

Finite Element modeling of Welding processes
Prof. Swarup Bag
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Lecture - 18
Heat source models in welding-II

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Quadruple-ellipsoidal heat source model

Follows
Gaussian distribution

- Non-symmetry in energy distribution due to
 - a moving heat source ①
 - welding of two different materials having different thermo physical properties ②
- ✓ Double ellipsoidal heat source model accounts moving heat source only
- Double ellipsoidal model is extended to Quadruple ellipsoidal heat source model to accounts non-symmetry energy distribution for dissimilar materials
- ✓ Consists of part of four ellipsoids
- ✓ Depth of penetration remain same
- Maintain C¹ continuity

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Hello, everybody. We have already discussed that ellipsoidal Heat source model or double ellipsoidal heat source model, and we have seen, we have seen that how this maximum intensity of the heat flux can be calculated exactly at the center point of the heat source.

And then in case of double ellipsoidal heat source, we are using the similar analogy; that means, to put some similar kind of the boundary condition and then we can estimate the distribution parameter independently along x, y and z direction. And in case of double

ellipsoidal heat source model and in these cases there is a merging of the two ellipsoid and to make it more convenient in case of the linear welding process.

Linear welding process means in the case, which cases the there is a heat source is moving one particular velocity to account the nonsymmetric energy distribution, that we consider the front part is one part of the one ellipsoid, and the rear part is a part of the another ellipsoid, and then merging together and getting a double ellipsoidal heat source model.

[FL] we can use the similar analogy, but we can extend this double ellipsoidal heat source model, particularly in case the when we try to join dissimilar materials; that means, two different material having different kind of the thermal properties as well as the linear welding process. It means that the welding heat source is moving one particular velocity in one particular direction.

So, in that cases, we are getting the nonsymmetric energy distribution from the two factor; one is that, because of the linear velocity on one direction and other is the material since material different material properties, because in case of dissimilar materials joining.

So, we see how what way the we similar analogy, the ellipsoidal or double ellipsoidal heat source model the same thing can be extended in case of the quadruple ellipsoidal heat source model. Because in this cases, part of the four ellipsoids are joined together and we can make the quadruple ellipsoidal heat source model.

So, in this case see, there is a ellipse 1, we can see that this is 1 ellipse, that having the minor axis the a_1 , b_1 ; so, this is the a_1 , b_1 and depth of penetration equal to c . Then we consider another ellipse. Then in this cases the in the ellipsoidal shape, the part of these ellipsoidals having the dimension or minor axis dimensions are like that; a_2 and then b_1 and depth equal to c .

Like that similarly, 3 third ellipsoidal in this case, this in this cases minor axis length are see a_2 , then b_2 , c and the fourth one that are having the dimension a_1 then b_2 then c . That means, with this part of the 4 ellipsoidals are joined together then we can make the quadruple

ellipsoidal heat source model, but why? There is necessary to taking the 4 ellipsoidal has to be merged in this particular case.

Because non symmetric energy distribution due to two parts; first is the, because of the a moving heat source and second one is that welding of the two different materials having the different thermo physical properties; that means, particularly in case of the dissimilar material two different materials if you want to join.

So, this side if you see that, this side indicates that first material; that means, one of the material and other side is having the second material, so, two different materials. And welding heat source is moving or welding is happening, in this particular direction. So, then it is this the non symmetric distribution with respect to one two different particular plane and that when you define two particular plane and then we merge the part of the four ellipsoidals and then we will get a one quadruple ellipsoidal heat source model.

So, double ellipsoidal heat source model accounts only the accounts the moving heat source only; that means, we can non symmetric energy distribution only the, because of the moving one particular direction. In this case double ellipsoidal is extended to quadruple ellipsoidal heat source model, that non symmetry energy distribution even for apart from the welding velocity even for the dissimilar materials also it can takes care of that.

Say it consists of the parts of the four ellipsoidal ellipsoids and then depth of penetration remains the same; all these cases to get and maintain the C^1 continuity. C^1 continuity means, at the smooth transition; that means the slope at this point is the same, in this case such that it looks like the smooth profile, even if we merge the parts of the four ellipsoidal in this particular case.

Now, we will look into this how the this heat source model can be developed or what how we can do the, we can assume the what way we estimated in case of the ellipsoidal or double ellipsoidal heat source model, the heat flux distribution; in the similar way we can estimate in case of the quadruple ellipsoidal heat source model.

parameter then we can estimate what is the total heat input one particular component for example, the first we consider the part ellipsoidal one. So, Q says for example, in this case is the heat input or volumetric heat for the ellipsoid 1 can be estimated like that.

So, Q_1 ; so, 0 to infinity other; so, 0 to infinity and x 0 to infinity q into $d x d y d z$. So, q indicates the volumetric heat flux, so, then elemental volume $d x d y$ and $d z$ and then we can estimate the Q_1 , so, only the part ellipsoidal 1 the total heat input or total volumetric heat can be calculated like this.

So, same analogy we can put this thing what we can estimated this thing q_m that integration 0 to infinity e to the power minus $A x$ square $d x$ same way 0 to infinity e to the power minus $B y$ square $d y$ 0 to infinity e to the power minus $C z$ square $d z$. So, from here we can estimate you see this expression, we have already seen 0 to infinity e to the power minus $A x$ square $d x$ equal to we can write, this is $\frac{1}{\sqrt{\pi}}$ by $\frac{1}{\sqrt{A}}$.

So, in this case, q_m into $\frac{1}{\sqrt{\pi}}$ into $\frac{1}{\sqrt{A}}$. Similarly, the second part it comes the $\frac{1}{\sqrt{\pi}}$ into $\frac{1}{\sqrt{B}}$, for the third part $\frac{1}{\sqrt{\pi}}$ into $\frac{1}{\sqrt{C}}$. Such that it becomes q_m into $\frac{1}{\sqrt{\pi}}$ and in this case, $\frac{8}{\sqrt{\pi}}$ root over of $A B$ and C . So, in this case $A B C$ is the distribution coefficient; so, distribution coefficient we can find the similar kind of exercise, what we have done in case of a double ellipsoidal heat source model.

So, in that case, we assuming that 5 percent intensity at the boundary is equal to the only the 5 percent of the maximum intensity from that point of view, we can estimate that value of the $A B C$. So, we have already shown that, A should be approx in this boundary if we put that kind of the boundary condition in this case, approximately A will be 3 by a^2 in this case, when we estimating this particular first one.

Similarly, the distribution coefficient B is approximately 3 by b^2 and C approximately, 3 by c^2 . So, then if we put all this value q_m and $\frac{1}{\sqrt{\pi}}$ and if you see, $\frac{8}{\sqrt{\pi}}$ root over of $A B C$; that means, in this case is $\frac{8}{\sqrt{\pi}}$ into 3 root 3 and this case is $\frac{1}{a b c}$ into c .

So, it comes in this that, q_m by $24\sqrt{3}$ and in this cases, $\pi \sqrt{\pi} a_1 b_1 c$; these are the expression. And from here we can estimate what is the maximum intensity q_m can be estimated like that; $24\sqrt{3}$ by $\pi \sqrt{\pi} Q_1$ and $a_1 b_1 c$.

