld penetration; but at the centre, it will decrease the weld width. So, that is the effect of the surface active elements.

So, in these two cases, we can differentiate also that in case of the in presence of the surface active elements, we can take this benefit that in presence of the surface active elements will enhance the weld penetration and that is also practically significant because when you try to join the two components, for example in case of TIG welding process.

So, TIG welding process, even GTAW welding process or TIG welding process, we cannot achieve much more penetration. But this is a in principle in GTAW process, in this normal penetrate the ratio of the width of the penetration is not much. But if you want to achieve very high penetration, even simple TIG welding system, in that cases it is possible to achieve just by simply adding some sort of the surface active elements.

Of course, without looking into the what are the effect of the surface active elements the material properties that is the different aspect and that is we are not discussing that aspect in this here also.

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So, we can take the benefit from the presence of the surface active elements or more mathematically, we can represent this for example, if this is the surface tension force is acting in this particular direction and this is the x axis represents the temperature. And in case of

pure iron, so it is following this in a as a function of temperature, it follows this kind of line straight line. But in case of the Fe-S system, it can follow this kind of a system. So, it is this curve something follow like that.

Now, this Marangoni shear stress or shear stress on the surface is acting basically when you are analysing the fluid flow analysis, definitely you have to look into the what are the value of the shear stress and then, because fluid flow is a viscous flow, then it is associated the shear stress and the we need to consider the shear stress value.

Now, from the surface tension force to shear stress, we can convert simply the $d\sigma$ by dx . So, σ in si unit, we can represent the N for Newton meter that is the indication of the surface tension force. Now, if you do simply derivative spatial derivative of the surface tension force $d\sigma$ by dx , then it is a it can be sorry, Newton per $d\sigma$ can be a newton per meter.

Then, once we do the derivative $d\sigma$ by dx , then we can say that the x another then it becomes that shear stress τ becomes Newton per meter square that is equivalent to the a shear stress. Now, $d\sigma$ by dx is the change of the surface tension force, surface tension gradient we can say that consists of the two parts; one is that $d\sigma$ by dT as a function, if we want to takes care of the effect of the temperature $d\sigma$ by dT and then dT by dx .

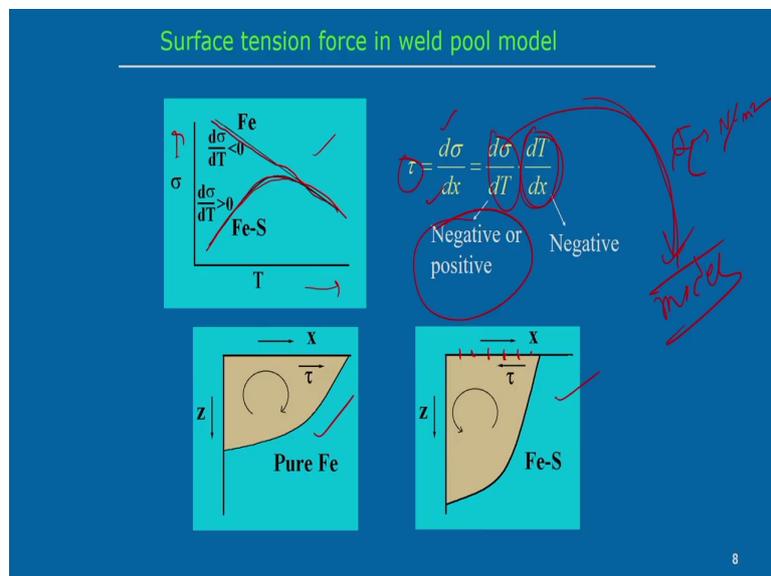
So, then first term is the called coefficients temperature coefficients of the surface tension force or second term, we can say that it is a temperature gradient. So, temperature gradient. So, this temperature gradient is always negative in the sense that the in weld pool also, the from the at the centre is the maximum temperature and when you move the outward periphery, that temperature actually decreasing and that solid liquid interface, then we can reach the temperature can be the melting point temperature.

So, therefore, $d\sigma$ dT by dx , we can consider as a negative and this $d\sigma$ by dT can be positive or negative depending upon the whether there is a presence of the surface tension, surface active elements or not. So, then this can be once we have this kind of data as a curve

as a sigma as a function of temperature, then we can represent the we can estimate the shear stress value and simply looking into this part.

So, then what is the d sigma by dT? That can be a effect of the or surface tension this temperature coefficient of the surface tension or surface tension sorry d sigma by dx. So, just gradient finally, we estimates d sigma by dT, this is the temperature coefficients of the surface tension that has to be modelled as a function of the surface active elements.

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Now, the one case that I have already shown this part that pure iron, this is the typical pattern and Fe-S, this is the typical pattern; but all depends on the what way we are modelling the surface tension a temperature coefficients of the surface tension force.

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Surface tension coefficient - as a function of temperature and activity of solute

$$\frac{\partial \sigma}{\partial T} = A - R_g \Gamma_s \ln(1 + C_s b_i) - \frac{C_s b_i}{1 + C_s b_i} \frac{\Gamma_s \Delta H^0}{T}$$

$\frac{\partial \sigma}{\partial T}$, A , Γ_s , C_s , b_i , ΔH^0 and R_g

surface tension gradient, adsorption coefficient, surface excess at saturation, segregation coefficient, activity of the 'ith' species, heat of adsorption and characteristic gas constant

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We can show some the temperature coefficients of the surface tension or surface tension gradient, we can see that $d\sigma/dT$ can be a function of all the a composition. This is a typical model of the $d\sigma/dT$. Say here you can see the A ; A is the A is called the Adsorption coefficient basically adsorption coefficient and then, R_g that this represents that surface excess at saturation.

Then, C_s represents the segregation coefficient; b_i represents the activity of the i th species, if it is a alloy system. Then, ΔH^0 is basically the heat of adsorption and the R_g is the characteristic gas constant. So, these are the all parameters and this is a typical surface tension coefficients, so all material parameter we have to consider.

And this if we see, the all material parameters associated with the to represents the coefficients surface tension model and here, you see all as a function of temperature T . So,

then once we model, we get all these parameters for a particular system; maybe we can say for particular alloy system.

