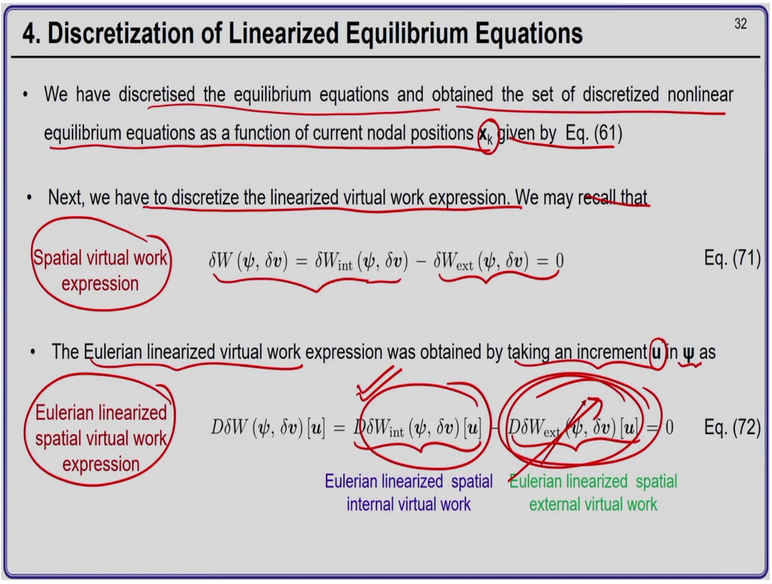
Computational Continuum Mechanics
Dr. Sachin Singh Gautam
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Lecture – 27 -28
Discretization

So, today we will see the Discretization of linearized virtual work expression. The last lecture we completed the discretization of the equilibrium equations ok. So, today we are going to see the discretization of the linearized equilibrium equations ok.

(Refer Slide Time: 01:06)



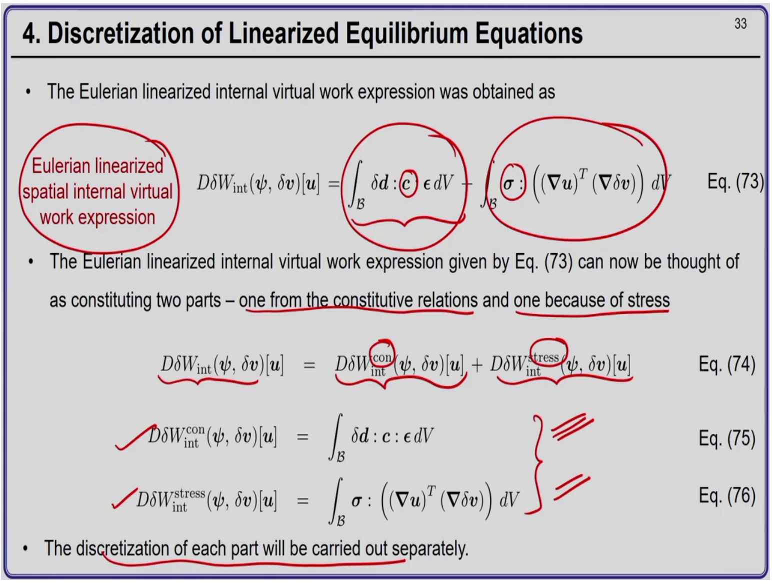
We have discretize the equilibrium equations ok and we had obtained a set of discretize non-linear equilibrium equation as a function of current nodal positions x k given by equation

61 ok. Now, what we have to do is we have to discretize the linearized virtual work expression ok.

So, you may recall that the spatial virtual work expression is given by the difference of the internal virtual work minus the external virtual work ok. And the Eulerian linearized virtual work expression was obtained by taking an increment u in current position psi and we obtain, it as the difference of the linearized virtual work expression internal virtual work expression and the linearized external virtual work expression ok.

So, this term over here is the linearized Eulerian linearized spatial internal virtual work. And this term is the Eulerian linearized spatial external virtual work. Now, as I mentioned in the previous lectures that in this course, we will treat that the external forces are independent of the configuration, which means that this particular term is 0. So, we only have to deal with discretization of the Eulerian linearized spatial internal virtual work ok.

(Refer Slide Time: 02:52)

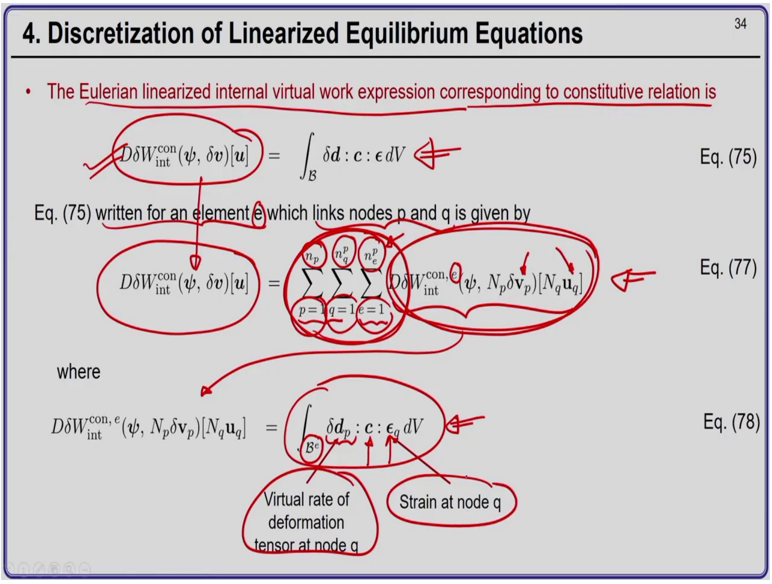


Now, the Eulerian linearized spatial internal virtual work composed of two parts ok. So, it is composed of two parts; this is the first part, and this the second part ok. So, the first part is because of the constitutive relations because, you see there is the spatial elasticity tensor out here.

And the another part the second part is because of the stress, you see there is a stress ok. So, we can write the linearized spatial internal virtual work expression, as the sum of the linearized internal virtual work expression, because of the constitutive relation and one because of the stresses ok.

So, these are the expression for the two parts ok. Now, we have to discretize each of these parts this one and this one that is equation 75 and 76 separately ok. So, we will first consider equation number 75 that is we will going, we are going to discretize the internal virtual work because of the constitutive relation ok.

(Refer Slide Time: 04:18)



So, the Eulerian linearized internal virtual work, expression corresponding to constitutive relation is given by equation number 75. So, I have reproduce its here now, this equation 75, when it is written for an element e which links two nodes p and q ok.

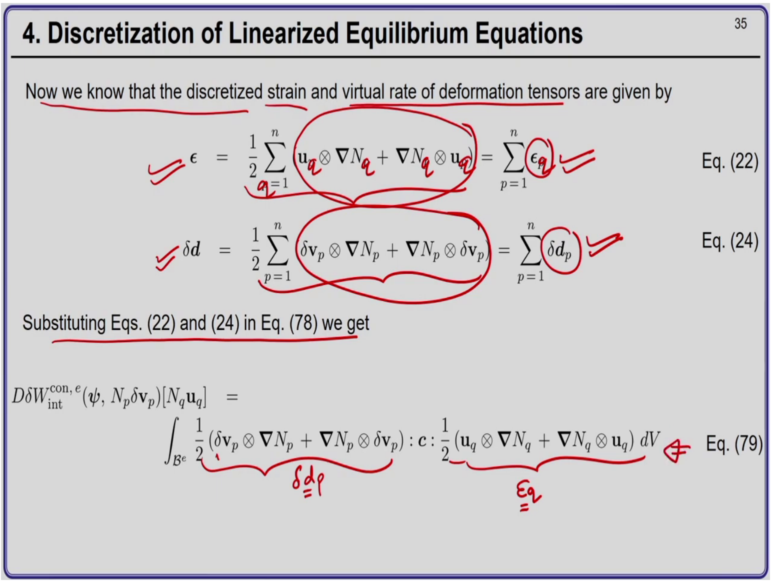
So, this can be written as summation over nodes p and q and element e of the linearized internal virtual work expression, for belonging to the constitutive relation, for a element e connecting nodes p and q by equation number 77 ok. So, now, if I can discretize this expression over here, then I can just take the summation ok. Actually the summation actually implies assembly operator its not merely a summation its the summation and assembly ok.

