

Principles of Mechanical Measurement
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Module – 05
Displacement Measurement
Lecture – 15
Inductive, Capacitive & Optical Devices

Good morning everyone, welcome back to the second lecture on the topic of displacement measurement. So, in the previous lecture we have learned about 2 of different methods for displacement or stress measurement, namely the potentiometer and also the resistive strain gauges. Both of them have same working principle that is as the length of the conductor changes, the corresponding resistance also changes, but they are employed in a different way. Potentiometer generally is used for larger scale application where a strain gauge can be much smaller in size and therefore, can directly be attached to the specimen where you want to have some kind of displacement or stress measurement and we can get extremely small deflection also recorded in the device.

So, today we shall be moving forward to learn about a few more such kind of devices, preferably electromechanical transducers; that is where the displacement is generally converted to some kind of electrical signal; like the two we have discussed in the previous lecture. They are also electromechanical transducers, but today we shall be learning about two three more electromechanical transducers and maybe one or two more which are not electromechanical, but something else let us see depending on the time how we can go for it, but as we always keep on doing.

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A little exercise

A potentiometer has a wire of length 8 m and the resistance of the wire is 20 Ω . It is connected in series with a cell of emf 2 V and an internal resistance 2 Ω and a rheostat. Find the value of the resistance in rheostat when the potential drop along the wire is 20 $\mu\text{V/mm}$.

$L = 8\text{ m}$
 $R = 20\ \Omega$
 $e = 2\text{ V}$
 $r = 2\ \Omega$
 $\frac{e}{L} = \frac{2}{8} = 0.25\text{ V/m}$

We shall be starting with a little bit of exercise. Now this problem I mentioned in the previous class itself on potentiometer. So, I hope you have already tried to solve it. So, I shall not be giving a complete solution for this just some rough idea about how to solve it because I do not have all the calculated values for this one, but just to see if the way I am trying to solve this whether it matches with your idea or not.

So, what is given here? Here we are talking about a potentiometer which has a length of 8 meter. So, you have L equal to 8 meter and the resistance of the corresponding wire where R is equal to 20 ohm. So, corresponding potential drop should be equal to oh sorry, we are yet to get the voltage here sorry ah, we are connecting this particular potentiometer with the cell of emf 2 volt. So, we are talking about the voltage of 2 volt instead of using volt let us use something that we have done earlier. So, e_x or v_x and there is something new that this cell itself has an internal resistance of 2 ohm.

So, we will be coming back to this later on [vocalized- noise], but before that the potential drop K should be equal to the excitation voltage divided by length. So, it is equal to 2 volt divided by 8 meter this much of volt meter that is 0.25 volt per meter. This is the potential drop for this ah, but now they are saying that the potential drop this is the potential for this. Now they are saying that this potential drop has to be limited to 20 micro volt per millimeter. Actually the calculation that we have shown here that is

wrong because here we are talking here we are not taking care of all the resistances, it is mentioned that we need to have this particular amount of potential drop.

So, let us calculate it in a different way, but before that what is the configuration that we have.

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A little exercise

A potentiometer has a wire of length 8 m and the resistance of the wire is 20 Ω. It is connected in series with a cell of emf 2 V and an internal resistance 2 Ω and a rheostat. Find the value of the resistance in rheostat when the potential drop along the wire is 20 μV/mm.

$L = 8\text{ m}$
 $R = 20\ \Omega$
 $E = 2\text{ V}$
 $K = \frac{V_0}{L} = \frac{IR}{L} =$

Here we have a cell and then we have the resistance. This is the resistance which is having a value of 20 ohm, but how the cell is looking like? The cell here has an internal resistance; that means, in a circuit actually you have this cell of emf 2 volt and then the this is the internal resistance for this cell, this is the 2 volt and this is this 2 ohm internal resistance, this is also present in the circuit and now we are adding another rheostat that is we are adding another rheostat to this circuit to complete this one.

So, we are having a 2 ohm internal resistance then a 20 ohm resistance for the register and we have to identify this particular rheostat register or sh or let us say write R rh we have to identify this value so, that we can get the total potential drop to be 20 micro volt per millimeter. Now we know that this K is equal to ex by L, now whatever the if we write this in terms of the current that is flowing through a general we write it as IR upon L. Now how much is I? Here it will not be equal to x it will be equal to the voltage drop that is we are having across this that you see this is V o. So, this will be actually this V o by r L. Now it is IR by L then.

Now, how much is I? Look at the circuit if we erase this one now in your circuit your I should be equal to the cell emf that you are providing by summation of all the resistances. So, I is replaced by e x into R by L and summation of all the resistances, how many resistances we have?

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A little exercise

A potentiometer has a wire of length 8 m and the resistance of the wire is 20 Ω. It is connected in series with a cell of emf 2 V and an internal resistance 2 Ω and a rheostat. Find the value of the resistance in rheostat when the potential drop along the wire is 20 μV/mm.

$L = 8\text{ m}$
 $R = 20\ \Omega$
 $e = 2\text{ V}$

$I = \frac{e}{2R}$
 $K = \frac{V_0}{L} = \frac{IR}{L} = \frac{eR}{L(20+2+R_{rh})} = 20\ \mu\text{V/mm}$
 $\approx 20 \times 10^{-3}\ \text{V/m}$

We have our 20 ohm resistance for the register plus 2 ohm internal resistance plus this R rh. This thing should be equal to that 20 micro volt per millimeter or if we can write in terms x into 20 into 10 to the power minus 3 volt per meter. So, from this the value of e x is given to be equal to 2 volt, capital is given to be 20 ohm from there you can easily identify the value of this R rh.

So, this is a procedure that you have to follow here. Be careful that here we are talking about not one, but three resistances, one corresponding to the register itself. Therefore, the potentiometer, one the internal resistance of 2 ohm corresponding to that cell emf and also the rheostat which we are adding this in the circuit to modify the potential drop.

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A little exercise

A potentiometer has a wire of length 8 m and the resistance of the wire is 20 Ω. It is connected in series with a cell of emf 2 V and an internal resistance 2 Ω and a rheostat. Find the value of the resistance in rheostat when the potential drop along the wire is 20 μV/mm.

$L = 8\text{ m}$
 $R = 20\ \Omega$
 $\mathcal{E} = 2\text{ V}$
 $k = \frac{V_0}{L} = \frac{IR}{L} = \frac{\mathcal{E}R}{L(20+2+R_{rh})} = 20\ \mu\text{V/mm}$

A strain gage ($R = 120\ \Omega$) is mounted on a steel specimen ($E = 200\ \text{GPa}$). On axial loading, the length of the rod changes by 3% and resistivity of the material changes by 0.3%. If $\nu = 0.3$, calculate gage factor. If the connected device identifies the change in resistance with an accuracy of $\pm 0.02\ \Omega$, estimate the uncertainty in stress & strain measurement.

$R = 120\ \Omega$
 $\mathcal{E} = \frac{\Delta L}{L} = 0.03$
 $\frac{\Delta R}{R} = 0.003$
 $\nu = 0.3$
 $S_g = \frac{\Delta R/R}{\Delta L/L} = \frac{\Delta R/R}{\mathcal{E}} = (1+2\nu)\frac{\Delta L}{L} + \frac{\Delta \rho}{\rho}$
 $= \frac{0.003}{0.03} \times 1.7 = 0.057$
 $S_g = \frac{\Delta R/R}{\mathcal{E}}$
 $\Rightarrow \mathcal{E} = \frac{\Delta R}{R S_g}$
 $\Rightarrow \mathcal{E} = \frac{0.02}{120 \times 1.7} = \pm 9.8 \times 10^{-5}$
 $\sigma = E \mathcal{E} = 200 \times 10^9 \times 9.8 \times 10^{-5} = \pm 19.6\ \text{MPa}$
 $\Rightarrow \sigma = \pm 19.6\ \text{MPa}$
 $\Rightarrow \mathcal{E} = \pm 9.8 \times 10^{-5}$

Let us now try to solve something for the strain gauge. There is a strain gauge that we have learned in the previous class. Let us try to solve the problem on that. So, quite straightforward problem is given here for the strain gauge we are having a resistance of 120 ohm then it is mounted on a specimen and the specimens is having Young's modulus of 200 Giga Pascal.