If we know all these parameters, then A , λ or this thing, then we can estimate the value of the temperature coefficient surface tension model and then, that we implement. We estimate the shear stress from this value and this definitely, this value as a function of temperature and $d\sigma/dT$ represents the temperature coefficients of a surface tension or surface tension gradient can be represented like in this way, we can represent the surface tension gradient.

So, once we make the surface tension model, surface tension $d\sigma/dT$ and put this $d\sigma/dT$ value depending upon the model. So, here some model is required and then, temperature gradient because temperature gradient at any discrete point, we can estimate what is the temperature gradient on the top surface of the weld pool.

And then because the temperature itself is the output from the solution of this equation. So, then looking into that, we can find out all these values, then we can estimate what is the shear stress value and then, that influence the material flow pattern.

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Allied welding process using surface active elements

Minor elements can be added to the weld pool by adjusting

- ✓ Chemical composition of the base material
- ✓ Spreading fluxes (halides or oxides) on the substrate material
- ✓ Using active gaseous addition (CO_2) to the argon shielding gas

Overall, addition of a small amount of minor elements to the base material significantly changes the weld penetration

Industrially A-TIG process has been developed

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So, then based on the affect of the surface active element, different allied welding process has been developed; one is the in these cases the minor elements maybe we can say the surface active elements can be added to the weld pool.

But three different ways that maybe this presence of sulfur or may be present within the chemical composition of the base material, that already existing with the base material and then, when you are try to do some welding, it is acting as a surface active elements.

So, that may be the one possibility and second, the spreading the fluxes. So, we can spreading the fluxes, halides or oxides on the substrate material. So, it means that suppose, we collect some kind of the halides or oxides in the powder form and then, powder form mixture that is a it is a make a solution with this thing and there is a (Refer Time: 25:30) on the surface. So, in that form, we can apply the surface active elements to the substrate material.

So, apart from that, we can use the that directly the add this surface active elements the for example, CO₂ can be added with the; CO₂ is the with the shielding gas. So, using the active gaseous addition to the argon shielding gas. For example, argon shielding gas, we are using in a TIG welding process, we simply add the CO₂ with this with the certain proportion, that have to be decided then what quantity of the CO₂ can be add to the argon gas, such that that oxygen will act as a surface active element.

So, these are the three different ways, we can incorporate the surface active elements in a particular welding system. So, overall finally, the overall addition of the very small amount of the minor elements or surface active elements to the base material significantly change the weld penetration and based on this principle, the industrially A-TIG; the Activated TIG welding process has been developed.

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Activated TIG Welding Process

- ✓ A-TIG is a variant of TIG Welding which involves application of thin coating (10-15 μm thick) of activated flux on the joint area prior to welding ✓
- ✓ Results in increase in penetration in single pass welding ✓
- ✓ Overcomes the limitations of the conventional TIG welding process

Advantages

- Enables single pass welding of higher thickness plates ✓
- Enhance productivity and reduce consumption of filler wire ✓
- Residual stresses reduced significantly (> 70%) and the weld joints are almost distortion free
- Significant reduction in the cost of fabrication (> 50%)

So, in activated TIG is a variant of the simple TIG welding process. We aware of this thing, the simple GTAW of TIG welding process which involves the application of the thin coating maybe 10 to 15 micro meter of the activated flux on the joint prior to the welding process, that can be done simply before the welding process, we just make a coating of the 10 to 15 micro meter thickness of the layer we can put.

And then, results in an increase in the penetration the single pass welding; that means, this basically this layer thin coating layer on the substrate, it acts as a in the form of surface active elements and then, it will enhance the weld penetration.

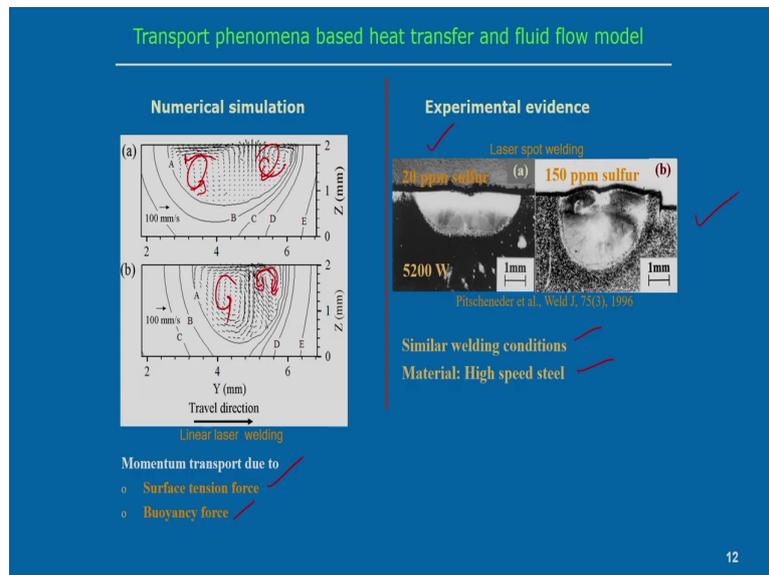
So, this is a very simplified way, the activated TIG welding process is developed. Overcomes the limitation of the conventional TIG welding process, the in conventional TIG welding process, if we use that we can simply cannot achieve the very high depth of penetrations.

So, that is why we can use a simply activated TIG welding process. Advantage single pass welding, we can achieve very high weld penetration, then it is not necessary the multi task to achieve the particular penetration. Enhance productivity and the reduce the consumption of the filler wire.

Definitely, if the filler wire may not be necessary to use in these cases if because you simply acting the involving the surface active force, we can achieve the very high depth of the penetration. Residual stresses can be reduced significantly and the weld joints are almost distortion free.

So, this is the perspective. This perspective depends on the this is the in general the advantage all these things; but it depends on the process itself. Then, significant reduction in the cost of the fabrication. Definitely, since we are not using any kind of the material deposition process or that thing. So, in this is a very simple process, but just then just we can reduce the cost of the process.

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We can see the experimental evidence the affect of the surface active elements also. So, here we can see there is the two figures are there; one is the this is the one case the effect of the surface active elements we can see that a laser welding process, the similar welding conditions, high speed steel and the it is a laser spot welding process; one case is 20 ppm sulfur, another case is the material content the 150 ppm sulfur.

