Fundamentals of Compressible Flow
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Module – 02
Wave Propagation in Compressible Medium
Lecture – 04
Wave Propagation in Compressible Medium - I

Welcome you all we are now in this course Fundamentals of Compressible Flow Module 2. So, in the first module we discussed about the fundamental aspects of fluid mechanics and gas dynamics that are used to cover in undergraduate syllabus. Now at higher level, in this compressible flow we will make use of those equations those properties and try to study the fundamental behaviour for the flows that are compressible in nature.

So, this is the first lecture on the module 1 and this title of this module is Wave Propagation in the Compressible Medium.

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At the end of the lecture we will come to know about some brief introduction to compressible flow and we will talk about some important parameters that are used specifically for compressible flow such as speed of sound, Mach number. And we also talk about wave propagation in the compressible medium means that under what circumstances we say the fluid can be treated to be compressible in nature.

Now, to address the fluid flow problems we will develop some fundamental flow equations that are routinely used for the flows that are compressible in nature, something about isentropic flow which is also equally important, because this isentropic flows are taken as the reference or basics for subsequent analysis. Now we will start this lecture.

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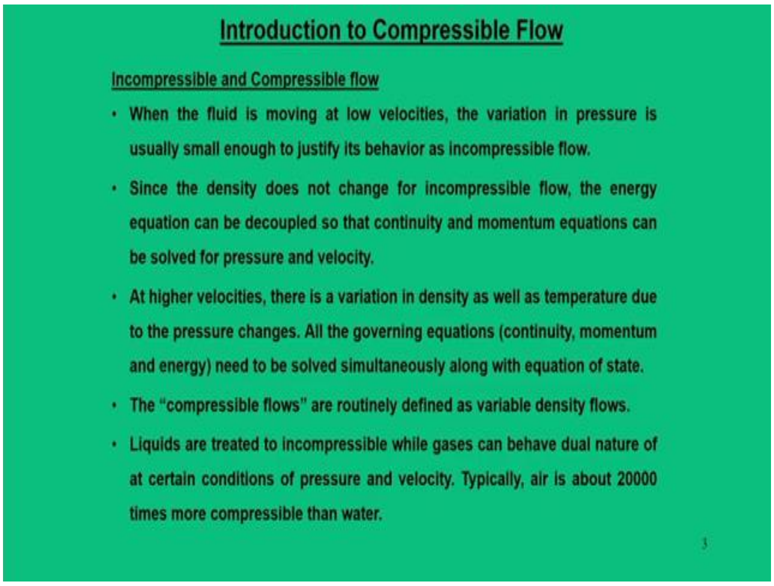

Now, we will revisit something what we discussed in the types of fluid flow that is incompressible and compressible flow, just to give some brief insight or revisit those concepts, I will just spell out some of the important aspects.

So, when the fluid is moving at low velocities the variation in the pressure is usually small enough to justify the behaviour of incompressible flow. That means to decide so that the flow to be compressible or incompressible, the pressure as well as velocities is important and for incompressible flow these pressure changes are very small.

Since the density does not change in the incompressible flow, the energy equations can be decoupled. So, that continuity and momentum equations are solved for pressure and velocity. Now, to address this flow phenomena, we normally encounter three major equations; continuity equations, momentum equations and energy equations, when the fluid is incompressible nature where density is a invariant quantity. So, it does not come as a specific parameter for calculation through these equations.

So, in normal circumstances the mass or continuity equation and momentum equations are solved simultaneously for velocity and pressure and to calculate the thermal energies or other types of energies or internal energy of the system we have to use the equation for energy equations explicitly or separately, but this is what we do in the incompressible flow.

Now, when you look at the compressible flow what we see is that at higher velocities the variation in the density as well as temperature to the pressure changes is large. So, in that aspects in terms of addressing those parameters we have to take two more other equations in addition to continuity momentum and energy we have to bring into two more equations that is equation of state and the entropy equations and that is specifically used when there is a directionality in the flow.

Hence the compressible flows are routinely called as variable density flows, the very first bottom line in this study is that for compressible flow we need the density changes to be a significant parameter.

Just to give some brief insight we say liquids are incompressible and in our case in a commonly use situations we say water whereas, gases are compressible in nature and so in our normal circumstances air is one of the fundamental gas. So, just to give you a number one can say that air is about 20000 times more compressible than water. So, this is how it makes the distinction between a compressible flow and incompressible flow.

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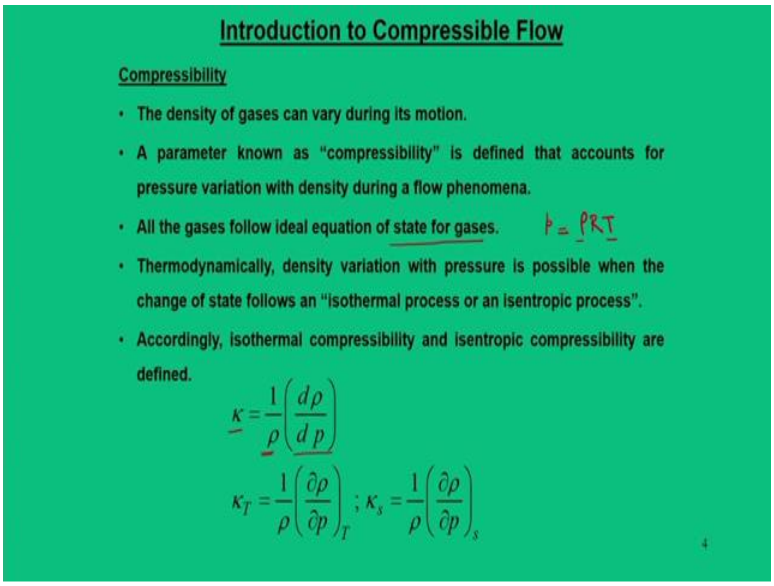

Now we will not deal about incompressible flow now onwards we will specifically talk about compressible flows, now to address this compressible flow the first fundamental parameter we are going to look at the is known as compressibility. So, it what it does is. So, it is given by this expression κ what it says is that the variation of density with respect to pressure.

$$\kappa = \frac{1}{\rho}\left(\frac{d\rho}{dp}\right)$$

$$\kappa_T = \frac{1}{\rho}\left(\frac{\partial\rho}{\partial p}\right)_T ; \kappa_s = \frac{1}{\rho}\left(\frac{\partial\rho}{\partial p}\right)_s$$

So, this is what the differential term which is used here variation of density with respect to pressure and this density change is taken as one of the parameter and with respect to its original density rho how this parameter changes. So, this is the fundamental definition of the compressibility term mathematically.

Now, let us use these equations logically for the flows to occur. So, as we say that since our situation is such that we will only use gases as a compressible medium. So, the very bottom line is that we can use the ideal gas or equation of state for the gas. So, if you say this equation of state we write $p = \rho RT$.