So; that means, this way we can estimate the maximum intensity, but we need to find out what is the value of Q_1 . Similarly, if we estimate the second quadrant the Q_2 ; the Q_2 can be estimated like that; so, Q_2 should be the total heat input for the second part of the ellipsoidal ellipsoid. In this case, Q_2 can be estimated like that, q_m into $\pi \sqrt{\pi}$ and then in this case, that it should be a_2 and then b_1 into c divided by $24\sqrt{3}\pi$, ok.

So, this will be the thing; similar we can estimate the Q_3 as well as the Q_3 third quadrant and Q_4 so, all can be its directly we can put this estimation $Q_1 Q_2$, the in terms of $a_1 a_2 b_1$ and b_2 you have to look into the figure, from there we can estimate. And in this case, c also related c also, but c is same for all this calculation.

Now, once we estimate this Q equal to something like that, then we can say that, what is a total heat input? So, total heat input is simply the Q_1 plus Q_2 plus Q_3 plus Q_4 this is the total heat input, or total volumetric heat with this particular geometric set, the $Q_1 Q_2 Q_3$. But we can assume that, but see, the what we can double ellipsoidal the total heat? We can approximately divide into two parts the 50-50 in the first ellipsoidal second ellipsoidal such that, similar kind of there is a we are merging the 4 ellipsoidal here.

So, we can say that Q_1 can be estimated like that; Q_1 is basically, we can see the one-fourth of almost of Q , but at the same time we can put some factor. Say $N_f 1$, this is the front part. Similarly, Q_2 can be something like that; $1/4$ of Q 25 percent of total heat input, the fractional heat, part we can introduce in this way also. So, this in this case, maybe we can say r_1 . Similarly, Q_3 equal to $1/4 Q$ into $N_f 2$ and then, Q_4 equal $1/4 Q$ $N_r 2$.

So that means, this fractional heat deposition $N_f 1 r_1 N_f 2$ or $N_f N_r$ like that only. So, I think Q_3 should be rear part. So, Q_3 and $N_r 2$ and this will be $N_f 2$ front 2. So, like that such that we can say that, $Q_1 Q_2$ something like that; that this is the $1/4$ into Q simply

this $N_{f1} + N_{r1} + N_{r2} + N_{f2}$ like that. Such that from here that is equal to Q_{total} heat input and in this case, we can get that $N_{f1} + N_{r1} + N_{r2} + N_{f2}$ equal to 4. This expression we can reach.

Here where we are assuming that we introduce the fractional heat deposit into the front part and rear part. The similar way in this quadruple ellipsoidal heat source model we are introduce the 4 different values of the fractional heat deposition; the two associate front part, two associated with the rear part.

Because there are 4 ellipsoidal we are merging and we are getting this thing, but in that way we can just proportionate the Q_1 is simply equivalent to one fourth of the Q that is a not, but it is not exactly non symmetric energy distribution, but not exactly the one-fourth 25 percent of this though, then we introduce this fractional heat deposition part such that we can reach this kind of the expression.

And, but if you see, in this case, that this Q_1 now Q_1 can be estimated like that q_m . What way we can estimate the q_m ? So, for any case, the Q_1 ; see for example, we consider the this calculation Q_1 equal to this value Q_1 . So, we say the Q_1 equal to q_m into $\pi \sqrt{\pi}$ and $24 \sqrt{3} a_1 b_1$ into c . So, from here we can say that, q_m equal to the maximum intensity $24 \sqrt{3} a_1 b_1 c$ into Q_1 divided by $\pi \sqrt{\pi}$ ok.

So, you see, in this case that, q_m equal to $\pi \sqrt{\pi} 8 \sqrt{}$ over of $a b c$, but if we look into this $a b c$ value a equal to 3 by a square so, 3 by a square so, this in this case, this is ok, but q_m equal to this Q_2 equal to this. So, q_m ok; q_m equal to already we have. We have already calculated the q_m equal to this $a b c$. So, look into this value ok.

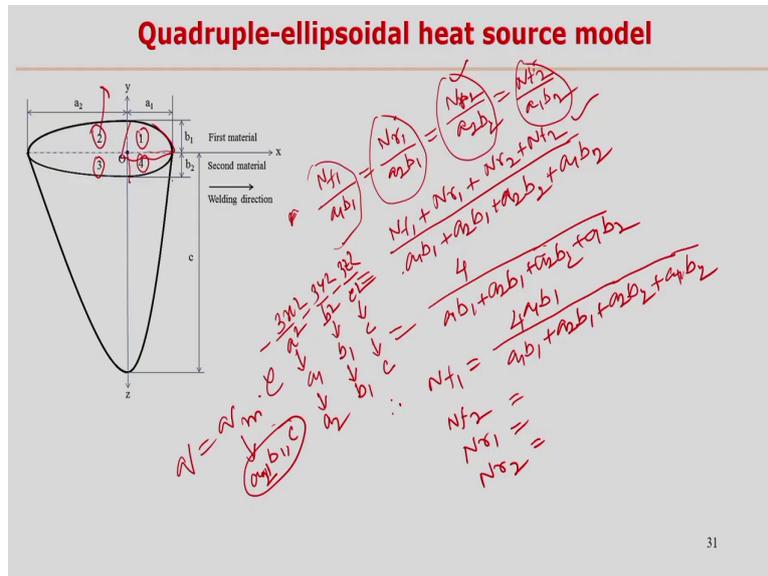
So, here then, q_m can be estimated by also; that means, other way also $24 \sqrt{3}$ and Q_1 can be estimated like that, 1 by $4 Q_1$, 1 by $4 Q$ into N_{f1} divided by 20 sorry, divided by $\pi \sqrt{\pi}$ into $a_1 b_1$ into c . So, therefore, we can see that, q_m equal to $6 \sqrt{3}$ into Q into N_{f1} divided by $\pi \sqrt{\pi}$ $a_1 b_1$ into c . Now, if you remember that, maximum heat flux intensity q_m equal to $6 \sqrt{3} Q_{nf1}$ by $\pi \sqrt{\pi} a_1 b_1 c$.

So, that kind of calculation we have already done in case of the double ellipsoidal heat source model, the see the maximum intensity we are getting the same similar fashion, but fraction of heat deposition may be a different in this particular case. So, now, this thing; if we estimate this maximum intensity at the center points, since you are merging the 4 ellipsoidal in this particular case so, maximum heat intensity should be same, if we calculate irrespective of what are the which ellipsoidal we consider.

So, that means, q_m should be same even if we calculate the heat flux intensity from the calculation of the ellipsoidal 1 or ellipsoidal 2 or 3 or 4, maximum intensity are the same for all the cases. So, from that point we can say that q_m even if you follow the estimate the q_m from here also, then you will be getting the similar kind of the exercise, but only the parameters will be different.

Such that if q_m same all the cases so, here the variable is the $6 \sqrt{3} N f 1$ in this case, $N f 1$ and this c is same for all the cases and $a 1 b 1$ will be the different.

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So, we can say that, ah q m say for the same so; that means, N_{f1} by N_{f1} by $a_1 b_1$ equal to N_{r1} then, $a_2 b_1$ same as, N_{f1} N_{r2} then $a_2 b_2$ equal to N_{f1} r_1 r_2 N_{f2} here it will be a_1 and b_2 . So, that means, it is a corresponds to 4 then a_1 and this parameter b_2 will be there, this is a_1 and this is b_2 .