So, I can do the assembly or summation over all the elements over all nodes q, over all nodes p to get the final discretize, internal virtual work expression corresponding to the constitutive relation. Notice that n p e here denotes the number of elements which connect to node p.

And n p q denotes the number of nodes q, connected to node p and n p denotes the total number of nodes in the finite element mesh ok. So, now, this expression over here can be written as integral over the current volume, the virtual rate of deformation tensor of node p double contracted with this spatial elasticity tensor c, double contracted with the strain corresponding to node q ok.

So, this is just equation number 75 written for node p and q and integrated over element e ok. So, this is the virtual rate of deformation tensor at node q. And this is the strain at node q ok.

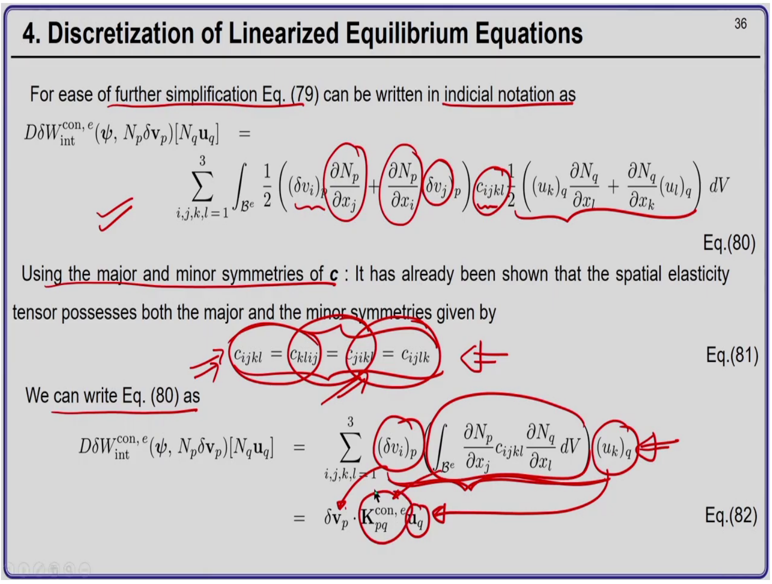
(Refer Slide Time: 06:56)



Now, from equations 22 and 24 we know that the discretize strain and the virtual rate of deformation tensors are given by following expression ok. So, this we have derived in our previous lecture. And now I can identify this term over here with the strain corresponding to node p ok. And I can identify this term as the virtual rate of deformation tensor corresponding to node p ok. So, now, I can substitute equation 22 and 24 I can substitute equation 22 and 24 in equation number 78 ok.

So, that is what we have done here this is nothing, but the del epsilon p del d p and this is nothing, but this complete expression is epsilon q ok. I can replace this p by q here its just the matter of convention and you will see this is epsilon q ok.

(Refer Slide Time: 08:18)



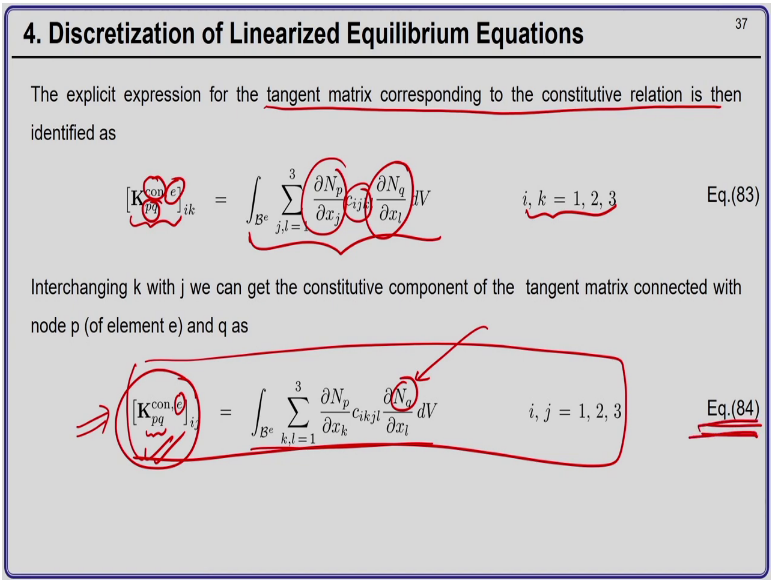
And now, for further simplification I can write equation 79 which is here that is in direct notation, I can write this in indicial notation like this ok. So, it will be del v i of node p into gradient of shape function of node p corresponding with x j. And the gradient of shape function corresponding to node p will x i del v j c i j k l 1 by 2 and this is the strain ok.

Now, I now can use the major and minor symmetry. So, during hyper elasticity discussion, we had shown that the spatial elasticity tensor c has the major and minor symmetries as it is shown in equation 81 here ok. So, if I use these major and minor symmetries 1 by 1 ok. I can reduce equation 80 to following expression ok.

So, I just have to use equation 87 I can replace first of all I can use this symmetry over here, then I can use this symmetry, this symmetry. And finally, I can use the other two symmetry and I can show that equation 80 ok reduces to following expression ok.

So, now I can convert this back to direct notation and I can write, this term over here as the virtual velocity vector corresponding to node p. This as the increment u corresponding to node q and this term inside this bracket can be identified as a matrix K.

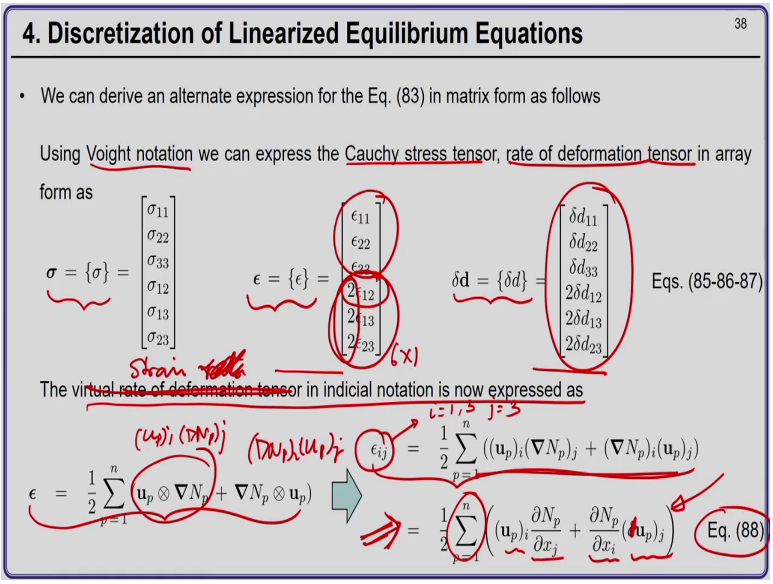
(Refer Slide Time: 10:28)



Which, we call as the tangent matrix corresponding to the constitutive relation ok. And the explicit relation for this tangent matrix K corresponding to the constitutive relations of element e for nodes p and q is nothing, but integral over the current volume ok, of del N p by del x j c i j k l del N q by del x l ok.

Now, to get more consistent ok, I can just interchange k and j I can get the tangent matrix corresponding to the constitutive relation ok. So, using this expression once I know my shape functions ok, I can find out this matrix ok. And this matrix will be of size 3 by 3 ok.

(Refer Slide Time: 11:26)



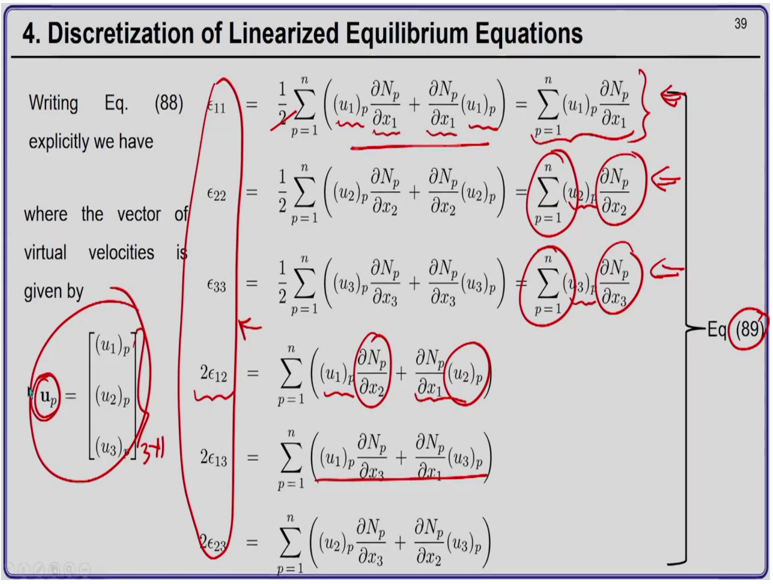
Now, in traditional finite element literature we have a alternate expression for equation 83 and that we can derive in matrix form ok. And we will see how that relation compares with our relation number 84 ok, relation given by equation 84 ok. So, let us start first we use the Voight notation and we can express the Cauchy stress tensor rate of deformation tensor and the strain tensor in array form as this ok.