Now, there is an axial loading has been applied on this ah. Here we are assuming that the strain gauge is also mounted on the axial direction only. So, that it can directly since this particular loading. Now the length of the rod has changed by 3 percent. Now what does this mean by? What is this 3 percent? This is basically this ΔL upon L to be equal to 3 percent or 0.03 and a resistivity of the material changes by 0.03 percent; that means, $\Delta \rho$ upon ρ is equal to 0.003 and it is also given that the Poisson ratio is equal to 0.3.

So, now how can we calculate the gauge factor? What is the definition of gauge factor? Gauge factor commonly represented by S_g or some similar symbol is ΔR upon R divided by ΔL upon L . Now ΔL upon L which can also be related to the strain that is directly given and R is also given, but ΔR is the that we have to calculate. How can we calculate ΔR ? You remember the derivation that we did in a previous class. So, ΔR upon R can be related to three factors. What are the three factors?

There it was $1 + 2 \nu \frac{\Delta L}{L} + \frac{\Delta \rho}{\rho}$. Remember the derivation for this how we calculated this, there are three components.

And the first part is related to the or the change in resistance because of the change in length, this is because of the change in cross section area, this is because of the change in the resistivity. So, you have all the values available. So, you put there $1 + 2 \nu \frac{\Delta L}{L}$ into $\frac{\Delta R}{R}$ is given as $0.03 + \text{change in resistivity}$ is equal to this leading to I have calculated the numbers for you, it is equal to 0.051 . So, your S_g is equal to 0.051 divided by 0.03 which leads to 1.7 . So, this is the first answer we have the gauge factor and now we have something very interesting it is given that the connected device here with the strain gauge we are connecting some device to identify the strain change in resistance something like a wheatstone bridge and the device itself measures this change in resistance in an accuracy of 0.02 ohm .

So, we have to identify the uncertainty that is involved in the final stress and strain measurement which is actually getting induced because of this uncertainty in resistance estimation. How can you estimate this? Let us first start with the stress. So, how we can get stress? We know that the Young's modulus is defined as stress upon strain or stress is equal to $\epsilon \times E$. Now how much is ϵ ? ϵ is $\frac{\Delta L}{L}$, but we can write in a different way, we know that S_g is equal to $\frac{\Delta R}{R}$ divided by ϵ . So, from there we can write ϵ is equal to $\frac{\Delta R}{R} \times S_g$. Why we are getting this ΔR into equation because ΔR is actually the one that refers to the change in resistance and so its uncertainty will be involved there. Now if we get that here. So, we have $\frac{\Delta R}{R} \times S_g$ this entire bracketed term if we take as constant because E is generally property of the material that is given gauge constant gauge factor can be taken as constant.

And R is the resistance of the strain gauge initial resistance which we do not have any information about the uncertainty involved there. So, you are also taking that to be a constant then if we differentiate $\frac{\Delta \sigma}{\sigma}$ $\frac{\Delta R}{R}$ will be how much will be $E \times R$ into $S_g \Delta R$; that means, the uncertainty involved in this estimation will be equal to $\frac{\Delta R}{R} \times S_g$ into uncertainty involved in ΔR that is equal to $E \times R \times S_g$ into uncertainty involved in ΔR estimation. Now we have or we should put a plus minus symbol before this because uncertainty can be plus or minus.

So, if we put the values your E is given as 200 GPa that is 200 into 10 to the power 9 always try to convert everything to SI units ah. So, it goes to Pascal R is given as 120 ohm S g we have just calculated as 1.7 and uncertainty is 0.02 it is given in absolute unit only. So, there is no need to convert anything. So, corresponding uncertainty will come to be plus minus 19.6 mega Pascal.

This is the uncertainty involved in the stress estimation and how much uncertainty is involved in the strain estimation? Now we can directly make use of this particular relation from there. So, this will become $1 \pm \frac{19.6}{E}$ into uncertainty to whole means stress estimation that is this plus sorry, this plus minus 19.6 divided by how much was E that was 200 into 10 to the power 9 10 to the power 9 be careful the numerator there has to be a 10 to the power 6 also because that is in mega Pascal. So, once we put the values it comes out to be actually I have not calculated the values, but we can easily calculate it from here.

So, it is 19.6 divided by 2 into 10 to the power minus 5 and if we calculate from there. So, it becomes 9.8 into 10 to the power minus 5 this is the uncertainty that is involved in the strain calculation. So, this way we can connect this one to something that we have learned actually in chapter number one, remember how we did that the uncertainty analysis there this is precisely the fall procedure once we have the relation here E R and S g are taken as constant.

So, ΔR is the independent variable and σ and the stress is the dependent variable. So, we calculate this formula and this is the one that we have derived there. So, making use of that rest is quite straightforward. So, this way we can also estimate the uncertainty involved in any kind of strain gauge measurement just from the simple knowledge of the accuracy of the corresponding wheatstone bridge or whatever instrument you are using for measuring the change in resistance that completes our discussion on the topic of resistive based instruments just one small thing that I would like to add I do not have too much space available let me make a small space somewhere here.

We have seen there the ratio $\frac{R_p}{R_m}$ should be very very small or should be very very small that is when R_p is very small then only we can neglect that loading effect and we can get a linear representation of a quite accurate zero order representation. So, R_p

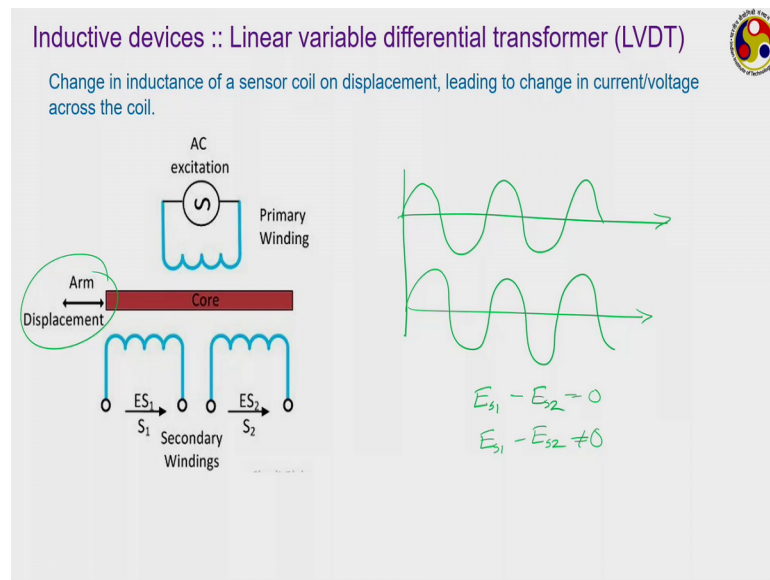
should be very very small on the contrary R_p should be large in order to have a high resolution that is another way actually we can without increasing the value of R_p you may think that we can also increase the value of the excitation voltage because what was a relation a relation was E_{out} by let me write once more here our relation was E_{out} by e_x is equal to x_i upon x_t .