So, like that similarly; that means, we are looking into the 1 this is correspond to that, maximum intensity is proportional to that, maximum intensity is proportional to that and for ellipsoidal 3 maximum intensity proportional to that, and for 4 it is like that. So, we can say it is equal to that, N_{f1} plus N_{r1} plus N_{r2} plus N_{f2} divided by $a_1 b_1$ plus $a_2 b_1$ plus $a_2 b_2$ plus $a_1 b_2$.

So, this way we can estimate, but this part is equal to 4 we have already seen this thing; $a_1 b_1$ plus $a_2 b_1$ plus $a_2 b_2$ plus $a_1 b_2$. So, any fractional or heat deposition we can find out

the N_{f1} equal to simply $4 a_1 b_1$ divided by $a_1 b_1 + a_2 b_1 + a_2 b_1 + a_2 b_2 + a_1 b_2$ so, like that.

Similarly, similar exercise we can perform this equal to this value and this third equal to this value, fourth equal to this value; such that we can reach that similarly, N_{f2} N_{f2} N_{r1} N_{r2} similar way, we can estimate this. That mean, it is maybe values of the or the in terms of $a_1 b_1$, but if you want to the fractional heat deposited of this calculation and fractional heat can be estimated from the known values of the $a_1 a_2 b_1 b_2$ value, but c is same for all the cases.

So, therefore, all these parameters need to be known, then we can clearly map if it is known to us, the $a_1 a_2 b_1 b_2$, then we can estimate the what are the fractional heat deposited in the different quadrant; basically first, second, third, fourth. And then, from that point we can estimate the heat flux and we can estimate the total heat input same exercise we can perform also.

So, this way quadruple ellipsoidal you see, that we can simply say that, all this calculation is actually coming from the from basic point of view, that if we understand the calculation in case of the ellipsoidal heat source model, so, we assuming the ellipsoidal they are assuming Gaussian distribution, and from the you can estimate what is the maximum intensity at the center point.

And we if we in case of double ellipsoidal heat source model, we can introduce some fractional heat deposit in the front and rear, because temperature actually, temperature get it in the front part of the arc and the rear part are different. So, that encounter the non uniform energy distribution.

So, to account this non uniform energy distribution, we just simply; we can introduce this fractional heat deposition in the front part and rear part and then we can estimate also, front part what is the fractional heat deposit in the front and rear part, in case of double ellipsoidal heat source model and same kind of analogy we can put the quadruple ellipsoidal model.

So, then we can estimate the fractional heat deposition accordingly, the intensity; that means, same heat flux intensity that we have already shown, q equal to q_m ; q_m can be calculated this thing, looking into the $N_{f1} N_{f2} N_{r1} N_{r2}$ value then q_m can be calculated, because then maximum intensity and e to the power minus $a x$ square; that means, $a x^3$ by a square minus $3 y$ square by b square minus $3 z$ square by c square.

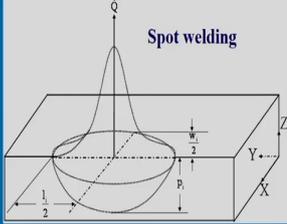
Now, q_m can be calculated from that each intensity should be the same irrespective of this thing now, suppose we want to estimate the heat flux distribution at the first ellipsoidal, may be for the this part. So, this volumetric part we can use only that part; the heat flux density distribution follow this equation q_m and q_m it should be the parameter a_1 , b_1 and c , this parameter has to be incorporated here, to estimate all other parameters.

And similarly, a should be replaced by in this case, a_1 and b should be replaced by b_1 and c is already replaced by the c . Similarly, if we want to estimate the heat flux, density distribution in the second ellipsoid or second quadrant then for the ellipsoidal 2 this part. In this case, we have to change that it should be a should be a_2 and in this case, a should be replaced by a_2 and b should be replaced by b_1 and c should be replaced by c .

So, in that way and of course, this q_m should be represented in the form of a may be in this case, a_2 b_1 and c that form we have to represent. So that means, if we understand clearly, that how the heat flux density distribution can be estimated, we can clearly estimate the all the heat flux density distribution for the different quadrant of a quadruple ellipsoidal heat source model.

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Adaptive heat source model



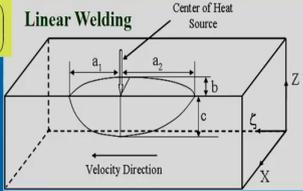
Spot welding

$$\dot{Q}(x, y, z) = \frac{6\sqrt{3}f_{DC}P_w\eta_{tot}}{\pi\sqrt{\pi}abc} \exp\left(-\frac{3x^2}{b^2} - \frac{3y^2}{a^2} - \frac{3z^2}{c^2}\right)$$

where

- $a = l_1$
- $b = \frac{W_1}{2}$
- $c = p_1$

Limitation:
predefinition of heat source parameters

Linear Welding

Strategy to implementation in FEM based analysis

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Now, we will discuss about the adaptive heat source model, but so far we have discussing the disc shaped heat source model or ellipsoidal heat source model and quadruple double ellipsoidal heat source model and quadruple ellipsoidal heat source model. In all these cases, you can understand that, we need to define the heat source parameters before start of the simulation of the welding process.

So, then we have to predefine all these parameter; parameters mean, the semi axis lengths of this ellipsoidal or double ellipsoidal or whatever heat source model we are considering that is necessary, then once we define that, geometric parameters of a particular geometric say for source model then accordingly, it calculates the what is the heat flux density distribution, and then we give this input and we are getting the from this finite element based model; we are getting the output as a temperature distribution.

But, it is very this is the limitation of this all kind of the heat source model. Now, to avoid this kind of the (Refer Time: 24:51) model, that adaptive heat source can be used, the concept for that adaptive heat source. So, in this cases, that distribution or heat source model with a similar nature. For example, in case of spot welding we can simply assume the ellipsoidal heat source model or in case of linear welding, we can assume the double ellipsoidal heat source model.

But, this parameters of the double ellipsoidal or ellipsoidal heat source model, it can be defined from the it can be continuously updated from the input from the simulation itself. For example, suppose in the start of the welding process so, in transient welding process on interacting the arc or laser so, small weld pool will be created; initially at a time various initial time, such that from that small we can from here when it gives a very weld pool, we can estimate what is the depth of penetration, what is the width.

Now, if we replace this a value semi axis length with the length b replace with the half width in this case, and c value the semi axis length or the ellipsoidal model replace with the penetration value. So, that a, b, c value will be decided exactly at the particular time step or particular time, what is the value of the weld pool dimension.

So, that start with this weld simulation, we start with the simulation with the only surface heat flux, then once it is developed some finite volume of the weld pool and from that when you get the finite volume of the weld pool from that, we are if we calculate what is the value of the weld dimension and that dimension we can feed with the value of the semi axis length of the ellipsoidal or double ellipsoidal heat source model.