So, the pressure, density and temperature are related with a characteristics parameter R, now looking at these equations if we want to see that how the pressure changes with respect to density then one must say that what happens to temperatures. So, obviously, so while looking at this equation differential equations and we want to make any changes in the density then we must do something about temperatures.

So, this brings thermodynamics into picture under what circumstances I can change the density with respect to pressure. So, for that reasons one assumption thermodynamic sense in one assumption says that I can hold temperature is constant.

So, this is where the κ_T is represented as compressibility at constant temperature. So, these differential equations now becomes two ordinary differential equation now turns to be to a partial differential equation where this κ_T is looked at the density change at constant temperature or for isothermal process.

But having said this so, we call this as a isothermal compressibility, but having said these, but important issues that is very difficult to change the density although it is mathematically feasible or thermodynamically feasible, but it is very difficult to change the density with respect to pressure while maintaining temperatures to be constants.

So, for that reason another way of looking at changing this compressibility is defining a term called as isentropic compressibility κ_s. So, here what the thermodynamic assumption says that instead of temperature you keep entropy to be constants. So, that is what the it gives you an indication that one can think of an isentropic process where the compressibility parameter can be addressed or in general density can change with respect to pressure while maintaining entropy to be constant.

So, this basic definition tells at that the how density can change with respect to pressures.

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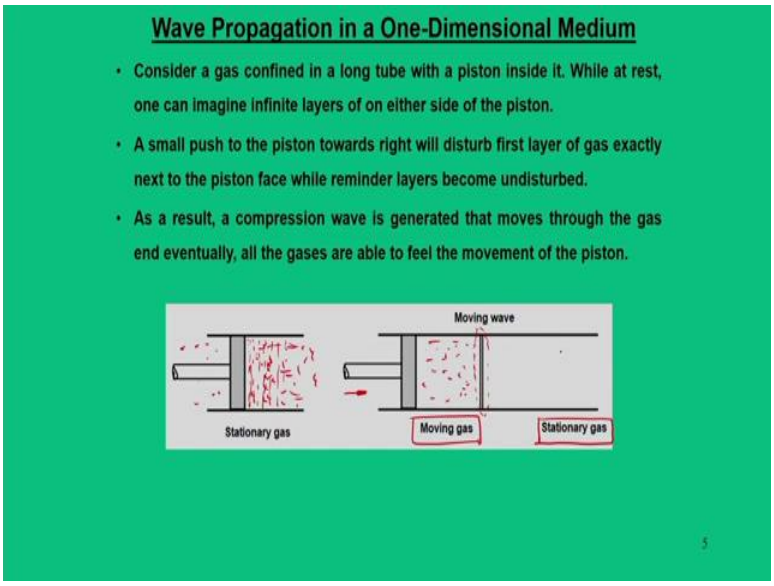

Now moving further, let us see that how density change with respect to pressure is addressed through isentropic process. So, for this reason let us consider the situation of what we call as wave propagation in one - dimensional mediums.

So, to bring out the simplicity we talk about a one - dimensional medium. So, for this one - dimensional medium the first thing what you see thermodynamically that we have a cylinder in which a piston is enclosed.

So, the entire idea is that the top part and bottom part or bottom wall of the cylinder we have, one side face we have the piston while the bottom surface of the cylinder is you think it to be a empty. So, that there is no physical wall.

So, ideally what we can see that this can be thought of consisting of a gaseous medium where lot of gas molecules we can find on either side of the piston and in the cylinder. So, statically so, in this situation we can say that gas is hypothetically at rest and piston is in equilibrium.

So, now what you do is that in the next instant you give a little bit of push to the piston now when you give a push to this piston what is going to happen these gases on the downstream side of the piston gets affected.

So, as if what we feel is that at the first instance, we say that the first layer of gases they gets affected subsequently they transfer the information about the movement of the pistons towards this next layer, subsequently this information keeps propagating to the down streams.

So, eventually what happens, the gas molecules which are just adjacent side of the piston gets disturbs and that disturbs the next layer of the gases. So, why it happens? To theoretically model this what we say is that there is a wave gets created and in the first instance we say let this wave be a compression wave. So, this compression wave keeps moving into in the stationary medium.

So, when it moves so what we see is that behind this wave whatever gases are there now they have make their changes because they got the information about the piston movement and, but as long as the wave does not pass through the stationary gas that is other side of the piston. So, they do not know about the motion of this wave; that means, the information what changes that happens through this moving wave, we can differentiate the entire stationary gas now as a moving gas other is stationary gas.

So, moving gas I mean the amount of the velocity which is given to the piston with same velocity the gases they try to move, while the other side of the moving wave, all the gases they are stationary. So, this is the philosophy how the information gets propagated through this movement of the piston.

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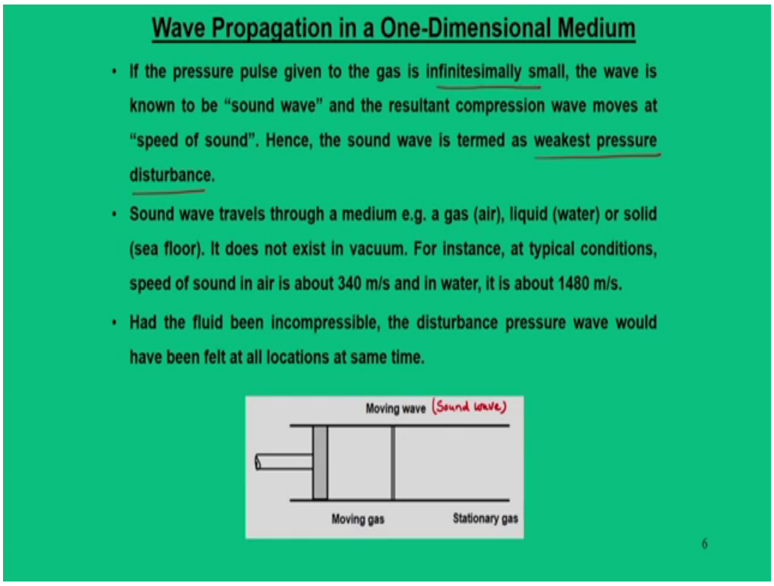

So, now in the beginning we say that this is a moving wave, now hypothetically you say that had this moving wave be a weakest disturbance. So, weakest disturbance means the disturbance is very small and since beginning we say it is a compression waves; obviously, it is the pressure parameter that is going to be a concern.

So, if this wave or the pressure pulse is happens to be very small then we call this moving wave as a sound wave or in other words we can say the sound wave is the weakest pressure disturbance. So, it travels always in the medium whenever we talk, disturbance is created, it moves at speed of sound.

Now, this is what we see that movement of the sound wave in a gaseous medium, even one can create a sound wave when the medium is liquid, we can have a sound wave when the medium is as well solid; that means, sound wave can travel in any medium that is gas liquid or solid, but it does not exist in the vacuum. So, it requires a medium to travel that is the reason in outer space we do not have a sound waves because there is no medium as such.