So, remember the strain tensor is written as epsilon 1 1 epsilon 2 2 epsilon 3 3 that is the normal component and then you have 2 epsilon 1 2, 2 epsilon 1 3, 2 epsilon 2 3. Remember this 2 is present over here because, there are 2 values of strain. Strain has actually 9 components and we are writing this as 6 cross 1 component. And because, epsilon 1 2 is same as epsilon 2 1 therefore, I put a 2 here to take into account both the strain values ok.

And now the virtual rate of deformation tensor, I can write similarly as this vector over here. So, once I have defined my vectors ok. So, this virtual rate of deformation tensor sorry strain tensor in indicial notation, we know as following relation ok. Now, in indicial notation I can write the same as this ok. So, you have. So, this is a dyad. So, this will be u p i del N p j and this will be del N p i u p j ok.

So, that is what you have written here. So, more explicitly in terms of the derivative of the shape function, I can write the component epsilon i j of the strain tensor as 1 by 2 summation over all the nodes of the elements u p i del N p by del x j plus del N p by del x i del so, u p j ok.

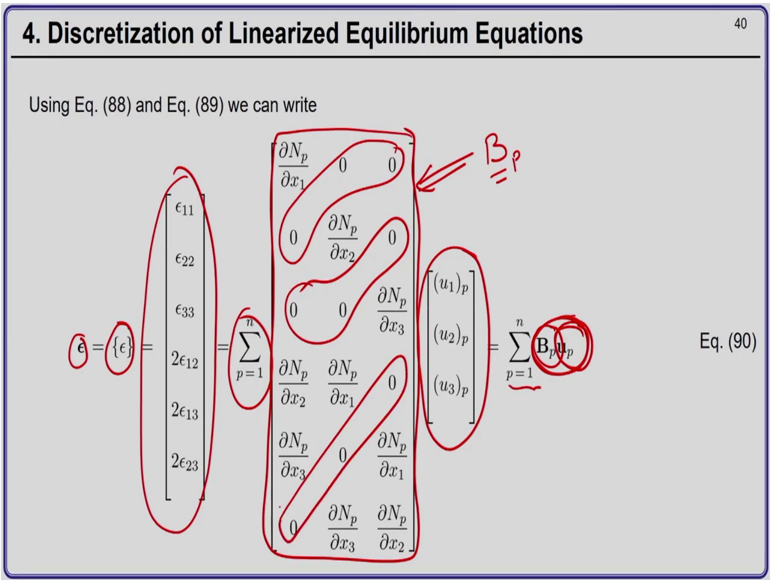
(Refer Slide Time: 13:57)



Now, equation 88 I can expand for each value of i and j ok. So, i goes from 1 to 3 and j goes from 1 to 3 ok. So, if I do this and I remember that the vector of virtual this vector of displacement is given by this 3 cross 1 vector. So, epsilon 1 1 I can write as 1 by 2 u 1 p del N p by del x 1 plus del N p by del x 1 u 1 p. So, these two terms are same so, it becomes twice of u 1 p del N p by del x 1 and this 2 cancels out with the other 2 and we get u 1 p del N p by del x 1 summation over all the nodes of the element e ok. So, that is your epsilon 1 1.

Similarly you can show epsilon 2 2 as summation over all the nodes of the elements u 2 p del N p by del x 2 epsilon 3 3 is summation over all the nodes of the element u 3 p del N p by del x 3 ok. And this twice of epsilon 1 2 is nothing, but u 1 p del N p by del x 2 plus del N p by del x 1 u 2 p. And similarly we have the other two components ok. So, this equation number 89 ok. So, on the left hand side I have the strains and using Voight notation, if I write them as a vector on the right hand side, I can write as a product of a matrix times a vector u p ok.

(Refer Slide Time: 15:48)

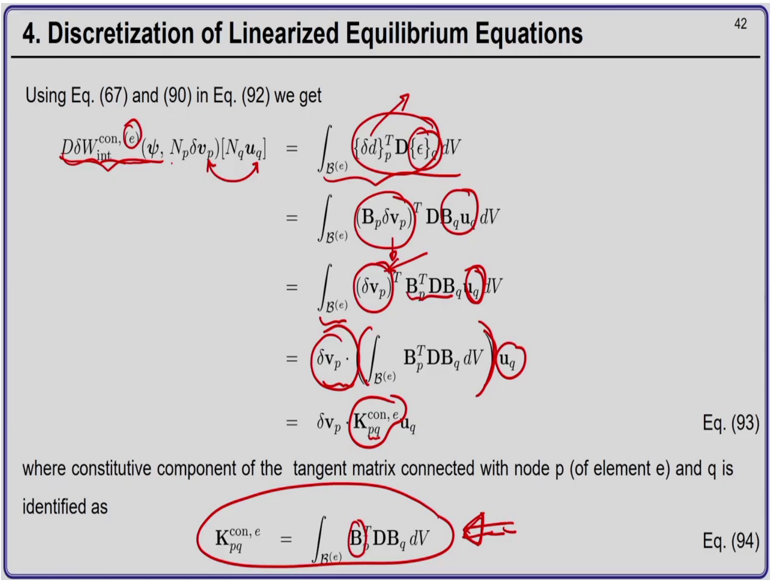


$$\epsilon = \{\epsilon\} = \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ 2\epsilon_{12} \\ 2\epsilon_{13} \\ 2\epsilon_{23} \end{bmatrix} = \sum_{p=1}^{n} \begin{bmatrix} \frac{\partial N_p}{\partial x_1} & 0 & 0 \\ 0 & \frac{\partial N_p}{\partial x_2} & 0 \\ 0 & 0 & \frac{\partial N_p}{\partial x_3} \\ \frac{\partial N_p}{\partial x_2} & \frac{\partial N_p}{\partial x_1} & 0 \\ \frac{\partial N_p}{\partial x_3} & 0 & \frac{\partial N_p}{\partial x_1} \\ 0 & \frac{\partial N_p}{\partial x_3} & \frac{\partial N_p}{\partial x_2} \end{bmatrix} \begin{bmatrix} (u_1)_p \\ (u_2)_p \\ (u_3)_p \end{bmatrix} = \sum_{p=1}^{n} \mathbf{B}_p \mathbf{u}_p \quad \text{Eq. (90)}$$

So, that is what we are going to do? So, this is a strain expressed using Voight notation which is shown over here is nothing, but summation over all the nodes of the element this matrix of the derivative of shape functions, with respect to current position times the nodal displacements of node p ok.

If I write this matrix as B p ok, then I can write my strain in terms of the gradient of the shape function times the displacement of the nodes ok. So, epsilon is nothing, but summation over all the nodes of the element of the strain gradient matrix B p times the nodal displacements u p ok.

