Computational Continuum Mechanics
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Lecture – 39
Finite Element formulation of Ductile Fracture in Coupled -Thermo-Elastoplastic Dynamic Contact Problems

So, welcome back. In today's lecture, we will do the next step after finite element modelling which is we will do the finite element discretization and we look into the finite different discretization that is we will see discretization in space and discretization in time.

And we will also see how the final Newton-Raphson procedure is setup, and I will just briefly mention what are the special things that you need to take account during the simulations ok. So, the thermal formulation and the contact formulation we will keep it for the next lecture and also some of the results that were obtained using this formulation.

So, you will get an overall picture of how the concept that we have discussed in this course could be applied for simulation of a much more complicated problem ok.

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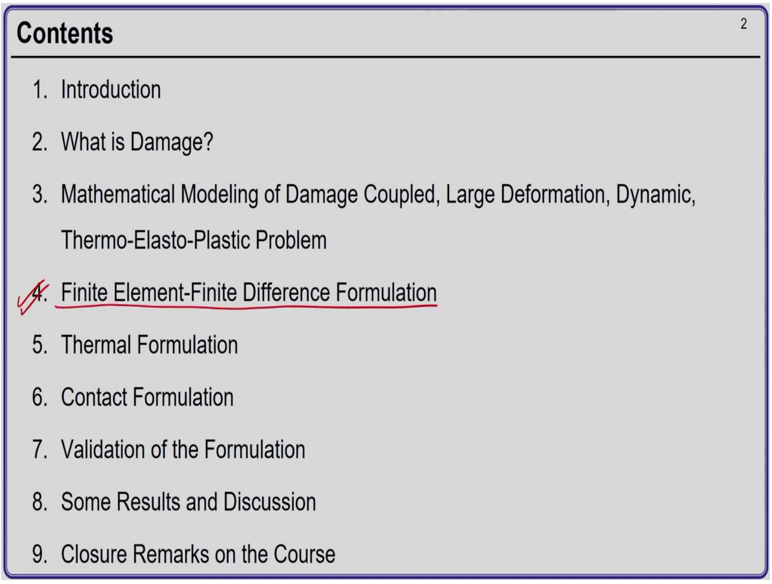

So, today we are just going to see the finite element and finite difference formulation ok.

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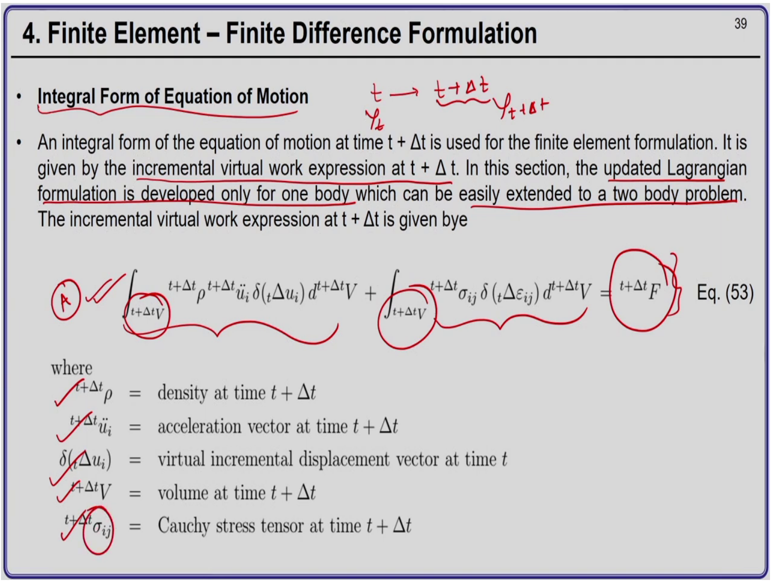

So, to start the finite element and finite difference formulation ok, so we first have to derive the integral form of the equation of motion ok. So, you need to remember that we are doing an updated Lagrangian formulation which means that we know everything at time t, and we want to compute the state of stress and the deformation of the body at time t plus delta t ok. So, we know psi t and we want to know psi t plus delta t ok. Psi is the current configuration ok.

Now, it is to get the integral form of equation of motion, it is given by the incremental virtual work expression at time t plus delta t ok. So, we use the updated Lagrangian formulation which is developed for one body in the previous slide, and we easily extend it to two body problems ok. So, we will do this for single body ok, and because we are dealing with a contact body, so, you can imagine this is written for body A, and then you can write a similar expression for body B ok.

So, you see this is the virtual work of the inertia forces at configuration t plus delta t ok. And this is the internal virtual work at t plus delta t ok. And these should be equal to the external forces at time t plus delta t ok. So, here you have density at time t plus delta t, you have the acceleration.

You have the virtual incremental displacement vector at time t ok. You have the configuration of the volume at time t plus delta t. This is the Cauchy stress at time t plus delta t ok.

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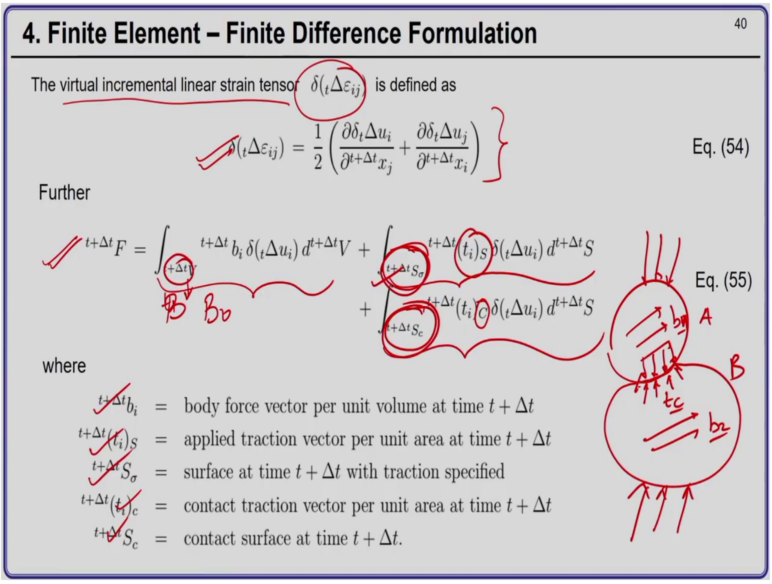

And then the virtual incremental linear strain tensor can be written using following equation ok. So, now the external forces ok. So, the external forces are nothing but the external forces because of the body forces ok.

So, the external forces because of the applied surface traction. And because we have contact problem, you will also have the external forces on each body because of the contact stresses or because of the contact traction. So, this C over here denotes the contact ok. So, you have two bodies ok. So, say you have these two bodies. So, a certain part, so you will have body forces inside the body. So, b 1 let us say b 1 and b 2.

You will you may have applied external tractions which are other than the contact tractions because the nature of the tractions on the surface and on the contact surface remains same, but we will distinguish between what is applied externally and what is because of the contact. And then when you separate these two bodies in contacts, this is body A and body B, at this surface each body will experience what is called the contact traction ok.

So, this body will have a compressive traction like this, and body B will have a compressive traction like this ok. And in addition, if the bodies are sliding over each other and there is friction, so there will be two kind of tractions. One is the normal traction which is because of the normal contact, and the tangential traction which is because of the frictional force ok.

So, you have the body forces, you have the applied surface tractions you have the surface ok. So, this is the contact surface, this is the; this is the applied traction surface, this is the contact surface ok. So, remember this formulation we have taken from Bathe. So, there is a difference in convention in which we write. So, initially we are writing for example, this as B, an initial surface is B 0.