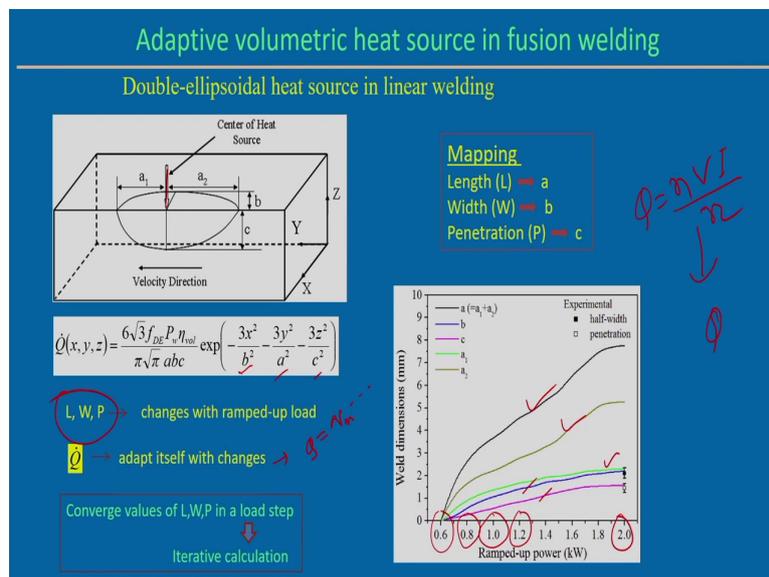
And then since these values are updating, because at a different time step the weld pool volume is gradually normally with respect to time the volume gradually increasing order. So, once it is in increasing order, then at the particular instant of time we can estimate the what is the value of this l, w and c depth of and it will be exactly, mapped with the what is the value of semi axis length of the volumetric heat source model, and then this keep on updating during the simulation process.

So, that is where, this adaptive in these cases, it is not necessary to predefine the values of the all the semi axis length during the simulation process. So, this is the concept of the adaptive heat source model, but in this case, we are assuming the similar kind of the distribution that means, Gaussian distribution and maybe we can assume the kind of geometric shape kind of ellipsoidal or double ellipsoidal cases.

In case of the spot welding process, we assume the ellipsoidal heat source model; in case of the linear welding process, we can assume the shape of the weld pool is equal to the that is the double ellipsoidal heat source model. So, then this kind of the strategy to implementation in FEM based analysis actually, this kind of analysis can be done.

If we want to develop some kind of the finite element base, heat transfer model and there it is easy to implement this adaptive volumetric heat source, such that its actually eliminates the pre defining the values of the weld pool dimension.

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Now, we can see that, double ellipsoidal heat source model a similar kind of the exercise we can see, we can or you can do also, see we can see that the center of the heat source. Here you can see and this is the particular shape of the weld pool. So, this particular shape of the which is defined by the volumetric heat, in terms of the different geometric parameters of a double ellipsoidal heat source model.

In this case, it is a 1, a 2 this front length and real length in case of double ellipsoidal heat source model and b is the width and c. So, then in this case, this b, a and c these parameters are continuously, updating with reference to the what is the value of the at particular instant of time, what is the value of the weld pool dimension.

So, therefore, this weld pool dimension L, W, P changes with the ramped up load and Q adapt itself with changes, because ultimately we are estimating the heat generation term. Q

we are estimating. Q equal to $q \cdot m$ into some expression, and then in the other way also, total heat input and then maximum heat intensity from that point.

So, in this case, we are introducing this value the internal heat generation term. So, basically, internal heat generation term \dot{Q} term is basically, gradually updating or changing with respect to time; so, but and this is changing, because the dimension of the geometric heat source model actually changes with respect to time.

And simply we are mapping with the at a particular instant of time, what is the values of the weld dimension, we are calculating in the particular instant point and that we are mapping in directly to the volumetric heat source term. So, this way we can do kind of the iterative calculation it can be followed, to get the converge solution of a particular time step and this way, it is possible to develop adaptive volumetric heat source term.

But the analysis in case of the transient state or analysis in case of the steady state are different, because in case of transient state analysis we can update all these value with respect to time, because each and every time step we are counting measuring the dimension and then that value we are updating to the heat source parameters.

But in case of the steady state load, there we just eliminate the effect of the time, so, in this case in quasi steady state situation basically, the what are the load is applied the heat flux or maybe, we can the heat amount of the heat this load in terms of the \dot{Q} term that load can be divided, then using the ramped up manner.

For example, that in quasi steady state analysis, we normally perform the analysis that not applying the all the heat flux at a single step, rather we divide the step, the several steps and ramping the power gradually until the complete ramp, for example, total power equal to say efficiency volt into amp.

So, we divide this total power ramping of the power into n number of steps for example, and we are ramping gradually, one single step was done then gradually we updated the next step, next step and until and unless we reach so, n number of steps until and unless we reach to the

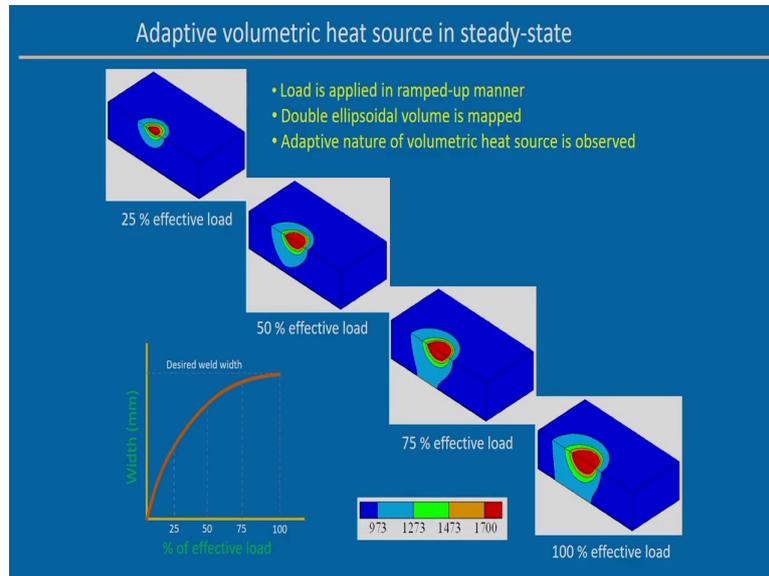
value of the total power input to the domain Q . So, that is why steady state analysis we normally, ramping of the power in that sense we generally solve the problem.

So, here we can see that, experimentally half width an penetration is something like that that ramped up power and the weld dimension. We can see, the ramped up power, weld dimension. The a_1 , a_2 this color indicates the a_1 and other one is the a_2 value and in this case, the b and c can be like that; b value and c value these are the experimental value reach this thing and a_1 plus a_2 this value total.

So, we can see that, ramped up power kilowatt for example, in this case is the total power is a kilowatt so, first step maybe we can use only the 0.6 kilowatt, then initially then we increase this power up to 0.8 kilowatt, then up to 1 kilowatt, then 1.2 kilowatt. So, like that we ramping up the power and then the similar analysis, what we did in transient analysis with respect to the time step here in case of the state quasi transient analysis, we do the similar kind of the analysis in the load step mode.

So, that is why, the adaptive volumetric heat source can be implemented and we can get; we can eliminate the requirement of the pre deposition of the heat source parameters.