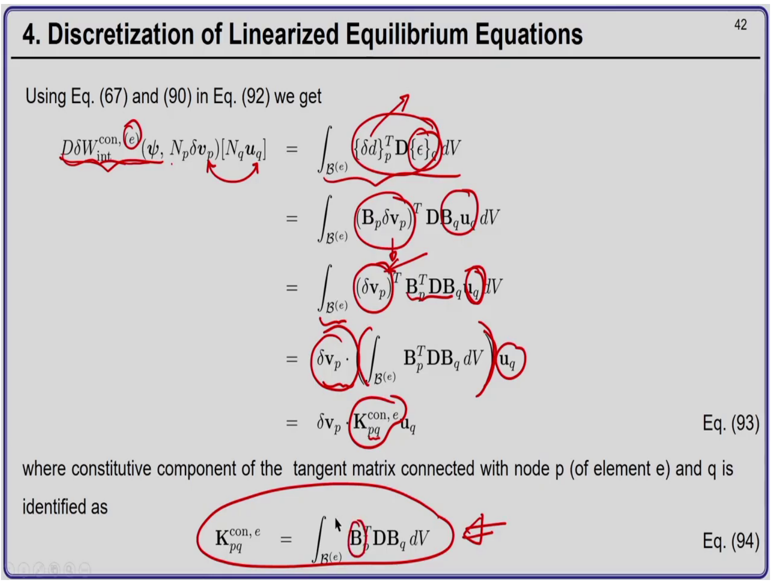
(Refer Slide Time: 16:54)



Now, this internal virtual work expression corresponding to constitutive relation ok, we again recall its written like this. So, for an element e it is nothing, but this integrant integrated over the volume of the element e ok. So, using Voight notation I can express this relation over here as del d transpose a matrix D into vector of strains epsilon integrated over the current volume of the element e, and we will see what this matrix D is all about ok.

So, now, from equation number 90 and 67 ok. So, equation 67 we have derived that the vector of the virtual rate of deformation tensor is nothing, but summation over the nodes of element of B p del v p ok. And this B p del v p I can write it as the vector of virtual rate of deformation tensor corresponding to node p. And similarly, the strain I can write from equation 90 as B q u q and integrate a and summed over all the nodes of the elements. And this B p u q I can write as vector epsilon corresponding to node q ok.

(Refer Slide Time: 18:48)

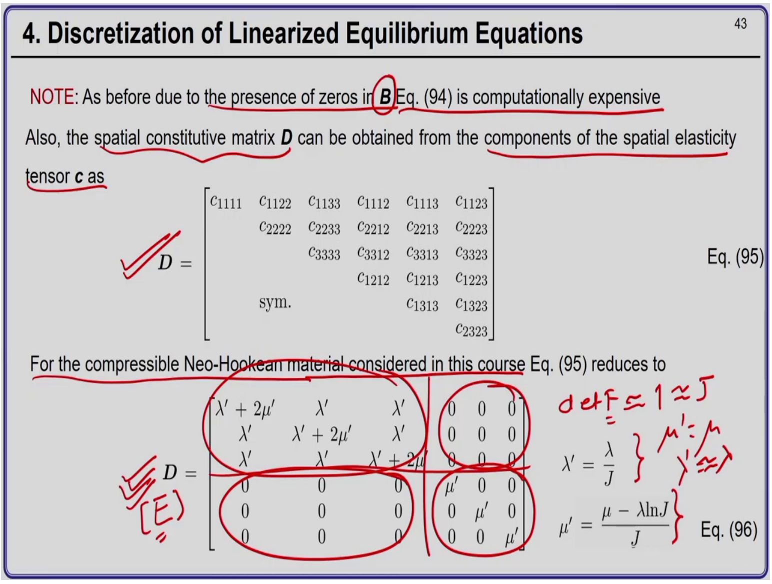


Now, if I use these here ok. So, I can write p and q as integral of this integrant over the current volume of the element e ok. And now, del d p transpose from equation number 67 is nothing, but B p del v p transpose. So, that is what we have substituted here and epsilon q is nothing, but B q u q so we write B q u q. Now, this I can open up and this is nothing, but del v p transpose B p transpose DB q u q integrated over the current volume of the element ok.

Now, virtual velocities are independent of the configuration. So, I can take them outside and it becomes del v p transpose integral of the remaining part is the vector. So, I can replace del v p transpose by del v p dot. And then I can take the nodal displacement out from the right hand side here, and I am left with the expression in the bracket. And this is nothing, but our tangent matrix corresponding to the constitutive relation of element e connecting nodes p and q ok. And this tangent matrix is given by equation number 94 ok.

Now, you note that the previous expression and the current expression there is a little difference ok. The difference is this p matrix over here ok, as you could see here so, B matrix over here contains a lot of 0s ok. So, you see there lot of 0s present here so, when you use equation 94 to compute the tangent matrix ok, then you will do a lot of 0 multiplication ok.

(Refer Slide Time: 21:16)

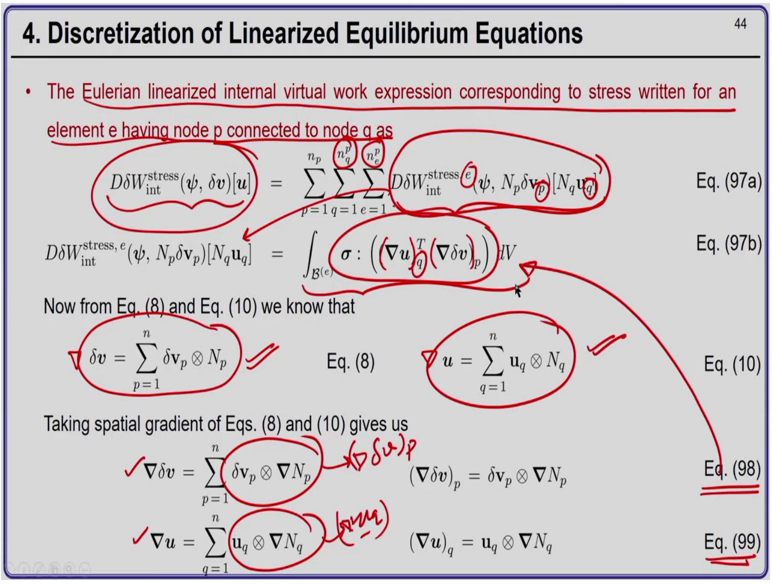


And this we note that because of the presence of 0s in this strain gradient matrix B, equation 94 is computationally very expensive ok. Now, we come to the form for the spatial constitutive matrix D ok. And this spatial constitutive matrix D can be obtained from the components of the spatial elasticity tensor c, as given by equation number 95 ok.

Where, in the present course we are considering a comprehensible neo hook material and the material that we discussed previously for that particular material this D matrix is given by following form ok. Notice that we have 0s and 0s over here and these correspond to the shear part ok. And these correspond to the tensile part ok.

Now, this lambda dash and u dash were given by following expression. And we also noted that for very small deformation determent of F was nearly equal to 1 that is j was nearly equal to 1. Therefore, mu dash was nearly equal to mu and lambda dash was nearly equal to lambda in that case, if mu dash is nearly equal to mu and lambda dash is nearly equal to lambda, then this D matrix is nothing, but your fourth order elasticity tensile that is discussed in the course on linear elasticity ok.

(Refer Slide Time: 23:03)



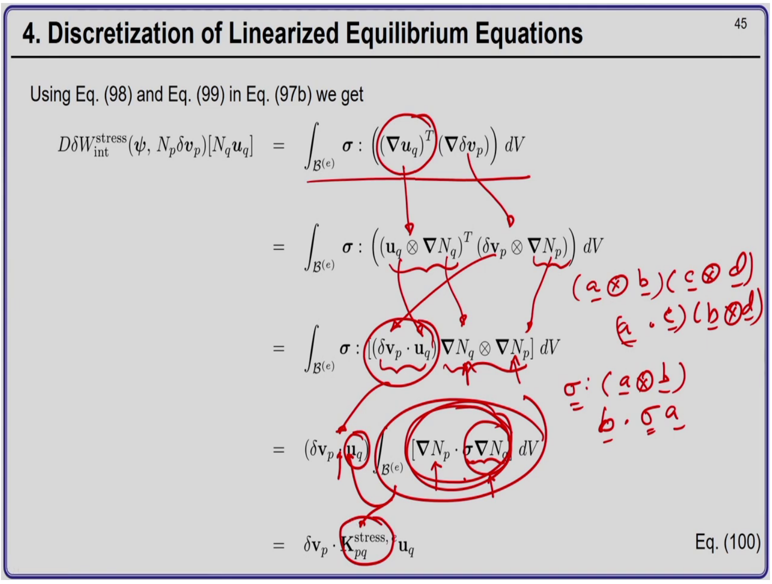
The Eulerian linearized internal virtual work expression corresponding to the stress, written for an element e having node p connected to node q is given by following relation ok. So, this is that Eulerian linearized internal virtual work expression corresponding to the stresses. And this is nothing, but the summation of the linearized internal virtual work corresponding to the stresses for an element e connecting nodes p and q ok.

