

Steam Power Engineering
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Lecture - 25
Examples: Steam Turbines

Welcome to the class. Today we are going to see the examples on steam turbines.

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The velocity of steam entering a simple impulse turbine is 1000 m/s and the nozzle angle is 20° . The mean peripheral velocity of blades is 400 m/s and the blades are symmetrical. If the steam enters the blades without shock, find the blade angles?

a. Neglecting the friction effects on the blades, calculate the tangential force on the blades and the diagram power for a mass flow of 0.75 kg/s. Estimate also the axial thrust and diagram efficiency.

b. If the relative velocity at the exit is reduced by friction to 80% of that at inlet, estimate the axial thrust, diagram power and diagram efficiency

a) $K_b = 0$ Turbine is impulse \rightarrow degree of reaction = 0
 $V_{r1} = V_{r2}$ $\beta_2 = \beta_1$

$V_{a1} = V_{r1} \sin \beta_1$ $V_{a2} = V_{r2} \sin \beta_2$ $V_{a1} = V_{a2} = V_a$
 $\therefore V_a = V_{r1} \sin \beta_1 = V_{r2} \sin \beta_2$ $V_{r1} \cos \beta_1 + V_b = V_{r2} \cos \beta_2$
 $V_{r1} \cos \beta_1 = V_{r2} \cos \beta_2 - V_b$
 $\tan \beta_1 = \frac{V_a \sin \alpha}{V_{r2} \cos \beta_2 - V_b} \rightarrow \beta_1 = \tan^{-1} \left[\frac{V_a \sin \alpha}{V_{r2} \cos \beta_2 - V_b} \right]$
 $\beta = \tan^{-1} \left[\frac{1000 \times 5 \sin(20)}{1000 \times \cos(20) - 400} \right] = 32.35^\circ \rightarrow \beta_2 = \beta_1 = 32.35^\circ$

$V_{r1} \sin \beta_1 = V_a \sin \alpha$
 $V_{r1} \sin(32.35) = 1000 \times 5 \sin(20)$
 $V_{r1} = 639.25 \text{ m/s} \Rightarrow V_{r1} = V_{r2} = 639.25 \text{ m/s}$

One example read that the velocity of steam entering a simple impulse turbine is 100 metre per second and nozzle angle is 20° . The main peripheral velocity of the plates is 400 metre per second and the blades are symmetrical. If the steam enters blades without shock find the blade angles; a. Neglecting the frictional effect on the blades, calculate tangential force on the blades and diagram power for a mass flow of 0.75 kg per second.

Estimate also the axial thrust and the diagram efficiency. If relative velocity at the exit is reduced by friction to 80% of it is inlet value estimate the axial thrust diagram power and diagram efficiency. So we will first plot the velocity triangle and then we will note whatever it is given to us. So let us say that this is V_b , this is V_1 , so this is V_{r1} and then this is V_{r2} and this is V_2 .

So what we have is, this is α , this is β_1 , this is β_2 okay. So noting this we can proceed for a, we are said that we have to neglect the friction factor. So $K_b = 0$. We have been told that the turbine is impulse turbine okay. So since the turbine is impulse we have degree of reaction

equal to 0. So $V_{r1} = V_{r2}$ since we have $K_b = 0$. Further we are also told that blades are symmetrical.

So $\beta_2 = \beta_1$, so with this point we can start then we will note that directly $V_{r1} \sin(\beta_1)$ which is basically, V_a, V_{a1} , parallel $V_{a2} = V_{r2} \sin(\beta_2)$ but what would happen $V_{r1} = V_{r2}$, $\beta_1 = \beta_2$, so $V_{a1} = V_{a2} = V_a$, so we will have $V_a = V_{r1} \sin(\beta_1) = V_1 \sin(\alpha)$. So this height, so for this we can just equate this much part and then we can have one more thing that $V_{r1} \cos(\beta_1) + V_b$.