Now, when we actually see the when this pressure moves at the speed of sound we can also quantify what is its speed. So, in down the line we will derive this expression, but for the sake of continuity I can say and estimate for a typical conditions the speed of sound in air is about 340 m/s and that speed of sound in water is about 1480 m/s; that means, speed of sound is higher in liquids.

Now, had this situation be a incompressible situation, the pressure disturbances would have felt at every locations at same time. So, there is no question of sound waves ok.

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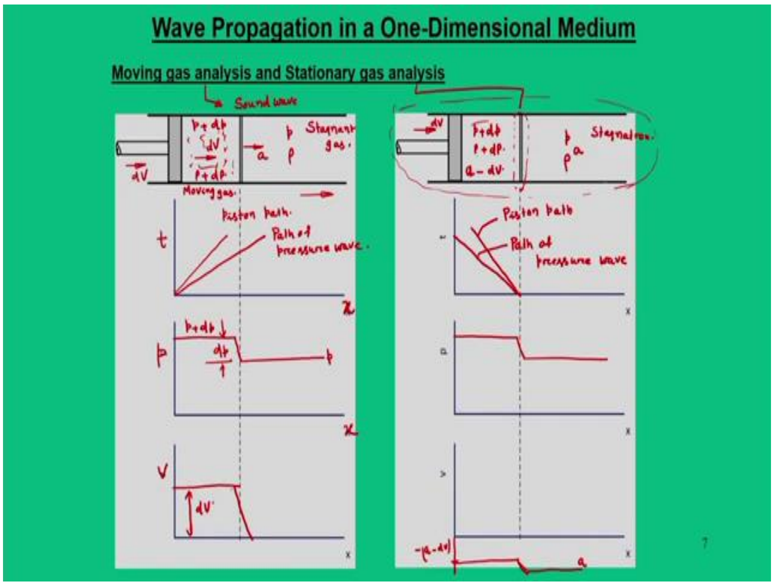

Now, we will bring about some mathematical orientation whatever I have explained through a analysis called moving gas analysis and stationary gas analysis and this falls under the domain that we are looking at a one - dimensional medium.

So, what we see is that. So, the first instance when you talk about the moving gas analysis we say that there is a moving wave it is moving at certain velocity. Now how does moving waves gets generated?

So, we give a small piston movement; that means, piston is given a velocity dV. So, when this piston is given a velocity dV the gases in one side of the piston they move at also velocity of dV and because of this is a moving gas, this gas this moves at speed of sound a. So, this moving wave or I can directly say it is a sound wave. So, it moves with a velocity a. Now let us see that, on this side we have stationary gas or stagnant gas, on this side we have moving gas.

Now, let us see that the when this gas does not see the sound wave its conditions are defined as pressure p density ρ and the other side of this, the corresponding parameter on other side would be p+dp because it is a very weakest disturbance and density also will change in this process that is $\rho + d\rho$. So, if I try to plot this an xt diagram where x stands for the distance along this piston and t stands for time; that means, we are looking at this wave at different time instance.

At one time instance when the wave is at this location, I draw a line. So, let us see that what happens. So, first thing it says that this line denotes the motion of piston. So, we call this as piston path. So, because this dV comes from this xt diagram through this piston path and the movement of this wave we can represent in this manner. So, we call this as path of pressure wave. So, this is how we see in xt diagram.

Similarly on pressure and distance diagram one can see that one side of the piston there is a pressure p+dp, but just at these vicinity of this moving gas, pressure again comes down to original pressure of the stagnant gas p.

Now, same thing when you draw this velocity versus x diagram. So, we can say that initially this piston has this velocity and all of a sudden this velocity drops to 0. So, this value is we say dV, this magnitude is dp. So, this is how we do in a moving frame of analysis. So, it means that I am looking at the wave sitting in the laboratory.

Now, let us see that I want to bring a stationary frame of analysis where the stationary gas analysis where we say we bring this wave to be stationary. So, all these plots in this side is the analysis for stationary wave which says that the wave is treated to be stationary. Now to address this how I bring the wave to be stationary so, effectively so, when I say this wave to be stationary I have to artificially initiate a flow that changes similar properties.

So, in the beginning stage we say same pressure and density and we say this to be a very thin region across which the property changes and in one side the pressure is p, density is ρ and velocity is a and in other side we say the pressure will be p+dp density will be $\rho + d\rho$, now if we say this wave to be stationary then; obviously, I should get a velocity $a - dV$ for the other side of the gas.

So, this is your stagnant situation. Of course, this side also flow is stagnant. The conditions at one particular time instance the condition will be p+dp, $\rho + d\rho$. So, I should subtract $-dV$ because the piston movement is originally in these directions.

So, the speed that happens will be a and $a - dV$. So, likewise one can draw from this particular point; that means, here we have to draw with this wave as the starting point. So, from there I can draw this piston path, I can also draw the moving fluid and path of wave. The pressure plot will remain same but velocity will be something different.

So, we have to bring a reference line which is much below this because the directions are opposite where we say it is $-(a - dV)$ and from there it will start, finally it will end with a, because directions are taken in the opposite directions when you look at in this frame of reference. So, I hope I make you understand about this concept and in the subsequent analysis we will now recall only this particular figure which will come into our equation.

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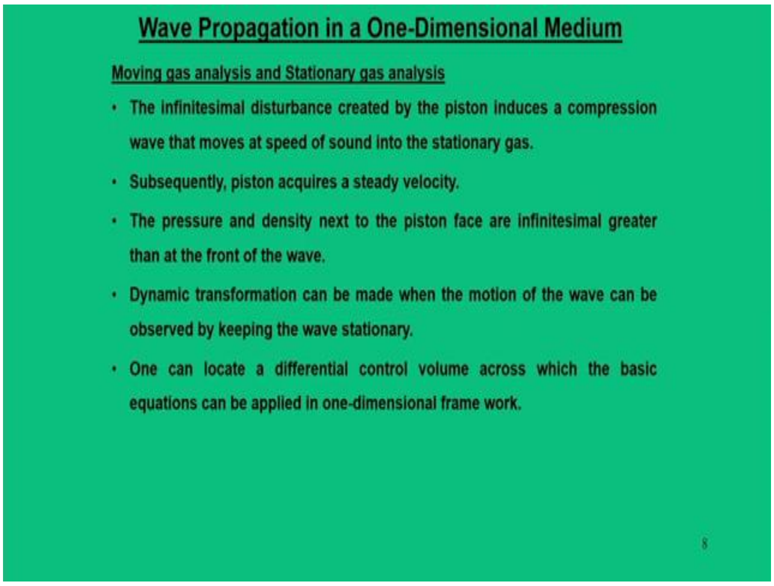

So, whatever I told I will just brief about it, say in moving gas analysis and stationary gas analysis, what is we looked at an infinitesimal disturbance created by the piston induces a compression wave that moves at speed of sound into the stationary gas.

