Fundamentals of Compressible Flow
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Module – 07
Lecture - 20
Compressible Flow with Friction and Heat Transfer

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Welcome to this course, Fundamentals of Compressible Flow. We are in module 7 that is Compressible Flow with Friction and Heat Transfer. Till this point of time we have analyzed the compressible flow in one dimensional medium and also two dimensional medium and we discussed about isentropic flow, normal shock, oblique shocks, expansion waves.

Now, after having done all these things, we are now going to study what will happen to a moving flow when we add or extract heat from the flow, this is one aspect. Second aspect is; if you increase the friction what is going to happen to this compressible flow.

There are some significant inferences that will get out of this analysis and in fact, in our earlier analysis we discussed about this compressible flow with heat transfer.

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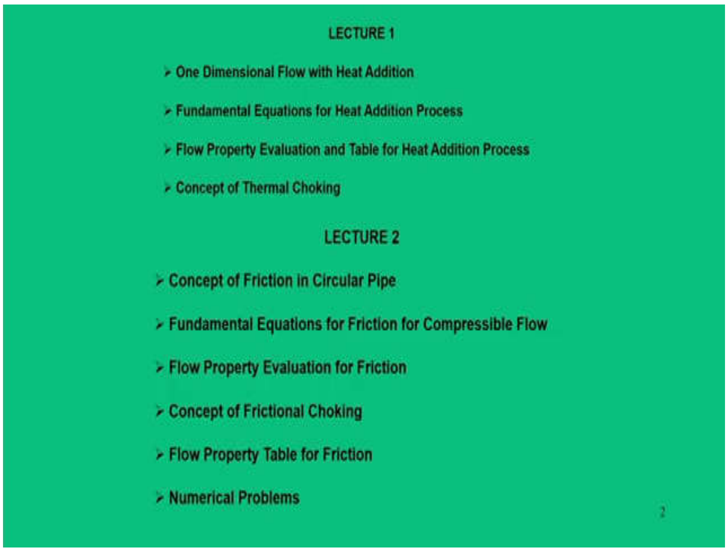

Now, here we will discuss about the compressible flow with friction. So, in the previous lecture we have analyzed the one dimensional flow with heat addition in a compressible flow. We found out its fundamental equations, then we evaluate the flow properties like pressure, temperature, density, etc at the inlet and the exit and based on these flow property evaluations we framed certain relations which are prevalent for one dimensional flow with heat addition process, but later on it was found that the equations are cumbersome to analyze.

So, based those equations we developed a table, what do you called as gas table for heat addition process. And while in the process of this discussions, we also introduced the concept of thermal choking; that means when the speed of the flow becomes sonic and this sonic condition is achieved through heat addition, then we call this flow to be choked thermally.

So, with same logic we are now going to analyze what is the effect of frictions. Now, when I say increase or decrease the friction I mean that we are adding some extra length to the flow so that friction becomes more. If I want to reduce friction, then you have to reduce the length of the duct. So, this is the concept of increasing and decreasing the friction. Until this process of time we are only talking about the one dimensional duct, but here we are going to be very specific that duct is a circular in nature; that means, it is a circular pipe flow.

So, in this pipe flow analysis we will frame these equations. There are specific reasons for this, because the flow in a circular pipe, most of the fundamental relations are known and that makes the basics for our analysis to deal with friction in the compressible flow.

Then based on these fundamental equations in a similar philosophy, we will discuss about the flow property evaluation. Then we will introduce another term which is known as frictional choking; that means, flow is choked or flow is brought to sonic conditions through friction.

And with this choking conditions and the fundamental equations, we will define a property table for the frictions, then based on our understanding we will solve some numerical problems. So, this is the summary of this lecture which we are going to analyze today.

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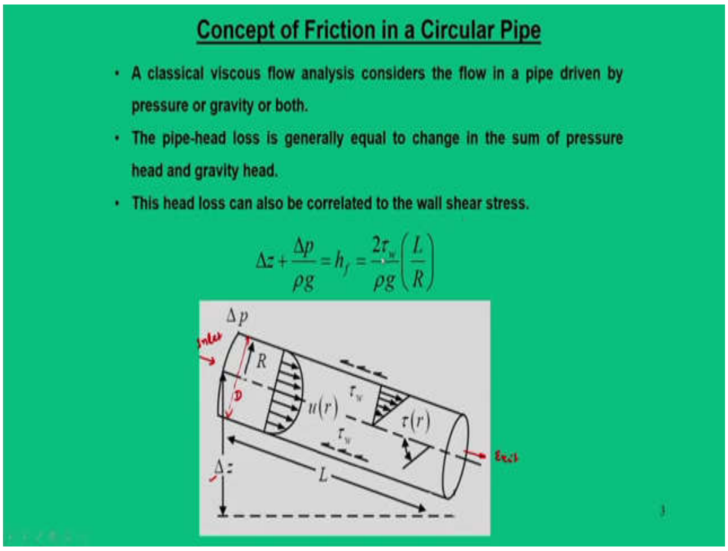

Now, just to give some brief introduction to the concept of friction in a circular pipe; we all know that the classical fluid mechanics problems involves a fully developed flow in a circular pipe. As shown in the figure, what do we have? We have inclined pipe. So, that there is a elevation difference between inlet and exit and this elevation difference is defined by Δz. So, if we say this is inlet and this is the exit.

So, this elevation is given by Δz and of course, since a flow has to occur, there is a pressure difference Δp. This the dimension of the pipe, it has a length L and radius R or maybe we can say diameter D. So, we can say it has a diameter D.

Now, for a fully developed flow in a pipe, we can say that the velocity profile which is u which is a function of r is shown in this figure and this velocity is maximum at the central line and it is 0 on the wall or inner wall and based on this things the velocity is maximum means; at that point of time at the centre line, the shear stress is 0.

