

**Principles of Mechanical Measurement**  
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**Module – 04**  
**Data Processing**  
**Lecture – 12**  
**Electromagnetic indicators**

Good morning, everyone. Welcome to week number 4 of our MOOCs course on the topic of Principles of Mechanical Measurement. Over last 3 weeks we have discussed about several initial concepts like in week number 1, you are introduced to the design of a basic measurement system; we talked exclusively or extensively about different kinds of errors which may get include during our measurement and also how to eradicate those errors.

Then, in the second week we have talked about the mathematical representation of any generalise measurement system and the response of zeroth order first order and second order measurement systems against some standard inputs and now, in the previous week we have talked about the analog to digital conversion or I should say the digitalization of measuring the systems because most of the measuring tools that we get nowadays have a digital interface and digital output.

So, once you have the output or readings from any particular measurement system then we generally need to go for some kind of data processing like and also several other associated steps like converting the data from one particular kinds of signals to another kind of signals which is conventionally called the transduction and also quite often you have to go for some kind of amplification or filtering of the measurement signals. We have to generally convert them any kind of inputs so that mechanical thermal whatever inputs you are getting most cases we convert them to some kind of electrical signals to get an electrical output which is the most easy to measure and all these comes under this data processing module.

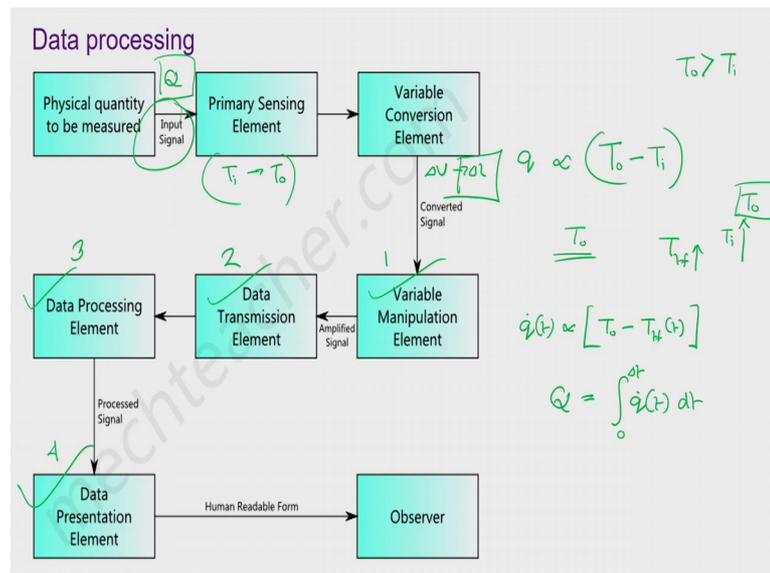
Of course, this is an extensive things which can include the most intricate electronics, concepts of electrical engineering, we can also include several completion aspects like the use of micro process processors, the use of several digital conversion techniques, image processing etcetera, but still you would like to keep this one to keep this one short,

mostly restricting ourselves to electrical measurements or electrical readouts how and this is the week where your first time going to get idea about some real measurements.

Because in all the previous week we have discussed previous three weeks in all the previous lectures we have discussed about only basic topics, but this week we are going to talk about measuring certain electrical quantities, converting or amplifying the signals and so that we can have a background about how we can measure different parameters, different output level parameters and with that concept we can go forward in the next week in the subsequent weeks about real measurements of different parameters.

I would like to keep this week be short we had quite extensive discussion in the previous three weeks each of having three or four lectures and I would try to finish this particular week in this two lectures, so that I can give a bit of relief to all of you.

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Now, if we look about any general kind of measurement process we are always try some kind of start with some kind of physical quantity and their physical quantity give certain input signal which is sensed by your primary sensing element and then that goes to some kind of variable conversion if required some kind of variable manipulation. Like just think about something that you already know as we have not talked about any real instruments so far so, I can give you example only from something that you already know. Just think about one mercury thermometer or something conversation you known as the liquid in glass thermometer.

So, there we would like to measure the temperature of something. Let us say we want to measure the temperature of the human body, then we are going to take that thermometer in contact with the body and then what is the first physical quantity we want to measure temperature, but what is going to be your input signal what do you feel? Input signal is not going to be temperature whether you are input signal will be in the form of the temperature difference between the body and the thermometric fluid; initial temperature of the thermometric fluid at least.

So, let us say if your body is at some temperature  $T_{\text{naught}}$  and thermometric fluid is initially at some temperature  $T_i$  then assuming  $T_{\text{naught}}$  to be greater than  $T_i$  then this we are going to get certain amount of heat being transferred from the body to the thermometric fluid which should be proportional to this temperature difference; that means, some amount of heat will be transferred which will be proportional to this temperature difference and this heat transfer will continue till the temperature of the thermometric fluid increases to attain a thermodynamic equilibrium.

The what I mean is that this  $T_{\text{naught}}$  that is a temperature of the body will remain same, but the temperature of the thermometric fluid let us say it is called  $T_{\text{tf}}$  which is initially  $T_i$  that will keep on increasing till this  $T_{\text{tf}}$  increases to go attain this higher value of  $T_{\text{naught}}$  and then only the system will be able to attain certain thermodynamic equilibrium or thermal equilibrium with the body.

