Aircraft Propulsion

Prof. Vinayak N.Kulkarni
Department of Mechanical Engineering
Indian Institute of Technology – Guwahati

Module No # 03
Lecture No # 15
Polytropic Efficiency

Welcome to the class now today we are going to see one more reality about the cycle and that reality we are calling it as polytropic efficiency. We have seen that till time that the non-idealist of turbine or pump was dealt with the turbine of pump efficiencies. And we call them as turbine and pump isentropic efficiencies turbine and compressor practically for our case as isentropic efficiencies.

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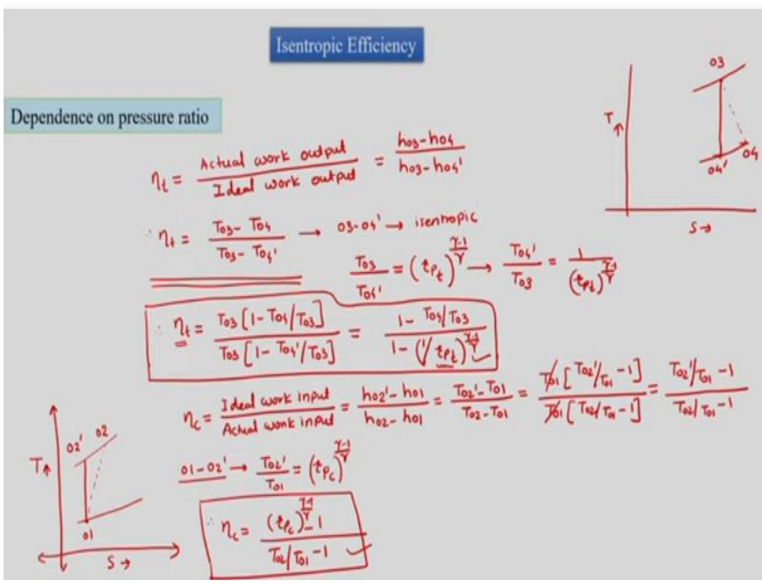

So let us see what did we use in case of turbine we said that isentropic efficiency is equal to actual work output divided by ideal work output. And this isentropic efficiency when we try to define in terms of the temperature which is total temperature. So we said that it is 03 and then it is coming down to 04 but it is 04' and 04. So this is ideal this is real. So this is isentropic 03 04' is isentropic and 03 04 is real.

So we said that h_{03} - h_{04} is real work output and incase of ideal h_{03} - $h_{04'}$. So we got efficiency for calorically perfect case as

$$\eta = (T_{03} - T_{04})/(T_{03} - T_{04'})$$

But we know that the process 03 to 04'is isentropic and then in this case we have got this as the relation and we have used it for many other calculations. But now we will see that how this equation has certain dependence on pressure ratio. Since 03 and 04'is isentropic ,

$$T_{03} / T_{04'} = (r_{p_t})^{(\gamma-1)/\gamma}$$

So similarly

$$T_{04'} / T_{03} = 1/(r_{p_t})^{(\gamma-1)/\gamma}$$

So for turbine isentropic efficiency we can take T_{03} common. So we will have $\{T_{03} *(1 - T_{04} / T_{03})\}/\{ T_{03} *(1 - T_{04'} / T_{03})\}$.This leads to

$$\eta = (1 - T_{04} / T_{03})/ (1 -1/(r_{p_t})^{\frac{\gamma-1}{\gamma}}).$$

So this is the formula for turbine efficiency which says that turbine efficiency is dependent upon the governing pressure ratio.