Now, basically when you deal with the friction, the friction is caused due to the shear stress in the wall. So, as you can see that for the flow, the opposing force is the shear stress on the wall as shown in this figure that is τ_w and in fact, it acts at the entire inner surface of this pipe and this shear stress is minimum or 0 at the central line and it is maximum at wall.

So, we call this as a wall shear stress and this distribution of shear stress with respect to radial direction is linear. So, with this philosophy when you deal with the head loss calculations h_f that is nothing but the total head loss in a circular pipe. We can represent as Δz that is the due to elevation and mostly another term that is due to pressure difference $\frac{\Delta p}{\rho g}$ and in fact, we have in our earlier classical fluid mechanics problem UG level, we correlated this head loss as $h_f = \frac{2\tau_w}{\rho g}\left(\frac{L}{R}\right)$ where is the L is the length of the pipe, R is the radius. So, this is how the head loss is correlated to wall shear stress. In fact, the main intention is that how to find out the wall shear stress?

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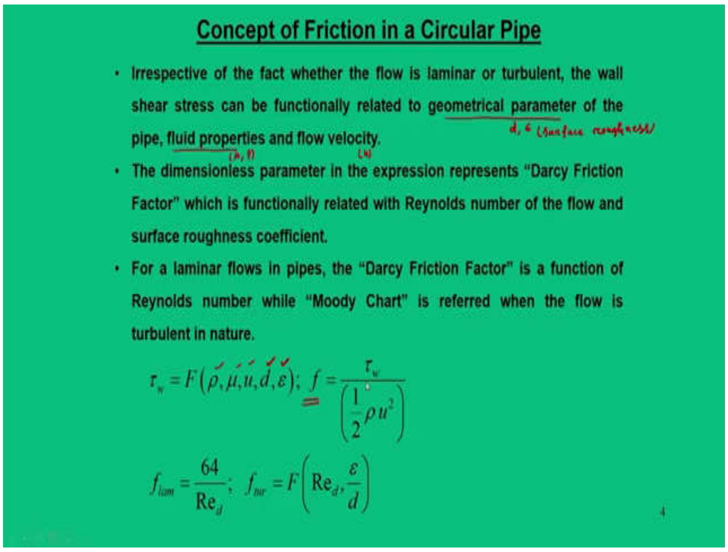

To find out the wall shear stress now, we have many models. So, the typical approach is normally we have to say whether the flow is laminar or turbulent. So, depending on this the wall shear stress can be functionally related to the geometrical parameters of the pipe, flow properties and flow velocity.

As you see in this equation, the τ_w is written as a function of ρ that is density, μ fluid viscosity, u fluid velocity, d diameter of the pipe, and ε which is the surface roughness. So, here as I said that geometrical parameters involve diameter and surface roughness. So, this is what the geometrical parameter. Fluid properties are μ and ρ and flow velocity is u.

Geometrical parameters are d and ε. So, ε is nothing but the surface roughness. So, this is how it is functionally related. Now, through a dimensional analysis, one can find out a non dimensional factor. Non dimensional factor, which is known as friction coefficient f and this friction coefficient is related to shear stress of the wall divided by $\frac{1}{2}\rho u^2$.

$$f = \frac{\tau_w}{\left(\frac{1}{2}\rho u^2\right)}$$

So, these this is a general expression to find out the friction coefficient factor. Now, later on based on many theories it was found that the friction factor is dependent on whether the flow is laminar or turbulent.

Now, when the flow is laminar, let us say the laminar value of friction factor is a fixed number which which is a function of Reynolds number and it is related as $f_{lam} = \frac{64}{\text{Re}_d}$, Reynolds number based on the diameter. So, that means once you know the flow Reynold number, this laminar value of friction factor is a fixed number which is $\frac{64}{\text{Re}_d}$.

But what happens when the flow is turbulent? So, that case we refer this functionally with respect to Reynolds number and another factor which is known as surface roughness factor, which is $\frac{\varepsilon}{d}$, where ε is the magnitude of the surface roughness and divided by diameter. So, it is a non dimensional number and typically this is obtained through a moody chart, when the flow is in turbulent nature.

So, this is how we deal with the friction factor and wall shear stress. In fact, for our compressible flow analysis these two are most important factor that is f and τ_w and we are going to see what is its effect when there is a compressible flow in a circular pipe.

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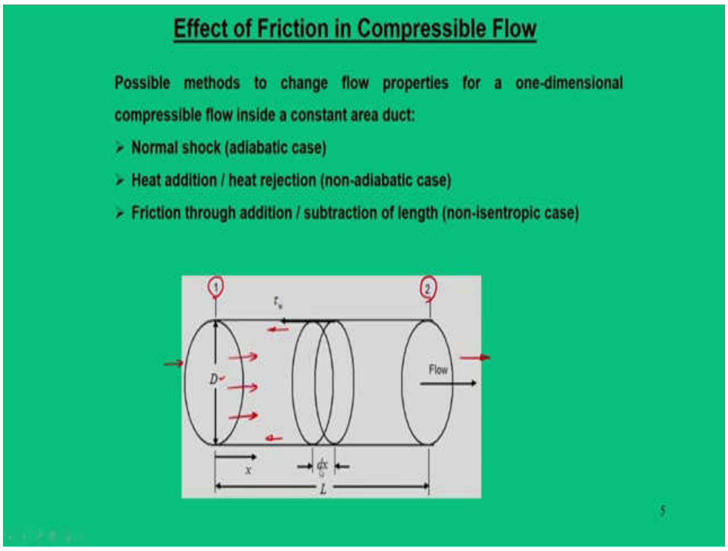

So, if you recall our previous analysis we developed some correlations for normal shock heat addition process and in fact, these two are the possible method to the change the flow characteristics in a one dimensional flow in a constant area duct.