But now it is written as t plus delta t v for body configuration at time t plus delta t. So, but anyway the idea remain same, the only the terminologies change, but the idea is same which we discussed in our chapter on kinetics where we discussed principle of virtual work ok.

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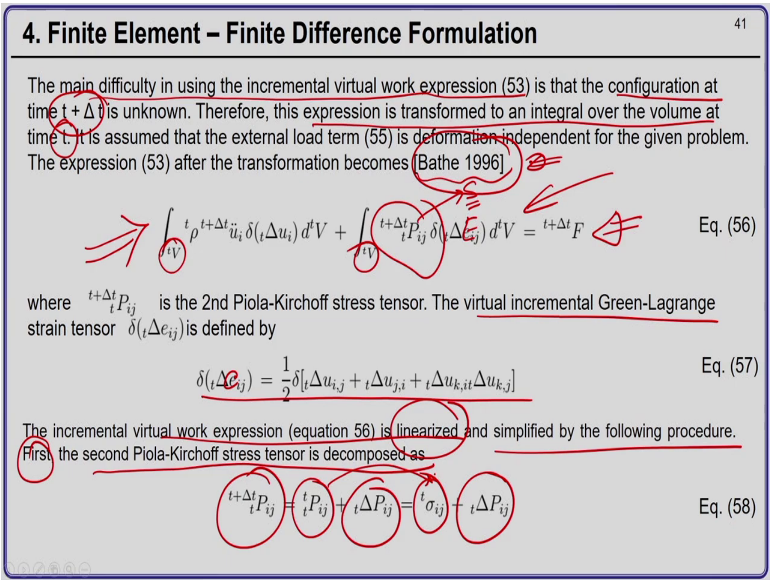

Now, the main difficulty as we already discussed previously in using the incremental virtual work is that we do not know the configuration at time t plus delta t. So, we cannot carry out those integration over the volume and the surfaces ok. So, therefore, these expressions need to be transformed to an integral over volume t and this we can do by the push back operations.

And once we do that ok, so here we have used one formulation which was given by Bathe ok. So, when you write the configuration ok, so the integral equation over volume t plus delta t you write at time t which is here, then you get the following expression. And instead of the Cauchy stress you get what is called the second Piola-Kirchhoff stress tensor ok.

So, remember this P we use for first Piola-Kirchhoff, but here this correspond to the second Piola-Kirchhoff stress tensor ok. And the virtual Green-Lagrange strain tensor comes out at the corresponding conjugate stress measure ok. So, here again instead of capital E, we are using the small e ok. Now, the incremental virtual work expression is a non-linear expression ok.

Now, in our previous discussion what we had done was we first did the discretization, and then we did the linearization, but here we follow Bathe. And as I mentioned in my those lectures also, there is another way you can do linearization first and you can do discretization later, so which is what Bathe follows. So, here also this current formulation, what we do? We use linearization first ok. So, for linearization, we use the following procedure ok.

So, for first the second Piola-Kirchhoff stress tensor which is here is decomposed as the second Piola-Kirchhoff stress tensor at time t plus the increment in the second Piola-Kirchhoff stress tensor ok. And the second Piola-Kirchhoff stress tensor at time t is nothing but the Cauchy stress tensor at time t plus then you have the increment ok.

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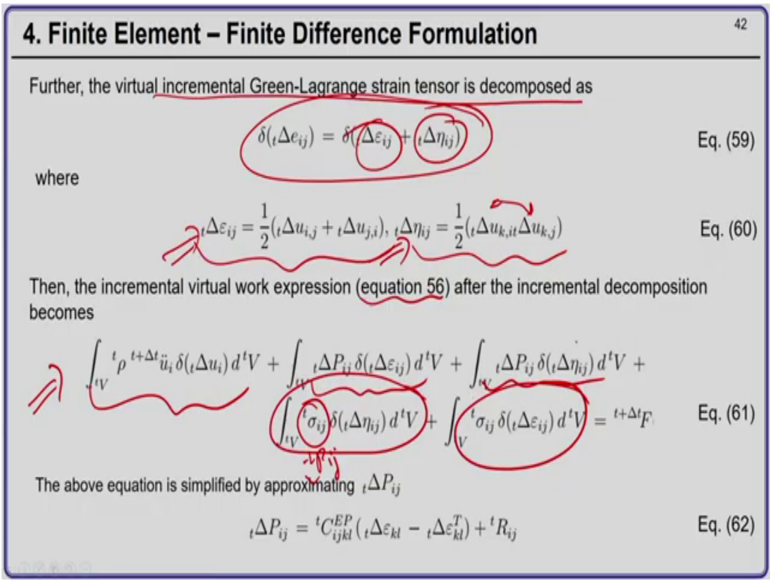

And the virtual Green-Lagrange strain tensor can be decomposed into what is called the linear part and the non-linear part, where the linear part is just like your linear strain tensor, and the non-linear part has the product of the gradient of the displacement at time t ok, incremental displacement at time t. So, now, if you substitute these expressions and those in the previous in equation 56 ok, so you will get following expression.

So, you will have the inertia forces, you have delta P times the linear part plus the non-linear part ok. And this was anyway your dP ij t. So, you have this expression, and then you have this expression ok. Now, you notice that ok. So, let me rub this.

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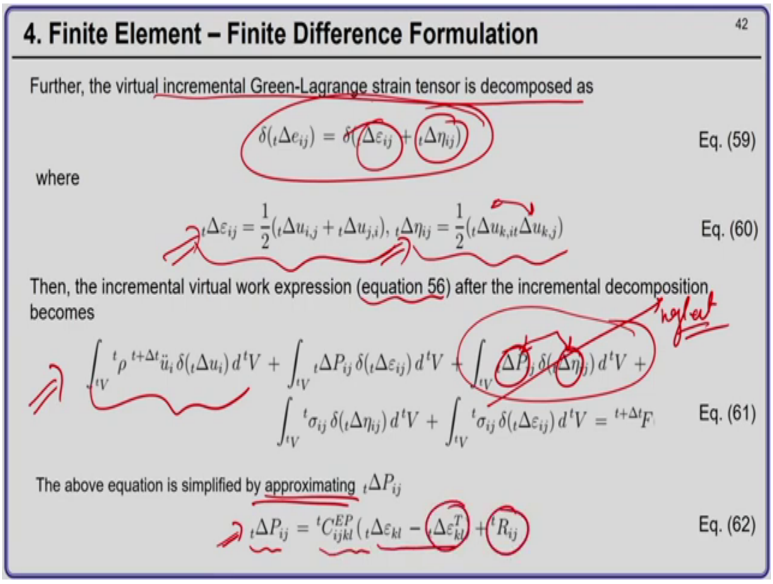

If you notice this term over here, this has a delta P here; it has delta eta here. So, product of two very small quantities gives you even smaller quantity. So, we can just simply neglect this ok.

So, this is what we do we are doing as linearization, we are neglecting the higher order terms ok. And then we can use the following approximation. So, this is the approximation that Bathe uses that t delta P i j is equal to C i j k l E P delta epsilon k l minus the thermal part plus R ok.