But other similar welding condition, the same parameter laser focused diameter a few all other forces parameters are same and material also same; high speed steel, but only difference is the material one is having the 20 ppm sulfur another is having the 100 ppm sulfur.

So, these are the difference. Other conditions are same. Then, we can expect that there is a huge difference in the weld pool shape. First case is we can get this kind of the profile. So, if

we can see the second, this kind of profile, other case is we can see that the depth of penetration is very high in the second case.

So, it means that definitely the only difference in this particular situation the activation of the surface active elements and their influence on this particular weld pool shape and basically, the in mathematical sense, we can explain the active surface active elements from the material flow pattern only.

So, in this case definitely we need to do some kind of the material flow analysis to understand this phenomenon. So, it means that the surface active elements here the influence the material flow pattern, then we are getting this kind of the profile and other cases the surface active elements is not acting, so we can get the material flow pattern in the different way.

So, that weld pool shape and size are completely different in these two cases. Even we are using the all parameters, all welding parameter are same. So, similar kind of the phenomena can be explained from the mathematical model by simple can be done the material flow analysis.

So, here is the importance of the consideration of the material flow pattern or fluid flow analysis in a welding problem. So, here you can see the numerical simulation perform this thing you can see the velocity vector, we have plotted each and every node point. We can see the in this case is the flow pattern is something like that, the flow pattern is working like this. So, we can get the this thing. But other cases, we can flow pattern is this way and I think ok flow pattern is this and other flow pattern is something like that ok.

So, one cases, we can see the flow pattern I think it is working in this direction and it is working this direction. But just opposite direction, we can see the velocity vector also. If you see this velocity vector pattern are completely different in these two cases so that we will be able to predict the effect of the surface active elements.

But to do the effect of the surface active elements, very carefully model the surface tension force or temperature coefficients of the surface tension. Now, in this case laser welding, we

are analysing these things even not only surface active elements acting on the GTAW process, it acts on the laser welding process.

So, here and in this case the momentum transport of the material depend at because of the effect of the surface tension force, other cases the effect of the buoyancy force because in laser welding process, we do not consider the electromagnetic force. So, the driving force, we when we are considering the material flow analysis, we should consider in laser welding only the surface tension force as and the buoyancy force.

(Refer Slide Time: 31:40)

The slide is titled "Implementation of Finite Element method" in green text at the top. Below the title is a white-bordered box containing the following text: "3D Finite element model (FORTRAN)", "o Heat transfer and fluid flow analysis" (with a red checkmark), "- Penalty finite element method for fluid flow", "- Fluid flow only for liquid zone", and "A frontal solver can be used (non-symmetric matrix)". Below this box is another white-bordered box containing: "o temperature dependent material properties", "o latent heat of melting and solidification", and "o composition dependent surface tension coefficient". Three red arrows point from the right side of the second box to the first box.

Implementation of Finite Element method

3D Finite element model (FORTRAN)

- o Heat transfer and fluid flow analysis ✓
- Penalty finite element method for fluid flow
- Fluid flow only for liquid zone

A frontal solver can be used (non-symmetric matrix)

- o temperature dependent material properties
- o latent heat of melting and solidification
- o composition dependent surface tension coefficient

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Now, we once we understand the effect of the aspects of the flow field basically the analysis of the flow field in particular to the welding process, now we should we look into the what way we can implementation of the finite element, how what way we can incorporate the effect

of the material flow pattern or what way we can develop the material flow pattern in using the finite element method.

So, to do that, we have to understand the first governing equation boundary condition. But before that 3D finite element model can be possible to develop even using the some kind of the language FORTRAN language or some code development is possible. But in this case, the this element or if it is the model consider the heat transfer and fluid flow, both analysis together and in this particular cases, we can use the penalty finite element method for fluid flow.

Definitely, there are other methods are available. But I am focusing in this particular flow analysis in the welding process, we can use the penalty finite element method. This is easy to implement in a finite element based model. So, fluid flow analysis only for the liquid zone that is the another parameter.

Because once we analyze this thing, we look into the domain also of the analysis. If you fix the domain, the where the fluid flow exist over a weld pool only, but the temperature distribution of the temperature exist not only within the weld pool, the outside of the weld pool, the solid domain also.

So, therefore, once you analysing the material flow, the it is analysis domain can be defined only on the weld pool. But the analysis of the temperature or an energy equation that exist over the whole domain of analysis that have to take care of that; so, therefore, fluid flow analysis only considering on the liquid zone.

Because it is possible to develop the flow analysis, if it is not possible to divide the zone which zone we should focus on the fluid flow analysis; but it can be done simply that artificially increasing the viscosity term in a welding problem, such that we will be able to predict the very low amount of the viscosity the flow field outside the fluid domain, basically solid domain.

Because if it is not possible to separate out the domain of the fluid flow analysis, but that can be overcome just by simply applying the large amount of the or very small a large quantity or large amount of the viscosity term to the solid domain and but appropriate term of the viscosity in the liquid domain.

That's way we can predict the flow field within the weld pool also. So, in this case, we will see that analysis can be done using simple frontal solver and this frontal solver assuming that non symmetric matrix or presence of the liquid flow field or flow field in particular sorry velocity field in particular direction.

So, then frontal solver can be used to solve this linearized equation, the shape of the equation and that will show that we will discuss about the frontal solver later on; but to develop the model, the convenient to consider the material temperature dependent material properties. Then, because it is very important, correctly define the material properties at particular temperature.

Then, latent heat of melting the solidification will consider and that will be able to predict basically the change of the phase from liquid phase solid phase phenomena and that will show the results also; what we have considered the latent heat of the melting solidification and finally, composition dependence surface tension coefficients that model is also required to develop for the fluid flow analysis.