Now, at one particular instant of time the piston attains a steady velocity and becomes equilibriums. So, when it happens the pressure and density next to the piston space face are infinitesimally greater than that of wave; that means, there is a slight increase in the pressure and density. So, in this process we can make a dynamic transformation when the motion of the wave can be studied by keeping wave to be stationary.

Now, to do the mathematical treatment we have to create a differential control volume across which the basic equation of motions can be addressed through one - dimensional framework.

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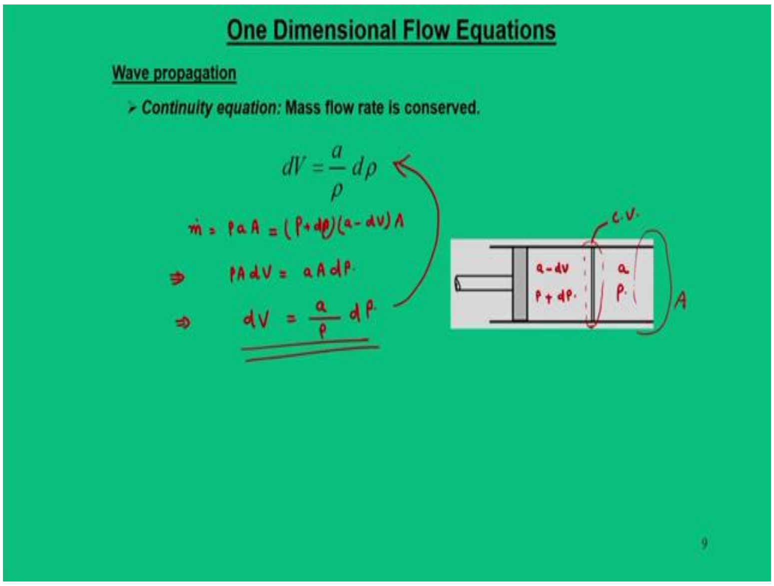

Now let us see how we are going to address. So, first thing we are going to address is the continuity equations so; obviously, when you talk about the continuity equations, we have to say is say this is how you have this control volume.

So, here are the parameters of interest are a, ρ in one side and in other side of the wave we have $a - dV$ and $\rho + d\rho$ and this thin region of control volume, we are going to see medium is one - dimensional, we assume that there is no mass or there is no heat and there is no work interaction. Having said this, the first equation that is going to be written is the continuity equations.

So, we can write $\dot{m} = \rho a A = (\rho + d\rho)(a - dV)A$ and we are saying this entire area to be A on both sides of the wave. So, if you simplify these equations what we are going to get is that you get a term $d\rho dV$ which can be neglected and this $\rho a A$ will get cancels and finally, what we get is $\rho A dV = a A d\rho$

And the end expression is which you are going to get is $dV = \frac{a}{\rho} d\rho$. So, this is nothing but this equation. So, this is the equation, which we get from the continuity equations.

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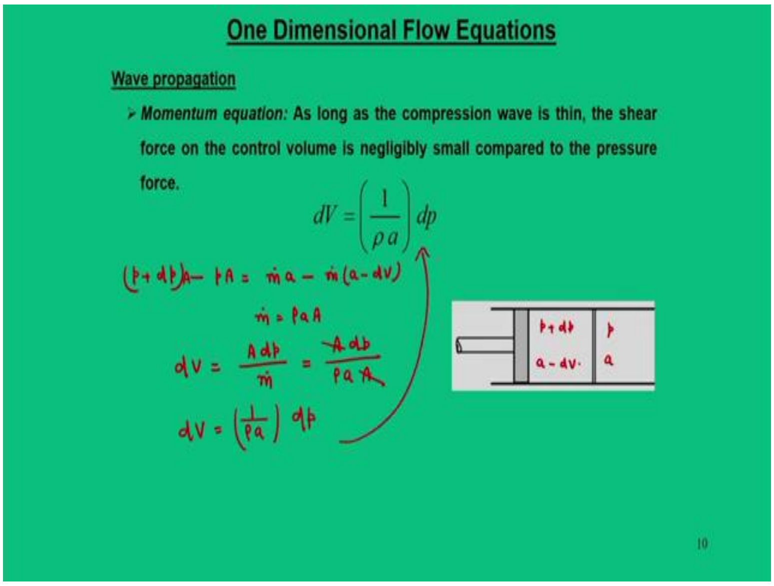

Moving back to the momentum equations here the parameter of interest will be p and a; here will be $p + dp, a - dV$. So, the momentum equations can be written as by considering the Newton's law for this one - dimensional medium. So, we say

$$(p + dp)A - pA = \dot{m}a - \dot{m}(a - dV)$$

So, one can simplify these equations. Also one can write $\dot{m} = \rho a A$ which is we get from the continuity equations.

After expanding these equations will get cancelled so many terms. So, there will be term that is going to cancelled is pA, pA will get cancelled; $\dot{m}a, \dot{m}a$ cancelled. So, ultimately what we get $dV = \frac{Adp}{\dot{m}} = \frac{Adp}{\rho A a} = \left(\frac{1}{\rho a}\right) dp$. So, this is what we get from this momentum equations.

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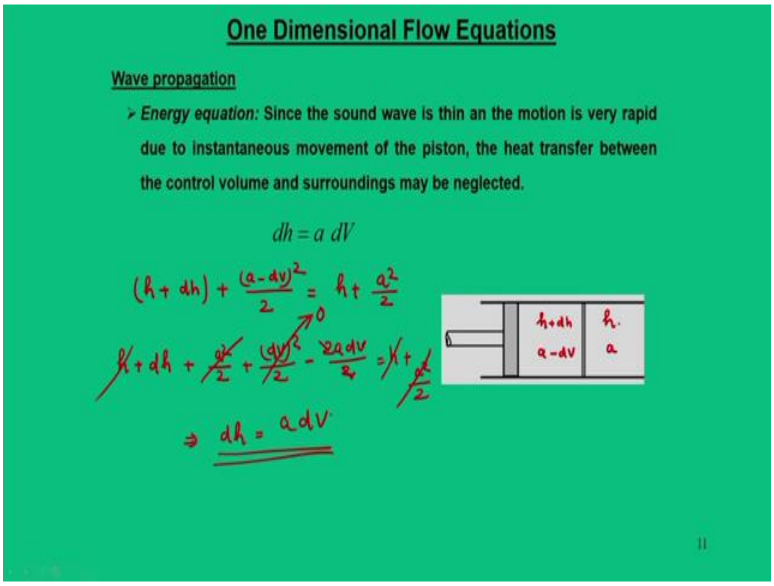

Now, moving further when you talk about energy equations, here I like to emphasize there is no work interaction, there is no heat interactions, the parameters of interest are $h+dh$ and h and where velocities are a and $a-dV$ and one thing we need to emphasize that we do not consider the elevation that is what we say the change in the elevation is negligible.