So, at any instant of time the temperature difference between the body and the thermometric fluid is  $T_{\text{naught}} - T_{\text{tf}}$  which is again a function of time with  $T_i$  being the initial temperature and the amount of heat that is going to get transferred we should not write a dot amount of heat being transferred is a function of time and that is going to be proportional to this temperature difference and that is going to provide this heat transfer is the one that is going to provide your input signal or if I write your input signal is going to be this  $q \cdot dt$  our  $q \cdot t$ . So, this heat transfer will continue till the temperature of the thermometric fluid attains this  $T_{\text{naught}}$ .

Now, let us say for a  $\Delta t$  period of time this heat transfer has taken place. So, the total amount of heat transferred to this if I rate notation now is over this period 0 to  $\Delta t$  and this is the way the amount of heat is being transferred. So, total amount of heat being sensed by this primary sensing element is  $Q$ , leading to a change in its temperature from

some initial value of  $T_i$  to the final body temperature of  $T_{naught}$ .

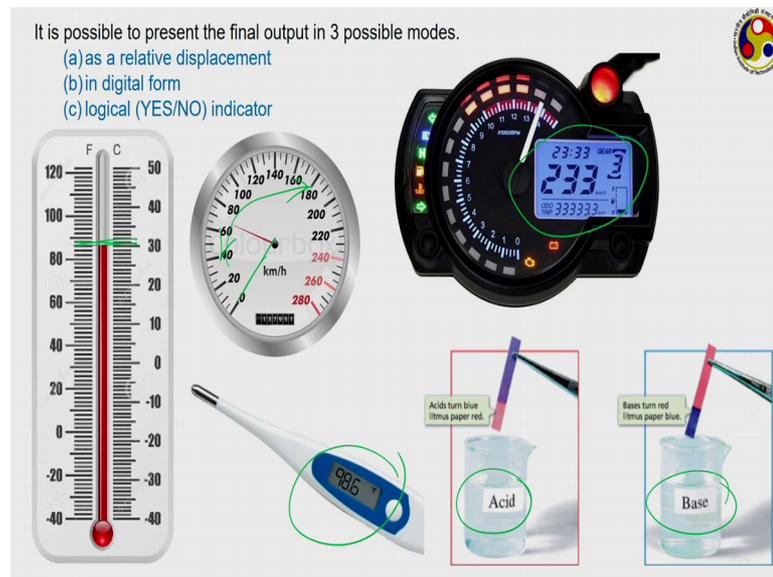
Now, once this primary sensing element has got this then heat will produce or whether heat will transduce this input signal to a certain form and that form will depend on the nature of the instrument and also its mathematical characteristic; like if we again go back to the example of thermometer then this heat actually is going to get converted to a change in temperature of the thermometric fluid and then that will get transduce to the change in volume of the thermometric fluid through the capillary column.

So, while your actual primary input is this amount of heat  $Q$  this variable will get converted to give some  $\Delta V$  or of the thermometric fluid or maybe some change in the length  $\Delta L$  of the thermometric fluid in the capillary tube. So, that is the variable conversion stage. This is where the mathematical model comes into play which relates this temperature difference to this  $\Delta L$ . Certain situations you may have to go for some kind of variable manipulation also like if the  $\Delta L$  is too small we may go for certain kind of amplification by changing the diameter of the capillary tube etcetera and finally, we get some amplifier signal.

And, now, this is the role where of the data processing unit that comes into picture. Data processing refers to all the stages like the transmitting the data actually variable manipulation can also be considered to be a function one of the functions of data processing. So, the data processing is going unit is going to first perform some kind of manipulation over the transduce signal, then transmits that to the recoding part if require some further data processing like if there is a kind some kind of feedback kind of options then this data processing element will do that feedback or certain other processing on this; sometimes we may have to go for some kind of integrating or differentiating action on this. So, all these will be done in the data processing and finally, the data presentation to the observer or sometimes recording.

So, all these four; 1 is this variable manipulation, then 2 is the data transmission, 3 is some further processing optional processing and the number 4 data representation and recording all these comes under the purview of this data processing unit and we shall be discussing just a few glimpses of each of this with particular focus on the data manipulation part and also the data presentation part.

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Now, it is possible that the final output like if I go back here this data presentation that we are talking about it is possible that the final data that will be presented to the observer or may be recorded to some kind of recording device that generally can be done in three possible modes that is your measurement system can give output in three possible modes; one – as the relative displacement, second in a digital form third I shall be coming shortly.

What do you mean by relative displacement? Like the thermometer. Here in the initial position of the thermometric fluid may be something like this. Then because of the heat being transferred from the body under consideration this there is a change in the column height. So, this much of deflection or displacement has taken place of the thermometric fluid inside the capillary tube which is proportional to the amount of temperature difference or in a way that gives you a measure about the temperature.

Another example of the same thing, but it is the digital form here instead of getting a deflection like in case of in the first example; now, once the thermometric fluid reaches this final position we have to compare this position sorry, we have to compare this position with the scale. Like here there are two scale shown on this thermometer one in Celsius one in Fahrenheit if we compared this tip of this thermometric fluid with the scale it is showing something just above the 30 degree Celsius and corresponding reading on the Fahrenheit scale.