Similarly in case of compressor we have said that this is 01 initial stagnation condition isentropically it will go to 02' but in reality it will go to 02 on a TS diagram. So with this we defined compressor efficiency which is isentropic efficiency as ideal work input divided by actual work input. So we wrote down the efficiency as

$$\eta = (h_{02'} - h_{01})/(h_{02} - h_{01})$$

So we can write down for calorically perfect gas as

$$\eta = (T_{02'} - T_{01})/(T_{02} - T_{01}).$$

Here as well let us take T_{01} common so $\{T_{01} (T_{02'} / T_{01} – 1)\} /\{ T_{01} (T_{02} / T_{01} – 1)\}$. So it gets cancelled and we get $\{ (T_{02'} / T_{01} – 1)\} /\{ (T_{02} / T_{01} – 1)\}$. But the process 02' 01 to 02' is isentropic. So we have $T_{02'} / T_{01}$ as $(r_{p_c})^{\frac{\gamma-1}{\gamma}}$. So we can get compressor efficiency as

$$\{ ((r_{p_c})^{\frac{\gamma-1}{\gamma}} - 1)\} / \{ (T_{02} / T_{01} - 1)\}$$

So this formula and turbine efficiency formula both formulas say that the isentropic efficiency of a turbine or compressor does not depend does not remain constant but it depends upon the governing pressure ratio.

And please look into the formulas we did not use a single pressure ratio for turbine and compressor. Since we are moving towards more reality and since we are moving towards more reality we have used pressure ratio as RPT for turbine and pressure ratio as RPC for compressor. And then this formulas where derived now we will see that how to cater this problem which leads with the fact that turbine and isentropic efficiencies are dependent upon their governing pressure ratios.

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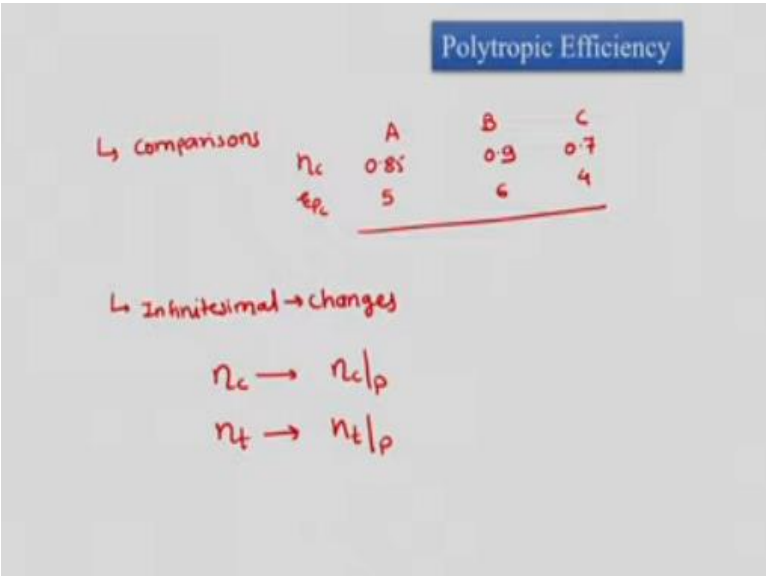

So what is the basic problem here basic problem is that if there are 2 compressors or 2 turbines then let us say that there is compressor A and compressor B. Isentropic efficiency of compressor A is 0.85 and point B is 0.9. A compressor works with pressure ratio 5 B compressor works with pressure ratio 6 and then there is also a C compressor which has efficiency 0.7 and it works with pressure ratio 4. So this is Compressor efficiency and this is pressure. Now I want to compare these three compressors and similarly there can be three turbines with three isentropic efficiency and three pressure ratios.

So if I want to compare three different entities four different entities which have different working ratio pressure ratios and which are different efficiencies then I cannot see who is good? And who is bad? Which that and who is better so for that fact I need to find out something which is more universal and that more in universal is polytropic efficiency which is related with not finite pressure ratio which is related with infinitesimal pressure for which there will be infinitesimal temperature change.

So this is why we have defined polytropic efficiency which lead deals with infinitesimal changes and as we were saying compressor efficiency isentropic as like this. So now we will see compressor efficiency polytropic we were using turbine efficiency isentropic as η_T we will say η_{TP} which is polytropic efficiency of turbine. So we will use this terminologies to find out what do you mean by if compressor and turbine polytropic efficiencies.