So, these two facts we have already discussed that how a normal shock or heat addition process can alter a compressible flow in nature, but here we will discuss about the role of friction. Role of friction I mean; whether I should increase the friction or decrease the friction, that is add or subtraction of length and in fact, when we are dealing with friction it is a non isentropic process.

So, what you see here is that; we allow a compressible flow which is entering in a pipe of diameter D and length L and we say that inlet condition is 1, exit condition is 2. Now, when the flow is entering into this pipe; obviously, the wall shear stress that opposes, this magnitude of this wall shear stress is τ_w.

What you are looking at a differential length dx in which we are going to analyze our fundamental equations. So, in a differential length dx we are going to revisit the fundamental equations.

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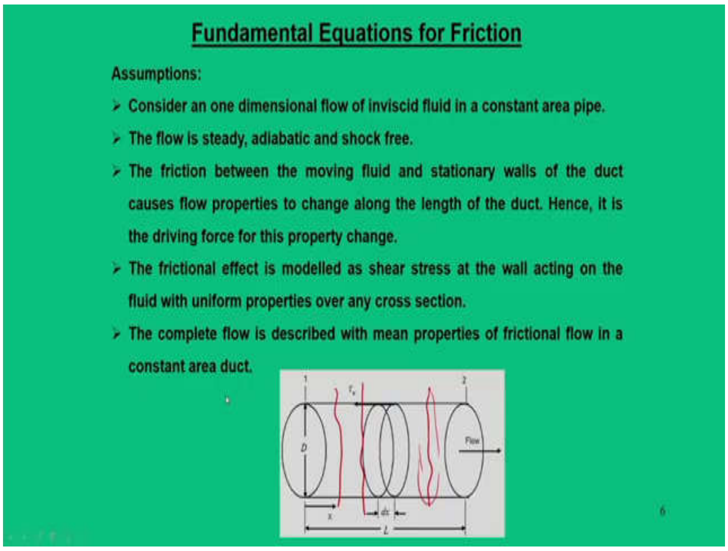

So, before you do with these fundamental equations, we try to say that there are certain assumption involved that is the flow is one dimensional in nature, but fluid is inviscid, but flow is compressible in a constant area pipe.

The flow is steady, adiabatic, and shock free. The friction between the moving fluid and the stationary wall of the duct causes the flow property to change along the length of the duct. So, the driving factor here is the friction, but here when you deal with this friction, it is modelled as a shear stress which is acting on the wall and the fluid is having uniform properties over any cross sections.

So, what I mean say; if you look at the fluid properties at any cross section, across this cross section the properties are uniform. So, that is the assumptions involved in our study and the complete flow is described with mean properties of the frictional flow in a constant area duct.

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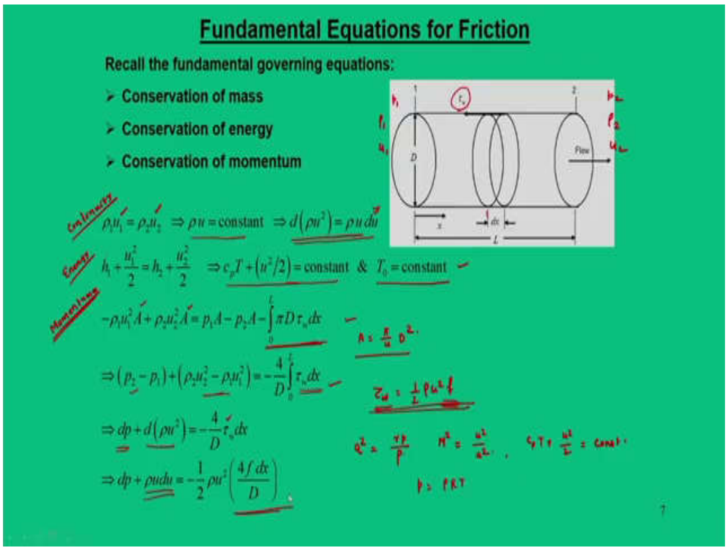

So, here we are going to revisit three fundamental equations; first is continuity. So, as you see that if you have $\rho_1 u_1$ and $\rho_2 u_2$, the continuity equation holds good $\rho_1 u_1 = \rho_2 u_2$ and since we are analysing in a differential section dx, so, I can express this term $\rho u = \text{constant}$; from this expression you can evaluate a term what will happen to $d\left(\rho u^2\right) = \rho u\, du$. So, these things we are going to use in this subsequent analysis.

So, next equation that we are going to study is energy. So, energy equation is not altered, because there is no heat added or there is no work is done on the fluid. So, we say $h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$. So, this says that $c_p T + \left(u^2/2\right) = \text{constant}$. So, our total temperature does not change that remains constant, but the most fundamental equation that we are going to use for friction is the momentum equations.

So, if you look at the momentum equations of 1 D flow, what extra term that is going to come from the surface force is the force due to this shear stress on the wall. So, it is this shear stress acts on the circumference. So, its corresponding force term would be $\pi D \tau_w dx$. So, we have to integrate the shear stress over the entire length. So, that is what the integral term is there.

So, apart from this there are pressure terms that is p_1, pressure is p_2 and area is a constant area duct. So, A is known. So, then we can use this momentum equations and try to simplify. So, when you simplify we have to use this concept that since it is a circular pipe, we have to write as $A = \frac{\pi}{4} D^2$.

When I use this $A = \frac{\pi}{4} D^2$, then these momentum equations gets simplified in this formula that is $\left(p_2 - p_1\right) + \left(\rho_2 u_2^2 - \rho_1 u_1^2\right) = -\frac{4}{D}\int_0^L \tau_w dx$.

So, here instead of writing in finite numbers, we are expressing this in a differential form. So, we say $dp + d\left(\rho u^2\right) = -\frac{4}{D}\tau_w dx$.

Then, $d\left(\rho u^2\right)$ becomes $\rho u du$, then we are going to use this τ_w as $\frac{1}{2}\rho u^2 f$. So, since this τ_w is difficult to obtain, so we have to relate this τ_w in terms of friction factor *f*. So, this is classical relations what you get from the pipe flow.