So, if we increase this e_x then also we can get a higher resolution I have the value of e_x the relation to be more, but practically that is also not possible because we also have to take care of total power that a potentiometer is dissipating and that is generally fixed most of the potential that you get they generally come with the power rating 5 watt 10 watt or something like this; that means, whatever power it is going to dissipate the voltage has to adjust accordingly.

Now, what is or rather the excitation voltage has to get adjusted accordingly and also the value of resistance. Now what is the relation? We know if P represents power that will be equal to x^2 by R that is the voltage that you can put in that should be equal to root over P into R where P is given or I should say $R P$. So, here this P is the power that is generally specified we generally cannot go to very high power level therefore, whatever may be the value of $R P$ e_x has to be adjusted accordingly and we cannot just put any high value of e_x .

So, that completes our discussion on potentiometer and resistive strain gauges. Let us move to a different category of device. The next device that we have in line is our inductive devices.

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Like in the previous case we made use of the change in resistance to measure displacements. Here we are going to make use of the change in inductance in some coil inductive coil to measure the displacement and the device that we are going to talk about here is known as LVDT or linear variable differential transformer, an extremely common method of displacement measurement can operate a very wide range of parameters or wide range of distance values and this is a very common measurement device in some very critical applications areas.

So, as the as I mentioned as the concept is the change in inductance of a sensor coil on displacement leading to the change in the current or voltage across the coil so, here the arrangement is something like this. The simple design of LVDT involves one permeable core and one side of the core we have a single primary winding it is quite similar to that similar to a transformer, only difference is that here instead of one we have two secondary windings.

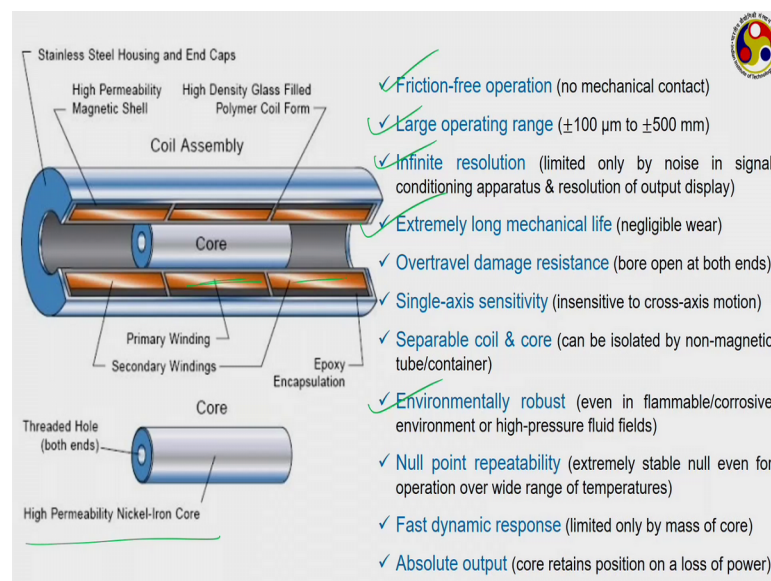
So, the core when they are in the normal position the core is located the center position or I should say the primary winding is located at the center position and both the secondary windings are equally placed with respect to the primary therefore, same amount of mutual inductance will be induced in both of them. Now the core is the one that is connected to a displacement sensor. So, whenever there is some kind of displacement the core itself will move there by changing the relative position of the

primary and secondary and accordingly the inductance or mutual inductance in the secondary coils their magnitude will change under normal condition like what we can see here.

Let us say if we plot with respect to time then if the primer is subjected to a sinusoidal voltage like this, both the secondary coils will also exactly follow the same. My drawing show some kind of phase lag here, but actually that is not there both of them will show the exact follow it exactly; however, whenever there is some kind of deflection then there will be some kind of phase lag induced in them and the voltage values will not be same in both the secondary windings there by leading to some output voltage value, which with proper design can be made directly proportional to the displacement of the core.

So, under normal situation we have this ES_1 to be equal to ES_2 when the core is in its normal position or sometimes referred to as the null position because we get this ES_1 minus ES_2 is equal to 0 in their situation; however, when the core is moved because of the displacement some displacement then ES_1 minus ES_2 that will not be equal to 0. So, that will give some kind of output voltage and that output voltage will be directly proportional to this displacement of the core that is the operating principle of LVDT or generally any kind of inductive devices.

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So, this is a typical configuration what we have here we have the core the core is generally made of highly permeable material something like nickel iron material and

there will be a threaded hole through this center of the core. So, the on core we have on one side we have the primary winding this is the primary winding and these are the two secondary windings. They are encapsulated inside a particular medium and generally both ends of this one this end and this end they are open. So, that in case extreme situation if it is subjected to some kind of some kind of displacement value which is beyond its limit the core will just move outside the shell there by preventing any kind of damage.

So, this is the simple construction of an LVDT transformer it offers a huge amount of advantages compared to the other displacement measuring device. The first big advantage is friction free operation. Here there is no direct mechanical contact between different components the primary winding, the secondary winding, and the core all are separated from each other and therefore, they can move without any kind of rubbing effect then they can give very large operating range, the range of such kind of device operation can be extremely large ranging from a few microns to about 500 millimeter and in certain special design cases it can also go up to even the value of 1 meter ah. Next is infinite resolution.

Now, the resolution of the device you know where theoretically is infinite because even with the smallest possible displacement there will be some kind of output voltage produced and therefore, its resolution is infinite you I hope you remember what how we define resolution, it is the change in output corresponding to the change in input or you can say resolution mathematical is change in output divided by change in input. Now whatever may be the smallest possible change in input that you can think about there will be a change in output.

That giving thereby giving the infinite resolution of the device, but practical cases the resolution will be finite because there can be two things, one there can be some kind of noise in incorporated by the signal conditioning approaches if there is any and secondly, which is much more relevant that is a voltage output voltage that of course, you have to measure by using some kind of voltage measuring instrument like a voltmeter. Now that may have certain kind of resolution and that is why mostly the resolution of the LVDT depends on the resolution of this voltage measuring device. Next extremely long mechanical life as we have mentioned earlier there is no friction.

So, there is no delay to orient here there is hardly any edging effect and we can have very very long life, very long means referred to really very long something in the range of 150 200 years that you are talking about. Over travel damage resistance as I have mentioned here both ends are open to the to this and therefore, if the core is made to travel more than it is supposed to there it no damage will be done the it can be protected and also as the core pops out of the system there will be of course, no voltage induction no induction and hence the devices stop working for that instant and then you can easily get the core back into its normal position and device will be back. Now single axis sensitivity then only the core moves only in one particular direction they are sensing displacement only along that direction it is often immune to any kind of transverse motion or vibration the core

and coils can easily be separated if required like suppose if we want the core to move in some highly pressurized fluid then we can easily separate the coils from the core by using some non magnetic material, non magnetic shell or tube. So, that the pressurized fluid and the core remains inside that non magnetic tube and the windings remain outside there thereby easily protecting the windings from this high pressure ah. Environmentally their robust even in flammable or corrosive environments also we can use this or like I just mentioned if we are working with a high pressure flow field there also they can easily be operated with null point repeatability is the null position of the null is very important. Because when it is not subjected to any kind of displacement then we should get a 0 voltage which we refer to as this null point or null position.