So, here as usual n p e is the number of elements connected to node p, n p q is the number of nodes q connected to node p ok. And then this term can be written as the integral over the current volume of sigma double contracted with the gradient of u transpose corresponding to node q and the gradient of virtual velocities of node p ok. Now, I can recognize ok, I can recognize that the virtual velocities and the displacement were discretized by equation 8 and equation 10. .

Now, if I take gradient on both the sides ok, if I take gradient on both the sides as shown here, I will get my expression for the discretize gradient of the virtual velocities ok, which is here and the gradient of the displacement which is here ok.

And now these terms in the bracket correspond to the gradient of the virtual velocities corresponding to node p. And this is the gradient of displacement corresponding to node q ok. Now, I can substitute equation 98 and 99, I can substitute them in equation number 97 b. And once I do this ok.

(Refer Slide Time: 25:12)

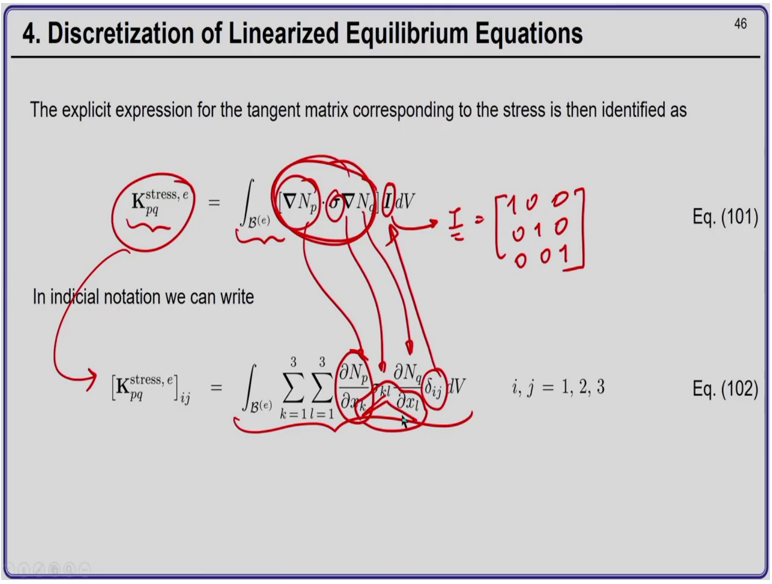


So, this is equation 97 b. Now, gradient of u corresponding to node q is u q tensor product gradient of N q transpose and this term is variation of virtual velocities so, the virtual velocities of node p tensor product gradient of shape functions N p ok. And now this is a tensor product b and c tensor product d ok. So, this is nothing, but a dot c b tensor product d ok. So, a is u q b is the virtual velocities corresponding to node p our b is the gradient of shape functions N q and d is nothing, but gradient of shape function N p ok.

And now, notice that this is a scalar this is a second order tensor ok. So, this is a scalar I can take it outside the bracket. And now, I have sigma double contracted with a tensor product b ok. So, this is a first order this is a vector this is also vector ok. So, sigma double contacted with b will be your b dot sigma a ok. So, this is what we have written here?

And remember sigma is second order tensor; the gradient of shape function N q is a vector. So, therefore, this is a vector and therefore, this vector dotted with this vector gives you a scalar. So, you have a scalar. So, you have to integrate a scalar over the current volume. And then I can write and because this is a scalar this is a scalar quantity, I can bring this here in between this dot product in dot symbol and displacement of node q and I can write this as a tangent matrix ok, corresponding to the stresses ok.

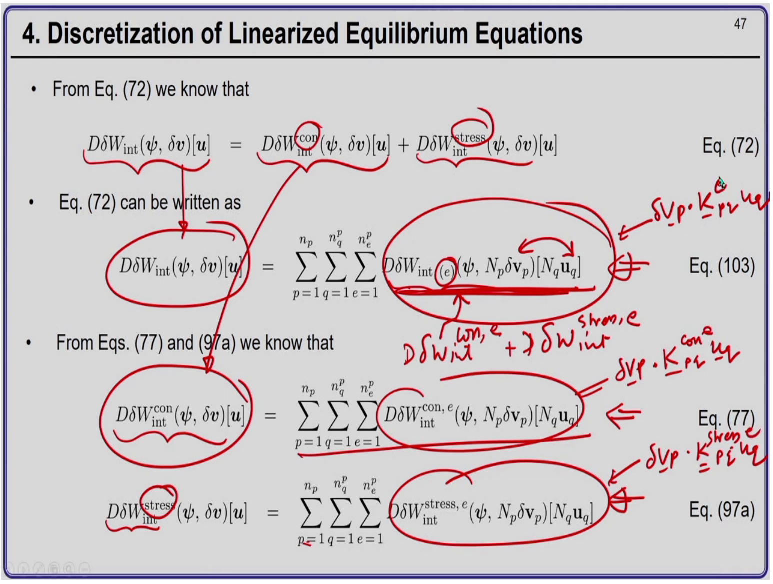
(Refer Slide Time: 27:43)



And the explicit expression for this tangent matrix corresponding to the stress is identified as, integral over the current volume, the scalar quantity, times the second order identity tensor I ok. So, this is a second order identity tensor that we have to have ok, you can obtain the tangent matrix corresponding to the stress using equation 101 ok.

So, in indicial rotation I can express my equation 1, equation 101 as following relation remember this delta i j is nothing, but this second order tensor ok. So, this gradient of N p is nothing, but del N p by del x j sigma is sigma k l and del N q is del N q by del x l ok. Remember this is a scalar this whole term is a scalar. So, we should have all the repeated index. So, k is repeated twice l is repeated twice with this in hand ok.

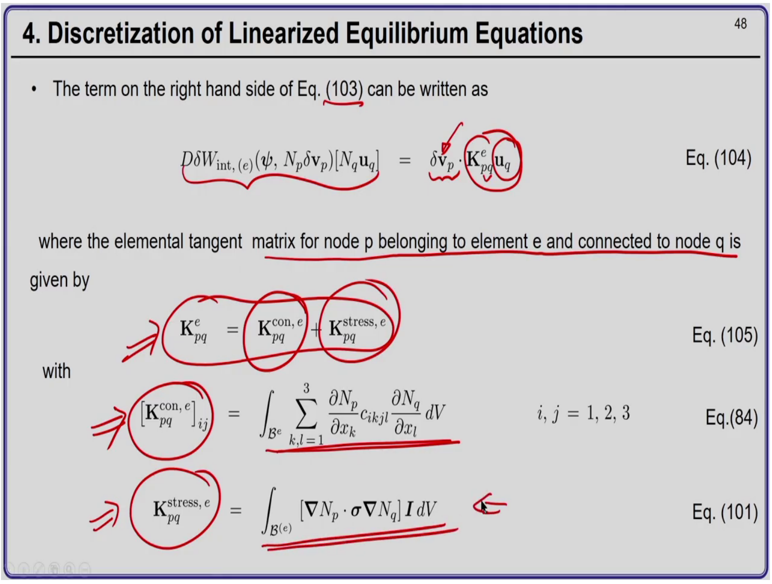
(Refer Slide Time: 28:53)



Now, the total linearized internal virtual work is the sum of the linear virtual work corresponding to the constitutive relation, plus the discretize linear virtual work corresponding to the stresses ok. Now, equation 72 we have we can write ok. So, the discretized linearize internal virtual work can be written as the discretized internal virtual work corresponding to element e connecting nodes p and q as equation 103 ok.

And we know that this is nothing, but D del W internal corresponding to constitutive relation plus D del W n corresponding to the stresses of e ok. So, this is the ok. So, this is here and this is here ok. So, the discretized linearized internal virtual work expression corresponding to constitutive is given by the equation 77. And the discretized linearized internal virtual work expression corresponding to these stresses, even is given by equation number 97 a ok.