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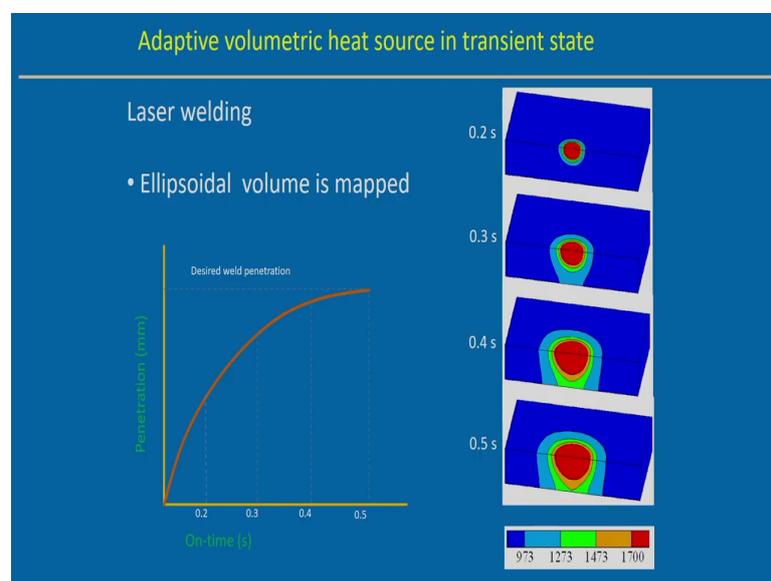


So, here we can see that, adaptive volumetric heat source in steady state situation you can see the load is applied in a ramped up manner and double ellipsoidal volumetric is mapped. Basically, we assuming the volumetric heat source, thus geometric shape is the double ellipsoidal model. So, but adaptive nature of the volumetric heat source is observed, we can see, the this is the weld pool development, it is around 25 percent of the effective load this development happens.

Now, 50 percent of the effective load there is a increment of the weld pool dimension, then the 75 percent of the effective load we can the increment of the weld dimension and with the 100 percent effective load we can get this and the so, this is the when 100 percent effective load, then it reach the completion of the load.

And then if we see that, how with and the with respect to the effective load the increasing we can see that, on 25 percent load there is increment of the width gradually increment 50 percent, then 75 percent and once reach to the 100 percent its, it there is a not further increment of the width, because it reach the quasi steady state situation. So, this way we can estimate the do the thermal analysis by assuming the adaptive volumetric heat source in quasi steady state model.

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Similarly, adaptive volumetric heat source in case of transient state also can be done similar way. For example, in laser welding and we in transient heat transfer model; we can assume the ellipsoidal volumetric is mapped, maybe it is possible to map some other volumetric heat source, but in this particular case, we have considered the ellipsoidal volumetric heat source is mapped.

So, for example, at 0.2 second, 0.2 second there is a development of the weld pool the red color actually, indicates the weld pool dimension remaining is heat affected zone similarly, at 0.3 second there is a see increment of the weld pool size, at 0.45 second and 0.5 second, 5 second we can get the certain value of the weld pool. So, in this case, we can see there is a weld pool development happening with respect to time that increment all this thing.

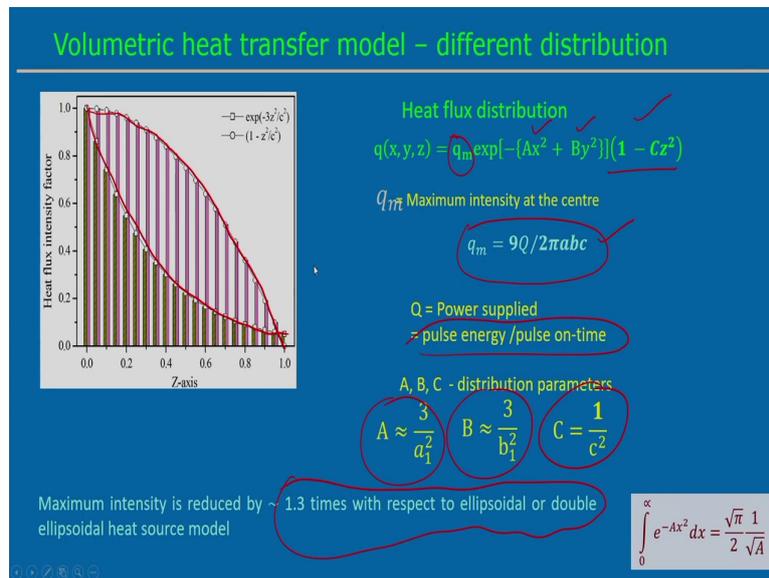
It means that, at 0.2 second the when we get this kind of the weld pool during the simulation at this point, we can measure what is the weld dimension so, then heat source will be accordingly. So, according to the weld dimension we can use this data, the weld dimension to the what is the value of the semi axis length in case of the geometric shape of the ellipsoidal.

Similarly, when it is raised to the 0.4 second, there is a increment of the weld pool dimension. So, then in that cases the ellipsoidal heat source model we use map the weld pool dimension with the dimension of the geometric shape of the ellipsoid. So, accordingly the there is a variation of the internal heat generation term or maybe heat flux applied to the domain there is a variation.

So, these way the volume adaptive volumetric heat source moves and once this reach 0.5 second, then we switch off then it, because we are estimating up to the in practical cases, if we assume that up 0.5 second we apply the heat flux, then it almost the equilibrium condition; that means, we can achieve the desire weld penetration up to this, when exactly the at 0.5 second the heat source is applied.

So, this way we can estimate the temperature distribution or simulation can be performed using the concept of the adaptive volumetric heat source model.

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Now, volumetric heat transfer model see we have already mentioned that the volumetric heat transfer model, there is a 2 way; first we have to define the geometrical shape, the which geometry shape already we have shown that ellipsoidal, double ellipsoidal model also, but at the same time, not only the geometric shape rather what are the distribution of the heat flux.

So, distribution it is not necessary always you to try to follow some kind of the Gaussian distribution. So, other kinds of the for example, parabolic distribution may also follow. So, now, if there is a change of the distribution, what way we can modify the heat flux calculation, that can also be done in this particular case.

So, we can see that how different distribution will affect the heat source parameter or what way we can estimate the heat flux density distribution; if the distribution is different from the Gaussian distribution. So, start with these things. For example, heat flux distribution can be

So, in these cases, in this case the Cz^2 is basically, the A, B, C is a distribution parameter. Now, maybe first look into this what is the total heat input we can estimate the Q equal to total heat input that, integration of 0 to infinity x axis, because Gaussian distribution y axis is 0 to infinity and, but it is not like that, 0 to c .

Because it is not infinite distribution may not the distribution; maybe not converse to the axis along the x and y axis that is why, you put the limitation from 0 to infinity, but it is having some finite value a and the z axis. So, that so, you can put only the finite value 0 to c . Then q_m into in this case, e to the power minus Ax^2 minus By^2 into in this case, 1 minus Cz^2 , $dx dy$ and dz .

So, we can estimate this way also; that means, 0 to infinity q_m the maximum intensity 0 to infinity e to the power minus Ax^2 dx the integration 0 to infinity e to the power minus By^2 dy and 0 to $C(1 - Cz^2)$, this is a capital C is the distribution parameter, small c is the depth maybe; in this cases, we can assume the ellipsoidal shape or double ellipsoidal shape also, but if we assume the ellipsoidal shape can be ellipsoidal, but distribution can be different.

So, in this case dz . So, now, if we do this thing, this we can see the q_m into e to the power minus Ax^2 dx 0 to infinity we can say that, root to power of π by 2 into 1 by root A . In case of this case and root over π by 2 into 1 by root B , but in this case, we can see it is a z minus Cz^3 by 3 and the limit is 0 to C .