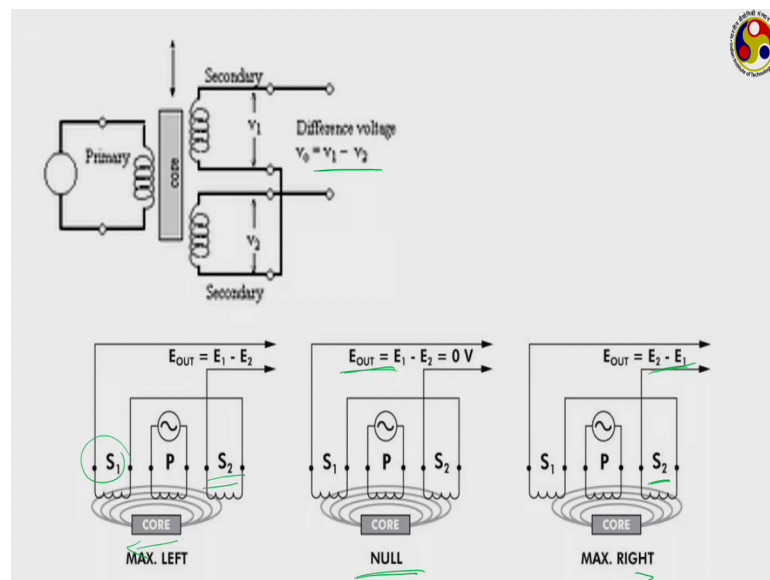
Now, the null position is highly repeatable even after several fluctuations even when the instrument is sub subjected to some fluctuation measurement repeatedly changing the position and then going back to null it is able to produce a null quite regularly with high accuracy. And also it does not suffer too much because of the changing the operating temperatures the null position generally is retained even at very high temperatures. The dynamic response is very fast it is limited only by the mass of the core itself and that mass generally is very very small.

So, we get quite fast response again the response that actually will be getting that depends on the resolution and the response of the voltage measuring instrument and finally, we are talking about absolute output here even if there is a loss in power during the operation whatever last position the coil was it will remain in that position and as

soon as the power is back it will start to operate back again. So, we always get an absolute output from this particular device. So, these are some of the advantages very important of this particular list we can talk about think about this friction free operation we have this large operating range infinite resolution and this long mechanical life these are all very very important points.

And of course, we also have to talk about the robustness of the device because of so many advantages and also the high accuracy that they are able to provide with such large level of resolution. Then we can easily use LVDT in all possible kinds of application starting from simple laboratory experiments for measuring the displacement of say a tool post on a lathe to something related to the space travel or satellites.

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However, in most commonly we do not use the voltages that way do not use the secondary windings that way rather what we do is referred to as the series of post-secondary. The series opposing secondary refers to this just look at this diagram here the opposite ends of the two secondary coils are connected with each other there by your reference voltage V_1 and V_2 sorry the output voltage V_1 and V_2 will always be opposite to each other thereby giving v naught equal to V_1 minus V_2 .

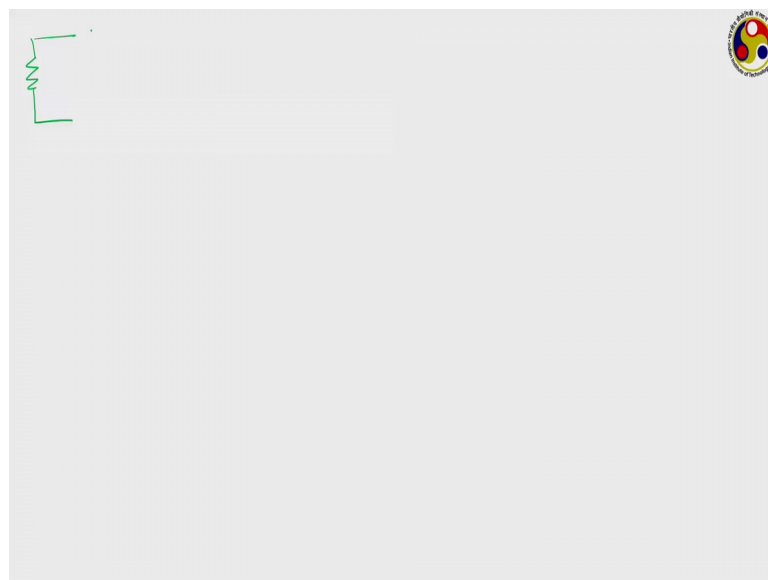
So, this is what we referred as the series opposed secondary this is the most popular configuration for LVDTs this is how they operate like when the core is at the null position just look at this first then both coils will be having same voltage induced and so

you will be having V_{OUT} equal to 0. When it moves towards the left side the core is moving towards left then the mutual inductance in S_1 will be larger than that induced in S_2 there by E_1 will be higher than S_1 you will be getting an net value of E_1 minus E_2 . Similarly when it move towards right the mutual inductance in S_2 will be more compared to S_1 .

So, E_2 will be higher than E_1 and you get E_2 minus E_1 therefore, not only the magnitude you also get a sense of the direction on which direction the core is moving or in a way your sensor is moving. The series of a secondary configuration is the one that is the most popular one. Let us know, but this is the situation that we are talking about when the output scales are I should say the output nodes are open means we are talking on open circuit voltage; however, practically we have to connect some voltage measuring instrument there thereby giving some kind of loading effect and we have to get an idea about the loading effect.

So, let us try to do that with some simple mathematics. So, initially we took the configuration something like this.

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So, this is your primary, let us say instead of drawing this way let us draw this way.

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The slide contains a circuit diagram and several equations. The circuit diagram shows a primary winding with resistance R_p and inductance L_p connected to an AC source e_m . The secondary side consists of two windings with inductances $L_{s1/2}$ and $L_{s2/2}$ connected in series-opposed configuration, with a total secondary voltage e_s and current i_s .

The equations shown are:

$$R_p I_p + L_p \frac{dI_p}{dt} - e_{ex} = 0$$

$$\Rightarrow (R_p + L_p D) I_p = e_{ex}$$

$$\Rightarrow I_p = \frac{e_{ex}}{R_p + L_p D}$$

$$e_{s1} = M_1 \frac{dI_p}{dt}$$

$$e_{s2} = M_2 \frac{dI_p}{dt}$$

$$e_o = e_{s1} - e_{s2} = (M_1 - M_2) \frac{dI_p}{dt}$$

$$e_o = (M_1 - M_2) \frac{D}{R_p + L_p D} e_{ex}$$

$$\Rightarrow \frac{e_o}{e_{ex}} = \frac{(M_1 - M_2) D}{R_p + L_p D}$$

$$= \frac{\left(\frac{M_1 - M_2}{R_p}\right) D}{1 + \left(\frac{L_p}{R_p}\right) D}$$

So, this is your primary side source the resistance of the primary circuit and the inductance of the primary circuit. So, this resistance let us say R primary this is L primary and this is your source say E excitation. So, a current I_p will be flowing through this and this is the core and.

Now, you have two secondary coils, this is the first secondary coil and its corresponding resistance and then we have the; we have we should have drawn the secondary voltage as well. So, this is the secondary side voltage and similarly on the other coil also we shall be getting something there, this is a series opposed configuration. So, when this side is open then we are going to get some voltage say e naught here between these two ah. Let us say that total resistance of the secondary is divided equally into the two circuit similarly the inductance also is divided into 2 as L_s by 2 and we get e_{s1} here and we are getting e_{s2} here.