There is no change in the elevation. So, for that potential energy term we will not come into account. So, for this reason we can write this energy equation beginning as $(h+dh)+\frac{(a-dV)^2}{2}$ that is one side of the gas, other side will be $h+\frac{a^2}{2}$ and this equation can be simplified $h+dh+\frac{a^2}{2}+\frac{(dV)^2}{2}-\frac{2adV}{2}=h+\frac{a^2}{2}$. Finally, simplifying we get $dh = adV$ and finally, we arrive at this relation.

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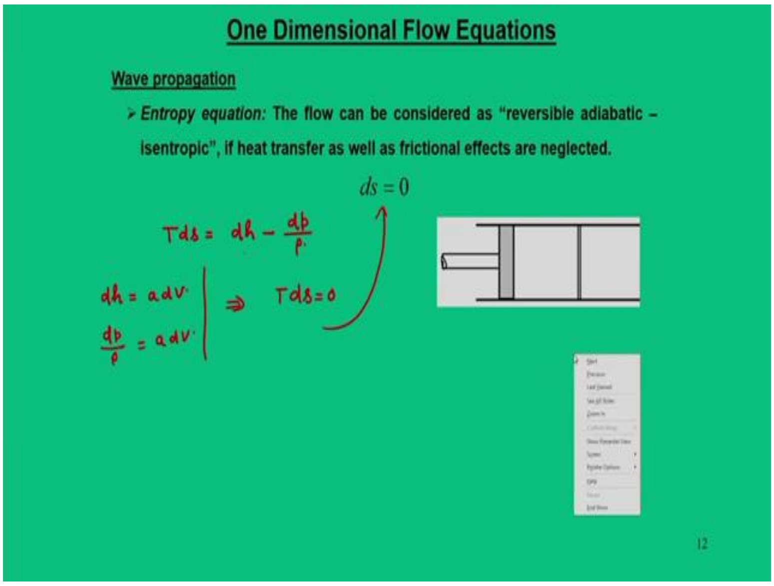

So, having said this we say that this we will now revisit the entropic equations to recall this entropy equation first thing what we see is that we recall this Tds relation, one of the Tds relation says that $Tds = dh - \frac{dp}{\rho}$.

Now, for this particular problem from the energy equation we get $dh = adV$ and from momentum equation we get $\frac{dp}{\rho} = adV$. So; obviously, when you put this equation we say $Tds = 0$ which means $ds = 0$.

So, this gives a very important inference, what says that entropy change for the whatever problem we say is there is no entropy change. Hence we say that when a weakest disturbance is created in the medium through a sound wave, the entire process can be treated to be isentropic. So, that is the emphasis what we are going to get from this analysis.

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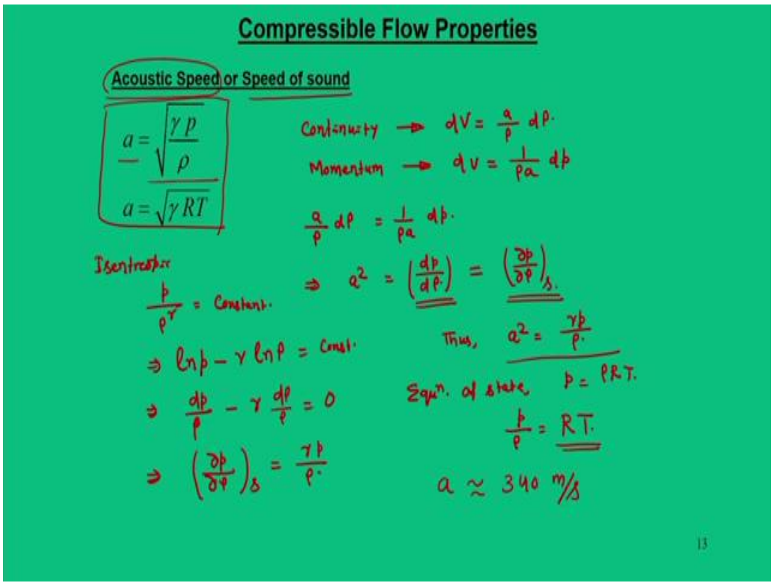

Now, we will again see that when we say that this process is isotropic process then we will see that what speed the wave moves. So, when these particular speed we call this as a speed of sound or many books also talk about this as a acoustic speed almost both are of same nature so, but most routinely used term is we say it is a speed of sound.

Now, let us derive the speed of sound which is a, we can relate this to term as $\sqrt{\frac{\gamma p}{\rho}}$ or we say that $a = \sqrt{\gamma RT}$. Now to do this what we have to recall now from continuity, we get $dV = \frac{a}{\rho} d\rho$ and from momentum equation we also get $dV = \frac{1}{\rho a} dp$.

So, when we recreate these two equation, what we get $\frac{a}{\rho} d\rho = \frac{1}{\rho a} dp$ then we can get a term as $a^2 = \frac{dp}{d\rho}$ and we also proved that although it is a differential equations. So, we also prove that when we define the speed of sound the process happens to be isentropic. So, these differential equations, a more appropriate way of representation would be in representing in a partial differential equation $\left(\frac{\partial p}{\partial \rho}\right)_s$.

Now let us evaluate what is this $\left(\frac{\partial p}{\partial \rho}\right)$. So, for an isentropic process we say $\frac{p}{\rho^{\gamma}} = C$. Now, this equation we can write in terms of logarithmic that is $\ln p - \gamma \ln \rho = C$. Now you differentiate, we say $\frac{dp}{p} - \gamma \frac{d\rho}{\rho} = 0$.

So, this becomes when we represent in terms of isentropic way, we can find out is $\left(\frac{\partial p}{\partial \rho}\right)_s = \frac{\gamma p}{\rho}$. So, thus this $a^2 = \frac{\gamma p}{\rho}$. So, this is how we get for an isentropic process, we say that is the square of speed of sound is function of pressure and density.

Next thing comes that for this equation can be further simplified by recalling equation of state. So, we say equation of state $p = \rho RT$. So, from this equation we can find $\frac{p}{\rho} = RT$.

So, when I say this and we can put it here and finally, we will end up with this important expressions speed of sound in two forms, one is $\sqrt{\frac{\gamma p}{\rho}}$ and other is $\sqrt{\gamma RT}$ and under atmospheric conditions when you put gamma for air and temperature to be 298K or 25ºC.

Then this we can evaluate the speed of sound and in most appropriate term we say the speed of sound is in the order of around 340 m/s under normal circumstances.