So let us derive polytropic efficiency for turbine before that we will find out the infinitesimal temperature changes what can bring in the what are brought in duty infinitesimal pressure changes. So let us find out that and then we will go for defining the compressor and turbine polytropic efficiencies.

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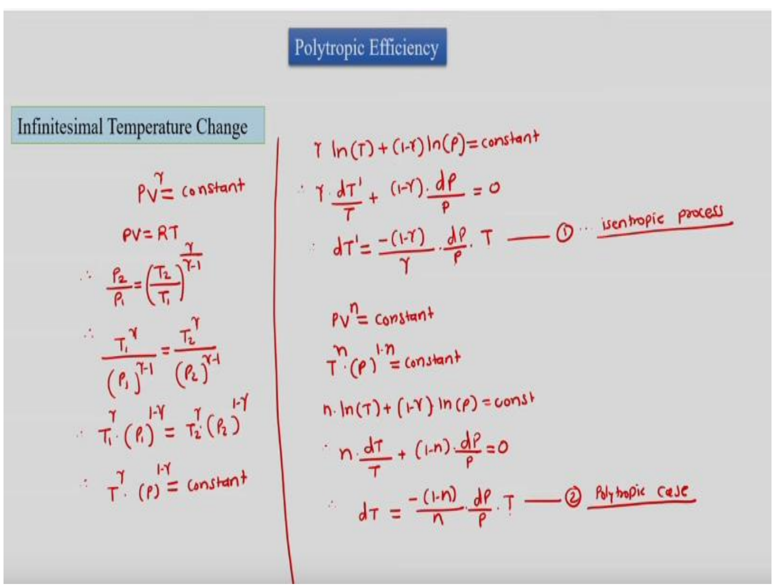

So we know that

$$(PV)^{\gamma} = Constant$$

for isentropic process we also know that $PV = RT$ for 1kg of gas as per ideal gas law. So we know that $P_2 / P_1 = (T_2 / T_1)^{(\gamma/\gamma-1)}$ which is the isentropic relation. So using this we can write down always that P_2 / P_1 bracket raise to we can write down as

$$(T_1)^{\gamma} /(P_1)^{(\gamma-1)} = (T_2)^{\gamma} /(P_2)^{(\gamma-1)}.$$

So we can write down $(T_1)^{\gamma} *(P_1)^{(1-\gamma)} = (T_2)^{\gamma} *(P_2)^{(1-\gamma)}$. So practically we can write down

$$(T)^{\gamma} *(P)^{(1-\gamma)} = constant$$

in an isentropic process.

So this is what we have got now considering this we can take log on both sides then we can get $\gamma*ln(T)+(\gamma-1)*ln(P)=constant$.

So let us differentiate this expression differentiating this expression since this is an isentropic process we will get $\gamma*(dT')$is it infinitesimal temperature change. Since the process is isentropic we are putting dash by original temperature T + $(1 - \gamma)*(dP)$ for given pressure change by original pressure at 0. So we can write down infinitesimal temperature change is

$$dT=-(1 - \gamma) / \gamma * dP / P * T$$

This is our equation number 1 which is for isentropic process.

Now we know that the process will not be isentropic process in reality will be polytropic since it is not going to follow $(PV)^{\gamma} = Constant$. But it would follow

$$(PV)^{n} = Constant$$

So the polytropic index n is equal to γ for isentropic case. But now we are not having them as equal since in reality we are having $(PV)^{n} = Constant$ then so for the same thing we can write down here

$$(T)^n \; *(P)^{(1-n)} = constant.$$

And we also write $n*ln(T)+(n-1)*ln(P)=constant$. So this helps us to give $n * (dT'/ T) + (1-n)* dP / P = 0$ after differentiation. So infinitesimal temperature changes okay it is not dash now since we are working with real temperature change. So infinitesimal real temperature change is dT and

$$dT = -(1 - n) / n * dP / P * T$$

Here we are saying that same pressure change has taken place and we are starting with same initial conditions P and T and for same pressure change.