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Governing equations and boundary conditions

$$\rho \left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} \right) = \nabla \cdot \boldsymbol{\sigma}_s + \mathbf{F}$$
$$\nabla \cdot \mathbf{U} = 0$$

Following Stoke's law as

$$\boldsymbol{\sigma}_s = \mu [(\nabla \mathbf{U}) + (\nabla \mathbf{U})^T] - P\mathbf{I}$$

P is the pressure, μ is the viscosity of the molten metal and I is the identity matrix

\mathbf{U} denotes the velocity vector, $\boldsymbol{\sigma}_s$ the total stress tensor, ρ the density of metal, t the time variable, and \mathbf{F} the body force per unit volume

Buoyancy force is expressed following Boussinesq approximation

$$\mathbf{F}_b^z = -\rho\beta (T - T_{ref})\mathbf{g}$$

β is the volume expansion coefficient, g is acceleration due to gravity, T_{ref} is the reference temperature, and T is the current temperature variable



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To start with that material flow pattern, we know that governing equation and boundary condition has to be defined to look into this thing. We can start with the governing equation of the momentum transport, can we start with this thing? The density velocity vector, \mathbf{U} is the velocity field, velocity vector then, here is the velocity vector and this velocity gradient also and t represents the time variable.

So, maybe we can consider the transient problem first and this is the stress tension in a analysis and gradient term also and this indicates the dot product and \mathbf{F} is the velocity body force vector and this is the this equation for momentum transport and then, $\boldsymbol{\sigma}_s$ the stress tension can be represented following the Stokes law that the. This is the viscosity term, the velocity gradient and here also velocity gradient, but transpose in the front; P is the pressure and \mathbf{I} is the identity matrix.

So, then putting the σ_s value here, we can develop the system of the equation; but and this second equation the indicate the continuity equation. So, therefore, conservation momentum and continuity equation, you have to solve here; but at the same time together, we have to solve not only the continuity on momentum equation, rather we have to solve the energy equation also.

Because from the energy equation, we will be getting the temperature distribution and once we get the temperature distribution, so it is able to predict the what is the value of the material property as a function of temperature. So, then coupling of this momentum and the continuity momentum equation with the energy equation is required in case of the transports phenomena-based heat transport fluid flow model.

But anyway, I am first I am focusing on this the momentum the only the flow field what we can develop this flow field also, we can develop the model using the finite element method. So, density of the material time, where F is the body force. So, remember the F represents the body force per unit volume.

So, body force has to be defined per unit volume. So, body force accounts in this particular problem, first is the buoyancy force. So, buoyancy force is a acts as a body force also. It is can be considered as a body force. So, this buoyancy force is a expression is the density, this is the volume expansion coefficients and T reference temperature; with this T is the temperature variable and T reference is the reference temperature, and g is the acceleration due to gravity.

So, then, we can define the buoyancy force; what we can act the buoyancy force is acting only in the z direction. Now, from this equation, it is possible to convert the x y and z direction, the momentum transport in x direction, momentum transport in y direction, momentum transport in z direction in a Cartesian coordinate system.

So, basically from here, we will be able to we will be getting the three the system of equation in x y and z direction respectively and now, same the continuity equation can also be represents the continuity equation will be getting the only one equation; but in one equation.

So, we can say that if you analyze this particular system of the equation basically from this x y and z continuity equation, momentum equation from x y and z direction and one equation will be getting from the continuity equation. So, there we are getting the 4 equations. But in this case 4 equations, the unknown variable. Basically, we are able to predict the what is the velocity field velocity field then.

So, once we are separating this governing equation in this x, y and z direction. So, then we will be able to get the u, v and w; u represent the velocity field in the x direction, velocity field in the y direction v and velocity in the z direction equal to w. So, basically three velocity u, v, w that is a output from this solution of the momentum and continuity equation.

But there is a another variable that is the pressure P term. So, this is the output of this thing; u, v, w this is the velocity field and the pressure field also. But if you look into the nature of the equation, so here you can see that in continuity equation is not associated any kind of the pressure term.

So, then although there are 4 equations and 4 variables, so we can get some kind of the solution the definite solution of the uv and pressure. But the solution becomes complicated in the sense that the pressure time is not associated with the fourth equation; continuity equation. So, therefore, some strategy has to be followed to get the to estimate what is the value of the pressure distribution. We will see in the due course of that, when you try to solve this particular equation.

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Electromagnetic force - analytical expressions

$$F_{em}^x = -\frac{\mu_m I^2}{4\pi^2 r_{eff}^2 r} \exp\left(-\frac{r^2}{2r_{eff}^2}\right) \times \left[1 - \exp\left(-\frac{r^2}{2r_{eff}^2}\right)\right] \left(1 - \frac{z}{t_{sh}}\right)^2 \frac{x}{r}$$

$$F_{em}^y = -\frac{\mu_m I^2}{4\pi^2 r_{eff}^2 r} \exp\left(-\frac{r^2}{2r_{eff}^2}\right) \times \left[1 - \exp\left(-\frac{r^2}{2r_{eff}^2}\right)\right] \left(1 - \frac{z}{t_{sh}}\right)^2 \frac{y}{r}$$

$$F_{em}^z = -\frac{\mu_m I^2}{4\pi^2 r^2 c} \left[1 - \exp\left(-\frac{r^2}{2r_{eff}^2}\right)\right]^2 \left(1 - \frac{z}{t_{sh}}\right)$$

I - RMS value of the applied welding current;
r_{eff} - effective radius of the welding arc
t_{sh} - thickness of the work piece being welded
μ_m - magnetic permeability of work piece material

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Now, apart from the buoyancy force, although it is a body force there is another body force will be acting during that is called the electromagnetic force. So, for simplicity, we can consider this simply analytical expression of the electromagnetic force that we can say that in along x direction along y and along z.

So, along x direction, we can say the electromagnetic force will; but all we are defining per unit volume. If you see per unit volume, see this is the yes and then, I is the RMS value of the applied welding current. So, then mu m is the magnetic permeability of the workpiece material and r effective, the effective radius; that means, so effective over is the fluid flow analysis in a particular what is the effective area over which the molten metal exist.

So, that is the effective area r effective and r is the variable here and then, other thing is the z direction and t sh is the thickness of the work piece welded; so, thickness of the work piece to

be welded and x is a variable, r is the radial distance basically and x is the variable along the variation along the x axis.

So, along the x axis, we can represent this is the electromagnetic force field. Similarly, y direction also, we can put and z direction, we can see we can estimate the electromagnetic force. This is a simple analytical solution. Otherwise, we have to solve the governing equation if you considered the electromagnetic field. So, that becomes separately, we have to solve this governing equation to get the electromagnetic force field. Because this is the one of the driving force.

So, once we estimate the electromagnetic force field, the x from this expression, we can put the each and the as a input as the body force and then, each and every node within the body force, we can give in the form of a driving force the F_x , F_y and F_z , that is the component of the electromagnetic force in the x , y and z direction.