Now, when you put these expressions we are now in a position to obtain an momentum expression, in this one that is $dp + \rho u du = -\frac{1}{2}\rho u^2 \left(\frac{4 f\, dx}{D}\right)$.

So, here the most important factor that we are going to say is that we are now get rid of this τ_w. So, these equations needs to be integrated and simplified. So, for that simplifications we are going to use the speed of sound expression $a^2 = \frac{\gamma p}{\rho}$. So, we can divide this particular expression by ρ. So, that we get dp/ρ here then we also have to use $M^2 = \frac{u^2}{a^2}$, then we have this energy equation $c_p T + \left(u^2/2\right) = \text{constant}$ and of course, equation of state $p = \rho RT$.

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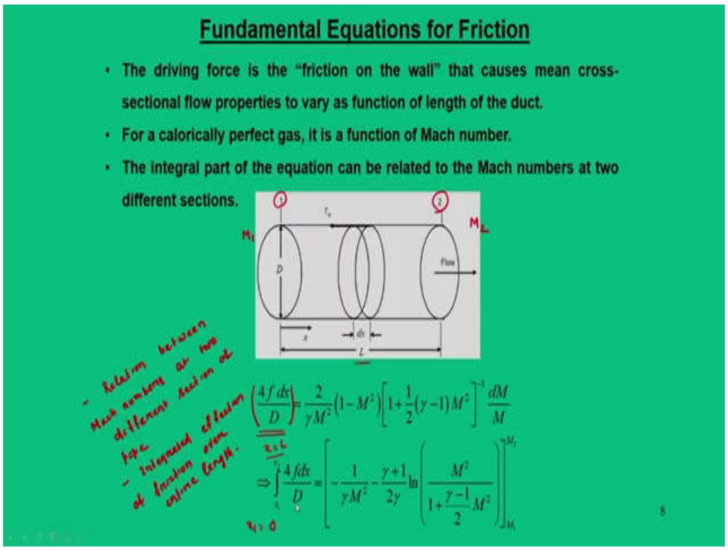

So, these equations after simplification, we are going to get a term in the form of Mach number. So, that means, in that previous equation the factor of interest is the term $\frac{4f\,dx}{D}$. This $\frac{4f\,dx}{D}$ is now related to Mach number that is given by this expressions.

$$\frac{4f\,dx}{D} = \frac{2}{\gamma M^2}\left(1-M^2\right)\left[1+\frac{1}{2}(\gamma-1)M^2\right]^{-1}\frac{dM}{M}$$

So, if you look at this flow in this compressible flow in the pipe, we are looking at two sections 1 and 2, if you have a Mach number M_1 and Mach number M_2 at the exit. So, Mach number M_1 is at the inlet and M_2 at the exit. The driving force is the friction; this friction is correlated to a parameter which is $\frac{4f\,dx}{D}$ and this factor needs to be found out as a function of Mach number.

So, here this particular expression tells us about relation between Mach numbers at two different sections of pipe and it will give you the integrated effect of friction over the entire length.

So, what we see here, this particular equation has a term $\frac{dM}{M}$. So, we can integrate this entire friction term that is integrated over $\int_{x_1}^{x_2} \frac{4fdx}{D}$.

Now, when you integrate the expression from right hand side, the Mach number has to be integrated from M_1 to M_2. And here you may say that x_1 is 0 and x_2 is equal to L. So, over the entire length L one is able to find the term $\frac{4f\,dx}{D}$.

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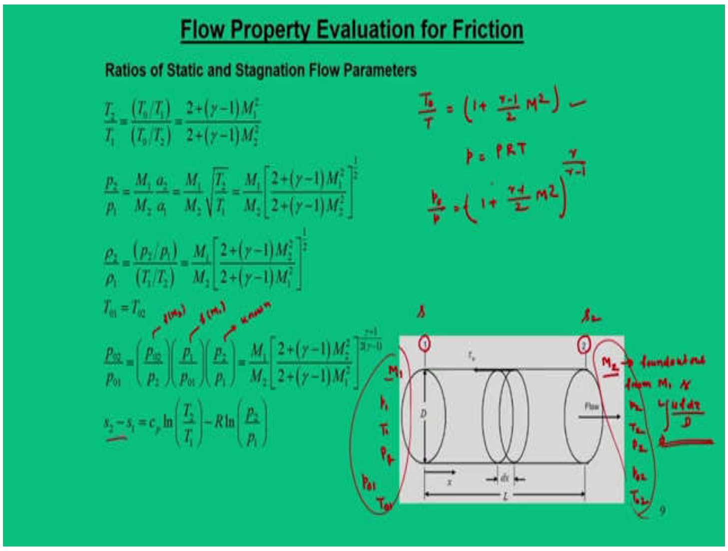

Now, once you do this, then we are in a position that we know all the parameters. So, based on these things since we know this friction, then you know the indent Mach number M_1, we can find M_2.

Now, once you find M_2, then static pressure and stagnation flow property parameters can be related. So, those parameters of interest in this figure we have one and two region. So, we already know M_1. We know M_1 is a known conditions, M_2 is already found out, how? From M_1 and the term $\int_0^l \frac{4fdx}{D}$. So, this particular term is known.

So, from these two we know M_2. So, once we know M_1 and once we know M_2, then one can able to find out the properties ratio. So, the first property ratio we can say p_1, p_2, T_1, T_2, ρ_1, ρ_2 also p_{01}, T_{01} and p_{02}, T_{02}. They are now given here.

What we say is that to calculate $\frac{T_2}{T_1}$ we can write them as $\frac{(T_0/T_1)}{(T_0/T_2)}$. So, here we can say except the flow inside the duct, all other parameters they are the static and stagnation properties are related to Mach number.