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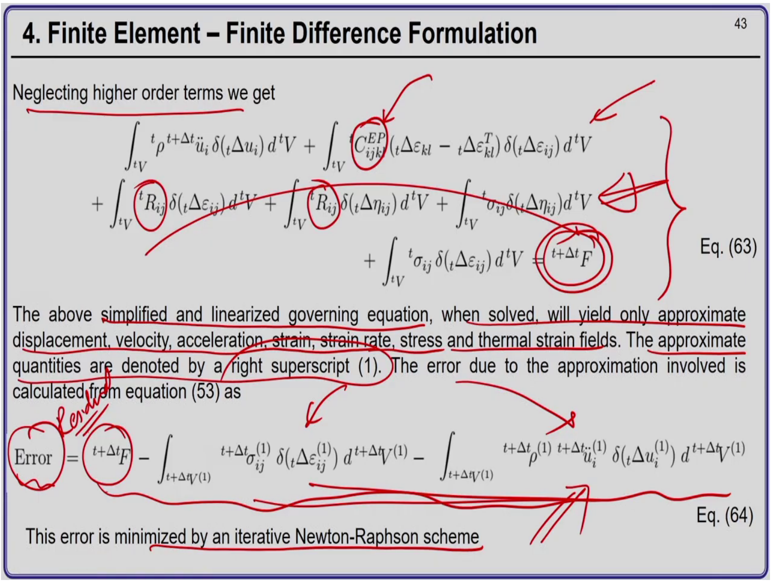

So, with this I am neglecting what I said was the higher order terms, you will get the following simplified and linearized governing equation ok. So, this equation when solved will yield only in an approximate displacement, velocity, acceleration, strain rate, stress and thermal stress field ok. So, everything is inbuilt here ok.

So, this R contains the effect of temperature ok. And strain rate this C i j k l E P has the effect of hardening, it has the effect of temperature, it has the effect of strain rate. So, everything is here inside this equation ok. And if you directly solve it, you will get only in an approximate solution which is not accurate.

So, these approximate quantities are denoted by right superscript 1 here ok. So, if you do this ok, you will write like this the error ok. So, this is the right hand side. If you take everything on the left hand side, then the difference of the right hand side and whatever is there on the left hand side gives you the error, and this error has to be minimized using a Newton-Raphson iterative procedure ok. So, this is what we called residual ok.

Remember we had this residual in the solution procedure which we discussed in the previous lectures ok. So, now, we have to solve this ok. And you have to minimize this residual, so that the external forces balance out the internal forces that is the internal forces and the inertia forces which are generated because of the externally applied forces like a body forces, surface traction and contact forces ok.

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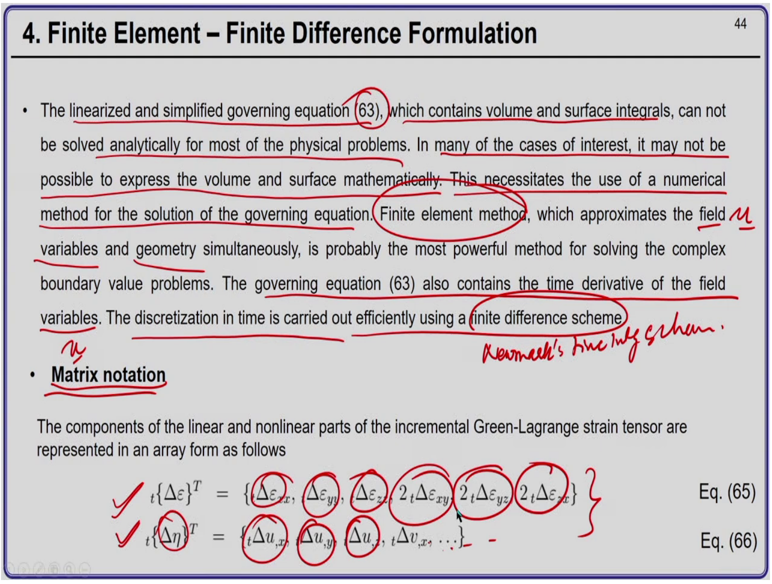

So, now the linearized and the simplified governing equation which is given by equation 63 contains volume and surface integrals and cannot be solved analytically for most physical problem ok. So, what we do, in most cases of interest, it may not be even possible to express the volume and surface mathematically ok.

So, in case of for example impact problem, the body is deformed so much that you cannot even recognize that the initial body has so say a cube will have become very deformed shape for example. So, this necessitates the use of a numerical method for the solution of governing equation, and we use the finite element method to approximate the field variables and the geometry.

So, the field variable in our case is the displacement. And then we are the governing equation also contains the time derivative of the field variable that is u because we have acceleration there. Therefore, the discretization in time we carry out using a finite difference scheme which in our case will be the Newmark's time integration scheme ok.

So, first we do the what is called the matrix notation, we define the matrix notation for the components of the linear and non-linear part of the incremental Green- Lagrange strain tensor are represented by the following array form ok. So, you have the normal components of the incremental strain, and you have the shear component ok. Similarly, delta eta is written like this ok. So, you will have as many terms as there are ok. So, we will have delta v y, delta v z, delta w x, delta w y, and delta w z ok.

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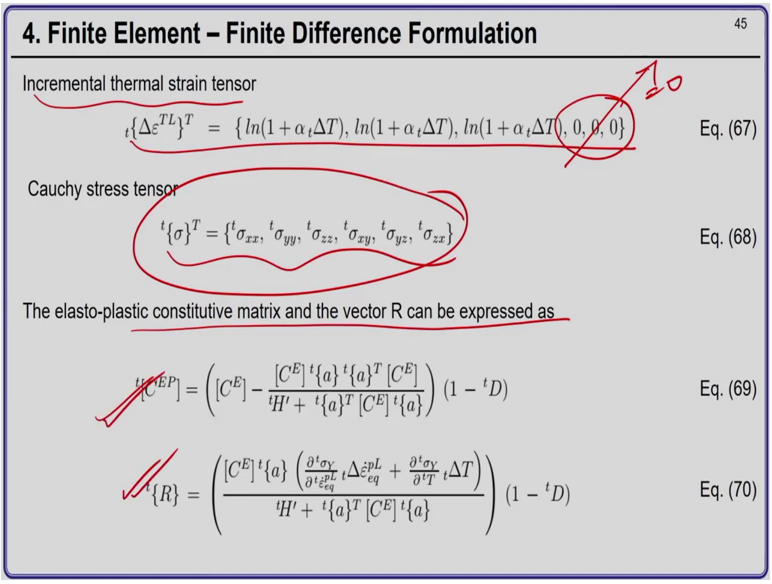

Now, the incremental thermal strain tensor can be written as following form, where the shear term are neglected, ok. Our shear terms are anyway not neglected, it is equal to 0. So, there are only normal strains thermal strains. The Cauchy stress using the Voigt notation we can write like this form ok. And then the elasto-plastic constitutive matrix and the vector R can be expressed in following form ok. So, this you can simplify yourself, this is what we write ok.