(Refer Slide Time: 30:45)

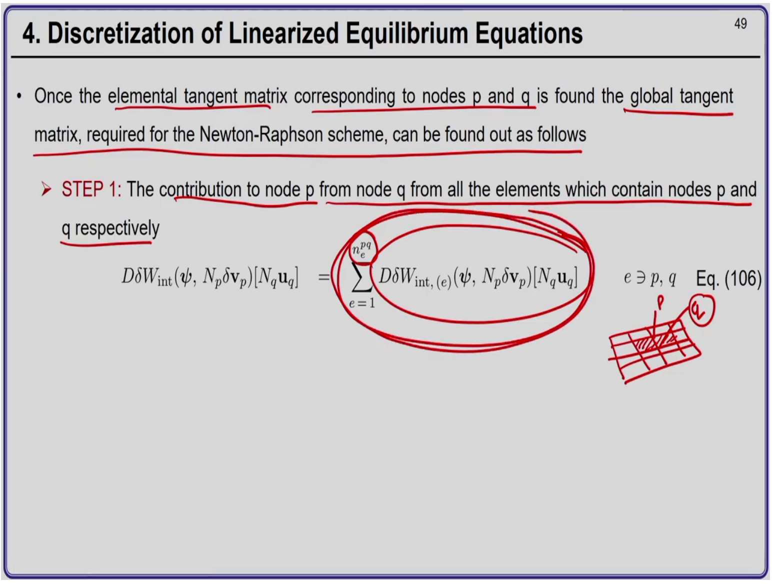


Now, I can write the term on the right hand side that is the right hand side of equation 103 ok. So, this I can write as the dot product of the virtual velocities corresponding to node p with the tangent matrix corresponding to element e connecting nodes p and q with displacement of node q ok.

So, I know that this is nothing, but del V p dot K con p q u q and this is nothing, but del V p dot K stress p q u q. And this I have written as del V p dot K p q u q corresponding to element e.

So, I can write the elemental tangent matrix for node p belonging to element e and connected to node q, as the sum of the constitutive tangent matrix and the stress tangent matrix ok. With the explicit expression for the constitutive tangent matrix is given by equation 84. And the explicit relation for the tangent matrix corresponding to the stress is given by equation 101 ok

(Refer Slide Time: 32:29)

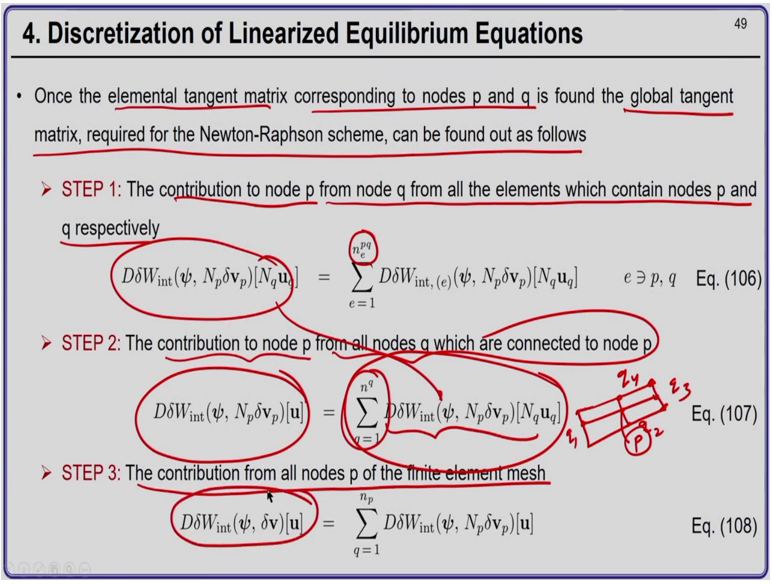


So, once the elemental tangent matrix which is this over here, this if we have found out corresponding to nodes p and q, then the global tangent matrix required for the Newton Raphson scheme can be find out as follows ok. In step one the contribution to node p from node q from all the elements which contain nodes p and q respectively is assembled ok.

So, you first assemble all the contribution to node p from node q ok, from the elements which contains both node p and node q. So, in a finite elemental mesh for example, you may have something like this and this is say node p and this is say node q. So, we have to first assemble the virtual work, internal virtual work from the discretized internal virtual work from all the elements which contain both node p and q.

So, for example, this element over here and this element over here, these two element contain both nodes p and q. So, we obtain the tangent matrix from these two elements and assembled to get the contribution to node p ok.

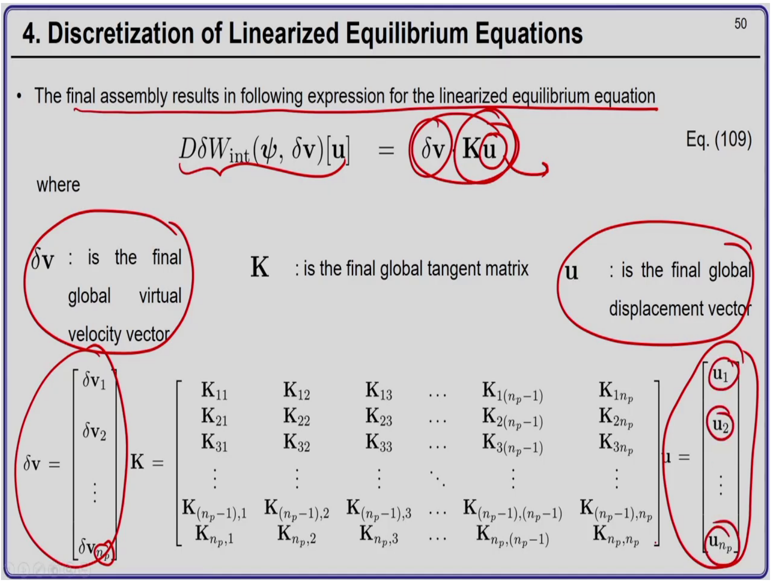
(Refer Slide Time: 34:00)



And then, next what we do? The contribution to node q from all nodes q which are connected to node p this is assembled ok. So, in a finite element setting this is your node p, then corresponding to node p you have q 1, q 2, q 3 and q 4 four nodes are connected. So, you take this contribution for each node which is connected to node p and then you assemble ok. So, this remember this summation sign means actually addition and assembly both ok.

So, once you have got the internal virtual work corresponding to the node p in a similar fashion, you do for all the other nodes in the finite element mesh. So, contribution for all the other nodes of the finite element mesh are then, assembled to get the final discretize internal virtual work for the mesh ok.

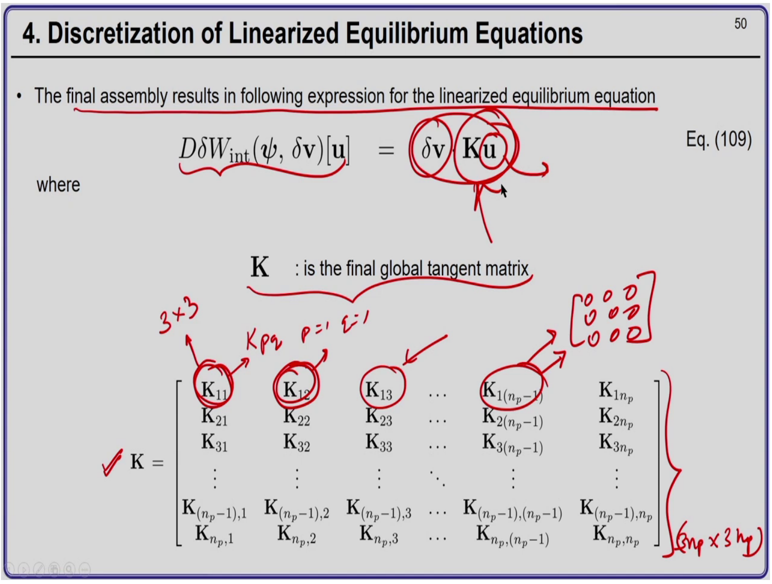
(Refer Slide Time: 35:12)



And this final assembly will result in the following expression. So, this discretize version of the linearized internal virtual work over the mesh is nothing, but velocity vector with the dot product with the global displacement vector ok.

Where the final global virtual velocity vector is given by this vector over here, where n p denotes the all the nodes present in the finite element mesh. And the global displacement vector that is the vector of unknown is the displacement of all the nodes present in the mesh and this tangent matrix ok. And this K what is called? The global tangent matrix ok.