So, we recall the expression $T_0/T = 1+\left(\frac{\gamma-1}{2}\right)M^2$. So, when we use these expressions, we are able to get the temperature ratio.

Similarly, one can find out the pressure ratio from the equation of state that is $p = \rho RT$; this equation of state we are going to use, another expression we are going to use that $p_0/p = \left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{\gamma}{\gamma-1}}$.

So, likewise in other situation the way we derived this pressure temperature ratio similar philosophy needs to be followed. So, we can find out total temperature ratio, total pressure ratio, total density ratio and one important thing is that the stagnation temperature remains constant, that we know from the energy equations, but we are able to find out the stagnation pressure ratio from already known conditions that is $\frac{p_{02}}{p_{01}}$ can be expressed as $\left(\frac{p_{02}}{p_2}\right)\left(\frac{p_1}{p_{01}}\right)\left(\frac{p_2}{p_1}\right)$. All these things are known, because this is a function of M_2, this term is function of M_1 and this is already known.

Now, once you know all these things we can also find out the entropy change $s_2 - s_1$. So, with this way we are able to find out all flow properties.

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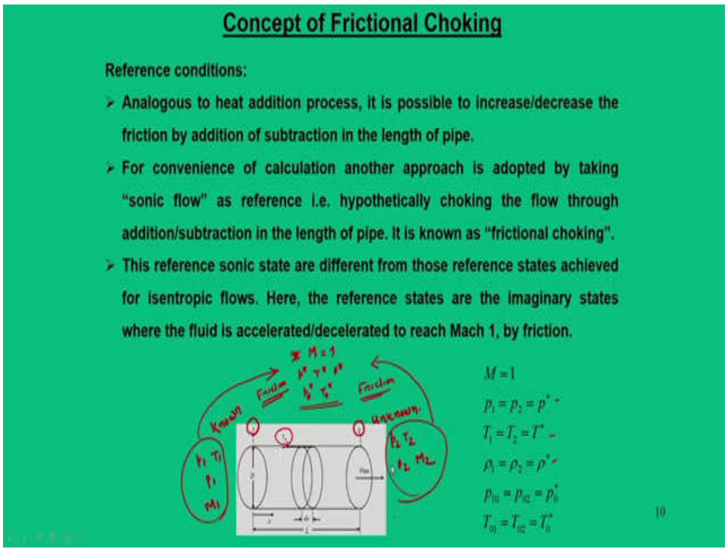

But the very basic point is that for a known conditions 1 and for a unknown condition 2; so, typically 1 is known condition, 2 is unknown condition means all flow parameters are known and all flow parameters of condition 2 are unknown.

So, our main job is to find out at least one parameter from the unknown region that parameter is to be evaluated through the expression of friction or shear stress. So, from that we are going to evaluate all other parameters, because all the property relations says that the equations can be useful only when the Mach numbers at 1 and 2 conditions are known, but which is not possible unless and until we find some other alternatives.

The other alternative is to define a reference conditions and in our all our study for convenience of calculations, we say that the sonic flow is treated as a reference.

Now, question remains that can I get a sonic flow by using the concept of friction? The answer is yes, which means that it is possible to choke the flow by hypothetical manner in which the length is added or subtracted to the pipe and such a concept is known as frictional choking.

So, in other words it is analogous to the fact that the similar way when you deal with heat addition process; the friction can cause the flow to reach the sonic state. So, for that things the reference conditions we define as star in which we say you have Mach number is supposed to 1 and all other conditions are defined as p^*,T^*, ρ^*, then we can say also p_0^*, we also say T_0^*.

So, if you say this is a reference condition, it means that any arbitrary flow parameters whether it is a condition 1 or condition 2, they can be brought to sonic state through friction. Now, when we bring them to sonic conditions irrespective of their own arbitrary conditions, the choking conditions value remains same.

So, this gives a trick to us that instead of going from region 1 to 2 directly through this pipe, our approach should be from known condition we should go to the choking conditions and again from the choking condition you get back your unknown parameters. So, this is the approach we are going to follow.

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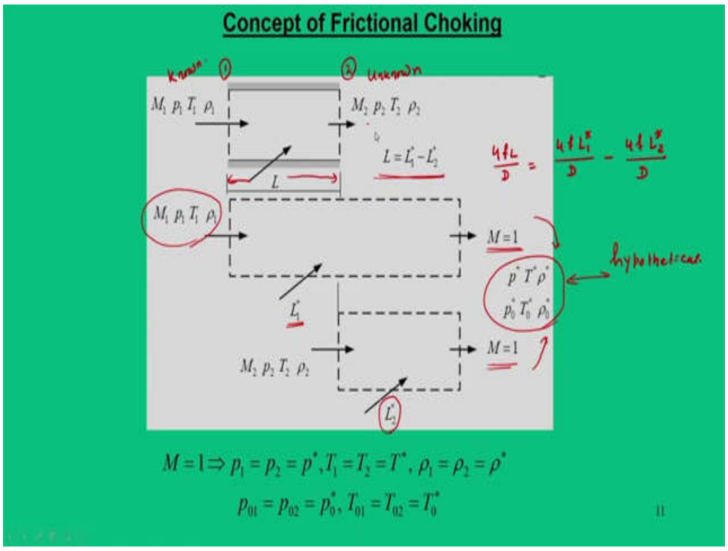

So, for that we can conceptually explain this particular fact in this manner that if we have a real situations 1 and 2 where the flow is entering in a duct or pipe of length L which is the actual length of the pipe and the condition 1 is known, the condition 2 is unknown.

Now, what you do is that from this known condition, you hypothetically add certain length to this pipe which is L_1^* so that the flow Mach number becomes sonic. So, this is a hypothetical situation. This we can do it for the condition 1 and similarly for condition 2, one can find out another term L_2^* so that flow becomes sonic.