So this is V_b and this is $V_{r1} \cos(\beta_1)$ and this is V_b , so this is totally plus. So for this plus we have equal to $V_1 \cos(\alpha)$. So we have $V_{r1} \cos(\beta_1) = V_1 \cos(\alpha) - V_b$ basically, $V_1 \cos(\alpha) - V_b$. We

can divide these 2 equations, this and this, and then we have $\tan \beta_1 = \frac{V_1 \sin(\alpha)}{V_1 \cos(\alpha) - V_b}$. So

$$\beta_1 = \tan^{-1} \left[\frac{V_1 \sin(\alpha)}{V_1 \cos(\alpha) - V_b} \right]$$

So now we have $\beta = \tan^{-1} V_1$ is given to us, which is 1000 metre per second, nozzle angle is

told to us as $\beta = \tan^{-1} \left[\frac{1000 \times \sin(20)}{1000 \times \cos(20) - 400} \right]$ is. So this β turns out to be 32.35 degree and

since blades are symmetrical $\beta_2 = \beta_1 = 32.35^\circ$. So we found out the angle. Now we can move ahead and then we can find out $V_{r1} = ?$ we know $V_{r1} \sin \beta_1 = V_1 \sin \alpha$.

So we know $V_{r1} \sin(32.35) = V_1 \times 1000 \times \sin(20)$. So we have $V_{r1} = 639.25$ and this we know since we have no friction $V_{r1} = V_{r2}$ and both are 639.25 meters per second. So now we know $V_{r1}, V_{r2}, \beta_1, \beta_2$ and then we can now calculate the desired thing.

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$$\begin{aligned} \Delta V_w &= \frac{V_1 \cos \beta_1 + V_2 \cos \beta_2}{\text{min}} \\ \Delta V_w &= 2 \times V_{r1} \cos \beta_1 \\ \Delta V_w &= 2 \times 639.25 \times \cos(32.35) \\ \Delta V_w &= 1080.07 \text{ m/s} \\ \text{Thrust} = \text{force} &= \dot{m} \times \Delta V_w = 0.75 \times 1080.07 \\ \text{Thrust} &= 810.05 \text{ N} \\ \text{Change in axial velocity} &= V_{r1} \sin \beta_1 - V_{r2} \sin \beta_2 = 0 \\ \therefore (\Delta V_a) & \\ F_a = \text{axial thrust} &= \dot{m} \Delta V_a = 0 \\ \text{Power (diagram)} &= \text{Thrust} \times V_b \\ P &= 810.05 \times 400 = 324.02 \text{ kW} \\ \eta_D = \frac{P}{\frac{1}{2} \dot{m} V_1^2} &= \frac{324.02 \times 10^3}{\frac{1}{2} \times 0.75 \times (1000)^2} \\ \eta_D &= 0.864 \end{aligned}$$

$$\begin{aligned} b) \quad k_b &= 0.8 \\ \frac{V_{r2}}{V_{r1}} &= 0.8 \rightarrow V_{r2} = V_{r1} \times 0.8 = 639.25 \times 0.8 = 511.4 \text{ m/s} \\ \Delta V_w &= V_{r1} \cos \beta_1 + V_{r2} \cos \beta_2 = 639.25 \times \cos(32.35) + 511.4 \times \cos(32.35) \\ \Delta V_w &= 972.06 \text{ m/s} \\ \text{Thrust} &= \dot{m} \Delta V_w = 0.75 \times 972.06 = 729.04 \text{ N} \\ P &= \text{Thrust} \times V_b = 972.06 \times 400 = 291.62 \text{ kW} \\ \text{change in axial velocity} &= V_{r1} \sin \beta_1 - V_{r2} \sin \beta_2 = \Delta V_a \\ \text{axial thrust} = T_a &= \dot{m} (V_{r1} \sin \beta_1 - V_{r2} \sin \beta_2) \\ T_a &= 0.75 \times [639.25 \times \sin(32.35) - 511.4 \times \sin(32.35)] \\ T_a &= 51.3 \text{ N} \end{aligned}$$

So for that we can go ahead and calculate $\Delta V_w = V_{r1} \cos(\beta_1) + V_{r2} \cos(\beta_2)$, but we know $V_{r1} = V_{r2}$, $\cos(\beta_1) = \cos(\beta_2)$. So it is $\Delta V_w = 2V_{r1} \times \cos(\beta_1)$, so we have $\Delta V_w = 2 \times 639.25 \times \cos(32.35)$ so we have $\Delta V_w = 1080.07$ meters per second. So we can find out thrust, or force which is equal to change in momentum. So it is equal to $\text{Thrust} = \text{force} = \dot{m} \Delta V_w$.