So, when the circuit is open then if we apply Kirchhoff's law on the primary side then what we are getting there is a current I_p flowing through the resistor R_p and also this inductor L_p then we can write $R_p I_p$ plus $L_p \frac{dI_p}{dt}$ minus e_{ex} is equal to 0. So, this is for the primary side this primary current which is flowing through this coil L_p that will induce something on the secondary side voltage how about the secondary open circuit there will be no current flowing through the secondary side.

Now, how much will be the mutual inductance if M_1 refers to the mutual inductance in coil number one then I should say e_{s1} will be equal to $M_1 \frac{dI_p}{dt}$ similarly the voltage induced in the secondary circuit e_{s2} will be equal to $M_2 \frac{dI_p}{dt}$ where M_1 and M_2 refers to the corresponding mutual inductance. M_1 is the mutual inductance in coil one secondary coil one, M_2 is a mutual inductance in secondary coil 2.

So, the net secondary voltage e_{naught} will be equal to $e_{s1} - e_{s2}$ that is $M_1 - M_2$ into $\frac{dI_p}{dt}$; however, there is no current flowing through the secondary circuit side and therefore, there will be no inductance back on the primary circuit corresponding to this and also no drop in the register, but this is the output voltage that you are getting. So, if we want to write a relationship between output and input then we can write this way let us use our earlier notation we can write this as $R_p + L_p \frac{d}{dt} I_p$ to be equal to e_{ex} ; that means, where d is the operator that we have used earlier e_{ex} divided by $R_p + L_p \frac{d}{dt}$. So, if we take it there we have now e_{naught} is equal to $M_1 - M_2$ into $\frac{d}{dt} I_p$ that is if we write $\frac{d}{dt} I_p$, then what we have we have $\frac{d}{dt} I_p = \frac{e_{ex}}{R_p + L_p \frac{d}{dt}}$ that is we have e_{naught} upon a e_{ex} should not be x it is e_{ex} as per our notation.

So, this is equal to again $M_1 - M_2$ to $\frac{d}{dt} I_p$ by $R_p + L_p \frac{d}{dt}$ and quite often we divide both numerator and denominator by with R_p to get $M_1 - M_2$ upon $R_p \frac{d}{dt}$ divided by $1 + L_p$ upon $R_p \frac{d}{dt}$. What kind of system is this? This is a first order system and in first order system how we characterize a first order system in terms of gain static gain and more importantly a time constant now what is your time constant here this L_p upon R_p is the one which is your time constant. So, you can write this as $1 + \tau \frac{d}{dt}$ with τ being L_p upon R_p .

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$$R_p I_p + L_p \frac{dI_p}{dt} - e_{e1} = 0$$

$$\Rightarrow (R_p + L_p D) I_p = e_{e1}$$

$$\Rightarrow I_p = \frac{e_{e1}}{R_p + L_p D}$$

$$e_{e1} = M_1 \frac{dI_p}{dt}$$

$$e_{e2} = M_2 \frac{dI_p}{dt}$$

$$e_o = e_{e1} - e_{e2} = (M_1 - M_2) \frac{dI_p}{dt}$$

$$e_o = (M_1 - M_2) \frac{D}{R_p + L_p D} e_{e1}$$

$$\Rightarrow \frac{e_o}{e_{e1}} = \frac{(M_1 - M_2) D}{R_p + L_p D}$$

$$= \frac{(M_1 - M_2) \omega}{1 + \omega^2 \tau^2}$$

$$\phi = 90^\circ - \tan^{-1}(\omega \tau)$$

$$R_p I_p + L_p \frac{dI_p}{dt} - (M_1 - M_2) \frac{dI_s}{dt} - e_{e1} = 0$$

$$(M_1 - M_2) \frac{dI_p}{dt} + \left(\frac{R_s}{2} + \frac{R_s}{2} + R_m\right) I_s + \left(\frac{L_s}{2} + \frac{L_s}{2}\right) \frac{dI_s}{dt} = 0$$

$$\Rightarrow \left. \begin{aligned} (R_p + L_p D) I_p - (M_1 - M_2) D I_s &= e_{e1} \\ (M_1 - M_2) D I_p + (R_s + R_m + L_s D) I_s &= 0 \end{aligned} \right\} \frac{e_o}{e_{e1}} =$$

So, the value of the resistance and the inductance in the primary circuit determines the time constant for this particular situation and output voltage is not given or rather output is open in circuit we can easily perform a frequency response because it is subjected to a periodic input.

So, you can always sub sup we can always subject this one to a periodic input like a sinusoidal wave and then we can get a solution. I am not going to go for the detailed derivation because such kind of derivations we already know how to get the from our discussion on the second chapter, but from this you can easily calculate the form of e naught by e ex into a form like this where we have M 1 minus M 2 by R p into omega divided by root over 1 plus omega tau whole square to angle of phi where omega refers to frequency and phi is the corresponding phase lag and what will be phi it will be 90 minus tan inverse omega tau this calculation we have done earlier.

You know how to calculate this not very important for I should say it is not mandatory for you to go for this derivation rather up to this is sufficient, but now instead of keeping the circuit open we are closing this by a voltage measuring device. So, we are connecting some R m through this where R m refers to the resistance of the meter now whenever you are connected we have closed this circuit then a current will be starting to flow through this. Let us say I s the current that starts to flow through this and is the current flows through this entire circuit then that will also put some inductance back to the

primary side that I mean because of the presence of this current is the secondary side will be now inducing something on the primary side.

So, we have to rewrite both the equations if you rewrite the equation for the primary side what we have. So, we have $R_p I_p$ as usual plus we have $L_p \frac{d I_p}{dt}$ as usual minus M_1 minus $M_2 \frac{d I_s}{dt}$ minus e_{ex} is equal to 0 this is the equation for the primary applying Kirchhoff's law where this particular term is coming because of the mutual inductance by coil one and two M_1 is an inductance imposed by coil one M_2 is the inductance imposed by coil two and is the current that is flowing through both the coils. Similarly for the secondary side if we right now for the secondary side M_1 minus $M_2 \frac{d I_p}{dt}$ is the current that has been introduced induced into this plus total resistance is R_{s1} for coil one R_{s2} for coil two.

And R_m is the resistance for the meter into I_s plus total inductance of both the coils L_{s1} and L_{s2} that will be equal to 0. So, we can convert both of them into a common equation like the there are two equations. So, if we write in our preferable form then we have for the first from the first equation we have R_p plus $L_p \frac{d I_p}{dt}$ minus M_1 minus $M_2 \frac{d I_s}{dt}$ is equal to e_{ex} that is from the first equation and from the second equation if we write oh sorry here it should be I_s . So, from the second equation if we write it is M_1 minus $M_2 \frac{d I_p}{dt}$ plus R_{s1} plus R_m plus $L_{s1} \frac{d I_s}{dt}$ is equal to 0.

So, you have two equations and you can solve this to get the expressions for I_s and I_p and from there you can get the expression for e_{naught} upon e_{ex} its entire nature you can identify from there. So, this is the mathematical part for an LVDT transformer we can easily if anyone is interested to know the frequency response you can easily calculate from this equations ah. Here interesting to know that the equations are given we can the phase difference or the output voltage and phase difference both will strongly depend upon the frequency of the input signal and with the frequency that will keep on varying both phase lag in this. Therefore, the system will provide accurate performance for only a certain situation.

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Inductive devices :: Linear/Rotary inductosyn

The diagram illustrates the principle of inductosyn. It shows five positions (A, B, C, D, E) of a slider (green) moving across a scale (yellow). The scale has a periodic rectangular pattern. The slider has a similar pattern. The relative displacement between the slider and scale is indicated by arrows and labeled as $1/4\text{Pitch}$, $1/2\text{Pitch}$, $3/4\text{Pitch}$, and 1Pitch . A graph on the right shows the degree of electro magnetic coupling as a function of the relative position. The graph is a sine wave with points A, B, C, D, and E marked. Point A is at the peak, B is at the zero crossing, C is at the trough, D is at the zero crossing, and E is at the peak. The y-axis is labeled 'Degree of electro magnetic coupling' and the x-axis is labeled 'Relative position'.