So, we have to compare the to the corresponding scale to get the magnitude to get the value whereas, in case of digital one we are directly going to get the value for this. Of course, this particular conversion is done internally and we are going to get the output not in terms of a displacement of something rather in terms of numbers or digits, that is what refers to the digital form.

Another example this is a common speedometer that you can have in different kinds of vehicles. Now, what kind of form of output that you are getting here? Initially when the vehicle is under stationary this indicator will be in some position like this that is zero and as you start the vehicle or keep on moving with this it will keep on rotating over this keep on getting deflected over the scale and its instantaneous position gives you the instantaneous velocity. So, this is the deflection type thing.

Whereas, we can also have a digital version of the same you will find several modern vehicles, modern cars can directly give you the velocity value just like this. So, both the modes are possible both the modes both the modes are used for to get the same kind of measurement, like we have examples of temperature and velocity measurements here where we have very common instruments to give both kinds of outputs.

But, there is a third form of output; that is also possible, which is a logical indication or yes – no kind of output. Any idea where you have seen this? I am not going to go for any kind of mechanical device because we are not going to use this one that much or we do not use the that much of for some specialise cases, but something you probably have seen or you definitely have seen in your chemistry laboratory.

Have you heard about litmus papers? That is yes or no kind of thing. Like you know that if we have a blue litmus paper and if we emerge that to certain kind of liquid, so, that its colour turns to red then that liquid has to be an acid. Similarly if a red litmus paper turns blue in contact with a certain liquid then that has to be a base. So, you can think this way you have a blue litmus paper if you are dipping this to several liquid samples that you have with you. If it is not changing in its colour then it is not an acid yours. So, your question is whether the liquid is an acid? Your answer is no.

However, if it changes its colour to red then that is an acid answer to the question is yes. Same about the base if you have red litmus paper if it turns blue in contact with something then that is a base otherwise it is not. So, that is an yes – no kind of answer of

course, when it is not it is not at all giving you any idea what is the nature of this. Like suppose a blue litmus papers is continuing to be blue in contact with certain liquid so, the you know certainly the that is not an acid, but that may be a base, that may be some kind of salt solution, that may be certain kind of alcohols or certain other organic fluids, but definitely not an acid. And, even when it tells you that it is an acid means, its colour changes to red you do not know the chemical composition of the acid it just tells that this is an acid, that is all. So, it is giving you a logical indication only yes or no.

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Analog electrical indicators 

Class	Operating principle	Subclass	Application field
✓ Electromagnetic	Interaction between currents and magnetic fields	Moving magnet Moving coil Moving iron	Dc voltage Dc voltage Dc and ac voltage
✓ Electrodynamic	Interactions between currents	—	Dc and ac voltage
✓ Electrostatic	Electrostatic interactions	—	Dc and ac voltage
Thermal	Current's thermal effects	Direct action Indirect action	Dc and ac voltage Dc and ac voltage
✓ Induction	Magnetic induction	—	Ac voltage
Electronic	Signal processing	Analog Digital	Dc and ac voltage Dc and ac voltage

Now, analog indicators there are different we can get analog output like we have seen couple of examples in the previous slide, the deflection of the mercury column or thermometric fluid column in a capillary, the deflection of indicator of an dial gauge, but in most of the cases you will find in most 70 – 80 percent instruments we generally transduce this output signal into some kind of electrical form, maybe voltage or current and then measure that voltage and current because measuring electrical signals or electrical quantities are quite easy. Like we shall be seeing discussing shortly

Therefore, we shall only be talking briefly about electrical indicators which gives you this analog output in terms of certain deflection of the scale. So, there are generally six different categories of electrical indicators that we can get. Electromagnetic which refers to the interaction between electric current and the magnetic field, and there we can have several subclasses like we can have a moving magnet type, we can have a moving coil

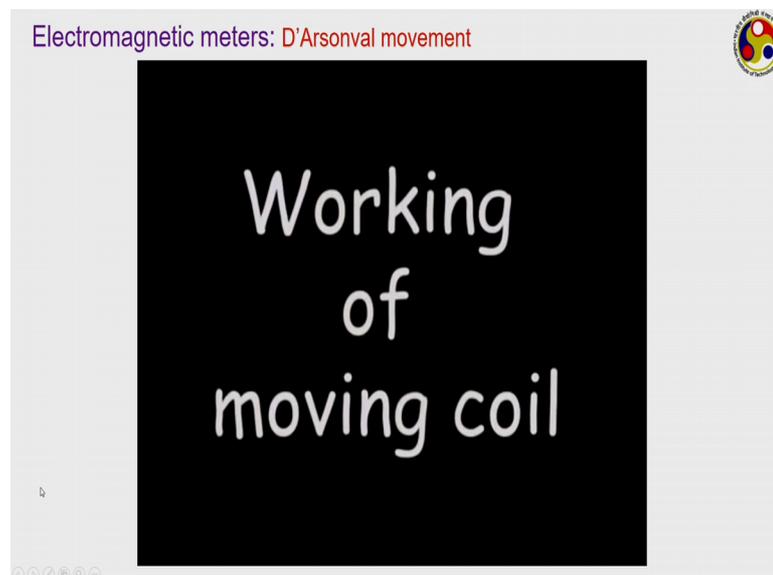
type or a moving iron type.