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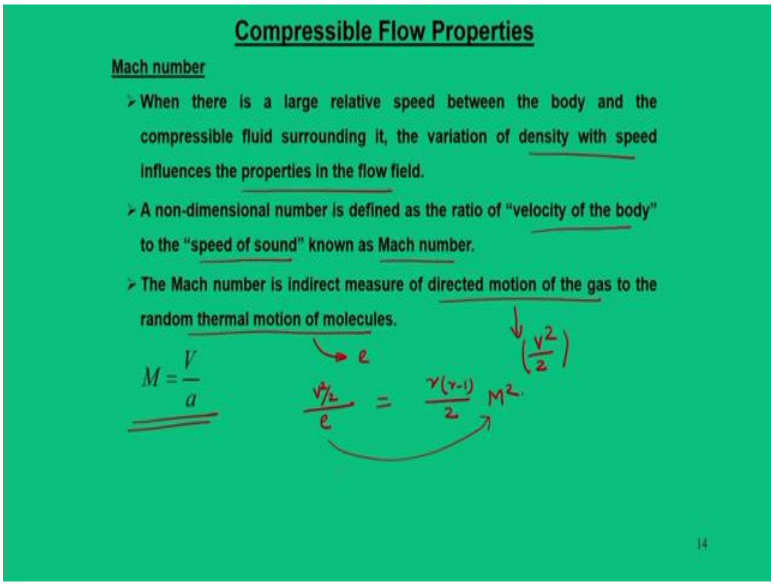

Now we will move further to define another properties that is called Mach number. So, when I define the speed of sound that is the disturbance that moves in a medium, but why that disturbance moves, but at the same time we say that your body is moving with certain velocity; that means, your moving gas also has certain velocity.

So, considering that when there is a relative velocity between the body and the compressible fluid, the variation of density influences the properties in the fluid. So, for which we define a non - dimensional number known as Mach number that relates the velocity of the body to the velocity of the sound. So, $M = \frac{V}{a}$.

Now interestingly this Mach number relates the direct measure of the directed motion. This directed motion is nothing, but that relates to the kinetic energy $\frac{V^2}{2}$ and also to the random thermal motion. So, this random thermal motion is with respect to internal energy. So, interestingly one can find out that ratio $\frac{V^2}{2}$ that is the kinetic energy to the internal energy. If you can calculate or simplify based on for a common calculations, this ratio happens to be $\frac{\gamma(\gamma-1)}{2}M^2$. So, essentially what I am trying to say here is that the ratio of kinetic energy to the ratio of internal energy is a function of Mach number. So, one can say the Mach number is also a indirect measure.

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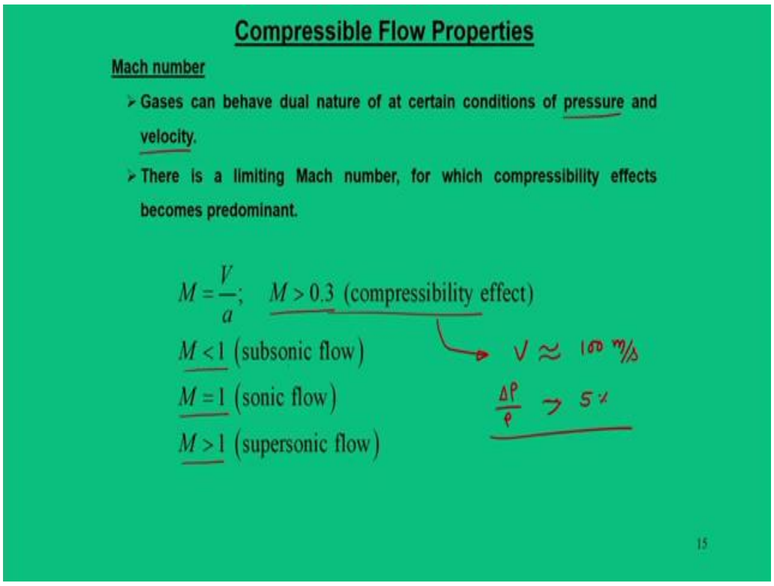

Now, moving further what we see in this for the gases, we say that the gases normally behave as a dual nature. Dual nature means under some situations, gases behave as a incompressible medium or gases can be considered as a compressible medium. So, what is that circumstances.

So, that essentially we see that these all depends on the pressure and velocity; that means, if the gas is moving at low velocity it may be treated as a incompressible medium because the pressure changes is small, but if the gas moves at very high velocity the pressure changes are very significant one cannot assume to be a incompressible medium.

So, for which we define this Mach number as a limiting case. So, a non - dimensional number that comes into picture is the Mach number which is the ratio of $\frac{V}{a}$ and it has been shown that if the Mach number is greater than 0.3, so, we can say the compressibility effect is predominant in the medium.

So, based on this the all high speed flows are decided in terms of Mach number. When the Mach number is less than 1 it is a subsonic flow, when the Mach number is equal to 1 it is a sonic flow, when Mach number is greater than 1 it is a supersonic flow. We will discuss about in this course in the subsequent lectures, but the most important point to be addressed here is that under what circumstances the compressibility effect comes into picture.

And this condition turns out to be fact that for this condition mach number your velocity is the hardly in the range of 100 m/s; that means, when your body is moving at more than 100 meter per second we expect a change in the density that is $\frac{\Delta\rho}{\rho}$ to be higher than 5%.

So, this is how the standard typical definition normal layman sense one can give that when the density changes are higher more than 5 percent the medium can be treated to be incompressible.

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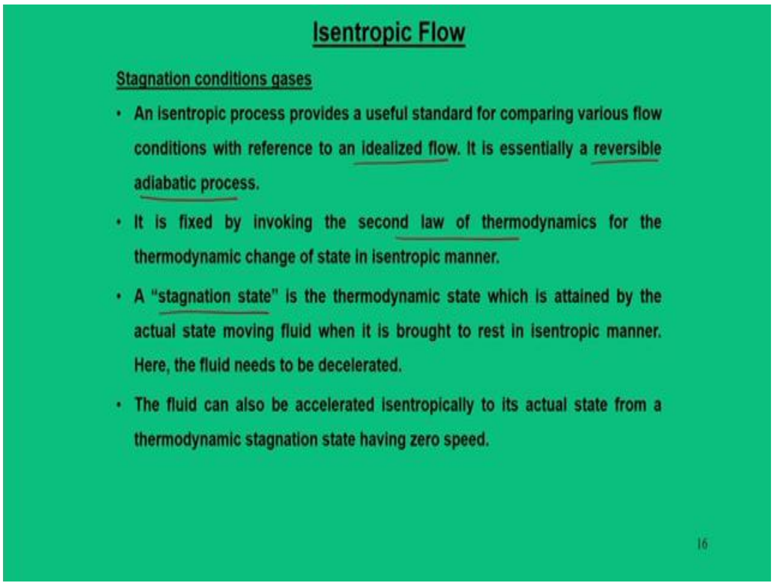

Now, moving further we will now move on to the next topic that is isentropic flow, since we say the medium to be isentropic when there is a disturbance occurs. So, based on the isentropic flow we will now define some properties which is known as stagnation properties.

And to define this stagnation conditions in a compressible medium we say that these are kind of hypothetical properties. So, before I explain further just to give some insight that in an isentropic process gives a useful standard for comparing various flow conditions with respect to an idealized flow and this isentropic process we call this as to be reversible and adiabatic process.