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Conservation of energy

$$\rho C_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) + \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{Q}$$

- ✓ Conservation equation of energy is solved in the complete solution domain containing the weld pool as well as the surrounding solid work piece material.
- ✓ The conservation equations of mass and momentum are to be solved within the weld pool only, which is contained by the solid-liquid (S-L) interface and the free surface of the weld pool.
- ✓ Since the deformation of free surface is small, we assume for simplicity that the free top surface of weld pool is flat.

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So, once the body force and the governing equation defined, then we need to solve parallelly that conservation of energy equation. So, that solution strategy can be different, but it can be sequentially that is the different aspect. But any way, we have to solve the conservation of the energy equation also.

You can see the transient state the conservation of the energy equation that see ρC_p , this term is there specific heat density and this because of the convective flow of the liquid; so, that influence the energy transport here also. Then, this u, v and w is represent the velocity field that comes the input from the velocity field and then, we are getting the temperature field also getting and \dot{Q} in the internal heat generation term. This is the governing equation.

Now, this term is here in when you analyze the combined fluid flow analysis, the transport phenomena heat transport fluid flow analysis, then this term comes extra; but if you consider only the heat conduction, then not necessary to consider this term, the energy transport because of the flow field. So, then that term is extra; here in this particular equation as compared to the only conduction based heat transfer model.

Therefore, once we solve the conservation of the energy equation, we need the information of the velocity field that we have to consider in this case. So, therefore, that is why we combining the momentum transport as well as the energy; both equation, we have to solve in case of the transport phenomena based heat transfer fluid flow model.

So, therefore, conservation equation of the energy is solved in the complete solution domain, containing the weld pool as well as the surrounding solid work. So, therefore, weld pool as well as the around the solid work piece material in the both cases we will be solving the this conservation of the energy equation. But within the weld pool, there is some u , v , w velocity components will be there.

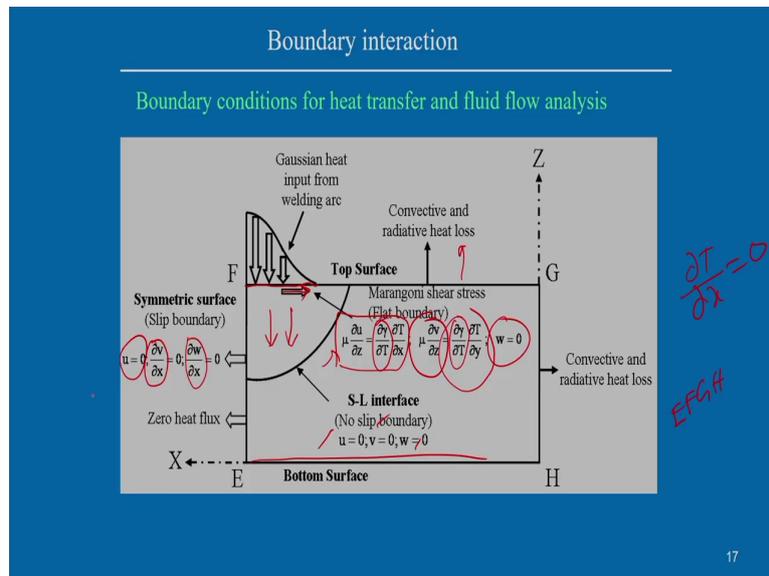
So, in transport of the energy is we will consider. But within the solid workpiece material, there is no velocity component. So, therefore, this component will not considered. So, that will not be comes into the picture, when we are solving the energy equation for the solid domain.

The conservation equation of the mass and momentum are to be solved within the weld pool only. So, definitely the mass and momentum equation, it is necessary to solve within the weld pool which is contained by the solid liquid interface. So, therefore, this domain is separated by the solid liquid interface and the free surface of the weld pool.

So, the between the solid liquid interface on the free surface weld pool that define the domain for the fluid flow analysis. Now, for simplicity, since the top surface the deformation of the free surface actually is very small. So, we can affect the free surface modeling also and we

can assume simply the flat top surface as a flat surface. Then, it will be easy to implement the solve the velocity field. So, that we can see also.

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Therefore, apart from this thing, we have to know that what are the boundary interaction. It is normally happens in this flow field also, when you are analysing the metal flow field. So, here, we can see this is the domain; the solution domain. We can see that EF that EF GH the on the one two-dimensional front I am showing.

EFGH represents the solution domain assuming this thing. Now, on the top surface, there is a gaussian heat input from the welding arc. So, heat input from here top surface that is considered when heat input from here. So, that can be considered as a boundary interaction.

So, that whatever heat flux, you have to incorporate through the boundary condition term. Now, from that surface there is a convection radiative heat loss other surfaces and the bottom surface, we can account; the contact surface, we can assume the or we can consider there is a convective radiative heat loss from the bottom surface as well.

So, then the loss, then this is the boundary condition for the energy equation. But boundary condition for the momentum transport is something like that on the top surface, there is a acting on the Marangoni shear stress. Only the Marangoni shear stress will be acting on the surface.

So, we have to consider the this is a surface phenomenon. So, this can be incorporate through the boundary interaction; so, boundary condition term. Now, but the this within this, there is a this body force that means in case of the buoyancy force, if you consider the electromagnetic force that has to be incorporated through the body force term. So, that already explained this thing. So, that is not the boundary interaction.

So, that can be incorporated through the body force term. Now, Marangoni shear stress is acting on the top surface. So, therefore, the flat boundary condition because since we are assuming the flat boundary conditions, so we can reach this kind of boundary interaction that viscosity, the shear stress this thing the on the top surface on the flat boundary. So, this is the one boundary interaction in terms of the shear stress. So, shear stress can be in the z direction and this one direction shear stress.

And the shear stress in other direction $\frac{\partial v}{\partial z}$, can be represented like this if you see the shear stress is acting the temperature surface tension and temperature gradient in the x, temperature gradient in the y direction in a three-dimensional problem and w equal to 0 because flow field in the this direction, w equal to 0 on the top surface.

These are the typical boundary condition for the momentum transport or momentum equation and for the solid liquid interface, no slip boundary condition. So, in the interface, we can see

the velocity u , v and w are 0; but at the symmetric surface if you see the symmetric problem, so symmetric surface we simply assume the zero flux.