So, here we can see that, q_m into π by 4 root AB and in this case, z that means, c small c minus 1 by 3 z^3 c^3 and then z minus c z^2 z^3 by 3. So, c minus capital C by small c^3 by 3. Now, this way we can estimate. Now, we should know what is the value of the distribution coefficient A, B , and C capital C .

A, B and capital C what we can estimate? A we have already shown that, A assuming the Gaussian distribution and assuming that maximum intensity at the boundaries is only 5

percent of the maximum intensity. From that point we can estimate A is 3 by a square similarly, B 3 by b square also, but C can be estimated like that.

Maybe at the heat intensity C at the equal to 0, say x equal to 0, y equal to 0 and z equal to small C. So, then how; what we can estimate? That q m into the e to the power minus A x square; that means, e to the power 1 e to the power 0 1 e to the power 0 1 also, minus B y square this is also 0 into 1 minus C z square. So, z equal to c. So, 1 minus C small c square.

And then in this case Q equal to 0 0 c. So, Q the value of the Q will be 0, because it exactly meeting in the z axis so, not it is Q is the intensity at the z equal to c will be 0 in this case. So that means 1 minus capital C small c square equal to 0. So, from here we can see the, C equal to 1 by small c square.

So, then this is one distribution parameter, this is second distribution parameter and capital C is the 1 minus small c square; that means, this is the approximate value, but this is exact value. Now, if you put this value here also, pi 4 root A B; root A B means, root over 3 into root over 3; that means, 4 into 3 and a into b.

And, this value pi a b and c ok. Now, small c into capital C equal to 1 by c square; that means, small c by 3 it means that, q m into pi a b and c minus c by 3 equal to two-third; that means, twice into twice c by 4 into 9. So, it means q m pi a b c divided by 1 ellipse, ok.

So, total Q should be 0 to c finite value, but it should be minus infinity to plus infinity, that will be there x and y should be so, that factor will come into the picture, because if we assume the ellipsoidal model, then it should be minus infinity to plus infinity and y the minus infinity to plus infinity plus infinity.

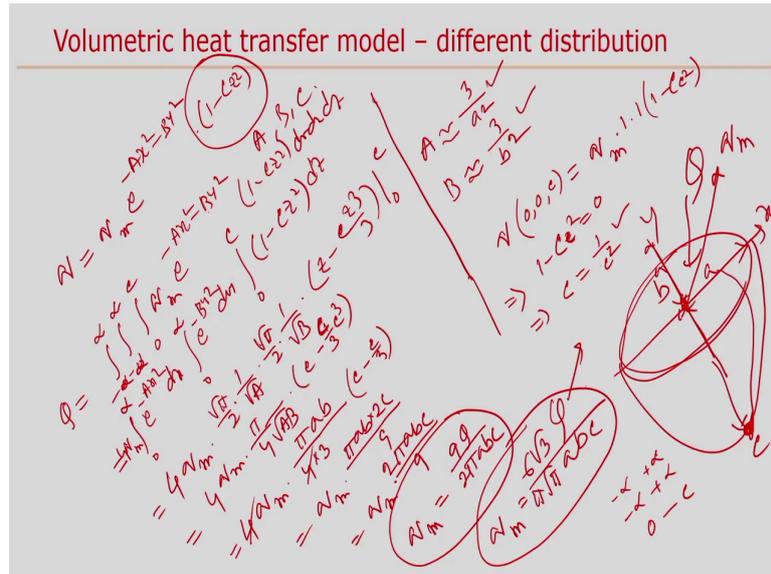
So, that way, we can estimate this thing; if you would consider this is the total and this is the volume, so, this is the c, this is a and this is b. So, this semi axis a, b, c then we can estimate the total heat input Q, it can be the a is the minus infinity to plus infinity and b for minus infinity to plus infinity and c for 0 to c.

So, that 0 to c so, in this cases 1 factor will be coming so, in symmetric 0 to infinity so, 4 will be coming into this picture so, $4 q_m$ and $4 q_m$ and then here $4 q_m$. So, then it could be like that only, the $4 q_m$ can be balance. So, then q_m into this 2 by this 4 will not be there, so, only 9 so, that twice by a b c by 9. So, it means that, q_m can be calculated like that, $9 Q$ by twice pi a b c.

So, this is the value of the maximum intensity at this point, this is the maximum intensity q_m maximum intensity; if we if simply we change the distribution in the z direction from not exponentially decaying, rather if we consider the distribution that is in parabolic in nature. So, then maximum intensity can be like that, $q_m 9 Q$ by twice pi a b c.

Now, if you remember in case of the ellipsoidal, it was like that; 6 ellipsoidal heat source model in that cases, we are assuming the distribution parameters are the same or maybe distribution are following the same kind of the equation; for or exponentially decaying in x, y and z direction. So, in that cases, we estimated that q_m was $6 \sqrt{3} \pi \sqrt{pi} Q$ by a b c ok, let us look into this thing.

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So, I think that distribution will be like that, q_m was $6\sqrt{3}$ by π root π . So, $6\sqrt{3}$ π will be the lower side, $6\sqrt{3} Q$ by π root π . So, in that case, this was the distribution in case of the we are assuming the all x , y and z direction the exponentially decaying, that means, but here in this cases the z direction distribution we just parabolic distribution, but it is exactly meeting the 0 exactly, at the at this point.

The distribution can be something like that; so, distribution is exactly 0, but that was not the case was Gaussian distribution. So, you see the objective is to show this kind of the calculation is that, even if you can change the distribution parameter also, and can change the distribution pattern also, then even you can estimate; what is the maximum intensity and from there you can develop some new heat source model depending upon the actual welding process so, that was the point.

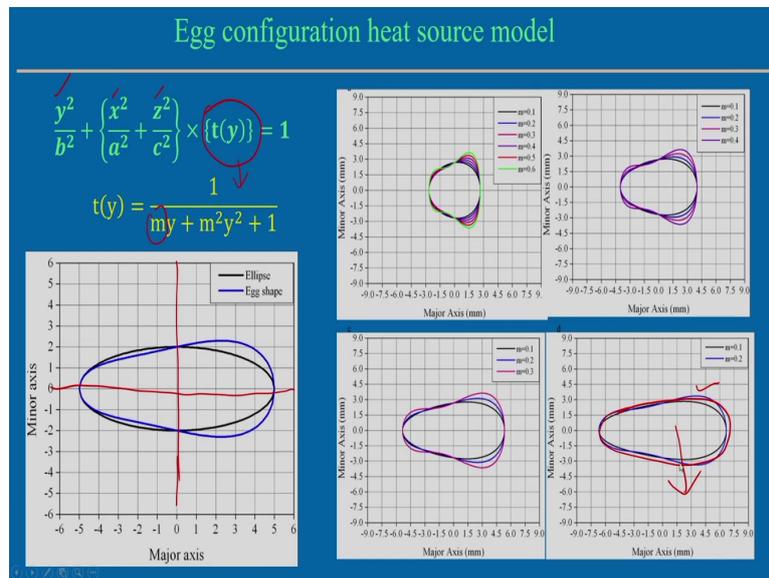
Now, if we compare the ellipsoidal heat source model and in this particular situation we can see that, q_m was that to $9 Q$ by twice $\pi a b c$, but Q is the power supply the pulse in this

cases, but definitely if it is a pulse welding process the pulse energy, per pulse on time equal to that and distribution parameter different. A is this one, B which is same as was the, because both the cases we are following the Gaussian distribution in x and y direction, but C is a parabolic distribution c square.