So, in all these things, by doing so we arrive at the flow properties value represented by star conditions and both are same; that means, both these hypothetical conditions are same. Now, when I say both hypothetical conditions are same then from this geometry of the figure one can find out the length L is related to L_1^* and L_2^* that is $L = L_1^* - L_2^*$.

Now, we are going to use this length in terms of friction. So, how you are going to do? In that case in the both sides we can just add one parameter $\frac{4fL}{D}$. We also can find out $\frac{4fL_1^*}{D} - \frac{4fL_2^*}{D}$.

Interestingly, these two terms are known to us; that means, from these particular conditions when the flow becomes choke, we can get this value the first term that is $\frac{4fL_1^*}{D}$ and the second term can be found out from these second situations that is from condition 2, when it is brought to sonic state.

So, we get these particular equations. So, this relation will help us to evaluate, how we can get all other parameters of unknown conditions.

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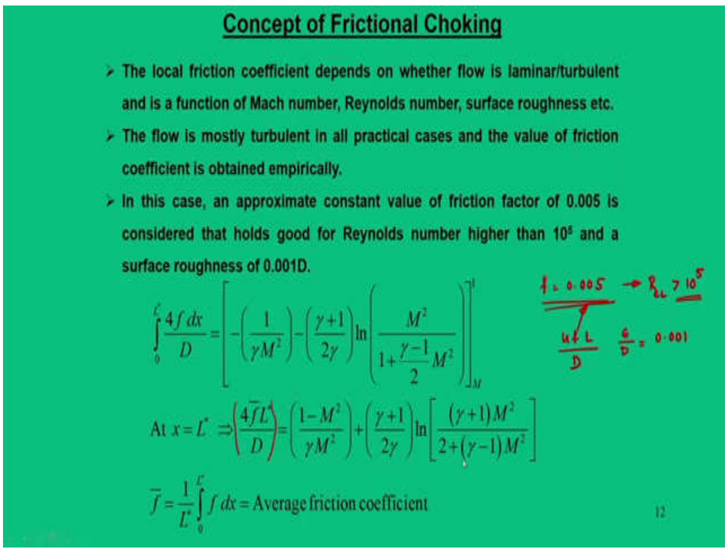

So, this is how we call this as a concept of frictional choking. Now, just to give some brief introduction about this frictional choking that the frictional choking is meant for the fact that flow needs to be sonic through addition or subtraction of length.

Now, when you add or subtract length, since the flow involves certain velocity, certain geometrical parameters, certain roughness parameters, flow Mach number. So, in all these, the flow can also be laminar or turbulent.

So, all these factor brings boils down to the fact that the local friction factor depends on whether the flow is laminar or turbulent and it is a function of Mach number, Reynolds number and surface roughness and in our most practical situations the flow is mostly turbulent. So, the friction factor coefficient is obtained empirically.

So, in this case an approximate constant value of friction factor $f = 0.005$ is considered for our analysis. So, as I mentioned in this expression that $\frac{4f\,L}{D}$; so, in this particular term f refers to the friction factor and that is 0.005 and since all the flow is turbulent so we say Reynolds number is based on length is 10^5 and the surface roughness parameter that is ε/d is 0.001.

So, based on that we say that instead of writing this particular term, this integrated effect we do it from 0 to L^*. So, the integrated number becomes $\frac{4\bar{f}L^*}{D}$. So, this factor happens to be uniform across a given cross section.

So, once you do that integration of the right hand side of the equation is now a function of Mach number. So, here we represent this $\bar{f}$ as average friction coefficient. So, likewise in this manner we are able to find out the term $\frac{4\bar{f}L^*}{D}$. So, that is how the concept of frictional choking was introduced. How L_1^* and L_2^* are determined?

$$\frac{4\bar{f}L^*}{D} = \left(\frac{1-M^2}{\gamma M^2}\right) + \left(\frac{\gamma+1}{2\gamma}\right)\ln\left[\frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}\right]$$

So, in this equation if I put L_1^*, I write here the right hand side as M_1, when I put L_2^*, on the right hand side I can represent this equation with M_2. So, in this manner one can find out.

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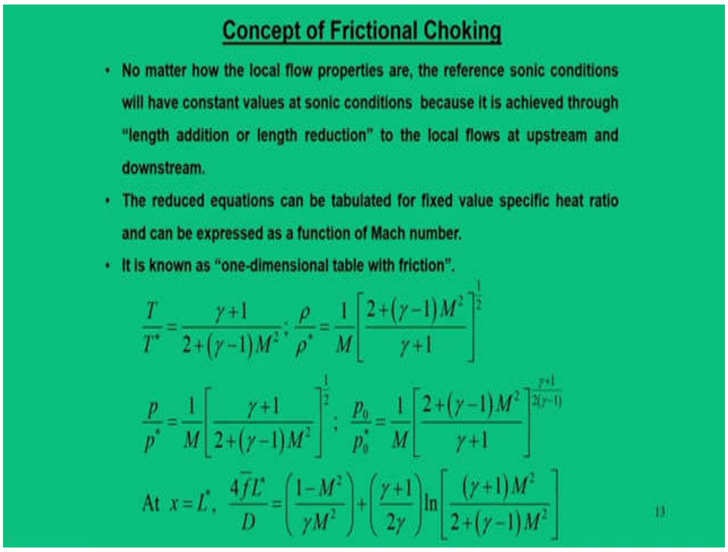

So, with this concept, the all other property relations can now be function of Mach number only. So, here the the superscript star represents the choke conditions and all other terms like pressure, temperature, density, they are any arbitrary flow conditions. But, what you see is that these equations needs elaborate calculations and although it is a direct relation on the function of Mach number; it needs elaborate calculations.

So, to simplify the analysis, what has been done; this equations are represented in a tabular form and that we call as one dimensional table with friction and this is used for compressible flow and all the parameters are now expressed as a function of Mach number.