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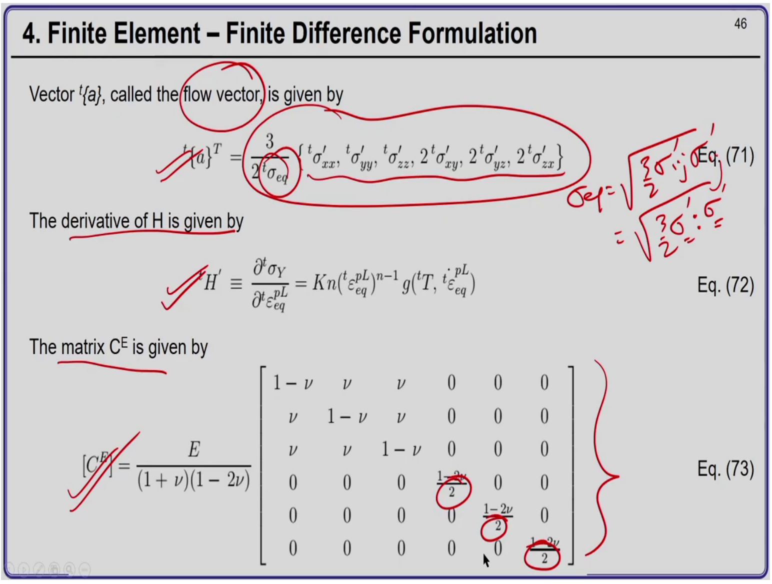

And this vector a is called the flow vector. If you read the theory of plasticity, you will come across this vector which is called the flow vector, its normal to the yield surface. And this flow vector is given in the following form.

It is a function of the deviatoric part of the stresses divided by the equivalent stresses ok, equivalent stress. And the, remember equivalent stress is nothing but root over 3 by 2 sigma dash i j sigma dash i j; or in direct notation, we can write this as sigma dash double contracted with sigma dash ok.

And the derivative of H is can be written like this ok. And the linear elastic constitutive matrix C E is written in the following form. Remember we have 2 here to take into account that 2 that we have in the strain in the shear part of the strain ok.

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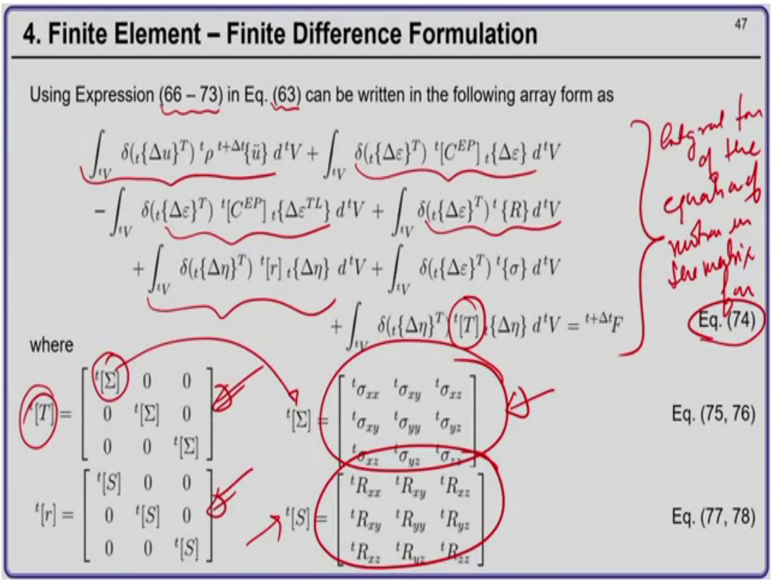

So, now if you substitute all those expressions 66 to 73 which we have in the previous slide in equation number 63, then you will get the following form ok. So, the integral form of the equation of motion can be written in the matrix form. So, this is the integral form of the equation of motion in the matrix form ok.

So, this is the matrix form where ok, so you have this inertia term over here ok, and then you have this is the coupled term because of temperature in the mechanical field ok. So, this is the purely, this is the thermal and mechanical also; this is thermal, thermal ok. And this is purely thermal field ok.

And then you have this matrix T here. And this matrix T has this following form and this matrix r has this form ok. Where sigma which you see here is nothing but the stress tensor in the full 3 by 3 form, and this S contains the tensor r in full 3 by 3 form ok.

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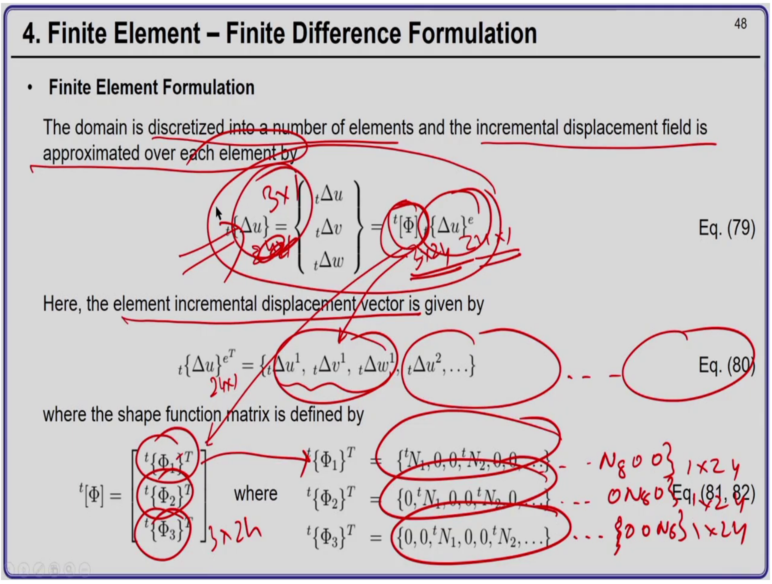

Now, to do the finite element discretization of our equation number 74. Now we can do the finite element discretization we have everything in the matrix vector form ok. So, what we do? So, first we discretize our domain into number of finite elements. And then the incremental displacement field is approximated over each element e using following form ok. Where this here over here phi over here is nothing but the shape function matrix ok.

And delta u e is nothing but the elemental incremental displacement vector that is the incremental displacement vector of node 1, then incremental displacement vector of node 2 all the way up to number of nodes that will have ok. So, for example, for a 8 nodal element delta u e will be a 24 cross 1 matrix ok. And this also will be a 24 cross 1, and this is a 24 cross 1.

Therefore, eventually we will have a, sorry, this is a 3 cross 1 at any point inside. So, there will be three displacements. So, this will be a 3 cross 24 matrix ok. And this matrix is nothing but composed of following sub vectors. And these vectors when you write can be written in terms of the shape functions of the N nodes of the element.

So, N 1, N 2 all the way up to you will have say 8 noded elements, so you will have till N 8 ok. So, this will be 0 N 8 0, and this will be 0 0 N 8 like this ok. So, this will be 1 cross 24; this is 1 cross 24; this is 1 cross 24. Therefore, this will be 3 cross 24 ok. So, 3 cross 24 here. And 24 cross 1 gives you 3 cross 1 which is the displacement at any point inside the element ok. So, it will have three displacement x, y and z ok, so that is what we have ok.

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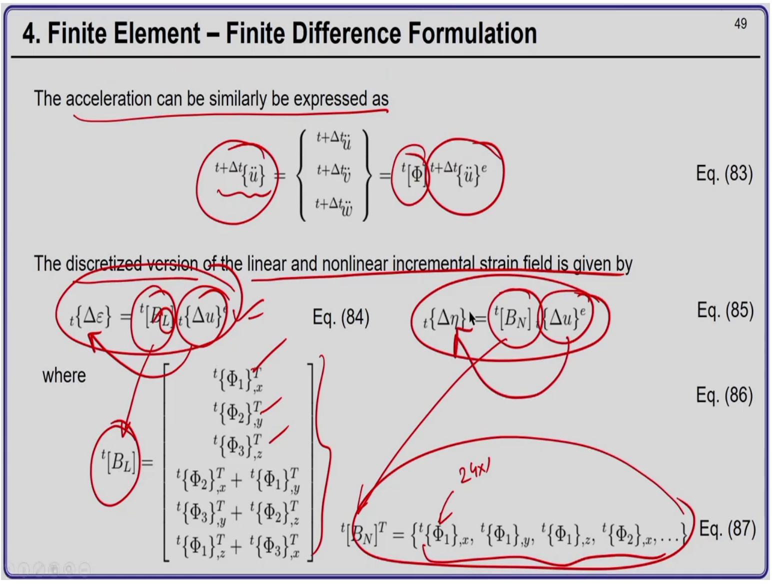

Now, the acceleration can be written by taking the double derivative of the increment of the displacement. And then using the finite element discretization the acceleration at any point inside the element can be written in terms of the shape function matrix times the acceleration at the nodes ok. And the discretized version of the linear and non-linear incremental strain field is given by following form ok.