(Refer Slide Time: 36:10)

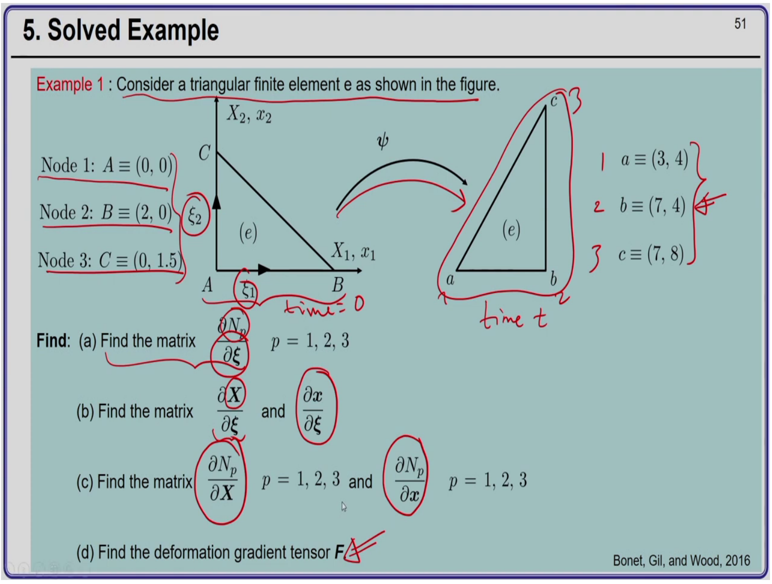


So, this global tangent matrix is composed of K 1 1 K 1 2 ok. So, K 1 1 is actually so remember we had K p q so, here p is 1 q is 1. So, K 1 1 means the contribution to the tangent matrix coming from node 1 interacting with itself. K 1 2 means the contribution to the tangent matrix coming from coming to node 1 because of a small change in node 2 ok. Like this you will get the final global assembled matrix.

And remember each of these matrix is of size 3 cross 3 ok. So, considering 3 degrees of freedom per node the total size will be 3 n p cross 3 n p. And remember a lot of these tangent matrices components maybe equal to 0 ok. So, for example, this may be a 0 by 0 0 0 matrix ok. So, from your connectivity matrix you can actually find out which of these components will be a 0 matrix ok.

So, once we have obtained our global tangent matrix, I can now move to setting up my Newton Raphson iteration ok. So, next we will do one small example to understand what we have discussed today. And we will do it for a simple to 2 D triangular element and we will derive most of these expression. So, that you understand what we have discussed today ok. So, next we do one example to understand the mathematical details that we discussed in previous slides ok.

(Refer Slide Time: 38:18)

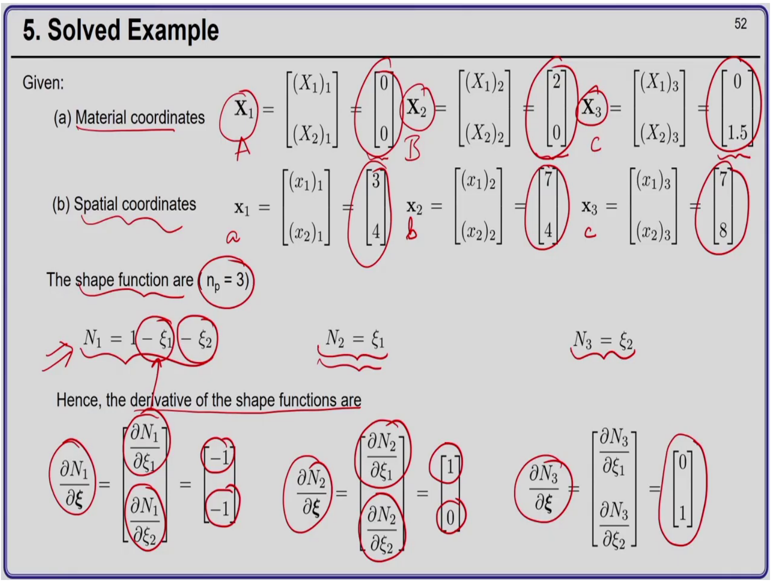


So, in this example you have a triangular finite element as shown in this figure ok. So, this triangular finite element deforms and it occupies this position say at time t ok. So, this say maybe this is at time 0 ok. And now, the nodal position of node A B and C are given here before the deformation and after the deformation the nodal positions a b c of node 1 2 and 3 ok. So, node 1 2 and 3 are shown here. .

And then, we have to find the matrix of the gradient of shape function with respect to the natural coordinates psi 1 psi 2. So, the direction of psi 1 and psi 2 are shown here and then, we have to find the derivative or the material coordinates with respect to the natural coordinates. And the directive or these spatial coordinates with respect to natural coordinates.

Then, we have been asked to find out the derivative of the shape functions of the node with respect to material coordinates. And also the derivative of the shape function of the nodes, with respect to the spatial coordinates ok. And finally, we have been asked to find out the deformation gradient tensor F ok. So, let us see how we can proceed? Ok.

(Refer Slide Time: 40:00)



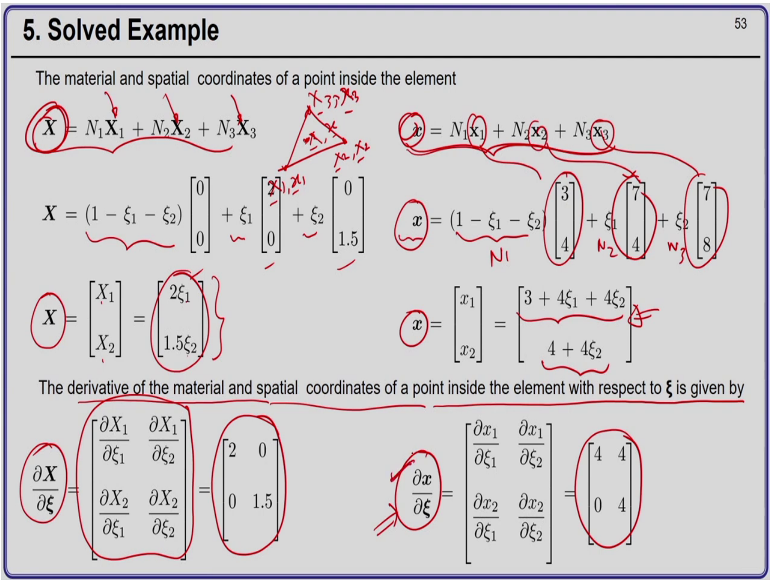
So, we have been given the material coordinates. So, let us say this is our X 1 this is 0 0 X 2 just which is 2 0. So, this is node A, this is node B and this is node C and this is 0 1.5 ok. And now, the spatial coordinates of the same nodes a b and c after the deformation are given to be 3 4, 7 4 and 7 8.

So, what we have done here? We have expressed the coordinates in array form ok. So, you see we have express the coordinates in array form and this is essential for finite element implementation so, that we can do these operations on a computer ok. Because, this is a simple example, we can do it by hand, but in actual situation we have to do it on a computer because, there are many many such finite elements ok. So, the shape functions of the 3 nodes. So, they are total 3 nodes and N 1 is 1 minus psi 1 psi 2 into is psi 1 and n 3 is psi 2 ok.

So, we can calculate the derivative of the shape function as del N 1 by del psi which is nothing, but del N 1 by del psi 1 and del N 1 by del psi 2 ok. So, if you look here del N 1 by del psi 1 ok. So, there are only minus psi 1 so, we get minus 1 del N 1 by del psi 2 there is minus psi 2 so, we get minus 1 ok.

Similarly del N 2 by del psi in array form is del N 2 by del psi 1 del N 2 by del psi 2 and N 2 is independent of psi 2 therefore, we have 0. And N 2 is directly pro directly equal to psi 1, so we get 1 ok. Similarly we can compute del N 3 by del psi and in the array form this is 0 1 ok. Once we have calculated the derivative of the shape function with respect to the natural coordinates, we next see that the material and spatial coordinates of a point inside the element.