Then, electrodynamic which talks about interaction between currents only no magnetic field involved; we can have electrostatic as well where it is electrostatic interaction; then thermal related to the thermal effect of current that is the joule heating or  $I^2 R$  loss which can have direct or indirect action; then we can have induction in the form of magnetic induction and finally, electronic signal processing in the form of analog or digital.

Out of this first three that is electromagnetic, electrodynamic, electrostatic and also this induction this for a commonly known as electromechanical meters or electromechanical devices. If the devices used measure certain voltage we call them voltmeter it used to measure current we call them ammeter, but the operating principle remains the same.

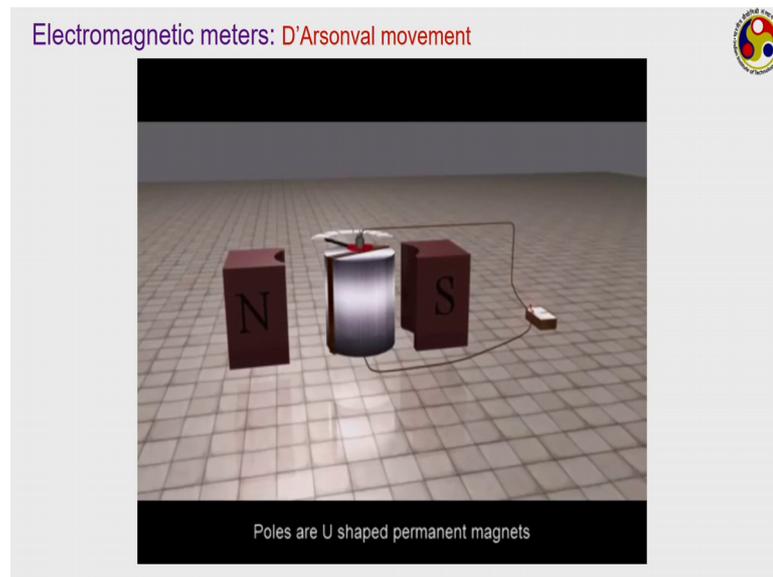
Let us just quickly touch upon the electromagnetic meters.

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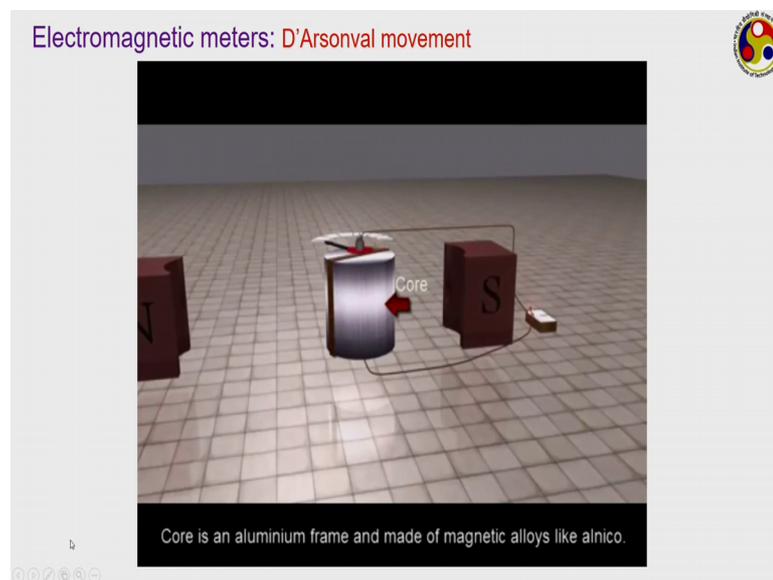
For electromagnetic meters the basic operating principle is based upon the D'Arsonval movement and what is that let us quickly see this YouTube video. In case of a D'Arsonval movement basically we have interaction between like in case of any electromagnetic case we have interaction between a magnetic field and current.

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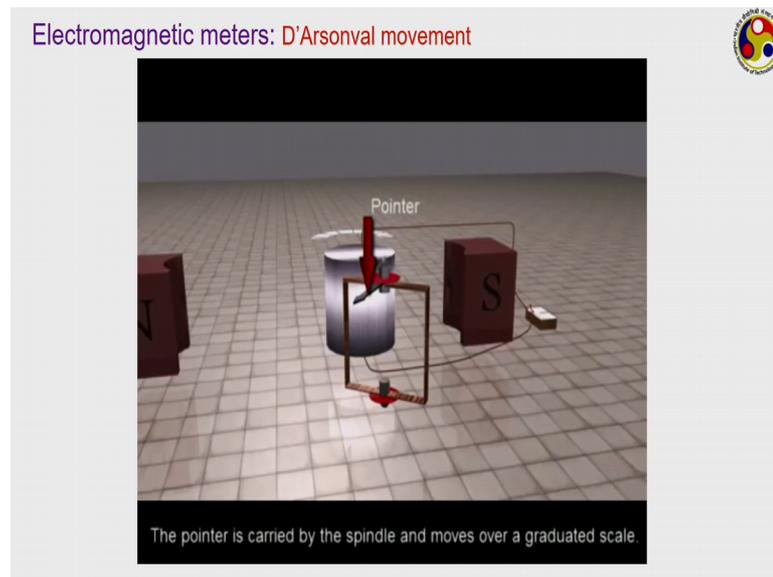
Like we have [seem/seeing]- seeing here we have a U shaped permanent magnet and these are the two poles of this. So, the north and south pole are placed.