So, delta epsilon e is like this, delta eta just like this ok, where the strain displacement matrix B L which is L here is for linear. So, B the linear part of the strain displacement matrix has the following form where phi 1, phi 2, phi 3 have been already described. And this non-linear part of the strain displacement matrix has this following form ok.

So, this again will be, so this actually is a 24 cross 1 ok. So, you will get what is called the big matrix over here, and this is also matrix. So, finally, you will get in from incremental displacement, you can compute the incremental strain. And from incremental displacement, you can compute the non-linear part of the incremental that delta eta ok.

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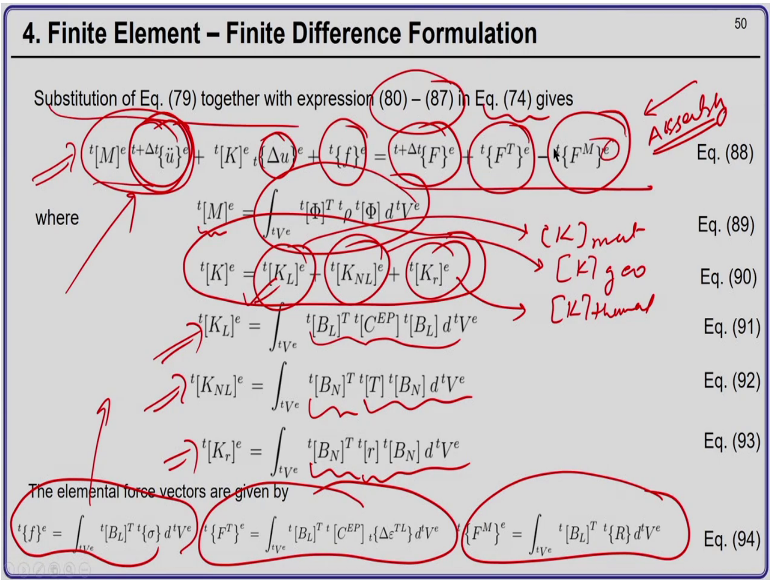

So, now when you substitute equation number 79 together with these expressions 80 to 87 in equation number 74. If you do this ok, you will get the elemental form for each element you will get the Newton's second law and which is given here.

So, mass into acceleration plus the tangent matrix K times the incremental displacement delta u plus the internal forces plus the external forces plus the external forces because of the thermal, thermal external forces, and then you will have what is called the effect of thermal forces on the stress fields ok. So, those three terms you will have ok.

So, remember we because this was already linearized. So, we are not getting mass into acceleration equal to net external force, we also have this tangent matrix time the incremental displacement ok. So, our objective is to find out this delta u ok. And if you know this delta u, then we can compute this also here ok.

So, we will see how we remove this t plus delta t u dot mu double dot also here t M is nothing but your elemental mass matrix ok. This is like N transpose a rho N this is the elemental tangent matrix. So, this has the linear part, the non-linear part. So, this is the what we saw in our previous discussion this was this K mat ok.

So, this was K geometric ok, the geometric stiffness. And this is the stiffness which comes because of the thermal effect ok. So, this is a linear part this is B transpose C B ok. This is the geometric part this is B N transpose T B N ok. And this is a thermal part which is B N transpose r B N ok. And the right hand side force vectors, so, this is your elemental internal force vector this is B transpose sigma.

If you recognize this has the same expression that we discuss in our formulation for hyper elastic material. So, this is the thermal part ok, and this is the coupled thermo mechanical part of the external right hand side force vector. Now, you can do the assembly ok. You can do the assembly over all the elements ok. So, now, everything is at element E and you can do the assembly over all the elements.

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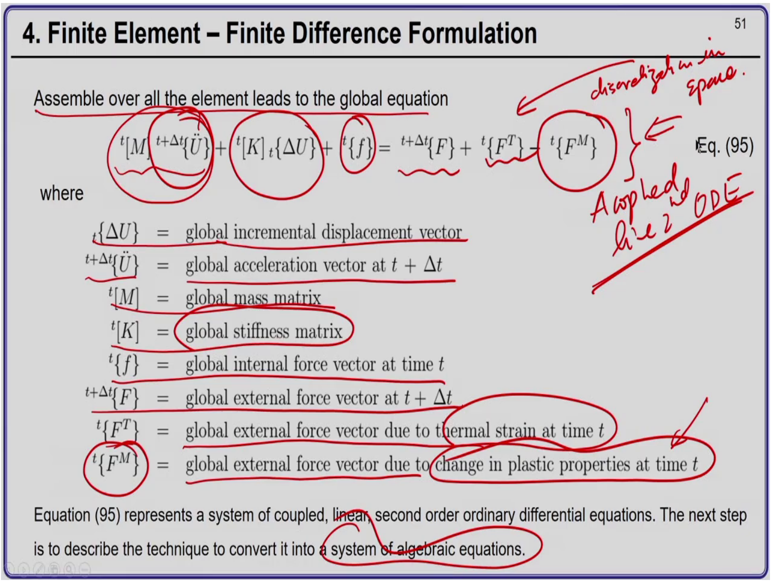

And if you do this ok, if you do the assembly over all the elements, the leads to the global system of non-linear equation ok, global system of equations given by M into U double dot this is the inertia terms M U double dot mass into acceleration plus the tangent matrix time K delta U plus delta F plus the internal forces at time t plus the external forces at time t plus delta T plus the thermal global external force due to thermal strain at time t, and this is the global external force due to the change in the plastic properties at time t because of the thermal effect ok.

So, here delta U is the global incremental displacement vector; t plus delta t U double dot is the global acceleration vector at time t plus delta t; t M is a global mass matrix; t K is the global tangent matrix; t f is a global internal force vector ok. And this is the global external force vector at time t plus delta t. This is nothing but the global external force vector due to thermal strain at time t.

And as I told already that F M is nothing but the global external force vector due to the change in plastic properties at time t ok, because of the temperature change our plastic properties are also changing, and also they are changing because of the severe plastic deformation ok. So, then this incorporates all such effects ok.

So, equation 95 represents a coupled a coupled linear second order ODE ok. And now what we have to do is to convert this coupled linear second order ODE into a system of algebraic equation ok. So, how do we do that?

We have to now go for approximation in time ok, discretization in time. So, this equation 95 completes your discretization in space ok. So, you have completed your discretization in space. Now, you have to do the discretization in time. And for this you use what is called the finite difference in time ok.