And we know we are given that \dot{m} is 0.75 into change in velocity is 1080.07. So $\text{Thrust} = \text{Force} = 810.05 \text{ N}$. So this is the tangential thrust. Now we can find out the axial thrust which is equal to $\text{axial thrust} = V_{r1} \sin(\beta_1) - V_{r2} \sin(\beta_2)$ but again $V_{r1} = V_{r2}$ and $\beta_1 = \beta_2$. So this is equal to 0, so we have this is basically, axial change in axial velocity which is ΔV_a .

So $\text{axial thrust} = F_a = \dot{m} \Delta V_a = 0$. So power and since we are finding out that power from the diagram, so which is diagram power is equal to the force, which is axial thrust into velocity which is V_b . So power diagram power $P = 810.05 \times 400 = 324.02 \text{ kW}$. So this is the diagram

power. So we can find out diagram efficiency which is equal to $\eta_D = \frac{P}{\frac{1}{2} \dot{m} V_1^2}$.

So here we can put it as $\frac{324.02 \times 10^3}{\frac{1}{2} \times 0.75 \times 1000^2}$, so we have diagram efficiency is equal to 0.864.

Now I can move ahead for the second part of the same example where we are told that

friction factor is 0.8. So we are told that $\frac{V_{r2}}{V_{r1}}=0.8$. So we have $V_{r2}=V_{r1} \times 0.8$. Our calculation of β , our calculation of V_{r1} is intact, it will be same as what we did for part a.

So we will carry forward with the same V_{r1} , which is basically, 639.25×0.8 , so $V_{r2}=511.4 \text{ m/sec}$. So we have $\Delta V_w = V_{r1} \cos(\beta_1) + V_{r2} \cos(\beta_2)$ and now this will be $639.25 \times \cos(32.35) + 511.4 \times \cos(32.35)$. So we have $\Delta V_w = 972.06 \text{ m/sec}$. So we have $\text{thrust} = \dot{m} \Delta V_w = 0.75 \times 972.06$ and this we can find out 729.05 Newton.

So $\text{power} = \text{thrust} \times V_b = 729.05 \times 400$. So this gives us magnitude 291.62 kilowatt. Similarly, we can find out change in axial velocity is equal to $V_{r1} \sin(\beta_1) - V_{r2} \sin(\beta_2) = \Delta V_a$. So we have axial thrust is equal to axial thrust maybe $T_a = \dot{m}(V_{r1} \times \sin \beta_1 - V_{r2} \times \sin \beta_2)$ we can find out T_a gives you all numbers which are known to us.

So we can put V_{r1} over here $T_a = 639.25 \times \sin(32.35) - 511.4 \times \sin(32.35)$, so axial thrust for us is 51.3 Newton. So this is how we can solve the example, which is for the impulse turbine. We will move ahead, in the next example and iterate that an impulse in turbine has number of pressure stages.

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An impulse steam turbine has a number of pressure stages, each having a row of nozzles and a single ring of blade. The nozzle angle in the first stage is 20° and the blade exit angle is 30° with reference to the plane of rotation. The mean blade speed is 130 m/s and the velocity of steam leaving the nozzle is 330 m/s.

- Taking the blade friction factor as 0.8 and a nozzle efficiency of 0.85, determine the work done in the stage per kg of steam and the stage efficiency.
- If the steam supply to the first stage is at 20 bar, 250°C and the condenser pressure is 0.07 bar, estimate the number of stages required, assuming that the stage efficiency and the work done are the same for all stages and that the reheat factor is 1.06.

$\alpha = 20^\circ$ $\beta_2 = 30^\circ$ $V_b = 130 \text{ m/s}$ $V_1 = 330 \text{ m/s}$
 $V_{r1} \sin \alpha_1 = V_1 \sin \alpha$
 $V_{r1} \cos \beta_1 + V_b = V_1 \cos \alpha$
 $\tan \beta_1 = \frac{V_1 \sin \alpha}{V_1 \cos \alpha - V_b} = \frac{330 \times \sin(20)}{330 \times \cos(20) - 130} = 0.621 \rightarrow \beta_1 = 32.07^\circ$
 $V_{r1} = \frac{V_1 \sin \alpha}{\sin \beta_1} = \frac{330 \times \sin(20)}{\sin(32.07)} = 212.5 \text{ m/s}$
 $V_{r2} = V_{r1} \times 0.8 = 170.04 \text{ m/s}$
 $\Delta V_a = V_{r1} \cos \beta_1 + V_{r2} \cos \beta_2$
 $\Delta V_a = 212.5 \times \cos(32.07) + 170.04 \times \cos(30) = 372.3 \text{ m/s}$

Each having a row of nozzles and single ring of blade. The nozzle angle in the first stage is 20° and the blade exit angle is 30° with reference to the plane of rotation. The mean blade speed is 130 meter per second and velocity of the steam leaving the nozzle is 330 meter per

second taking the blade friction factor as 0.8 and nozzle efficiency as 0.85, determine the work done for stage per kg of steam and the stage efficiency.