1. The induction pattern, in the rectangular form, is printed on both Slider & Scale in the Linear type and Stator & Rotor in the Rotary type Scale.
2. When the alternating current is fed to the Slider (Stator) patten, the voltage is energized to the Scale (Rotor) pattern due to the inductive coupling action.
3. The energized voltage will vary as the relative position of Scale (Rotor) and Slider (Stator) changes.

Now, we move to the next device that is also inductive in nature and its operating principle is somewhat similar to LVDT, but with a difference. Here instead of two you are having just one secondary side there is a primary side and there is a single secondary side, but they are not like normal coils rather they are given some special shapes the induction and also where LVDT is mostly used for linear displacement measurement this inductosyn can also be used for rotary displacement measurement rather they are more commonly used for rotary displacement measurement only.

Now the induction patterns that we are generally dealing with in inductosyn they are given rectangular forms, in any inductosyn we generally have two parts one stationary part and one moving part. If we are talking about a linear differential transformer then the stationary part is known as scale moving part is called slider stationary for linear displacement we call them scale and the moving part is slider whereas, for rotary case more conventional term is the stator and rotor, stator the stationary one, rotor is the moving one.

Now both stator and rotor or slider scale they are given the form of rectangular forms which are printed on certain surfaces when current is fed to the stator side or the slider side the voltage is energized to scale on the slider or stator side, I think we have I have made a mistake while writing this on the slider or rotor side this should be rotor side the moving part where alternating current is fed the voltage is energized to the scale pattern

due to the inductive coupling action and the energized voltage will vary as a relative position of the stator and rotor that keeps on changing.

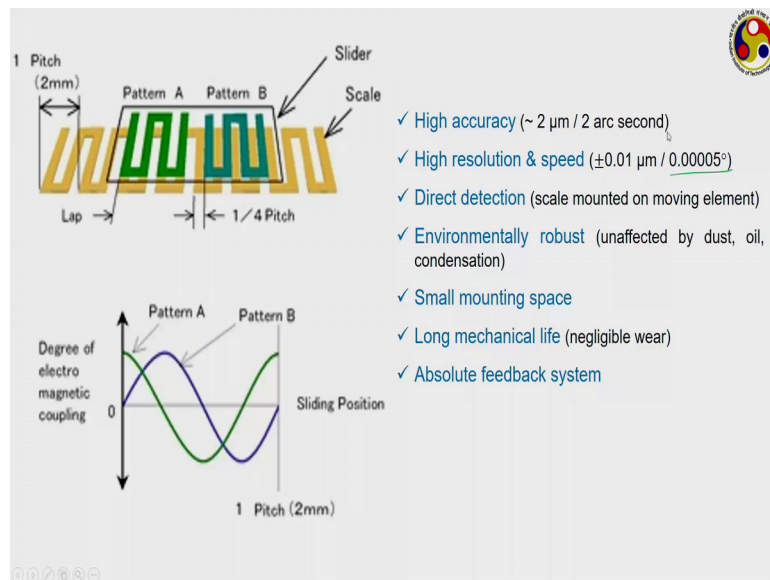
You will get more idea from this structure that we are trying to say. So, this is the initial position the position a refers to initial position look at this we have a repetitive rectangular pattern a pattern something like this for both the scale and slider. The scale is a very long structure, slider is much smaller one which has the same nature, but much smaller in total length and initially there perfect the slider is positioned perfectly over the scale. And if you can see the direction of flow of current are parallel in the same and they are parallel to each other and because of this the degree of electromagnetic coupling is the highest at this particular position.

So, giving a very high output voltage value now because of the displacement of whatever specimen that we are using for if there is one-fourth pitch movement of the slider then now in this case the direction of current flow in the scale will be negated by the direction of the same in the slider there again thereby giving 0 electro kinetic coupling with further movement when moves half pitch then the they are moving again in the same direction. But actually overall direction of flow of current is opposite between stator and rotor or slider and scale.

So, it becomes coupling becomes maximum in a negative direction for three for the three-fourth pitch displacement a further one-fourth pitch displacement giving a total displacement of three-fourth pitch we are back to 0 and finally, when that the movement is full one pitch then we are back to again to the final value or the whatever we started with that a maximum degree of coupling. So, the degree of curve electromagnetic coupling between stator and rotor or slider and scale that depends upon their relative position and as the position keeps on changing the coupling also changes and the corresponding output voltage that can directly be related to the displacement; generally using a linear relationship.

So, inductosyns are very accurate device again quite easily operatable and quite easily be understandable for above the work principle.

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Sometimes instead of one we can use two different patterns like this look at the positions they are generally one-fourth pitch apart and because of this while we get sinusoidal response from one we get a cosine response from the other thereby covering both kinds of possibilities. Inductosyns give us a quite high accuracy high accuracy in the level of just 1 or 2 microns or if we are looking for rotary second rotary displacement it can be as small as 2 arc seconds or you get very high resolution of the level of plus minus 0.1 micron and also very high speed look at the resolution in case of rotary scale we are talking about 10 to the power minus 5 degrees. So, extremely small resolution we can get from this direct detection a scale mounted on the moving element.

So, directly we can get the value of the displacement environmentally they are robust hardly unaffected generally by the presence of dust in the surrounding air any kind of oily or water oily or moisture particles present there and they are generally very small in size thickness of an inductosyn can be hardly in the range of 15 to 20 millimeter therefore, the mounting space required for them is also much smaller compared to an LVDT. Finally, their mechanical life is also very long quite similar to LVDTs. There is also no direct mechanical contact the coupling is only by electromagnetic forces and therefore, they are not subjected to any kind of wear and tear and they can have very long mechanical life something in there any of again 200 years ah. Finally, it is an absolute feedback system.

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Capacitive devices :: Capacitance pickups

Change in capacitance because of the change in the dimensions of the capacitor.

- ✓ High accuracy ($\pm 0.01\%$)
- ✓ Large operating range ($\pm 100 \mu\text{m}$ to $\pm 10 \text{mm}$)
- ✓ Long mechanical life (negligible wear)
- ✓ Environmentally robust

Conductive Plates
Dielectric (ϵ)
Distance between plates: d
Overlapping area of plates: A

Capacitance (C) = $\frac{\epsilon A}{d}$

ϵ = permittivity of dielectric
 A = overlapping area of plates
 d = distance between plates

(a) (b) (c)

So, these two are inductive devices now quickly we shall be talking about. One capacitive device which is also called capacitance pickups where we use the idea of capacitance to measure the displacement concept is quite similar to the previous two cases instead of resistance or inductance here the capacitance changes because of the change in the dimension of the capacitor. Now theory of capacitor is all known to you when two conductive plates are placed very close to each other the distance then we can have a capacitance something like this where A refers to the area and d refers to the distance. The distance generally distribute the place the distant signal can be very very small or hardly in the range of 0.01 millimeter.