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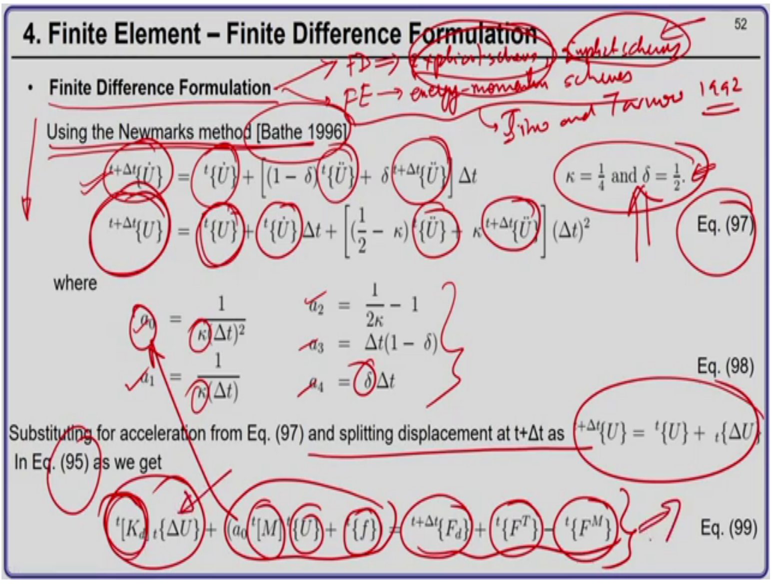

So, just to note that, you can either do finite difference formulation in time or you can also do. So, in the current discussion, we are using the finite difference formulation, but you can also use the finite element formulation ok.

So, finite element formulation will give you what is called the energy-momentum schemes ok. So, these schemes will have some criteria, where the energy and momentum of this process can be conserved, if it is a conservative system or it can be dispersed dissipated at a particular rate ok, but this is a little difficult to understand and difficult to apply ok.

So, if you wish, you can go and look into the literature. There are lot of papers which are research which is available starting from the work of Simo and Tarnow in 1992 ok. So, you can see this paper and all the papers which are referring this. So, in this present discussion, we use the finite difference in time ok. So, finite difference in time means we use the procedure where the derivative of the expressions in time are approximated using certain known quantities at or unknown quantities at the current or the previous step ok.

So, in finite difference, you have what is called the explicit schemes, and we have the implicit schemes ok. So, implicit schemes means that you would need to do what is called the Newton-Raphson iteration procedure.

Explicit scheme on the other hand you do not need the Newton-Raphson iterative procedure ok. So, explicit scheme are only conditionally stable, while the implicit schemes are at least for linear problem they are unconditionally stable, but for non-linear problem they can be unstable, but however you can take a much larger timestep ok.

So, if you are doing some very high velocity impact kind of problem, then definitely we use explicit scheme because there to find out the tangent matrix K ok, so the tangent matrix K is a very tedious process.

So, we do not want to do that. But for certain problems where we do not require we do not have strain rate at that level ok, then we can use what is called the implicit scheme. But implicit schemes can be used even for impact kind of problem, it would be just like a it will be accurate ok, because we will be using Newton-Raphson anyway.

So, here we use what is called the Newmark's method and the approximation for which is given by Bathe and the formulation also the derivation is also given. So, here the velocity at time t plus delta t is approximated in terms of the velocity at time t, acceleration at time t and the acceleration at time t plus delta t ok. So, using Taylor series expansion, you can write this.

And then the displacement at time t is written in terms displacement time t plus delta t is written in terms of displacement at time t, the velocity at time t, the acceleration at time t, and the acceleration at time t plus delta t. Again you can obtain this using Taylor series expansion ok.

So, here a 0, a 1, a 2, a 3 are some constant that will be used which depend on this constant kappa and delta and kappa delta are called the Newmark's constant, and they take the value of 1 by 4 and 1 by 2 respectively. And these values are derived from the unconditional stability property for linear system ok.

For non-linear system, you have to change this value according to a certain relation which is given in Bathe where you can have certain amount of dissipation numerical dissipation ok. Now, if you substitute equation number 97 and splitting your displacement at time t plus delta t like this ok, you can write equation 95 in the following form ok.

So, you can write the matrix K d times the incremental displacement plus a 0 M into U plus f equal to the right hand side force vector where a 0 is here ok. So, this is your a 0. So, these quantities are at time t, and therefore, they are known. The forces at time t plus delta t are usually known, the external forces are known what. And even these forces are at time t, so we know that and this K d is computed based at time t ok. So, that also we can compute.

So, only thing which cannot be known right now is delta U. Therefore, if you can solve equation 99, you will get your solution ok.

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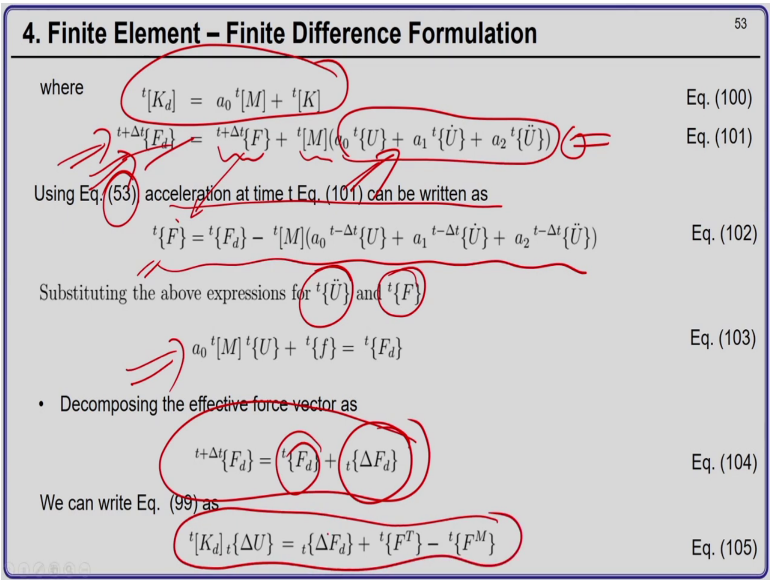

So, if the K d is nothing but a 0 M plus K and if t plus delta t F d is nothing but the external force plus M times this quantity over here. This is nothing but the acceleration approximation ok. Now, at time t, you can write the forces like this ok. So, now, if you use equation number 53, acceleration at time t equation 101, you can write like this ok. If you write this at time t, you can write the following expression.

And when you substitute for the expression of the acceleration at time t and the force external force at time f, you can write this equation ok. And if you decompose the effective force vector which is this here, as the effective force at time t plus the incremental effective force, we can write equation 9 as following form ok.

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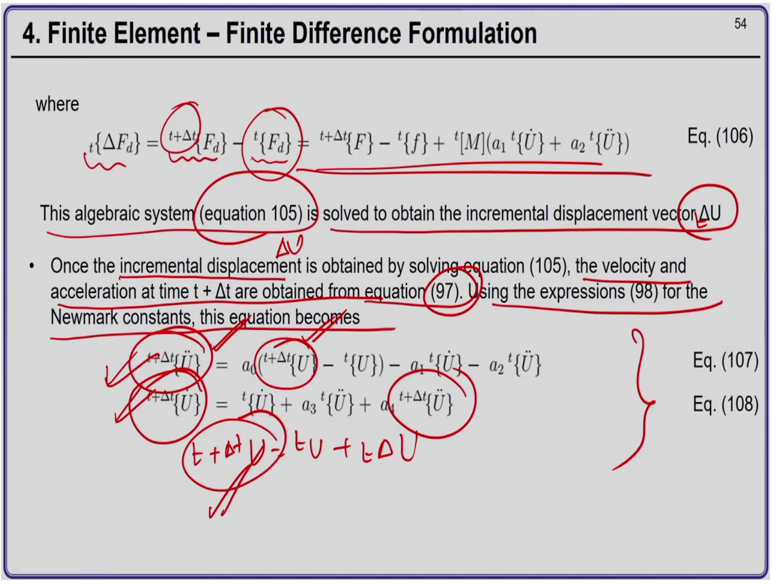

Where this delta F d, where this delta F d is given by the force at time external force at time t minus the external force at time external force at time t plus delta t minus the external force at time t, and this is what we have ok. So, this algebraic system of equation given by equation 105 is solved to obtain the incremental displacement ok. So, equation 5 is what you solve to get delta U.