Now, maximum intensity is reduced by 1.3 times with respect to the ellipsoidal or double ellipsoidal heat source model. If you change the distribution parameter there is a change in the maximum heat flux intensity and of course, the distribution also will change, depending upon this thing.

So, we can see also the distribution pattern, if they are parabolic in nature then it is the distribution pattern follows, in this particular way, but if it is exponentially decaying then distribution pattern can follow like this. So, that is why the heat flux density distribution can also be vary and in that case, depending upon the problem itself the welding problem we can analyze or we can develop some other heat source model whenever required.

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Now, we can discuss the egg configuration heat source model, because so far we are discussing the merging of the 2 ellipsoidal model, then we getting the 1 double ellipsoidal heat source model or quadruple ellipsoidal heat source model, but is that possible to something the geometrics of the single geometrics can be defined or by changing the distribution parameter also, or by changing the distribution also is that possible to develop some kind of the heat source model?

So, we have tried in that direction also just simply modifying the egg, that is called the egg configuration heat source model. In this case, it is possible to develop some modification of the ellipsoidal equation. So, we start with this thing the y square by b square x square by a square plus z square by c square equal to 1, so, that is the simple equation of the ellipsoidal.

Now, this ellipsoid equation can be changed, by simply introducing this term $t y$. So, $t y$ can be a function t can be represented the functional form something like that, is a function of y this function and it can be have several expression or several functional form also. One of the functional form is for example, $1 + m y^2 + y^2$ plus 1. So, if we follow this functional form, then it is possible to modify the shape of the ellipse.

This figure also, we can look into this figure also the black border indicates the ellipse. So, the in this particular shape this ellipse the symmetric kind of shape, in this particular plane, but the same say we simply modified by introducing the $t y$ term. And there is a one constant term m so, then from there we can find out that, ellipse can be modified to the different shape, although same major axis and minor axis are the same the shape can be change.

So, that if you look into this change of the shape, which is almost resembles to the moving heat source problem can be accommodate the similar kind of the shape. So, that is why here we have tried to in that way, the shape can be change by introducing the one extra parameter.

So that means, ellipse shape can be represented can be egg shape heat source model and that different egg shape heat source model you can see that, the different value of the m . If you consider different value of the m or different major and minor axis length the leaves this thing.

For example, this is the case, m equal to 0.1 for m equal to 0.2 this is says, we are getting this kind of the shape. So, this shape is very much resembles to the moving heat source problem. So, there is this shape can be used or can be considered this geometric shape as a form of a volumetric heat, and then new heat source can be developed using this volume at using this particular concept.

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Egg configuration heat source model

$$2Q = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} q(x, y, z) dx dy dz$$

$$q(x, y, z) = q_m \exp[-(Ax^2 + By^2 + Cz^2)]$$

$$A \approx \frac{3}{a^2}; B \approx \frac{3}{b^2}; C \approx \frac{3}{c^2}$$

$$q_m = \frac{4BQ \times \sqrt{ABC}}{\pi \sqrt{\pi} (2B + m^2)}$$

$$q_m = \frac{36\sqrt{3}Q}{\pi \sqrt{\pi} abc (6 + m^2 b^2)} \quad m = 0$$

$$q_f(x, y, z) = \frac{36 \times \text{sqrt}(3) \times Q}{\pi \times abc \times \text{sqrt}(\pi) \times [6 + b^2 m^2]} \times e^{-\left[\frac{3y^2}{b^2} + \left(\frac{3x^2}{a^2} + \frac{3z^2}{c^2} \right) \times \left(\frac{1}{my + m^2 y^2 + 1} \right) \right]}$$

$$q_r(x, y, z) = \frac{36 \times \text{sqrt}(3) \times Q}{\pi \times abc \times \text{sqrt}(\pi) \times [6 + b^2 m^2]} \times e^{-\left[\frac{3y^2}{b^2} + \left(\frac{3x^2}{a^2} + \frac{3z^2}{c^2} \right) \times \left(\frac{1}{-my + m^2 y^2 + 1} \right) \right]}$$

So, here the similar exercise what we have done in case of the ellipsoidal or double ellipsoidal heat source model, same exercise can be done also. We can start with this thing the heat flux intensity distribution something like that, q_m equal to exponential $a x^2 + b^2 y^2 + c^2 z^2$ same.

Similar heat flux density distribution we can assume, maybe we can assume the Gaussian distribution then we can estimate what is the total heat input the same way, this calculation and from that the distribution parameter we are getting almost similar, this similar to the Gaussian distribution, then we can estimate the q_m . q_m is the maximum heat flux intensity at the center point.

So, we can see the put this q_m can be represented the other parameters a, b, c and the semi major minor axis length, and then we can we reach this kind of expression. Now, if you see if

simply, m equal to 0 then if m equal to 0, then it is equal to the that expression of the q m is equivalent to the what we have done in case of the ellipsoidal heat source model.

So, that is this x m heat source model can also be developed, in this cases the intensity is there and then similarly, the front part what is the flux density distribution same way we can see, this is the t y term and this, because this is the distribution for the front part and similar heat flux density distribution in the rear part can be expressed this thing, but in this case the instead of m y we have written the minus m y.

So, that changing the, simply changing the distribution and based on that we can develop the x m heat source model, which is over which it is not necessary to separate out into a two different ellipsoidal and merging to this thing, rather we can define the heat source in a single geometric shape and that was the purpose of this egg configuration heat source model.

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Conical heat source model

$$r_0(z) = r_e + \frac{r_i - r_e}{z_i - z_e} (z - z_e)$$

$$Q_v(r, z) = \frac{9 \times P \times \eta \times e^3}{\pi \times (e^3 - 1)} \times \frac{1}{(z_e - z_i) \times (r_e^2)} \times \exp\left(-\frac{3 \times r^2}{r_0^2}\right)$$

Similarly, the conical heat source model also developed depending upon the practical situation, for example, in case of laser high penetration laser welding process. So, there is a need to heats the steep temperature gradient exist along the depth direction. So, to get our intensity and the depth direction, sometimes we can it is possible to make it the geometric profile, following the truncated cone also.

So, if we assume the geometric shape is truncated cone, then the shape can be the distribution assuming the Gaussian distribution only on the surface Gaussian distribution in the heat flux on the surface and from that point of view, the volumetric heat source term or volumetric heat source can be generated, that is called the conical heat source model.

In this case, if you see the parameters the this is the truncated cone the geometric shape instead of the ellipsoidal or double ellipsoidal, we can use assuming the conical shape, truncated cone. The parameter z the distance from the bottom surface to reference to that and the radial distance r_0 at the intermediate position and there are two different radial distance r_e and r_i ; this is the r_i and here is the radial distance with the r_e .

So, then in terms of that, we can estimate the volumetric heat flux this ηP , P is the power, here P is the basically, equivalent to Q here also and then, $\eta^3 \pi (r_e^3 - r_i^3)$, so, this is the efficiency term. So, this way we can this thing, that constant term. So, this indicates the maximum intensity and this maximum intensity but this maximum intensity remains constant.