If steam is applied in the first stage is at 20 bar 250 °C and the condenser pressure is 0.04 bar estimate the number of stages required, assume that the stage efficiency and the work done are same for all stages and that the reheat factor is 1.08. So we start, so for that let us draw again the velocity triangle. This is V_1 , this is V_{r1} , this is $V_{a1}, \beta_1, \alpha, \beta_2, \Delta$, this is V_{r2} .

Now we are given that $\alpha=20^\circ$, we are also told that $\beta_2=30^\circ$, $V_b=130\text{ m/sec}$ and $V_1=330\text{ m/sec}$. So these are the things with us now we have also drawn the velocity triangle. So let us move ahead with part a. So here as well, we can write down the expression as $V_{r1} \sin(\beta_1) = V_1 \sin(\alpha)$ and the also we can know $V_{r1} \cos(\beta_1) + V_b = V_1 \cos(\alpha)$.

We can put the V_b or here and then we can divide and then what we got last time we will get

the formula which is $\tan \beta_1 = \frac{V_1 \sin(\alpha)}{V_1 \cos(\alpha) - V_b} = \frac{330 \sin(20)}{330 \cos(20) - 130}$ and this gives us $\tan(\beta_1)$ as 0.627 and which gives us $\beta_1 = 32.07^\circ$. So we can have V_{r1} equal to basically, what we know

V_{r1} from this equation we can write down $V_{r1} = \frac{V_1 \sin(\alpha)}{\sin(\beta_1)}$.

So we can write down β as $\frac{330 \sin(20)}{\sin(32.07)}$ and then this gives us V_{r1} as 212.5 m/sec. So

$V_{r2} = V_{r1} \times \text{friction factor} = 212.3 \times 0.8$, so we have $V_{r2} = 170.25\text{ m/sec}$. So we have got this all, so now we can move $\Delta V_w = V_{r1} \cos(\beta_1) + V_{r2} \cos(\beta_2)$. So we have $\Delta V_w = 212.5 \times \cos(32.07) + 170.25 \times \cos(30)$ and this gives us $\Delta V_w, \beta_2$ is given as 30° .

So now in this we can find out ΔV_w and this turns out to be 372.3 m/sec and we will go ahead and find out rest of the things till time we have found out ΔV_w blades are not symmetrical in this case that is why we do not have β 's as equal, so we have found out β_1, β_2 is already given, now we have found on changing tangential velocity.

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$$\begin{aligned}
 \text{Tangential Thrust} &= \dot{m} \Delta V_w \\
 \text{Power} &= \dot{m} V_b \Delta V_w \\
 \therefore P &= 1 \times 130 \times 327.33 = 42.55 \text{ kJ/kg} \\
 \eta_D &= \frac{P}{\frac{1}{2} \dot{m} V_1^2} = \frac{2 \times 42.55 \times 10^3}{1 \times 330 \times 330} = 78.15\% \\
 \eta_{stage} &= \eta_j \times \eta_D = 0.85 \times 0.78 = 0.66 \\
 \text{b) } \eta_T &= \eta_j \times \eta_D \times \text{Reheat factor} \\
 \eta_T &= \eta_{stage} \times \text{Reheat factor} \\
 \eta_T &= 0.66 \times 1.06 = 0.7041 \\
 h_1 &= 2902.3 \text{ kJ/kg} \quad \therefore S_1 = 6.5466 \text{ kJ/kg} \\
 s_2 &= s_1 + x_2 s_{fg} \rightarrow s_1 = s_2 \\
 \therefore s_1 = s_2 &= s_f + x_2 s_{fg} \Rightarrow x_2 = 0.7757
 \end{aligned}$$