(Refer Slide Time: 42:18)

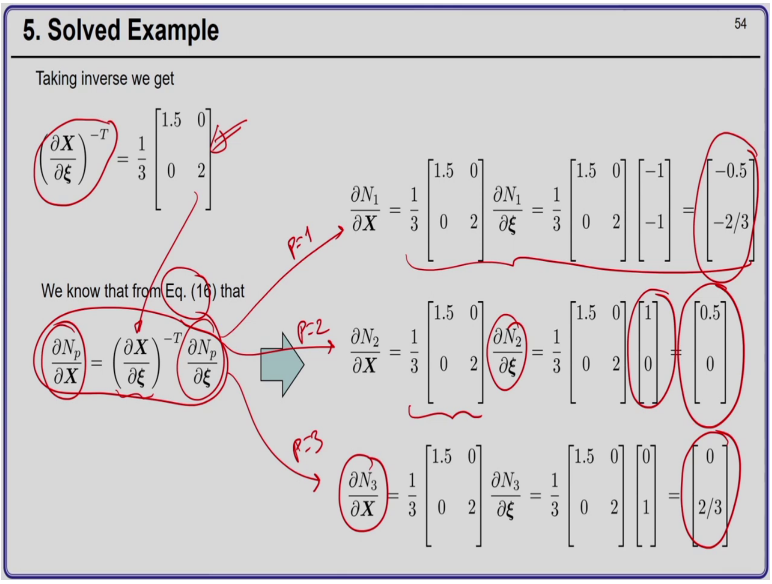


So, you have a triangular finite element and you want to get its material and spatial position with respect to its nodal say X 1, x 1, X 2 x 2 X 3 x 3 ok. So, you can get the position ok, current coordinates and the reference coordinate of any point inside the element with in terms of its nodal values ok. With this in hand I can substitute N 1, N 2, N 3 and the coordinates X 1, X 2, X 3 in the expression ok. And I can get the coordinate of any point inside the finite element in terms of psi 1 and psi 2 ok.

Similarly I can do it for the current spatial position and, I can get the current spatial position inside the same point in terms of the natural coordinates like this ok. So, these are the coordinates of the nodes these are the coordinates of the nodes. And these are the shape functions N 1, N 2, N 3 ok. So, I get the current spatial position of the point in terms of the natural coordinate psi 1, psi 2, psi 3. .

Now, I can compute the derivative of the material and spatial coordinate of a point inside the element, with respect to the natural coordinates using the formerly following formula. So, del X by del psi is given by this matrix and I already know that X 1 is 2 psi 1 and X 2 is 2 psi 2. So, I can get the matrix del X by del psi as 2, 0, 0, 1.5. A similar procedure when applied for the derivative or the spatial coordinate with respect to the natural coordinates, I get 4 4 0 4 ok.

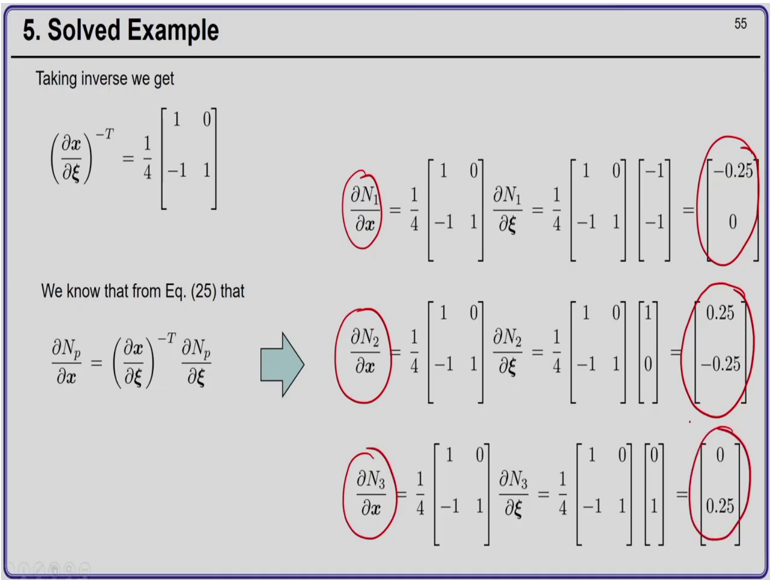
(Refer Slide Time: 44:53)



So, now, once I have del X by del psi and del X by del psi I can compute the inverse transpose remember we need this. So, I can compute the inverse transpose and with this formula ok, the equation number 16 the derivative of the shape function with respect to the material coordinates is nothing, but del X by del psi inverse transpose del N p by del psi. And now, I have my del X by del psi inverse transpose like this. So, that I can substitute and del N p by del psi, I have it from the previous slides ok.

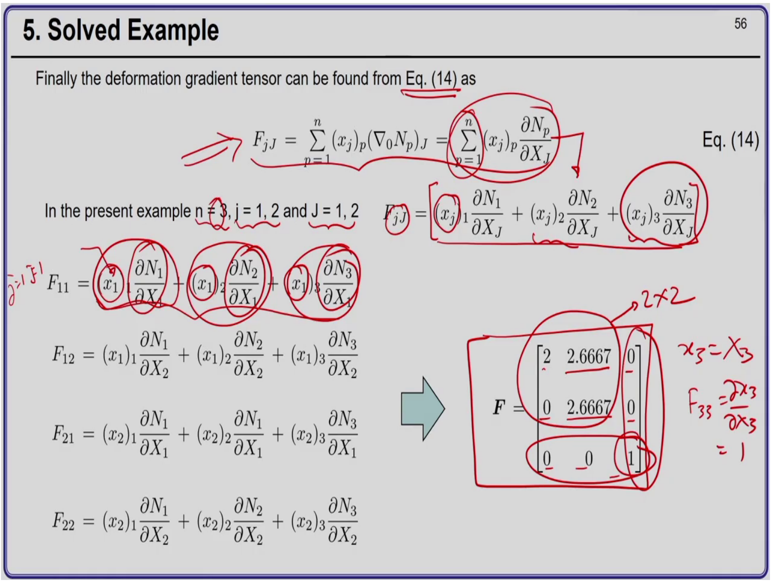
So, for P equal to 1 I can substitute and I can get the expression del N 1 by del X as minus 0.5 minus 2 by 3 for P equal to 2 so, this is P equal to 2 I can substitute del N 2 by del psi which is 1 0 and I get the del N 2 by del X. So, derivative of the second shape function, with respect to material coordinates at 0.5 0. And for the third node I can compute the third derivative, I mean the derivative of the third shape function with respect to material coordinate as 0 2 by 3.

(Refer Slide Time: 46:24)



Once you have done this I can follow a similar procedure and I can get the derivative of the first shape function, with respect to the spatial coordinates as 0.25 0. The derivative of the second shape function with respect to spatial coordinates as 0.25 minus 0.25. The derivative of the third shape function with respect to spatial coordinates as 0 0.25 once we have this in position ok.

(Refer Slide Time: 46:55)



I can now using equation number 14; I can compute my deformation gradient tensor using the formula given here ok. So, if I now see my n the total number of nodes in the element are 3 my small j varies from 1 to 2, because its a 2 dimensional problem and capital J varies from 1 to 2.

Now, if I expand this term explicitly in terms of p. So, I remove the summation sign and I get following expression. Now, using different values of j and J I can compute the different components of the deformation gradient tensor ok. So, let us say if I put small j as 1 and capital J as 1 this is my expression ok. So, what this means is? x 1 1 del N 1 by del X 1 ok, that is we have here. Then we will have x 1 2 del N 2 by del X 1 that is we have here and we have x 1 3 del N 3 by del X j ok. So, this x 1 1 means is the x 1 coordinate of node 1, this is the x 1 coordinate of node 2, this is x 1 coordinate of node 3 ok.

And we have already calculated the derivative of the shape functions with respect to the material coordinates from the previous slide and when we substitute and when we do all these operations, we get the deformation gradient tensor as 2 2.6667 0, 0 2.6667 0, 0 0 1. Remember I have added the last row in the last column with because, for the sake of completeness otherwise our answer will be this 2 by 2 matrix ok. We can also think x 3 was capital X 3.

So, F 33 was nothing, but del x 3 by del X 3 which is nothing, but because there is no deformation in the third direction therefore, the deformation gradient tensor will come out to be 1 ok. So, with this we come to the end of this particular module on discretization ok. Next we move to our final module, which will be on the solution procedures for solving the equations that we have obtained ok.

Thank you.