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And, in between you are putting core; core means some kind of soft metal core, generally we have an aluminium frame.

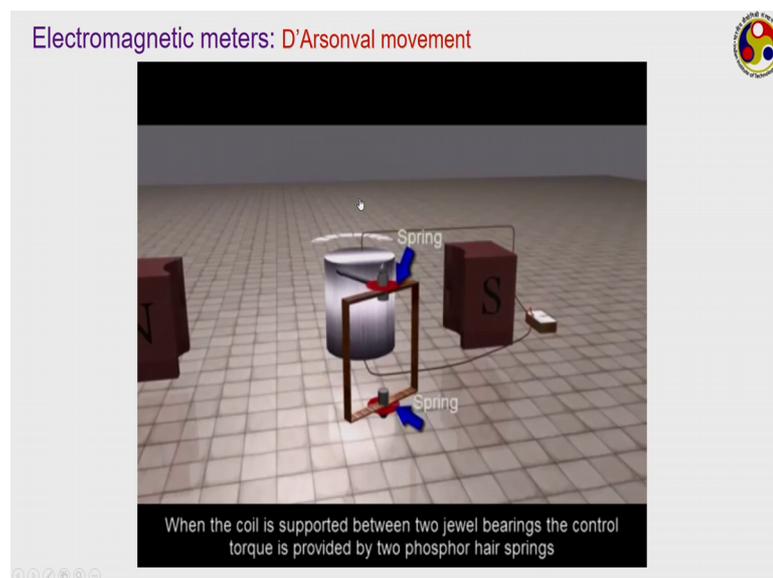
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And, some magnetic oil on top of this we have this particular frame which is housing the pointer, which is going to move over a certain scale to give you the final reading.

So, the pointer is carried by the spindle and it moves over the scale. The scale is shown somewhere here on this video.

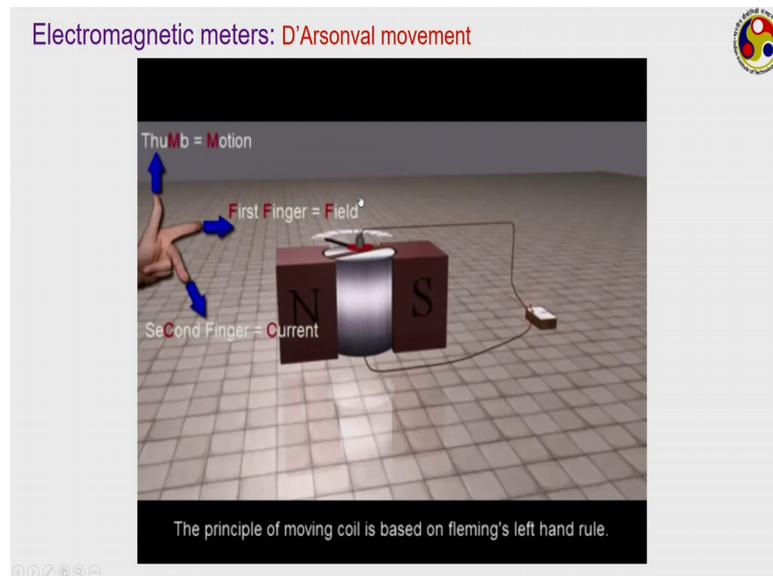
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We will shortly be seeing the option of the scale also. The coils are wrapped around this particular frame and there are jewel bearings or there are supporting something else to support this and also there are a couple of springs to limit the operation or I should say

the movement, restrict the movement.

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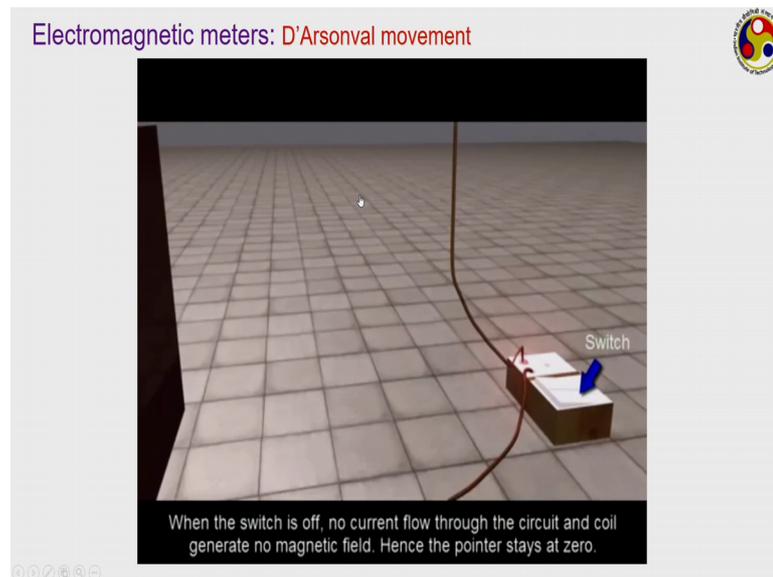
Then, we have the Fleming's left hand rule to detect operation you know that if the index finger is showing the field and the middle finger is showing the current that thumb is always going to give you the direction of motion.

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So, accordingly we have we are going to use this principle. This is the scale that I just mentioned about.