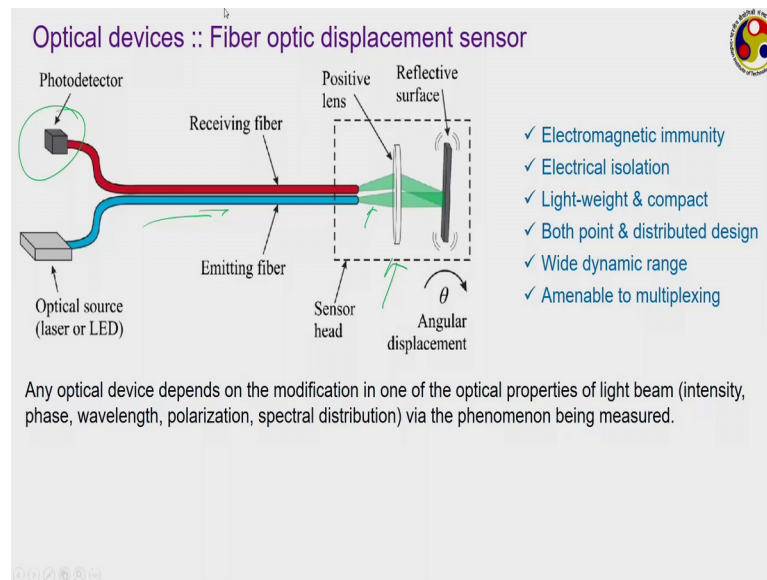
Now, the capacitance can be changed in different possible ways like in this mathematical relation involved three terms we can involve we can buy to measure the displacement we can change any one of them like if we are looking to measure very small displacement then we can talk about the change in the value of d . Of course, the d should be very small. So, if you are looking for very small displacement measurement the distance between the plates can be changed and the pickups will be done in this particular direction as shown here in this a whereas, if you are looking for much larger displacement measurement then the area will be modified. So, in the first case you can see as the displacement is changing your Capa modified value of Capa C prime will be equal to epsilon A by d plus some delta d .

So, it is actually a non-linear relationship because capacitance changes with distance in inverse proportion whereas, when we are talking about a change in area then C prime is equal to ϵ into A minus ΔA divided by d . So, it is a linear relationship similarly we can also change the position of the dielectric medium itself. So, thereby change in the value of ϵ it is also somewhat like a linear relationship. So, that is also possible it is less applied compared to the area related thing because their effect is quite similar, but the basic idea is when you are looking for very small displacement measurement we look for a change in a value of d the distance between plate whereas, we are looking for much larger displacement measurement.

We go for either change in the area or change in the position of this dielectric medium. Capacitance pickups can also give very high accuracy of the level of 0.01 percent they have a large operating range starting from 100 microns to something in the range of several millimeters and they can have very long mechanical life because the wear and tear is negligible of course, wear and tear is more compared to the inductive Inductosyns or LVDTs, but still that is also sufficiently long mechanical life that we can get from this and finally, they can be environmentally very robust just like the inductive devices.

So, these are the different kinds of electromechanical transducers that we get where we directly change any kind of electrical properties either a resistance or inductance or capacitance and try to relate that to displacement. Let us quickly talk about one or two more transducers where we are not trying to change any kind of electrical property, but still we are getting an electrical property as output example can be the optical devices.

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Optical devices are very common for measurement of several kinds of parameters pressure, temperature, flow, and displacement and motion are also exception. The idea particularly with the there are several kind of optical devices.

Here we are talking about the fiber optic displacement sensors. Here the idea is very simple. Firstly, we have an optical source which can be a laser on led laser or led ah. There are two optical fibers, so the light that is produced by this laser source that goes through this emitting fiber and once it strikes the specimen wherever you want to do some kind of measurement there is a change. Any optical device depends on a modification of one of some of the optical properties of this particular light beam. So, now, this light beam after reflection will be coming back through the receiving fiber and will be detected by this photo detector depending on whatever phenomenon that is going on here between the immediate and reflected lights.

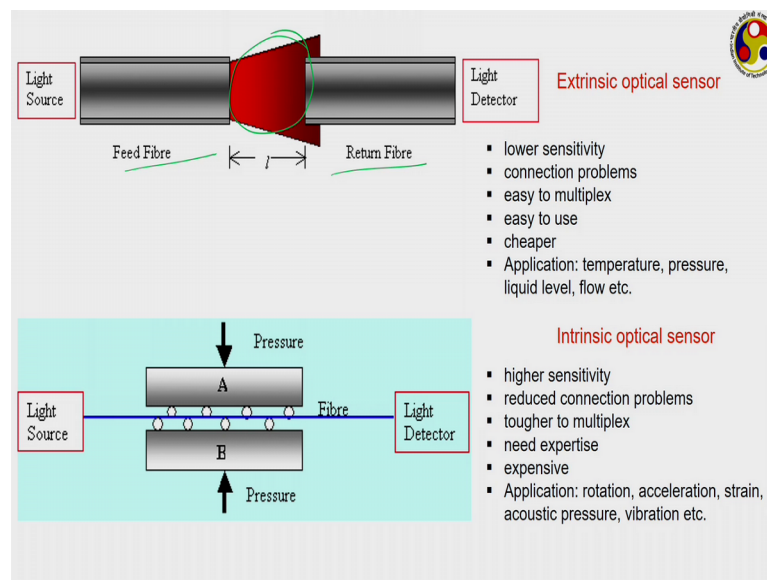
So, there will be some change in some of the optical properties. There are generally five optical properties which are more common that intensity, phase, wavelength, polarization, or spectral distribution. Different devices make use of different properties, but intensity and wavelength generally are the most common one depending on the nature of the phenomena and the property will get modified and that can be related to the concerned parameter which we are trying to measure like the displacement in the present

case. So, they are electromagnetically immune optical sensors there is no electric there can be completely electrical isolation lightweight.

And compact they are extremely accurate and one big advantage we can have both point and distributed design LVDTs Inductosyns strain gauges they all require certain kind of areas. So, they are not suitable for point measurement, but here in optical devices you are talking about one light beam which can be as thick as thin as a point and therefore, we can easily go for a point measurement as well as well as we can have distributed measurement and they can give wide dynamic range also we can easily go for multiplexing with the optical devices. So, there optical devices, because of all these advantages are very popular in any kind of measurement systems.

For generally in all optical sensors can be classified into two categories, one is the extrinsic optical sensor where the light actually goes out of your feed fiber then some changes take place and then only it return to the second fiber; that is in this is a zone where the light is actually outside your device.

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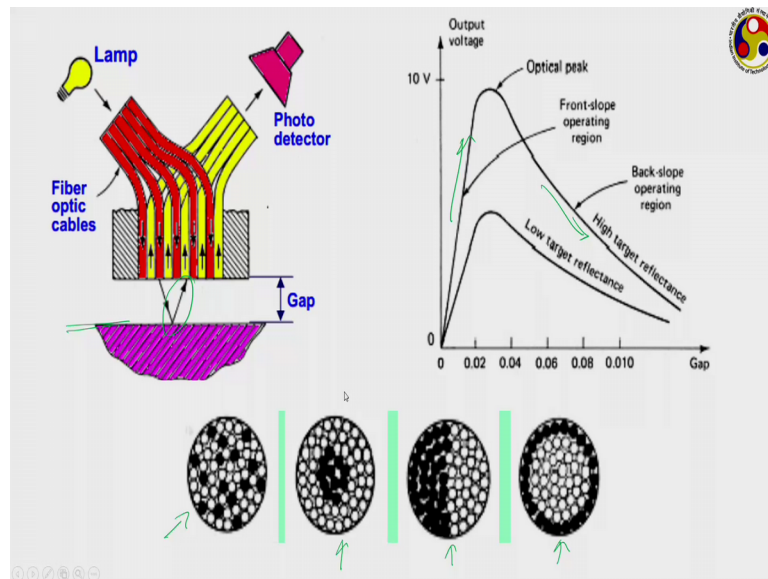


The other intrinsic where the light never goes outside rather during its passage through the device itself there is some modification in its properties. So, this second one is intrinsic. Extrinsic one generally has a lower sensitivity and leads to connection problems, but they are very easy to use we can easily go for multiplexing and they are

cheaper whereas, intrinsic devices have much higher sensitivity and much lesser connection problem.