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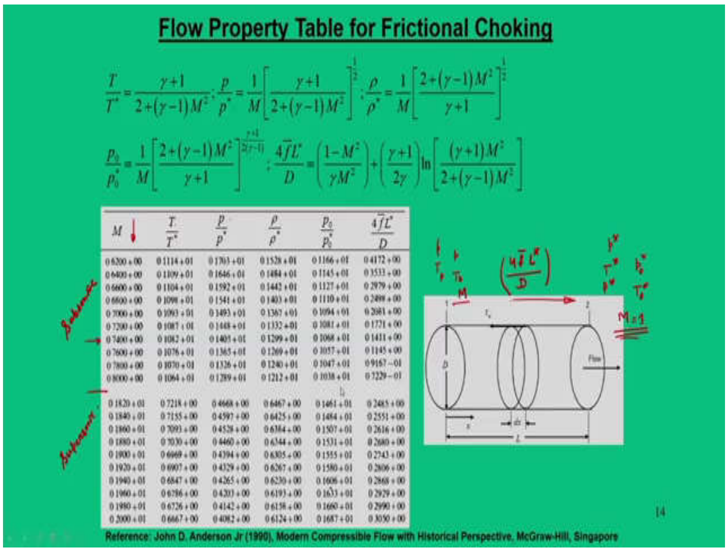

So, in this way the table is introduced. When we introduce the table here what it says is that in first column Mach number is written. So, here I have just defined in two regions; one is subsonic region, other is supersonic. So, this entire column is about Mach number. So, if I say any 1 or 2 conditions are there, so based on the Mach number, one can go in the vertical column and for a given Mach number one can find out all the values $\dfrac{T}{T^*}$, $\dfrac{p}{p^*}$, $\dfrac{\rho}{\rho^*}$, $\dfrac{p_0}{p_0^*}$.

So, here you say p, T, ρ, p_0, T_0 and here we say p^*, T^*, ρ^*, p_0^*, T_0^*. Here your Mach number is M; your M is equal to 1. How this flow becomes 1? This is governed by the parameter $\dfrac{4\bar{f}L^*}{D}$. So, with this friction term, the flow becomes sonic. So, all the relations now are in a tabular form.

In fact, this table is derived from this book John D Anderson, modern compressible flow in historical prospective. So, it is the McGraw Hill Publications. So, I hope one can utilise this table to solve any numerical problems.

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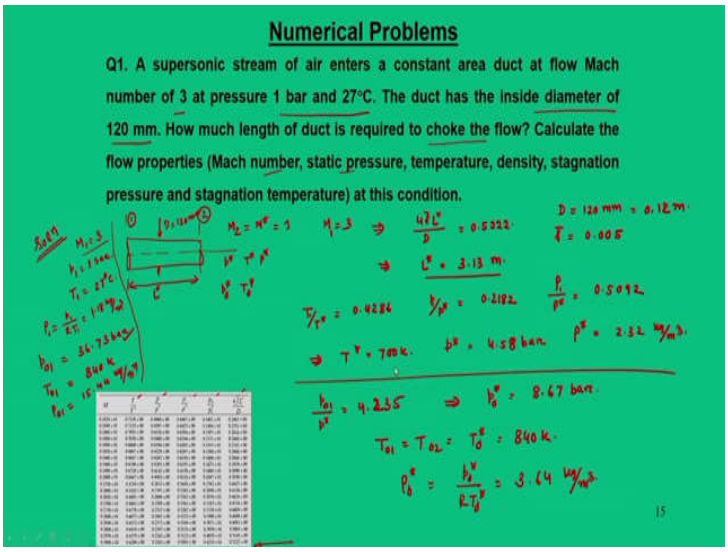

So, let us see that how this particular table can be used to solve some problems. So, a similar problem we have solved in a heat addition systems. We are now trying to solve the similar problem with respect to friction. So, there the parameter that was given is heat added to the flow. Here, the parameter that is given to us that addition of length.

So, the problem governs like this that a supersonic stream enters a constant area duct and in this case we will say this duct is a pipe, having inside diameter of 120 mm. So, the inlet conditions are known at Mach number 3 pressure 1 bar temperature 27°C.

So, you can start the solution. To start the solutions one need to schematically represent the problem. So, we have a circular pipe. Now, in this circular pipe what is required is condition 1. We say that the flow is the downstream side is the choke. So, the length has to be L^*, other parameter of that is given diameter 120 mm.

Now, the inlet conditions we say M_1 is equal to 3, p_1 is 1 bar, T_1 is 27°C. So, when you know pressure, temperature, we can find density, that is $\rho_1 = \frac{p_1}{RT_1}$; that number we can find out as 1.18 kg/m^3.

So, once you know p_1, T_1, and ρ_1 and Mach number, then we can evaluate p_{01} from isentropic relations as 36.73 bar and we can also get T_{01} as 840 K and ρ_{01}would be 15.44 kg/m^3.

So, basically we know all the inlet conditions and we say they are known conditions. What is unknown? We want to find out what is M_2. So, M_2 is nothing but M^*, because we say the flow is choked. So, we require p*, T^*, ρ*, p_0^*, T_0^*, to do that what you have to see that for a flow which is supersonic in nature, if this flow needs to be choked.

So, for that Mach number when I refer this table, extract of the table one can find out that this last line corresponds to Mach 3. So, I can take the extract of this ratio for M is equal to 3, I can write this particular terms. We say first thing we require $\frac{4\bar{f}L^*}{D}$ is 0.5222. So, this is of importance and in our case you have D is equal to 120 mm that is 0.12 m and $\bar{f}$ we can say it is 0.005. So, an average value of this.

So, this will tell you that what is the value of L^*, that would be 3.13m. So, we require about 3.13m to choke this supersonic flow. So, you have to also find Mach number and static pressure. So, you have to take all these ratio, $\frac{T}{T^*}$ as 0.4286, $\frac{p}{p^*}$ as 0.2182, $\frac{\rho}{\rho^*}$ as 0.5092.