We are having differential temperature changes one in isentropic case and one is polytropic case. So we are differential pressure changes now having this we will come back and define our polytropic efficiencies for turbine and compressor.

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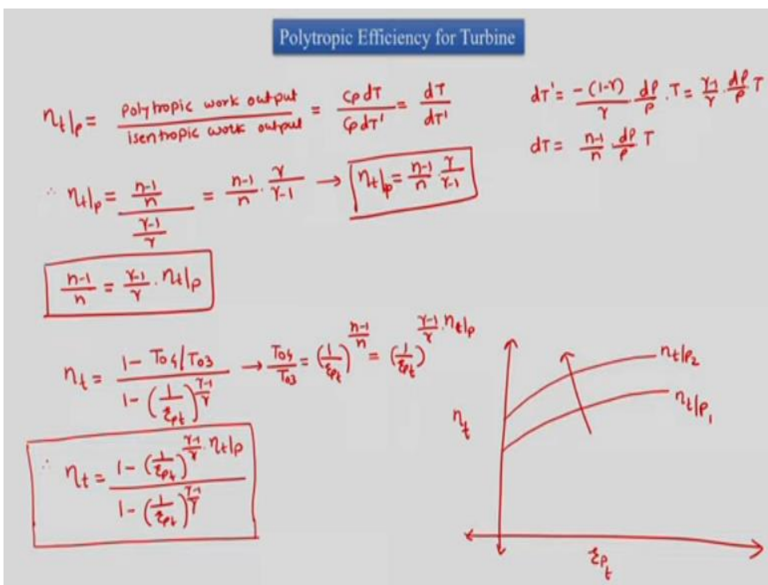

So let us define polytropic efficiency for turbine us is poly tropic work output divided by isentropic work output. So practically we can see that it is equal to in case of infinitesimal we can say that Cp dT which is which leads to infinitesimally real temperature change and Cp dT' which has infinitesimal isentropic temperature change so this is practically dT/dT'.

So if we go back and see the expressions for dT expression for dT' initially we derived as $(\gamma-1) / \gamma * dP / P * T$.. So this can be written as $(\gamma-1) / \gamma * dP / P * T$ and dT was similarly $(n-1) / n * dP / P * T$. So we can replace this and we can find out turbine efficiency polytropic is equal to

$$\eta_{tp} = ((n-1)/n)*(\gamma/(\gamma-1)).$$

Now this polytropic efficiency is for infinitesimal change and this polytropic efficiency therefore independent of the pressure ratio and independent of the initial state. So this polytropic efficiency once defined we can find out the polytropic index. So practically we needed (n-1)/n. So that can be written as $(\gamma-1) / \gamma * \eta_{tp}$.

So this helps us to give this further we can also write

$$\eta_{tp} = ((n-1)/n)*(\gamma/(\gamma-1))$$

so this is the formula. So now we can go back and see what the isentropic efficiency of turbine and we had got a formula as $(1 - T_{04} /T_{03})/(1 -1 / (r_{p_t})^{(\gamma-1)/\gamma})$. but what is T_{04} /T_{03}. Since it is a polytropic process it is $(r_{p_t})^{(n-1)/n}$. So it is $(r_{p_t})^{(\gamma-1)/\gamma * \eta_{tp}}$.

So we can use this and write down

$$\eta_t = (1- (r_{p_t})^{(\gamma-1)/\gamma * \eta_{tp}})/ (1- 1 / (r_{p_t})^{(\gamma-1)/\gamma}).$$

Now this is how if we know polytropic efficiency of a turbine for a pressure ratio. We can find out its isentropic efficiencies so for a given polytropic efficiency of turbine isentropic efficiency will vary.

So we can plot isentropic efficiency of turbine with pressure ratio of turbine for different polytropic efficiencies and then we can see that this is a variation of isentropic efficiency for a given polytropic efficiency of turbine. And then this is for other polytropic efficiency of turbine where polytropic efficiency is increasing in this direction we are working here with pressure ratio 1 to start.