So, zero flux in case of the temperature should we know the $\frac{\partial T}{\partial x}$ temperature in case of $\frac{\partial T}{\partial y}$ if it is a along the x direction. So, $\frac{\partial T}{\partial x} = 0$, it indicates the zero flux the normal to the surface. But slip boundary condition, we can symmetric surface slip boundary condition, we can put that $\frac{\partial v}{\partial x} = 0$; $\frac{\partial w}{\partial x} = 0$, this is the other component of the velocity normal this thing.

But u should be 0, normal to this at this symmetric surface. So, these are the typical boundary condition associated with the both momentum transport as well as the energy transport.

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Boundary conditions

(a) At time $t = 0$, no velocity field exists i.e. $u = 0$, $v = 0$ and $w = 0$.

(b) The solid-liquid interface is considered to host a no-slip boundary condition i.e. $u = 0$, $v = 0$ and $w = 0$

(c) A symmetry boundary condition (typical slip boundary) is assumed along the axis of symmetry is expressed as $u = 0$, $\frac{\partial v}{\partial x} = 0$ and $\frac{\partial w}{\partial x} = 0$

The top surface of the weld pool is assumed to be flat with the following velocity boundary conditions:

$$\mu \frac{\partial u}{\partial z} = f_L \frac{\partial \gamma}{\partial T} \frac{\partial T}{\partial x} ; \mu \frac{\partial v}{\partial z} = f_L \frac{\partial \gamma}{\partial T} \frac{\partial T}{\partial y} ; w = 0$$

γ is the temperature dependent surface tension coefficient and f_L is the volume fraction of liquid metal along the weld pool top surface.

The dragging force exerted on the top surface of the weld pool by the arc plasma is usually much smaller in comparison to the surface tension driven forces and hence, neglected.

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We can explain also here boundary interaction. So, at the time t equal to 0, if a transient problem, so no velocity field will exist. So, if it is, so the initial condition you can put. And then, at the solid liquid interface is considered to host the no-slip boundary condition that we have already explained u , v and w it will be 0.

A symmetric boundary condition typical the slip boundary is assumed at the symmetric boundary the u equal to 0, $\frac{\partial v}{\partial x}$ equal to 0, the velocity gradient $\frac{\partial w}{\partial x}$ is the flux basically velocity in the gradient or in terms of the flux also equal to 0. All these terms are 0.

On the top surface, we can put assume the flat surface and following the velocity boundary condition, we can put this thing. So, simply you are making the this shear stress value in terms of the surface tension force; but the f_L and f_L term is introduced here because in this case the f_L represent the volume fraction of the liquid metal along the weld pool surface.

So, volume fraction of the liquid metal with partially filled with the if particular element is partially filled with the liquid metal or partially filled with a solid metal. So, accordingly, we can decide what is the value of the f_L value; the volume of the fraction of the liquid on the weld pool top surface.

So, that we have to look specifically that is important of the boundary; otherwise, f_L equal to 1, completely liquid zone or f_L equal to 0 and it is completely solid zone. Even if we do not consider the f_L , it is the solid zone also; but it is the interface can be passes through the part of the element, then we have to account the f_L that should be between 0 and 1.

Now, driving principle, the dragging force exerted on the top surface of the weld pool by the arc plasma because arc plasma can create some kind of the dragging force; but that is much smaller in comparison to the surface tension. So, surface tension driven forces. So, therefore, this arc plasma force can be neglected also, that will simplify the problem in the domain.

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Boundary condition (Transport)

Conservation of mass, momentum and energy

Mass: $\frac{\partial u_i}{\partial x_i} = 0$

Momentum: $\rho \left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = f_i + \frac{\partial}{\partial x_j} \left[-P\delta_{ij} + \mu_{\text{eff}} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right]$

Energy: $\rho C \left(\frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} \right) = \frac{\partial}{\partial x_i} \left(k_{\text{eff}} \frac{\partial T}{\partial x_i} \right) + Q$

Boundary Conditions

Top surface:

$$\mu_{\text{eff}} \frac{\partial u}{\partial z} = f_L \frac{dy}{dT} \frac{\partial T}{\partial x}$$

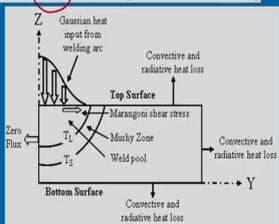
$$\mu_{\text{eff}} \frac{\partial v}{\partial z} = f_L \frac{dy}{dT} \frac{\partial T}{\partial y}$$

$$w = 0$$

Energy: $k_{\text{eff}} \frac{\partial T}{\partial n} + q_s + h(T - T_0) + \sigma \epsilon (T^4 - T_0^4) = 0$

S-L interface \Rightarrow No-slip boundary condition

Symmetric surface \Rightarrow Zero flux



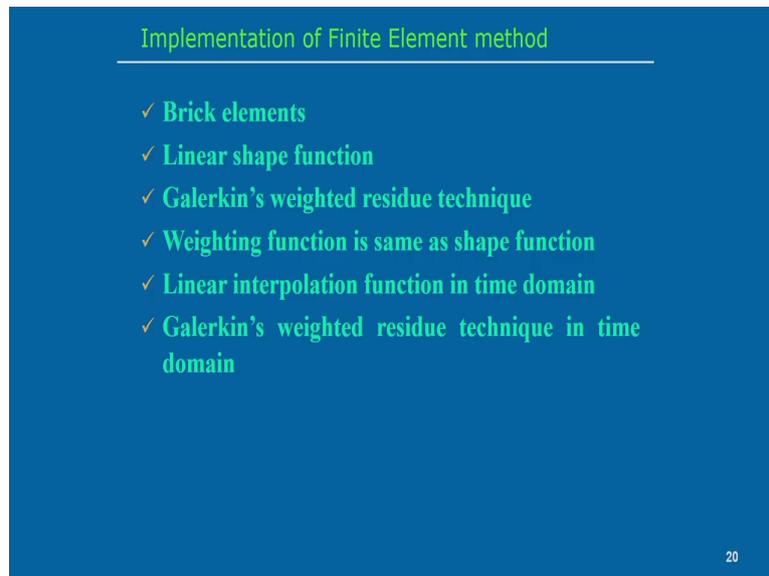
In impact, compact form, we can see the boundary condition also. See this is the momentum equation, energy equation you can solving in this particular, then conservation of the mass, here mass momentum energy equation, then boundary conditions on the top surface is this one and solid liquid interface no-slip boundary condition, symmetric surface zero flux.