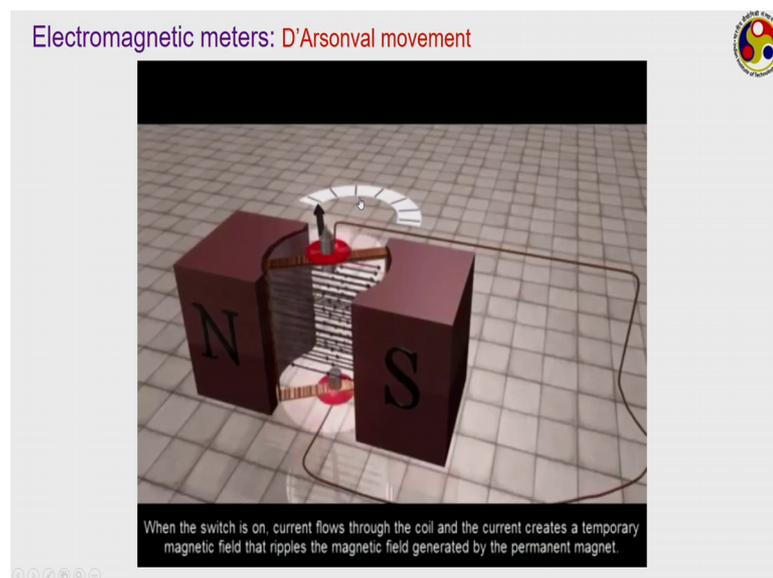
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Now, if we switch on the power, then it is of course, I should mention we are talking about the measurement of direct current only or DC current or voltage.

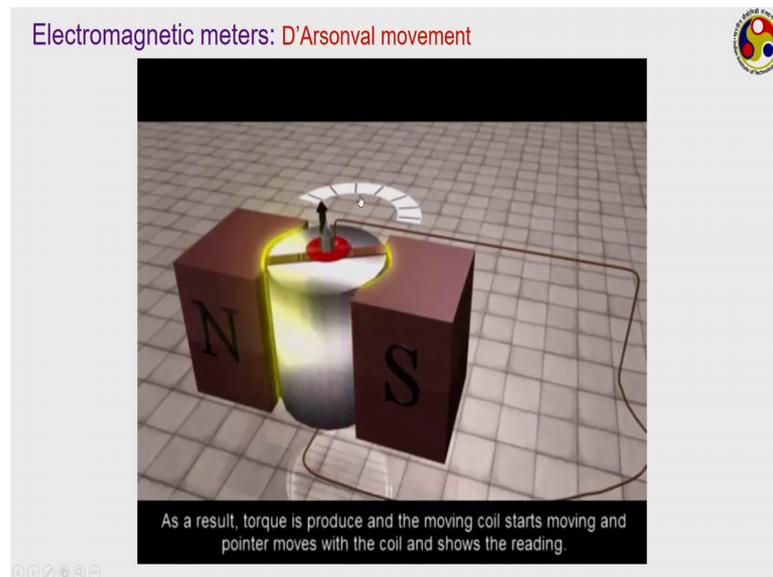
Now, when the switch is off there is no current flowing through the circuit and therefore, there is only a single magnetic field generated by the permanent magnet, but no magnetic effect from coil.

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As soon as the electricity is switched on, then current starts flowing through the electric coil and because of that it creates a magnetic field of its own.

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And, that magnetic field has an interaction with the permanent magnetic field which is already there and the interaction between these two leads to rotation leads to a formation of a torque and the moving coil starts to move which is indicated by the movement of this indicator over this scale. So, this is how an electromagnetic meter works.

So, you can see this is principally a kind of electric meter or I should say a current meter which gives you a measure of the current because a proper design the torque produced because of the interaction of these two magnetic fields can be made proportional to the current and if you want to use this one for measurement of voltage, then we have to do certain additional arrangement like we shall be seeing shortly.

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$$\left. \begin{matrix} I \\ N \\ B \\ l \end{matrix} \right\} F = NBIL$$

$$\rightarrow T_i = Fd = NBdIl = k_i I$$

$$k_i = NBdl$$

$$T_i = T_s$$

$$NBdIl = k_s \theta$$

$$\Rightarrow \theta = \left( \frac{k_i}{k_s} \right) I$$

$$h \Rightarrow \text{displacement } \lambda = h \theta$$

$$\Rightarrow \lambda = \left[ \left( \frac{k_i}{k_s} \right) h \right] I \Rightarrow \lambda \propto I$$

- current meter (voltmeter/ammeter)
- ohmmeter & volt-ohm-milliammeter (multimeter)
- meter system preceded by amplification
- × hysteresis
- × repeatability
- × mechanical friction
- × pointer-bearing movement
- × linearity errors in spring

But this is just picture of the same.

Now, if we say that I is the current DC current that is flowing through this DC refers to direct current, but just as we are talking about current and voltage in both DC and AC sense. So, we are going to use the term DC current though it does not sound good. So, let us I is the current that is flowing through the coil, N is the number of turns of the coil that is there on the spindle in the core, then B is the magnetic field intensity and l is the length of the magnetic the coil which cuts the length of the electric coil which cuts the magnetic field also called active sides l is the length of that.