So, once you solve for incremental displacement delta U ok, the velocity and acceleration at time t plus delta t can be obtained by equation number 97 ok. And using the expressions 98 for the Newmark constants, you can compute ok.

So, you can get t plus delta t U as t U plus t delta U ok. So, this should be t, t delta U. So, you can get t plus delta t U. And using equation number 107, once you have t plus delta t U, you can find out the velocity acceleration at t plus delta t U, t plus delta t.

And once you have the acceleration at t plus delta t, you have displacement at t plus delta t, you can get the velocity as t plus delta t, where you will use the acceleration at t plus delta t ok. So, from equation 107 and 108, you can get the velocity and acceleration at t plus delta t. So, once you know the displacement and velocity and acceleration at time t plus delta t you know everything ok.

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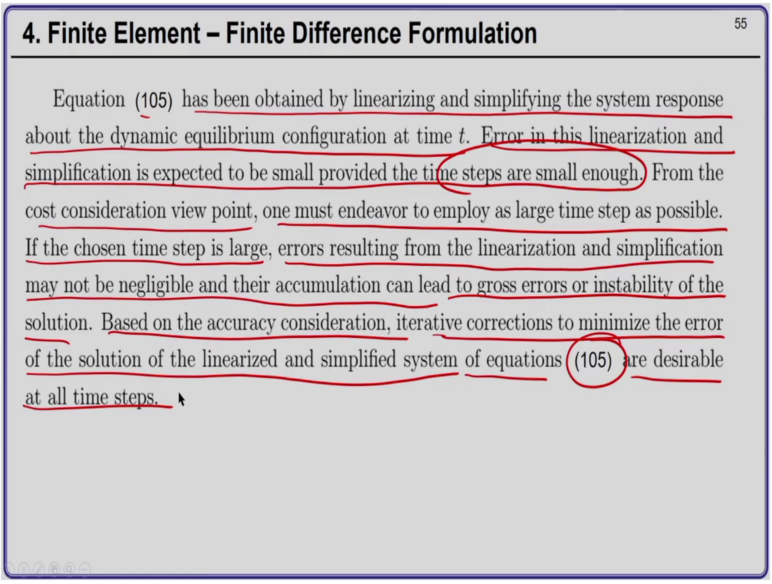

Now, this equation 105 has been obtained by linearizing and simplifying the system response about the dynamic equilibrium position at time t ok. So, this error in this linearization and simplification is expected to be small provided the time steps are small enough. From the cost consideration viewpoint, one must endeavor to employ as large time step as possible.

If the chosen time step is large, errors resulting from the linearization and simplification may not be negligible and their accumulation may lead to gross error or instability of the solution. So, based on the accuracy consideration, iterative corrections to minimize the error of the solution of the linearized and simplified system of equation given by equation 5 are desirable at all time steps ok.

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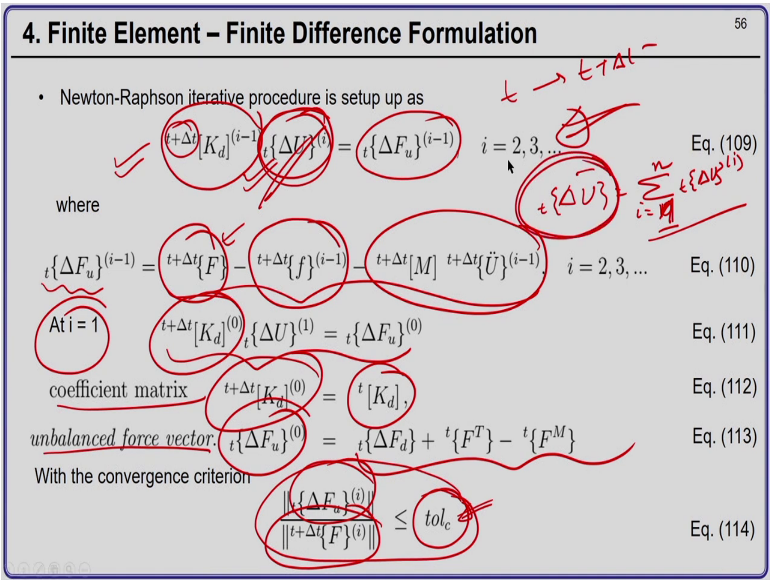

So, what we do? We have to set up the Newton-Raphson iterative procedure. And we set up the Newton-Raphson iterative procedure like this ok. So, the K effective tangent matrix evaluated at i minus 1 ok. And this is for approximation for t plus delta t times the incremental displacement vector for iteration i is equal to the residual force vector corresponding to iteration i minus 1 ok. So, this delta F u at i minus 1 is given by following form ok.

So, this is the external force. This contains the external traction plus the contact force. So, this is the body forces at i minus 1, and this is the mass into acceleration at i minus 1. So, for the first iteration, we have the equation 109 can be written like this, where the coefficient matrix or the tangent matrix is approximated by this here. And the unbalanced force vector or the residual vector is approximated by following expression ok.

So, you can solve this equation 109 iteratively over a particular time increment going from t to t plus delta t, and you can get the value of delta U ok. So, here the displacements are updated in a incremental manner, rather than iterative manner we update the displacement in a incremental manner ok.

Now, our convergence criteria that we use, so, we had discussed three types of convergence criteria. So, we use the residual of the unbalanced force vector divided by the norm of the external force vector at time i and if this is less than the specified tolerance that is the tolerance maybe this is what the user will give we can get the solution ok, not always, but in correct setting you will get the solution ok.

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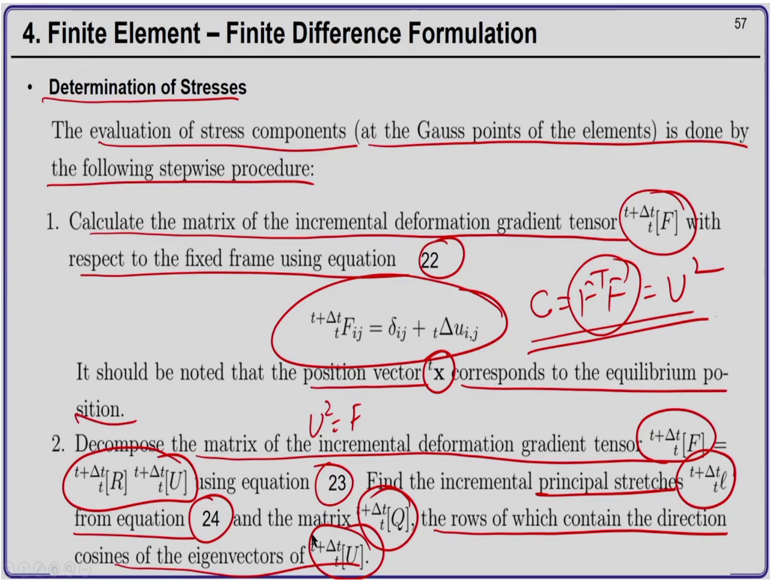

Now, how do you determine the stresses once you have determined ok? So, once you have determined this delta U, what you do? So, your delta U t will be summation i goes from 1 ok, i goes from 1 to say n, n is the number of Newton-Raphson steps that you do will be d delta U i ok. So, at the end of ith iteration, you will have ok, you will have the incremental displacement. Now, from this incremental displacement how do you find out the stresses ok.