So this is how we are having polytropic efficiencies usefulness to find out isentropic efficiency at any pressure ratio. And then we can compare two turbines which are operating at different pressure ratios we can bring them to one pressure ratio and then compare their characteristics. So this is how we can compare now we can go and find out what is polytropic efficiency of a compressor.

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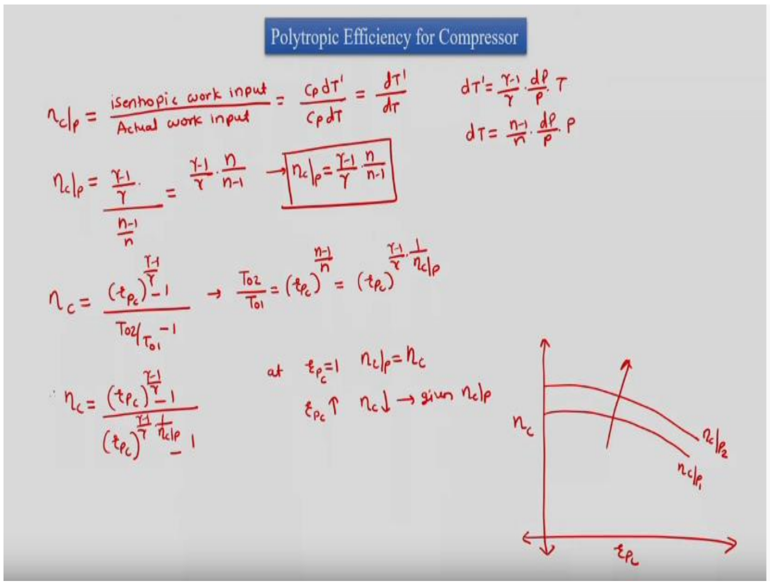

So same way we will define polytropic efficiency of a compressor is equal to isentropic work input / actual work input. And we know that we can write down for calorically perfect and infinitesimal case $(Cp\ dT') / (Cp\ dT)$ then say it is dT'/ dT. We again can know that dT' we had defined as $(\gamma-1) / \gamma * dP / P * T$ and dT was similarly $(n-1) / n * dP / P * T$.

So this is what we can use and find out what is polytropic efficiency of compressor and then we can get it as $(n/(n-1))*((\gamma-1)/\gamma)$ If we see the previous slide it was $((n-1)/n)*(\gamma/(\gamma-1))$ for turbine which has changed to $(n/(n-1))*((\gamma-1)/\gamma)$ so polytropic efficiency of compressor is

$$\eta_{tc} = (n/(n-1))*((\gamma-1)/\gamma).$$

Then this we can use for further calculation of compressors isentropic efficiency and we have defined it as $(r_{p_c})^{(\gamma-1)/\gamma}. / (T_{02}/T_{01}-1)$. But what is T_{02}/T_{01}? It is $(r_{p_c})^{(n-1)/n}$. but what is $(n-1)/n$ it is $(r_{p_c})^{(\gamma-1)/\gamma*1/\eta tc}$.

So we can write down

$$\eta_c = ((r_{p_c})^{(\gamma-1)/\gamma}. -1)/((r_{p_c})^{(\gamma-1)/\gamma*1/\eta tc}-1)$$

So what we can get is at r_p = 1 we will have polytropic efficiency of compressor is equal to its isentropic efficiency and if r_p compressor increases then compressors efficiency decreases for given polytropic efficiency of compressor.

And hence we can see that how does it vary if we see then we can plot compressors as isentropic efficiency verses pressure ratio and then it is like this where this is for compressors polytropic efficiency 1 this is for compressors polytropic efficiency 2 and polytropic efficiency is increasing in this trend.

Now if there are two compressors or three compressors which are working at different pressure ratios they have different compressor isentropic efficiencies then we can bring them to one pressure ratio and then compare or in principle if we know their polytropic efficiency then we can compare the polytropic efficiencies to identify a better compressor.