So, when you say the process to be reversible adiabatic flow and we get this information from the second law. So, considering this as one of the input and consequence of the second law, in our compressible flow situations we defined some states which is called as "stagnation states". So, the stagnation state is a state where the fluid is treated to be a reservoir. So, fluid happens to be in a state where as if it is lying in a reservoir, but there is a thermodynamic meaning to the state.

So, here we will now give some these thermodynamic meanings to the state as a stagnation state for the fluid; that means, any fluid might be moving, but it will have a hypothetical state which is known as stagnation state. How this hypothetical state will achieve, when the fluid is isentropically slowed down to 0 velocity.

So, this is how the thermodynamic definition of a stagnation state is all about. So, the fluid can be accelerated isentropically to axial states from a thermodynamic stagnation state having 0 speed. There is another angle one can interpret that one can accelerate a fluid from a stagnation state. Either a moving fluid can be brought to rest or stagnantal fluid can move to a active state.

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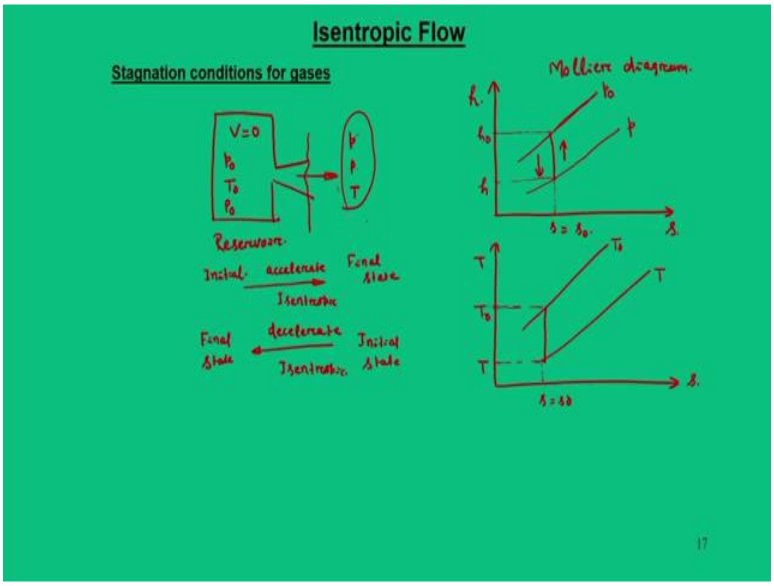

To define these things what I am trying to say here that we now say that as if we have a reservoir having certain conditions of pressure p_0 temperature T_0 density ρ_0 these are the properties. So, to study the stagnation conditions, we define these things in a standard way as p_0, T_0, ρ_0, but it happens to be at 0 velocity.

Now, we want to create a flow from this reservoir. So, we say it is a reservoir. So, one can imagine that this fluid is at very high pressure high temperature and density such that it can induce a flow to happen in certain passage. So, we are allowing this flow to happen in one of the passage by creating a hole into this reservoir and we allowed it to go to a state.

So, this is how we defined this active state; that means, p, ρ, T all these parameters we say as active state. So, active state means as if this flow gets generated from this reservoir.

So, hypothetically what it says is that the fluid can accelerate from reservoir to arbitrary state or fluid can decelerate from its active state to the stagnation state. So, if this is your initial stage, one can accelerate this fluid to final state. We can accelerate the fluid and this process is isentropic, other angle is that if we assume this to be an initial state and we come to the final state. So, here we have to decelerate the fluid and the process is also isentropic.

So, in other words what we are saying any arbitrary fluid state has a thermodynamic situation which is known to be a stagnation state and that condition is reached through an isentropic process and the corresponding notations what we give we for pressure arbitrary state is denoted as p corresponding stagnation state is denoted as a p_0 similarly, T_0 and ρ_0. This is how we view this.

Now let us see that how it means to us in terms of a fundamental diagram which we normally represent for an isentropic flow what we call as Mollier diagram. This Mollier diagram has x axis as entropy and y axis as enthalpy and on this Mollier diagram thermodynamically one can represent constant pressure lines. So, this constant pressure lines for our case we can say because in our case our two constant pressure lines one is p, other is p_0.

So obviously stagnation state will have higher pressure. So, it is denoted as a p_0 and what we say is that when we are going from either static state to stagnation state or from stagnation state to the static state. So, in this case we get the first case we either you can come this way or you can go this way. So, on this process we say entropy remains constant.

So, for this process one can also draw the temperature entropy diagram here the curves will be little bit steeper this is also $s = s_0$ and the temperature terms we can say T_0 line, this is T line. So, this is how one can say we call this as a static temperature and we call this as a stagnation temperature, here we call it as a static enthalpy and here we call it as a stagnation enthalpy and this is how the concept behind the static temperature, static pressure and static enthalpy.

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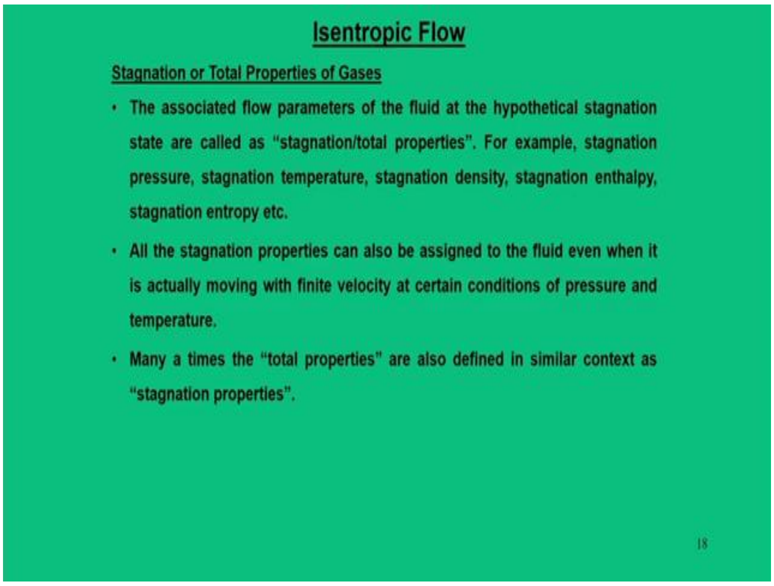

So, whatever I told if we want to just summarize what we can say that the associated fluid parameters of the fluid in a hypothetical stagnation state are called as "stagnation properties." So, one can have stagnation pressure, stagnation temperature and stagnation density. All these stagnation properties can be assigned to the fluid when it is actually moving with a finite velocity at certain conditions of pressure and temperatures. Many a times the "total properties" are also used similar to "stagnation properties."