And this apart from that, there is energy because of the energy with mathematically in one equation we can form that is a boundary condition for the energy by solving for solving the energy equation.

So, here conduction at the boundary, heat conducted at the boundary, surface flux, heat input, heat loss by convection, heat loss by radiation. So, all this kind of the boundary interaction

basically forms in the in case of this combined effect of the when you consider the fluid flow and temperature distribution in case of the welded welding problem.

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Implementation of Finite Element method

- ✓ Brick elements
- ✓ Linear shape function
- ✓ Galerkin's weighted residue technique
- ✓ Weighting function is same as shape function
- ✓ Linear interpolation function in time domain
- ✓ Galerkin's weighted residue technique in time domain

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Now, when we talking about the implementation of the finite element method, then first we know the we have to define the geometry already you have shown the boundary condition. This is the whole domain and that domain have to discretize using the even in brick element, if we assume we are assuming here the brick elements, so in a three dimensional shape.

Then, brick elements assume the linear shape function shape function is a linear shape function we consider, then following the Galerkin's weighted residue technique. And definitely, the when you made the formulation with the governing equation and boundary condition, then weighting function in Galerkin weighted residue technique is the same as the shape function we consider.

And even that is a spatial domain; but even temporal domain also linear interpolation function in the time domain also, we can consider. Same similar kind of the Galerkin weighted residue technique in time domain is also followed and for this discretization of the this governing equation along with the boundary condition, when you in a finite element domain.

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Finite element discretization

- ✓ Discretized following the Galerkin's method of weighted residue technique
- ✓ Solution domain is discretized using eight node isoparametric brick element
- ✓ The reduced integration penalty method in finite element methodology is followed to solve the mass and momentum equations
- ✓ The penalty method is designed such that the continuity equation, which actually represents a constraint condition, can be eliminated from the solution process

$$P = -\lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right); \quad \frac{P}{\lambda} + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) = 0$$

λ is the penalty parameter (also referred to dilatational viscosity in the context of fluid flow analysis). The value of λ is set equal to a large number so that it can satisfy the continuity equation



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Now, finite element discretization, we can say discretization following the Galerkin weighted residue technique and the solution domain using the eight noded isoparametric brick element, that we have already explained in the when you try to basics of the FEM processes also.

Then, but one thing is that in this case reduced integration penalty method in finite element methodology is used. So, reduced integration penalty method because see once we consider

the now when you try to integrate the over the volume, the volume integration with the and then, this integration, then in that cases it is the necessary to consider.

Some kind of the integration maybe we can use the regular integration or sometimes you can reduce integration. So, depending upon the what is the number of integration points, when we are estimating the numerical estimating the volume of a particular element. So, therefore, that is very important because it can induce some amount of the spurious solution during this process.

So, therefore, when we looking into the penalty finite element method. So, some in some point of time, we have to consider reduced integration method. We will show that. So, therefore, penalty method is designed; we in these cases, where the penalty finite element method, we are considering. So, penalty method is designed in such a way that it can be incorporates to the continuity equation. So, you can see the continuity equation which actually represents as the constant condition that can be eliminate from the solution process.

So, basically simply if we use the link the penalty method, the P that is for example penalty value is designed the P which is equal to the by incorporating the continuity equation and there, we can use the penalty parameter. λ is a penalty parameter. So, P can be P equal to minus λ into this. So, this you know the this terms is coming from the continuity equation or mass conservation equation.

So, from there, we introduce the penalty term such that we can get this equation and then, this can be represent as a constant condition. Implementing this specially at the solution of the governing equation.

So, this penalty parameter also refers to as the dilation and viscosity in the extent of the fluid flow analysis and this penalty λ value can set its intentional is very high value, large number such that it can satisfy the continuity equation.

Because see if this λ tends to infinity very large value, so P by λ , it tends to 0. So, when P by λ tends to 0, then it becomes 0. So, it basically satisfying the continuity

equation. So, that is the purpose of introducing the penalty finite element method, that link with the continuity equation.

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Finite element discretization

The transport equation of momentum is nonlinear due to presence of the product of velocity and its gradient in the convective term.

To avoid the nonlinearity, the velocities in convective term are made independent from the velocity variables that linearise the momentum equation.

Considering the elemental shape function, the unknown variables (velocity and temperature) within the element are expressed in terms of the corresponding nodal variables as

$$u = \sum_{i=1}^8 [N_i] \{u_i\}$$

$$v = \sum_{i=1}^8 [N_i] \{v_i\}$$

$$w = \sum_{i=1}^8 [N_i] \{w_i\}$$

$$T = \sum_{i=1}^8 [N_i] \{T_i\}$$

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Now, the momentum transport equation is non-linear, basically if you look into this thing due to the presence of the product of the velocity and its get it in the convective term. So, if you look into the convective term governing equation, this term. So, this becomes non-linear the. So, velocity term is there u and as well as this thing.

So, the and as well as the velocity gradient both terms are there. So, therefore, that creates one kind of the solution methodology because it is non-linear due to the presence of the product of the velocity and velocity gradient in the convective term. So, that to avoid this kind of nonlinearity, so linearizes the equation is required.

So, then in that cases, we can make the this value of the $u \frac{\partial u}{\partial x}$ for example. So, in the finite element solution, we make the u is independent of this velocity gradient such that the nonlinearity arises during the finite element discretization using the linear shape function that can be avoided.

Made independent from the velocity variables that linearize the momentum equation. So, then you just simply use value of the sum u_0 value which is independent as variable such that u can be as a function of the shape function and the velocity term. So, that u_0 is independent of that.

So, in that sense simple way, we can linearize this convective term is a momentum equation. So, therefore, considering the elemental shape function or other point I just want to mention also, even if we linearize the equation. So, u_0 and v_0 and w_0 in this three term, linearize the equation ok in such a way that can be linked with the Reynolds number, we can then fix the value of the Reynold's number can be the input this particular.