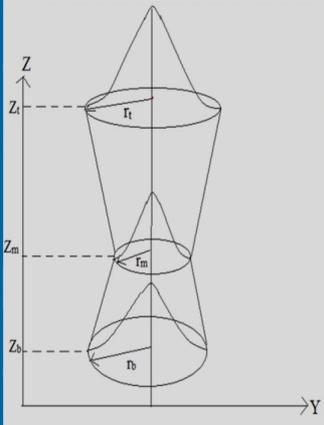
And then other parameters varies $z = z_i$ as a function of r_e square and exponential 3 into r_i square radial distance r_i is the variable and r_0 is the at any intermediate position, what is the value of r_0 . So, r_0 is actually, varying with respect to z . So, r_0 as a function of z can be expressed in terms of the other parameters the limiting the truncated cone r_e , there the upper side and the lower parameter. So, r_e and r_i and z_i and z_e . In terms of that this z is the particular height situation.

So, r_0 as a function of z can be expressed. So, this heat flux then along the depth direction this there is a variation of the r_0 , r_0 can vary and accordingly the distribution can also vary. So, this is all about the conical heat source model.

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Hour glass type heat source

To represent the shape of full penetration welds, an hourglass like heat source with Gaussian power density distribution is developed.



Volumetric shape consists of two combined frustums

$$\dot{Q}(Z,r) = \frac{3d\eta P}{\pi[1 - \exp(-d)] \cdot (Z_1 - Z_m)[r_m^2 + r_m r_1 + r_1^2]} h(t) \exp\left[-d \frac{(x - Ut)^2 + y^2}{r_{eff}^2}\right]$$

for upper cone ($Z_0 \geq Z_m$)

$$\dot{Q}(Z,r) = \frac{3d\eta P}{\pi[1 - \exp(-d)] \cdot (Z_m - Z_b)[r_m^2 + r_m r_2 + r_2^2]} h(t) \exp\left[-d \frac{(x - Ut)^2 + y^2}{r_{eff}^2}\right]$$

for lower cone ($Z_0 < Z_m$)

But same in the hour glass type heat source model also developed by using the merging of the two different truncated cone, in such a way, that to represent the shape of a full penetration weld, because in laser welding process sometimes we can get the different heat source profile of the weld zone.

So, to account the full penetration weld zone sometimes, we can use the hourglass type of the heat source, it is the flexible enough, but it starts from this thing.

If you see merging of the two conical truncated cone and at the same following the Gaussian distribution on the at the center point, and then we can express this thing. For example, this is the for upper cone z_0 greater than z_m for the upper cone, this is the expression for the maximum intensity.

And in this case, the $h(t)$ term is introduced that. If you want to develop some kind of the pulse laser welding process, you want to develop the heat source model, because pulse here in case of the pulse laser; the pulse energy can vary as a function of time also. So, accordingly we can put, because when there is a pulse off time, then simply $h(t)$ becomes 0 and pulse on time, then $h(t)$ become 1.

And such that, this can be taken care of the application of the pulse energy during this pulse energy process, but if it is a continuous mode simply $h(t)$ becomes 1 and, but this indicates the maximum heat at this at the center point; the what is the heat total heat, the at the center point that we can estimate here also.

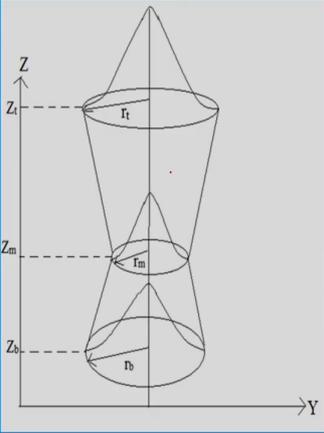
And then, sorry heat flux the center point and if you see the, it depends the all the parameters and d is the distribution parameters. So, d can be 3 in case of the; if we assume the only 5 percent energy falls at the boundary or particular in case of the GTAW process or arc welding process, but d can also be, d can also be 2 approximately, in case of the laser welding process.

So, we can takes care of that the simply, d is the distribution parameter and u is the weld velocity. So, that we can takes care of here and we can develop the heat source model. So, this is the for the upper cone and this expression can be lower cone. The similar concept we can use, but just by analyzing, but just concept of the conical heat source.

And in case; so, it is particularly more fit, in case of the laser welding process, where the high depth of penetration and full penetration depth in that case, we can use the hourglass type of the heat source model.

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Hour glass type heat source



d = distribution coefficient
 P = power
 η = efficiency

Effective radius of the cones

Upper truncated cone:

$$r_{eff} = \frac{Z_o - Z_m}{Z_t - Z_m} (r_t - r_m) + r_m$$

Lower truncated cone:

$$r_{eff} = \frac{Z_o - Z_b}{Z_m - Z_b} (r_b - r_m) + r_m$$

So, here you can see, the d already mentioned distribution parameter P power is the efficiency and the upper truncated cone, this is the effective radius and lower truncated cone is the effective radius. So, this effective radius, because this effective radius is actually varying if you look into this expression also, that effective radius this one the effective radius, so, this effective radius is actually not fixed, this effective radius is actually varying.

So, at the different at the function of the, at different height the effective radius can be vary also. So, then this effective radius is the variable in this case. So, effective radius can be estimated in terms of the other parameters in these two cases. So, this way, we can takes care of the different kind of the heat source model and depending upon the application or depending upon the welding process.

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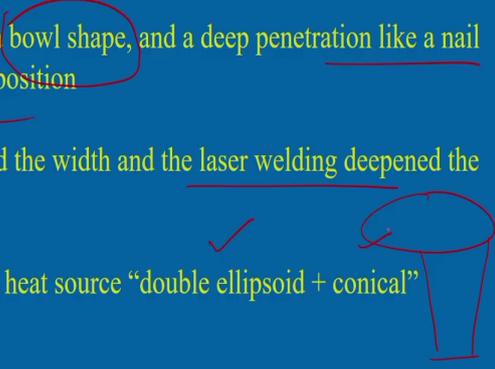
Hybrid heat source model

Hybrid laser arc welding process

Upper position like a bowl shape, and a deep penetration like a nail shape at the bottom position

Arc welding widened the width and the laser welding deepened the penetration

Example: Combined heat source “double ellipsoid + conical”



So, similar exercise can also be done, if there is a need of the hybrid heat source model developed in particular to the hybrid laser arc welding process. So, the interaction of the arc as well as the interaction of the laser together, then we can use some kind of the hybrid heat source model.

For example, the upper position like a bowl shape; so, then bowl shape, in case of the arc welding process interaction of the arc to represents and deep penetration like a nail shape. So, deep penetration like a nail shape is accounts the laser welding and bowl shape accounts the arc welding.

So that means, arc and laser welding together the hybrid welding process, when you are using then if you want to develop some kind of the heat source model for particular the hybrid

welding process, then it is possible to merge the top part in the ellipsoidal and remaining part can be conical.

So, this merging these two and accordingly, we can define the intensity of the double ellipsoidal heat source model or intensity we can calculate the intensity according to the conical heat source model. Merging these 2, then combine heat source can only be that double ellipsoidal and conical that is called the hybrid heat source model.

So, in this way, when decide the heat source model; it actually just by looking into the profile of the weld pool and accordingly, we can decide the different heat source model. So, next class we will be discussing the remaining part of the other kind of the heat source model, what we can develop specifically, the keyhole model laser welding process, or and in case of the, solid state welding process, in these two cases we will discuss. So, now, I stop here.

So, thank you very much for your kind attention.