So, this will give me T^* about 700 K. Because we know p_1, T_1, and ρ_1, so, we can say p* 4.58 bar, ρ* would be 2.32 kg/m^3.

So, similarly we also can find out $\frac{p_0}{p_0^*}$ that is 4.235. So, this will give you p_0^* 8.67 bar, because p_{01} is 37.63 bar. So, we know that $T_{01} = T_{02} = T_0^*$ as total temperature does not change 840K.

So, we know p_0^*, T_0^*, then you can find out ρ_0^* is equal to $\frac{p_0^*}{RT_0^*}$. So, this is about 3.64 kg/m^3. So, likewise we are now able to find out what is the value of the choking parameters in the downstream situations.

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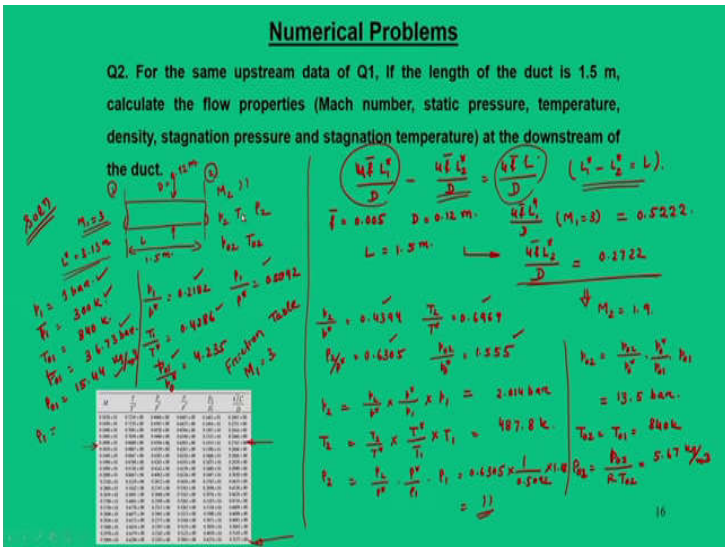

Now, moving further we are now in the next problem that for the same data, for the question 1; if the length of the duct is 1.5 metre, we have to calculate the flow properties.

So, what is it says is that, in the last problem, we found that to choke this flow of supersonic inlet conditions M_1 is equal to 3 we require L^* as 3.13 metre, but here our second question does not say that it is 3.13 metre, but it is actual length is 1.5 m; that means, L is 1.5 m. Diameter remains same 0.12 m and; obviously, the flow will not be Mach number 1 at 2.

So, you have to calculate M_2, p_2, T_2, ρ_2, p_{02}, and T_{02}, but the known conditions which are us is p_1 as 1 bar, T_1 as 300 K, then T_{01} is 840 K, p_{01} is 36.73 bar; so, I am just extracting those numbers which we found from the last problem.

So, ρ_{01} is 15.44 kg/m^3. Now, for this Mach number, we also know p 1 by p star that is 0.2182; that means, from this last column we say $\frac{T_1}{T_0^*}$ as 0.4286, $\frac{p_1}{p_0^*}$ as 0.2182, $\frac{\rho_1}{\rho_0^*}$ as 0.5092 $\frac{p_{01}}{p_0^*}$ is 4.235. So, this is known condition that we get from this friction table for Mach 3.

So, now let us see that we know that $\frac{4\bar{f}L_1^*}{D} - \frac{4\bar{f}L_2^*}{D} = \frac{4\bar{f}L}{D}$. So, this we get $L_1^* - L_2^* = L$. So, from this equation we can rewrite this. So, we know $\bar{f}$ 0.005 D is equal to 0.12m and first term $\frac{4\bar{f}L_1^*}{D}$ that is with M 1 is equal to 3 happens to be 0.5222 and here you know L is equal to 1.5m.

So, from these equations we can evaluate what is the value of $\frac{4\bar{f}L_2^*}{D}$. So, this number happens to be 0.2722.

Now, for this particular friction term $\frac{4\bar{f}L_2^*}{D}$ as 0.2722; so, closely this number comes to this, where M_2 becomes 1.9. So, we can use this particular row. So, this will imply M_2 is equal to 1.9.

Now, when I say M_2 is 1.9, I can say $\frac{p_2}{p^*}$ all the number I can note down 0.4394, $\frac{T_2}{T^*}$ 0.6969, then $\frac{\rho_2}{\rho^*}$ 0.6305, $\frac{p_{02}}{p_0^*}$ is 1.555.

So, likewise once you get this, then now you are an able to find out what is p_2 that is nothing but $\left(\frac{p_2}{p^*}\right)\left(\frac{p^*}{p_1}\right)p_1$. So, this will give you 2.014 bar. Similarly, $T_2 = \left(\frac{T_2}{T^*}\right)\left(\frac{T^*}{T_1}\right)T_1$.

So, we get 487.8 K, $\rho_2 = \left(\frac{\rho_2}{\rho^*}\right)\left(\frac{\rho^*}{\rho_1}\right)\rho_1$. So, one can calculate this number. So, you get this, then we can say $p_{02} = \left(\frac{p_{02}}{p_0^*}\right)\left(\frac{p_0^*}{p_{01}}\right)p_{01}$.

So, in this way I can evaluate this number is 13.5 bar. $T_{02} = T_{01}$, which is remain same 840 K and $\rho_{02} = \frac{p_{02}}{RT_{02}}$ that number is 5.67 kg/m^3.

So, we are now in a position to calculate all this number. So, likewise this particular problem tells the fact that when a length is given how you are able to achieve these downstream conditions.

Now, to achieve this downstream conditions; we have to bring this in flow, fast to this choking conditions. From this choking condition, we have to come back to the downstream conditions. This is how you are going to solve for the problems that involving friction. So, with this I will conclude this talk for today.

Thank you.