And then, we are solving for u is the variable of this equation, then we are solving for the this u value take the variation; but do not consider this as a u variable, u can be independently and such that you can link with this some constant value. For example, it can be link with the Reynolds number and we define the Reynolds number particular solution.

Such that in this particular way, you can simplify the momentum equation or linearize the momentum equation; so, therefore, considering the elemental shape function, they are known variables. So, that means, the velocity and the temperature within the element can be expressed like that also. The unknown variables u , v , w and T as a function of this shape function and the u is the u_i is the defined the value of the velocity at the each end node point.

So, therefore, i equal to 1 to 8 because we considered the brick element; so, having eight noded brick elements, so that is why i should be varying from 1 to 8. So, this way, we can take we can consider this as a variable in the form of a shape function and the definite value of the this velocity component at the different node point.

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Finite element discretization

The momentum conservation equations in the x-, y- and z- directions can be written in linearised form

$$\begin{bmatrix} [M^e] & 0 & 0 \\ 0 & [M^e] & 0 \\ 0 & 0 & [M^e] \end{bmatrix} \begin{Bmatrix} \frac{\partial \{u\}}{\partial t} \\ \frac{\partial \{v\}}{\partial t} \\ \frac{\partial \{w\}}{\partial t} \end{Bmatrix} + \begin{bmatrix} [\bar{C}^e] & 0 & 0 \\ 0 & [\bar{C}^e] & 0 \\ 0 & 0 & [\bar{C}^e] \end{bmatrix} \begin{Bmatrix} \{u\} \\ \{v\} \\ \{w\} \end{Bmatrix} + \begin{bmatrix} [\hat{K}_{11}^e] & [\hat{K}_{12}^e] & [\hat{K}_{13}^e] \\ [\hat{K}_{21}^e] & [\hat{K}_{22}^e] & [\hat{K}_{23}^e] \\ [\hat{K}_{31}^e] & [\hat{K}_{32}^e] & [\hat{K}_{33}^e] \end{bmatrix} \begin{Bmatrix} \{u\} \\ \{v\} \\ \{w\} \end{Bmatrix} \\
 + \begin{bmatrix} 2[K_{11}^e] + [K_{22}^e] + [K_{33}^e] & [K_{21}^e] & [K_{31}^e] \\ [K_{21}^e] & [K_{11}^e] + 2[K_{22}^e] + [K_{33}^e] & [K_{23}^e] \\ [K_{31}^e] & [K_{32}^e] & [K_{11}^e] + [K_{22}^e] + 2[K_{33}^e] \end{bmatrix} \begin{Bmatrix} \{u\} \\ \{v\} \\ \{w\} \end{Bmatrix} \\
 = \begin{Bmatrix} \{F^{2D}\} + \{B^{2D}\} \\ \{F^{2D}\} + \{B^{2D}\} \\ \{F^{2D}\} \end{Bmatrix}$$

$$[M]\{\dot{U}\} + [\bar{C}]\{U\} + [\hat{K}] + [K]\{U\} = \{F\}$$

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And then, we can reach this finite element discretization, if we the from the governing equation and then if you look, follow the Galerkin weighted residue technique these things, the what way we have done in case of the heat conduction equation. So, similar kind of the exercise you can do in case of the fluid flow analysis also.

So, individually considering the x, y and z momentum transport equation along with the continuity equation and plus we can incorporate the penalty term also P, we can induce the continuity equation in the form of a penalty term. Then, finally, you can reach this expression is something like that the this expression the in the form M because there are x, y and z component.

So, then x and z component the velocity in these cases, the velocity component will be their u, v and w. Then, we can reach this expression that C, u, v, w and the in terms of the K. So,

manipulation these things, we can reach this equation. But finally, the equation formed in this way $M \dot{U} + \bar{C} K$.

So, this expression of the C is this thing different; K is different and K kappa and K is different in these cases. So, all this kind of manipulation of the governing equation along then we can reach this thing and F from the body force also, it accounts this thing F is the body force vector.

So, this is value linear the form of this U and here is the \dot{U} ; that means, this is the transient problem. So, here we can see that we can getting the \dot{U} , \dot{U} further represent as a derivative of with respect to time also. But here, we can reach this is the simple form of the equation.

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Finite element discretization

$$[\hat{K}_{ij}^e] = \int_{\Omega^e} \lambda \frac{\partial N_i}{\partial x_1} \frac{\partial N_j}{\partial x_1} d\Omega$$

$$[K_{ij}^e] = \int_{\Omega} \mu \frac{\partial N_i}{\partial x_1} \frac{\partial N_j}{\partial x_1} d\Omega$$

$$[M_{ij}^e] = \int_{\Omega^e} \rho N_i N_j d\Omega$$

$$[\bar{C}_{ij}^e] = \int_{\Omega} \rho \left(u^0 N_i \frac{\partial N_j}{\partial x} + v^0 N_i \frac{\partial N_j}{\partial y} + w^0 N_i \frac{\partial N_j}{\partial z} \right) d\Omega$$

$$[M] \{\dot{U}\} + [\bar{K}] \{U\} = \{F\} \quad [\bar{K}] = [\bar{C}] + [\hat{K}] + [K]$$

- [M] - Mass
- [C] - Velocity dependent convective transport
- [K] - Viscous diffusion
- $[\hat{K}]$ - Penalty term
- {F} - Body force and surface tension force

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Now, this equation can be represented in this form also K kappa this in this form. The lambda this thing K simple K , M and C . You can see the M is the basically a mass account, the mass part because it is accounting the lambda, that penalty parameter it introducing this thing.

Then, C is the velocity dependent convective transport term. So, here we can see the u_0 , v_0 is you defined and then, we can use expression and then, K is the penalty term also. K is the; this K kappa is a penalty term and but K is the viscous distribution. This K is the viscosity term is there. So, viscosity disappear and this K represents the this thing penalty term.

But mass is represents this equation; the density term is there. So, we can use the mass term, then viscous diffusion term, penalty term as well as the body force and surface force also, we can see the right hand side. The body force and the surface tension force is comes into this category there.

Apart from the body force and along with the application of the boundary condition also. Then, we can reach this is the system of equation. Now, we have to think that what way we can discretize in the finite element. Now, we will discuss the remaining part maybe in the next part of this particular lecture, so.

Thank you very much for your kind attention.