Because you do not have to get the light back into the second one the light always remains inside your device itself, but they are tougher to multiplex they need expertise for operation and they are much costlier compared to the extrinsic versions. So, the extrinsic versions are generally used for measurements of rotational motion, acceleration, strain, vibrations etcetera whereas, extrinsic optical sensors are used in temperature, pressure, or liquid level measurement, flow measurement we shall be seeing this application in the concern modules.

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Now, for displacement measurement this is a simple picture of the possible way we can do it the optical the lamp is produces certain light as the light passed through your device and say this is a specimen where you want to measure the displacement or you want to sense the position of this then the light strikes the surface and reflects back and this reflected light is sensed by the photo detector. Now as the gap changes that time the property of the reflected beam that also keeps on changing accordingly ah.

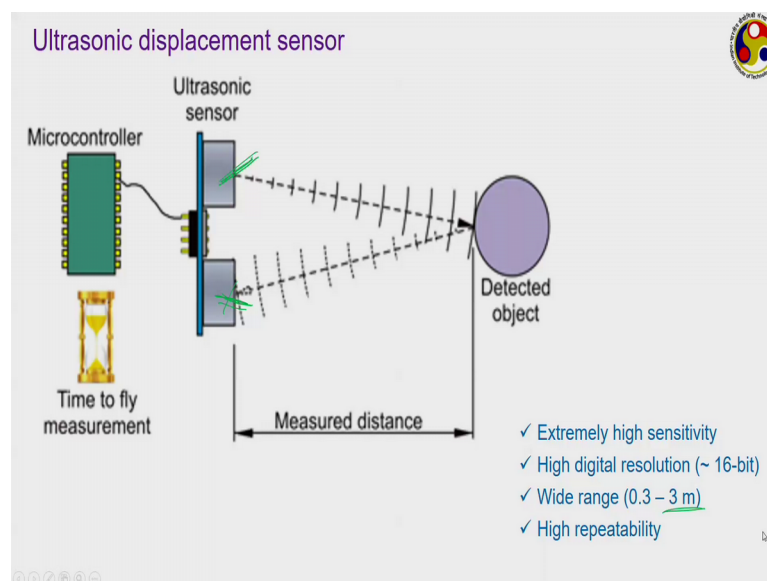
Generally we get a relation of output voltage to this gap in this way this photo detector actually converts the difference between the supplied light and the reflected light to certain kind of electrical signal again generally preferably volts voltages in general in a range of millivolts. So, that output voltage with a gap shows this kind of nature we can

have one front slope operating regime in this or we can have a back slope operating region where we have high target reflectance, depending on the reflectance of the target we can operate either on the front slope or in the back slope.

The optical fiber can have a cross section view like this there are different possible scenarios like here this white ones are the one through which the light is supplied whereas, the red ones are the ones which are receiving. So, this is a quite random one this one here you can see all the received receptors are located at the center whereas, in this case all the receptors are located at the periphery. These are more hemispherical kind of thing half is supplying, half is reflecting ah. This way the there are several other designs of optical fibers are also possible, but the principle of optical detection of any parameter remains quite similar just what we have discussed here for displacement.

We shall be seen much more about this optical thing in later works, may be in particularly in the chapter of flow.

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One final topic for today's discussion that is ultrasonic displacement sensor. This is something probably you have heard about this ah. Just think about the echo like when you go to a newsstand at somewhere maybe close to a valley and there is a hill far away from you, if you shout something then it echoes back, now how much time it requires for the echo to come back depends upon the distance from you and the hill or the sound reflector that is precisely what we do in any kind of sonic base instrument sonic based

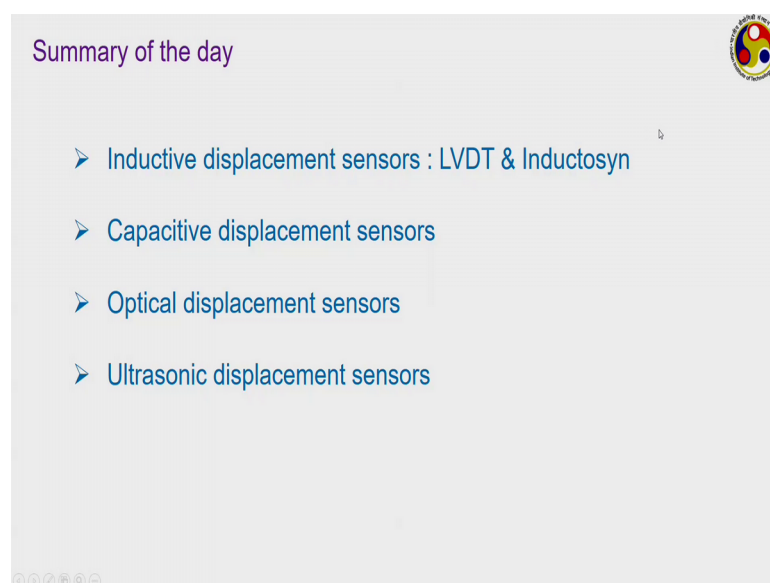
displacement sensor. Here the ultrasonic sensor produces a signal which goes to the object which is we want to locate.

And then it comes back the time required for this signal to start from this point and then come back at this point is a measure of the distance from the sensor to the object ah. They have very high sensitivity, this particular principle is used in several vehicle based applications like in ships to identify the position of the seabed or ocean bed, they easily use this kind of sensor, the signal is supplied and the signal gets reflected signal goes down through the water gets reflected at the ocean bed and then comes back from there, they can easily sense how much is the depth of the water there. They have again very high sensitivity and they have high digital resolution. Here actually one thing you can see as soon as the signal is produced from here then the sensor will stop till it is detected back only when it is detected back then only you can supply another one.

So, it is quite suitable for digitalization because we can have a certain gap it is not for continuous operation rather it is an intermediate operation only and we can have very high deterioration of the order of 16 bits that you are talking about. Their operating range can be high actually I have written here three meter, but it can be much more than that like estimation of the depth of ocean bed and they have high repeatability as well.

So, that takes us to the end of today's discussion.

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A presentation slide titled "Summary of the day" with a logo in the top right corner. The slide lists four types of displacement sensors: Inductive displacement sensors (LVDT & Inductosyn), Capacitive displacement sensors, Optical displacement sensors, and Ultrasonic displacement sensors. The slide also features a navigation bar at the bottom left.

Summary of the day

- Inductive displacement sensors : LVDT & Inductosyn
- Capacitive displacement sensors
- Optical displacement sensors
- Ultrasonic displacement sensors

Today we have discussed about four different kinds of instruments first is the inductance based measurement where the inductance of a coil changes with displacement and there we have particularly talked about LVDT and a bit about Inductosyn then we have talked about three other devices very quickly, one is capacitive based sensors or capacitance pickups where the capacitance changes depending on the displacement they may have talked about the optical displacement sensors very briefly and also the ultrasonic displacement sensor. So, that is it for the day in the next lecture we shall be discussing about couple more displacement sensors and we shall specifically focusing a bit on rotary displacement or angular displacement sensors. So, please wait till that moment.

Thanks for your attention today.