$$\begin{aligned}
 h_2' &= h_2 + x_2 \cdot h_{fg} = 2032.29 \text{ kJ/kg} \\
 \eta &= \frac{\Delta h_{ideal}}{\Delta h_{stage}} = \frac{h_2 - h_1}{\Delta h_{stage}} \\
 h_2 - h_1 &= \eta_T \cdot (h_1 - h_{2s}) = 0.7041 \times (2902.3 - 2032.29) \\
 h_2 - h_1 &= 612.57 \text{ kJ/kg} \\
 (h_2 - h_1) &= \Delta h_{ideal} \\
 \therefore \eta &= \frac{\Delta h_{ideal}}{\Delta h_{stage}} = \frac{612.57}{42.55} \rightarrow \text{Power diagram} \\
 \eta &= 14.39 \text{ or } 14\%
 \end{aligned}$$

So tangential thrust is equal to basically, $\dot{m} \Delta V_w$ and then $power = \dot{m} V_b \Delta V_w$, so power is equal to \dot{m} , let us assume 1 kg per second, V_b is told as 130 and ΔV_w we have just find out 327.33 and then this power is 42.55 kilojoule per Kg. So we can have diagram efficiency

$$\eta_D = \frac{P}{\frac{1}{2} \dot{m} V_1^2}$$

So we know it is 2, these 2 will go in the numerator 2 into 42.55 divided by \dot{m} is $1 \times 330 \times 330$. So this is in kilowatt joule we can make it into joule 330×330 and this gives us diagram efficiency as 78.15%. Now we can find out stage efficiency and stage efficiency is nozzle efficiency into diagram efficiency or blade efficiency. So nozzle efficiency is 0.85 and diagram efficiency is 0.78.

So we get it as 0.66, so this is the stage efficiency. Now we will go to the part b, in part b needs turbine efficiency in total that is equal to nozzle efficiency, blade efficiency into reheat factor. So we have found out this answer, stage efficiency into reheat factor. So turbine efficiency or internal efficiency is $\eta_T = \eta_{stage} \times \text{Reheat factor} = 0.66 \times 1.06 = 0.7041$, we can find out enthalpy that let to the nozzle from the given temperature and pressure condition.

That is equal to 2902.3 kilojoule per Kg, similarly, we will also note down entropy under same equation which is 6.5466 and S_g corresponding the same saturation pressure we can find out under steam level and then we can say that $S_2 = S_1$ basically, dash and then from here

we actually need $S_1 = S_2'$. So we have basically, $S_1 = S_f + x_2 S_{fg}$ and this gives us S since S_2' is known and then we know S_f liquid saturation entropy for the condensation pressure.

Then this is the latent entropy for the condensation pressure and then we know x from here which is x is 0.7757, knowing this we can find out ideal enthalpy h_2' at the exit of the nozzle so rest of the enthalpy is converted into kinetic energy. So this is level enthalpy with the flow. So this is equal to $h_f + x_2 \times h_{fg}$ and then we can find out this as 2032.29 kilojoule per kg.

So number of stages is equal to $\frac{\Delta h \vee \dot{c}_{total}}{\Delta h \vee \dot{c}_{stage}}$. So $\Delta h \vee \dot{c}_{total} = \frac{h_2 - h_1}{\Delta h \vee \dot{c}_{stage}}$, here $h_2 - h_1$ is equal to turbine into $h_2 - h_2'$. So $h_1 - h_2'$. So we know this efficiency is 0.7041 into turbine efficiency is this into h_1 and h_1 is basically, the enthalpy at the entry to the nozzle and that is 2902.3. We are given that this is going to have boiler at 20 bar and 250 °C.

So considering these 2 conditions we can find out h_1 , this is the condition for 2', this is the condition for 1. Now in the state 1 we can find out h_1 and we also found out h_2 . So it is minus 2032.29. So we have $h_2 - h_1 = 612.57 \text{ kJ/kg}$. So this is the $h_2 - h_1 = \Delta h \vee \dot{c}_{total}$. So we have

$$n = \frac{\Delta h \vee \dot{c}_{stage}}{\Delta h \vee \dot{c}_{total}}$$

So Δh stage, Δh total divided by Δh stage and $\Delta h \vee \dot{c}_{total}$ is 612.57 and Δh stage is 42.55 and basically, this is the power of diagram, so $n = 14.39$, which is almost equal to 15. So we need 15 number of stages for this turbine. So here we have solved this example on impulse turbine.