Then, if we combines to then the force that is being produced because of this interaction is from the simple knowledge of electromagnetism is  $NBI$  into  $l$  and that will lead to the formation of a torque. Let us say the torque is  $T_i$ , the corresponding torque will be  $F$  into  $d$  that is  $NBdl$ , where  $d$  is just the width of the coil that we are talking about quite often this  $d$  and  $l$  is also combined together to indicate the total cross section area total volume of the coil that we are going to use. So, this  $T_i$  is the mechanical torque that is exerted on the coil because of the interaction into magnetic fields.

Now, as the coil rotates around its own axis there are couple of springs. Let us say the spring is having a spring constants of  $K_s$  that the springs provide a restraining torque. So, the corresponding restraining torque  $T_s$  can be given as  $K_s$  into  $\theta$  if  $\theta$  is the movement or the angle over is the coil has rotated. The springs are trying to restrict the

motion or restrict motion of this coils and  $\delta$  is the amount of rotation. So, corresponding restraining torque will be  $T_s$ . The pointer will reach a balance situation only when these torques are equal to each other that is when  $T_i$  equal to  $T_s$  that is  $NBdl$  is equal to  $K_s \text{ into } \delta$ . Quite often this  $T_i$  we write as  $K_i \text{ into } I$  because if you see everything else where this  $K_i$  is equal to  $NBdl$ . Here all are constants  $N$  is the number of turns,  $d$  and  $l$  refers to the dimension of the coil,  $B$  is the magnetic field and as you are talking about a permanent magnet that is also fixed.

So, we can write this  $\delta$  is equal to  $K_i \text{ upon } K_s \text{ into } I$  that is directly proportional to the current that is happening or that is flowing through this coil. And, now if the length of this particular pointer is  $h$ , then for small angle of rotation we can also write that the displacement experienced by this pointer the displacement of the pointer  $\lambda$  will be equal to  $h \text{ into } \delta$ ; of course, when this angle  $\delta$  is small or in a way we can write  $\lambda$  is equal to  $K_i K_s h \text{ into } I$ .

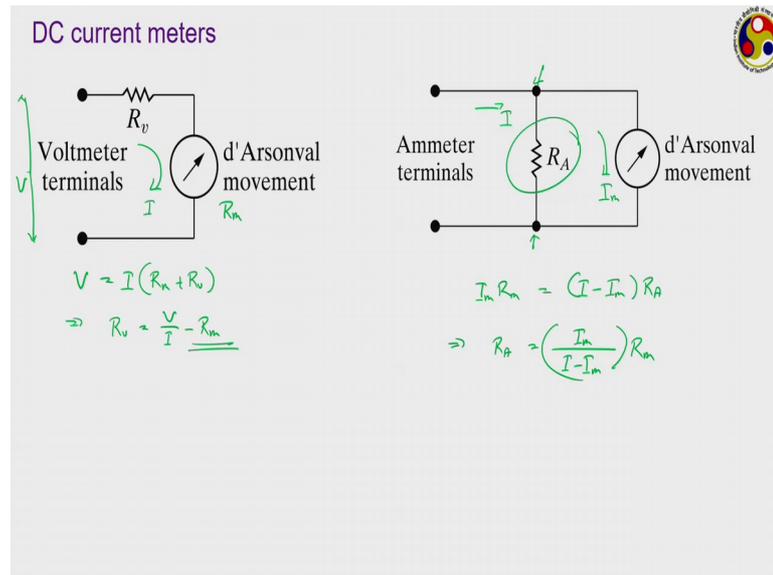
So,  $h$  again being a constant; that means, this  $\lambda$  is proportional to the current that is flowing through it. So, this is a that is what I mentioned it as a current meter because the deflection of the pointer or displacement of the pointer is directly proportional to the current and generally this in constant that is specified from the manufacturer because everything else are known a priori and accordingly just from the knowledge of the pointer movement we can get a measure of the current for this.

There are several as you can make use of the same meter like we can use as a current meter to measure either voltage or current voltmeter or ammeter. We can also use it to as a multimeter; multimeter means which provides you multiple measurements simultaneously by small alteration in the internal circuit which can give you volt, ohm or milliammeter that is voltage resistance or current simultaneously or just simple ohm meters. We can also have meter system preceded by amplification. These all kinds of designs are possible with just addition or removal of other electrical components in series and parallel with this particular instrument.

However, there are a few losses that can often creep in the analysis that we have shown like there can be hysteresis affect, there can be repeatability related issues, mechanical friction of the pointer as it is moving over the scale, pointer bearing movement itself can have its own issues; I should means I here refer to the bearing of the pointer that is where

the pointer is connected that bearing can lead to loss of frictions and also there may be linearity errors in the spring particularly after sustained use over a long period of time. Still this is the most convenient easy and also accurate way of measuring any kind of electrical quantity, voltage or current. And, that is why we use it as both voltmeters or ammeters particularly when measuring DC quantities.

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When we want to measure voltmeter then the meter is connected across the meters is connected in the parallel to the location where you want to have the measurement whereas, in case of ammeter is connected in series with a shunt resistor.