So, this is what you do once you have found out the displacement. So, determination of the steps is carried out using the following procedure ok. So, the evaluation of stress component at the Gauss points of the element is done by the following stepwise procedure.

Remember, here we use we are using a different kind of objective stress rate ok. So, we calculate the matrix of incremental deformation gradient tensor F with respect to the fixed frame given by equation number 22. And we use the following formula. And it should be noted that the position vector x corresponds to the equilibrium position ok.

Now, we decompose the matrix or the incremental deformation gradient tensor using right polar decomposition into an orthogonal part and a symmetric part R and U using equation number 23.

And from U square equal to ok, so C is F transpose F and this is equal to U square ok. So, we do find out F and then we find out F transpose F to get us U square and then from there we find out the principal stretches from equation number 24. And we also determine the matrix Q, the rows are which are contain the direction cosines of the eigenvectors of the right stress tensor U ok.

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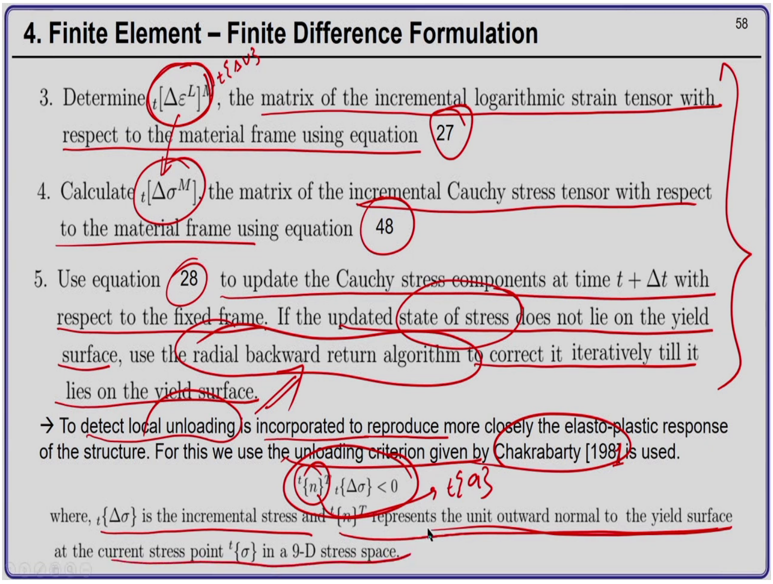

Then from the incremental displacement t delta U, we find out the incremental strain delta epsilon L. And this is the matrix of the incremental logarithmic strain tensor with respect to the material frame using equation number 27. And from this incremental logarithmic strain tensor, we find out the matrix of incremental Cauchy stress tensor with respect to the material frame using equation number 48 ok.

So, using equation number 28, we update the Cauchy stress tensor at time t plus delta t with respect to the fixed frame. And if the updated state of stress does not lie on the yield surface, we then have to use the radial backward return algorithm which we are not discussing in this particular lecture because it is itself is a big topic.

So, once you find out that the updated state of stress does not lie on the yield surface, then we use the radial return backward radial return algorithm to bring these stresses back to the yield surface, and we correct it iteratively till the stresses lies on the yield surface this is the consistency condition that we discussed. Now, this completes the procedure for finding out the stresses.

Now, during the deformation there can be what is called the local unloading, the point may the Gauss point may experience unloading the decrease in stresses or the forces because of which it may go from plastic state to elastic state which is called the unloading.

Now, to detect that unloading ok, what we have to do, we have to incorporate a particular procedure. And for this we have incorporated by using the unloading criteria which is given by Chakrabarty in his 1981 ok, sorry this is 1, 1981 book, where it is given by n transpose delta sigma should be less than 0.

Here delta sigma is the incremental stress, and n represent the unit outward normal to the yield surface. And remember this is nothing but your flow vector that we have ok. Remember a, that was the flow vector. So, this n is nothing but the flow vector. So, the flow vector that we described is used here ok. And n represent the unit normal outward normal vector to the yield surface at the current stress points sigma, in the 9-D stress space ok.

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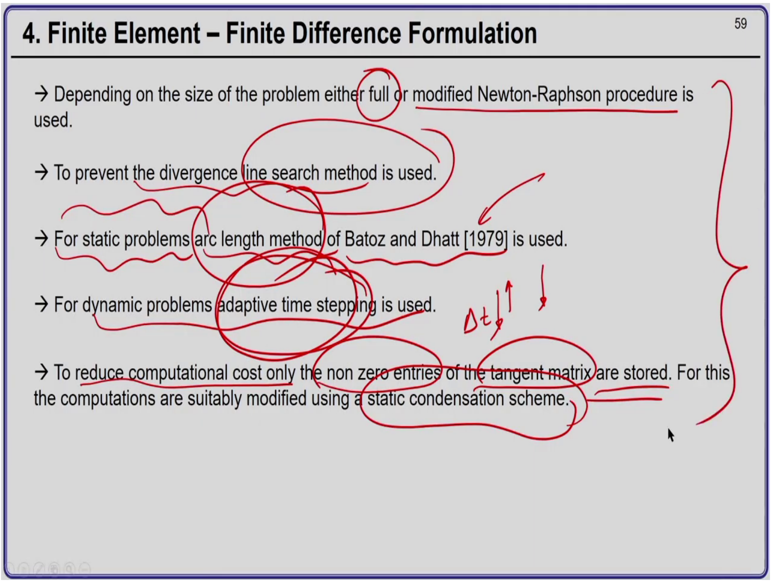

Some of the other salient features that we have to use is depending on the size of the problem ok. If it is a very big problem, then we may like to go for modified Newton-Raphson procedure instead of the full Newton-Raphson which means that the tangent matrix is not computed at each and every step, this also we have discussed in our discussion on Newton-Raphson method ok.

And if the Newton-Raphson procedure experiences some kind of divergence, so we have to incorporate some line search method ok, we have to handle those divergence. For static problems ok, we have to incorporate and static problems which are dominated by where displacements are specified not the forces we have to use what is called the arc length method. And here in this formulation, we use the arc length method proposed by Batoz and Dhatt in 1979.

If you are solving a dynamic problem, then definitely depending on the convergence rate, you have to use what is called the adaptive time stepping which means that the delta t can go up, it can go down depending on the rate at which your Newton-Raphson procedure is updated.

So, if your Newton-Raphson procedure fails and the line search also fails, then the next step is to use the adaptive time stepping to maybe decrease the delta t, and then redo the complete step from t to t plus delta t ok. And the last step ok, the last salient feature that you have to use is to reduce the computational costs only the nonzero entries of the tangent matrix have to be stored.

Remember for a very big problem, your K matrix will be only containing up to 5 to 10 percent nonzero values, the 90 percent to 95 percent of the values will be 0. So, we do not have to store those values. So, what we do? We have to use what is called a static condensation scheme. And we just store only the nonzero values of the tangent matrix for this the code needs to be suitably modified ok.

So, with this we have carried out the finite element finite difference discretization of the integral equation of motion. So, the next step is to do the thermal formulation and the contact formulation, and then we look into some of the validation problem and the results; and of the tact, we will conclude this lecture ok. So, with this, I will end today’s lecture ok.

Thank you.