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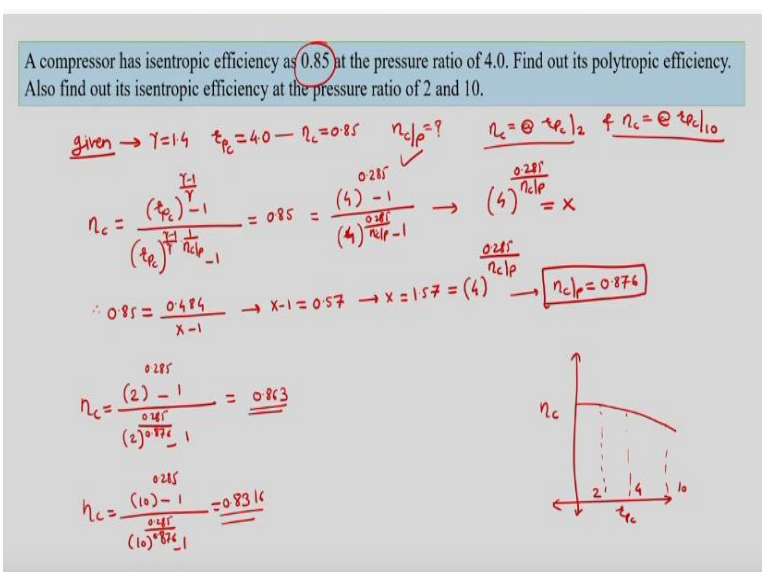

Now here is an example which states that a compressor has polytropic efficiency 0.85 at pressure ratio of 4. So sorry compressor as isentropic efficiency 0.85 at pressure ratio of 4 so find out it is polytropic efficiency also find out it is isentropic efficiency at pressure ratio of 2 and 10. So what are the given things here? Here we are given with the fact that we will take gamma as 1.4. We are said that pressure ratio compressor is equal to 4 and then corresponding isentropic efficiency is 0.85.

We are supposed to find out compressors polytropic efficiency and then isentropic efficiency at pressure ratio of 2 and compressors isentropic efficiency at pressure ratio of 10. It is a straightforward example we know we have just now derived the formula for compressors isentropic efficiency and this formula we can use and this formula state that

$$\eta_c = ((r_{p_c})^{(\gamma-1)/\gamma}.\ -1)/((r_{p_c})^{(\gamma-1)/\gamma*1/\eta tc}-1)$$

and this is given as 0.85.

So 0.85 = $(4^{0.285}-1) / (4^{0.285*1/\eta tc}-1)$. So we can take rearrange the terms and then we can get from here 0.85 suppose let us say that $4^{0.285*1/\text{ polytropic efficiency of compressor}} = x$. So we have $0.85 =(0.484 /(x-1))$. So we have $x-1 = 0.57$ and then this gives us x as 1.57.

But we know $4^{0.285*1/\eta tc} = x$. We can take log on both sides and then we can find out what is compressors polytropic efficiency and then that would give us 0.876. Now we know what the compressor polytropic efficiency then this is found out we need to know these two things but these are very straightforward.

Now we need to find out compressors polytropic efficiency and isentropic efficiency at pressure ratio $(2^{0.285}-1) / (2^{0.285/0.876}-1)$. And this gives us 0.863 the notable point are here we had seen that for a compressor the polytropic efficiency decreases with pressure sorry isentropic efficiency decreases for a given polytropic efficiency with increasing pressure ratio. So we were at suppose 4 we were at pressure ratio 4.

Now from that we are coming to 2 so it should increase and it is evident it is 0.863 which has increase point from 0.85. Now we need to evaluate the same for pressure ratio 10 and for pressure ratio $(10^{0.285}-1) / (10^{0.285/0.876}-1)$. and it will come 0.8316 which is somewhere here so it has decreased. So this is what intuitively we should have known that if efficiency is higher if efficiency is known at one pressure ratio so at lower pressure ratio that it will be higher and at higher pressure ratios it will be lower. We can find out similar things for the turbine also however we will stop in the today's class. Thank you.