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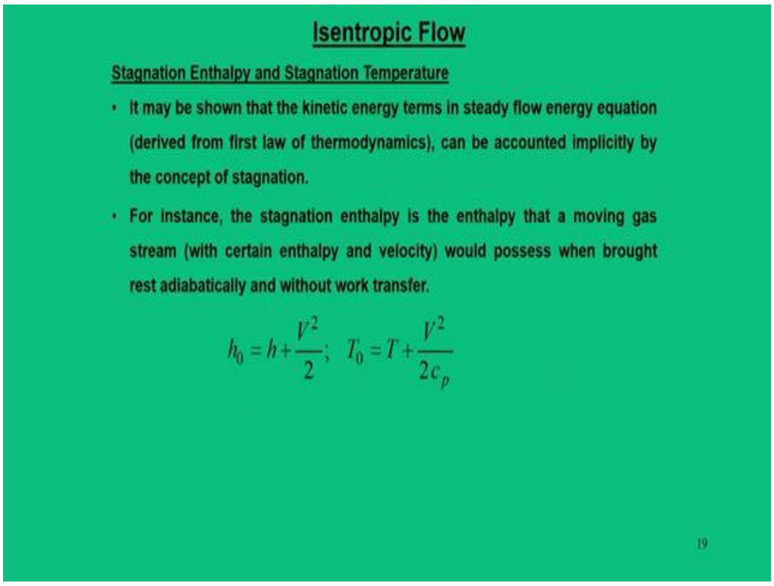

One other important inferences that we get from this stagnation enthalpy and stagnation temperature is that the term associated with velocity. For stagnation enthalpy we say $\frac{V^2}{2}$ and for stagnation temperature we say $\frac{V^2}{C_p}$ what we see is a dynamic term. So, if I say stagnant enthalpy or stagnation temperature implicitly this dynamic term is already taken into account.

So, this is how, what is the advantage of talking about stagnation temperature or stagnation enthalpy or stagnation pressure in addition to its static value, because the velocity components are already incorporated in these equations.

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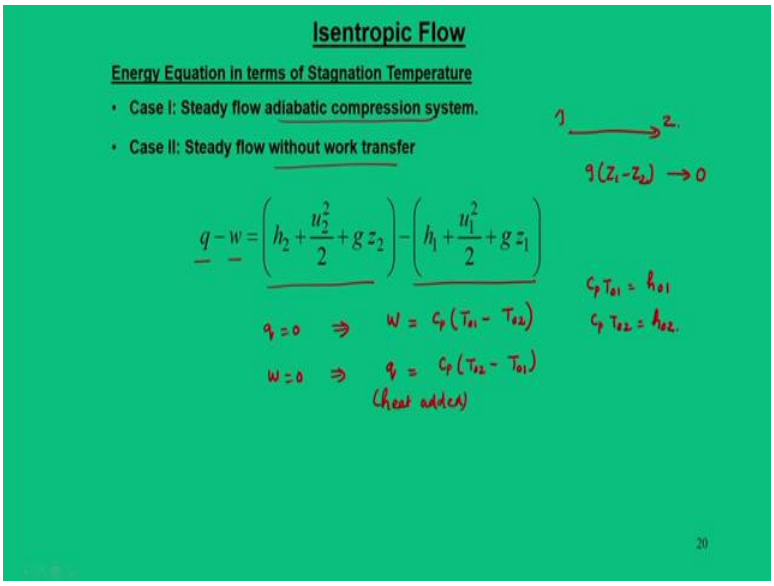

Just to moving further whatever analysis we do we did it for the last case. Now, we can just revisit that analysis just to say that just to analyze the energy equations in terms of stagnation temperatures.

So, the first thing what we are going to study that if the flow is steady, for an adiabatic compression system if the flow is steady without work transfer. In all our analysis we never consider work transfer and heat transfer, just to say for the sake of work transfer and heat transfer and we defined this stagnation properties how these equations we can address.

So, for that we talk about a process one which goes from 1 to 2, now when it is when some system goes from 1 to 2 we can write the energy equations in this form.

$$q - w = \left(h_2 + \frac{u_2^2}{2} + gz_2\right) - \left(h_1 + \frac{u_1^2}{2} + gz_1\right)$$

Now in this equation if both 1 state and 2 states are at same elevations, we can neglect $gz_1 - gz_2 = 0$ and all the analysis we consider for unit mass of the fluid and when it goes from 1 to 2, the heat transfer and work transfer are related through these energy equations that is q - w.

Now, from the first case when the flow is steady, but it is a adiabatic compression systems, so, we get q to be 0 and this will turn out to be $w = C_p(T_{01} - T_{02})$ where we say $C_pT_{01} = h_{01}$ and $C_pT_{02} = h_{02}$ and in other situation, the second case when it is without work transfer so, we say w to be 0. So, the heat added $q = C_p(T_{01} - T_{02})$.

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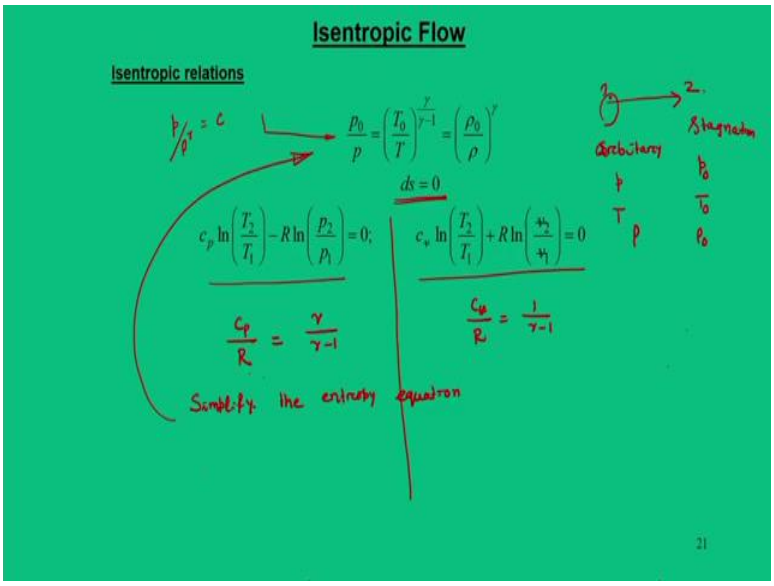

Now, the last part of this topic is the isentropic relations which is a very commonly written as for an isentropic process where $\frac{p}{\rho^{\gamma}} = C$. So, for that process a commonly used relations we can write $\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{\rho_0}{\rho}\right)^{\gamma}$

So, here the thermodynamic process which goes from 1 to 2. Here we consider 1 as arbitrary state, 2 as stagnation state. So, when you say arbitrary state we say p, T and ρ and when you say stagnation state we say p_0, T_0, ρ_0.

So, by recalling this ds to be 0 and this equation as this entropy equation as this, one can find out these relations with respect to ds by 0 and using these equations one can write and also we have to use the relations like $\frac{C_p}{R} = \frac{\gamma}{\gamma - 1}$ and we say $\frac{C_v}{R} = \frac{1}{\gamma - 1}$.

$$C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right) = 0 \qquad C_\upsilon \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{\upsilon_2}{\upsilon_1}\right) = 0$$

And once we simplify the entropy equations we can frame these most important relations what we call as the static versus stagnation relation. So, with this I conclude for this lecture for today.

Thank you.