So, if we talk about the voltmeter let us say the meter resistance itself is  $R_m$  and the current flowing through this circuit is  $I$  then if the across the terminal that is in the voltage that is available across this terminal is  $V$  then we can easily write  $V$  is equal to  $I$  into  $R_m$  plus  $R_v$ ; this  $R_v$  is often referred to as series multiplying resistor. So, therefore,  $R_v$  is equal to  $V$  upon  $I$  minus  $R_m$ . So, this  $R_m$  should be very high that is resistance of the meter should be high, so that we can avoid any significant current flowing through the meter itself.

Whereas, when go to go to the case of ammeters following the same principle if  $I$  is the current flowing through the primary circuit and  $I_m$  is the one that is flowing through the meter then  $I$  applying the equality of voltage drop between these and this terminal we can write that  $I_m R_m$ , that is the potential drop across the meter should be equal to the

current flowing through the others in the shunt resistance that is  $I_m$  into  $R_m$  which gives you  $R_m$  is equal to  $I_m$  divided by  $I_m$  into  $R_m$ .

So, this way we can easily calculate the resistance required either for the voltmeter or ammeter. For voltmeter we generally want this meter is has to be as high as possible to restrict the movement through this movement or restrict the amount of current that is flowing through the voltmeter whereas, in case of ammeters we generally want the meter resistance or I should say the meter resistance plus the shunt resistance combination to be as low as possible so that there is hardly any voltage drop while flowing through this.

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AC current meters



$$P_{DC} = I^2 R = \frac{E^2}{R}$$

$$P_{AC} = \frac{1}{T} \int_0^T i^2 R dt = \frac{1}{2\pi} \int_0^{2\pi} i^2 R d(\omega t)$$

$$= \frac{1}{T} \int_0^T \frac{e^2}{R} dt = \frac{1}{2\pi} \int_0^{2\pi} \frac{e^2}{R} d(\omega t)$$

$$I_{rms}^2 = \frac{1}{T} \int_0^T i^2 dt = \frac{1}{2\pi} \int_0^{2\pi} i^2 d(\omega t)$$

$$i = I_0 \cos \frac{2\pi t}{T} = I_0 \cos(\omega t) \quad \rightarrow I_{rms}^2 = \frac{1}{2\pi} \int_0^{2\pi} I_0^2 \cos^2(\omega t) d(\omega t)$$

$$\rightarrow I_{rms} = \frac{I_0}{\sqrt{2}} \approx 0.707 I_0$$

$$e = E_0 \cos(\omega t) \quad \rightarrow E_{rms} = \frac{E_0}{\sqrt{2}} \approx 0.707 E_0$$

This same thing can be extended in case of AC meters as well. Like in case of AC meters say for DC the total power dissipated while current flows through a resistor can be mentioned can be written two ways one can be  $I^2 R$ ; where  $R$  is the resistor  $I$  is the corresponding current or if  $E$  is the voltage drawback according or across this then it can also be written as  $E^2 R$ .

Now, when your writing the same thing this is for DC writing the same thing for AC then the current actually changes with time. So, we can write this one as integral 0 to  $T$   $i^2 R dt$ ; where  $T$  is the time period and  $i$  is the profile of the current that you are considering or we can also write it in terms of voltage as  $e^2 R dt$  where again  $t$ . So, same time period depending on which one is known to us either the current profile or volt voltage profile we can make use of any one of them. Quite often we can also

write this one as something like  $I^2 \pi \int_0^{2\pi} i^2 R d\omega t$ . Similarly, this one can also be written as  $\frac{1}{2\pi} \int_0^{2\pi} e^2 R d\omega t$ .

So, if we combine both the forms with the corresponding DC form then what we can write. So,  $I_{rms}^2$  will be equal to  $\frac{1}{T} \int_0^T i^2 dt$  that is  $\frac{1}{2\pi} \int_0^{2\pi} i^2 d\omega t$ . If we assume a common form like quite commonly we will find that this small  $i$  or is given as some reference will be  $I_{naught} \cos$  of  $2\pi$  small  $t$  by capital  $T$  where capital  $T$  is the time period that is  $I_{naught} \cos$  of  $\omega T$  then correspondingly  $I_{rms}^2$  will be equal to  $\frac{1}{2\pi} \int_0^{2\pi} I_{naught}^2 \cos^2 \omega t d\omega t$ .

And, if we solve this  $I_{rms}$  will become equal to  $I_{naught}$  divided by root 2 that is roughly 0.707  $I_{naught}$  and the same way solving this we can also get  $E_{rms}$  is equal to  $E_{naught}$  by root 2 if of course,  $E_{rms}$  or  $I$  should say small  $E$  follows a profile similar to this that is when small  $e$  is equal to  $E_{naught} \cos$  of  $\omega t$  in that case it is also become 0.707  $E_{naught}$  that is the rms value will be only 70 about 71 percent of the peak value. And, we can using then AC meter using following the principle for any DC our measurement we can get the value of the rms or peak current  $I$  following the simple relation.

So, this way we generally use any kind of voltage or current measuring meters in using the principle of D'Arsonval movement.

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Impedance meters: Wheatstone bridge (DC)

- Deflection mode
- Null mode

Relation for a balanced bridge:

$$I_1 = \frac{V_s}{R_1 + R_2} \quad I_2 = \frac{V_s}{R_3 + R_4}$$

$$V_{cA} = I_1 R_1 = \frac{R_1}{R_1 + R_2} V_s$$

$$V_{dD} = I_2 R_3 = \frac{R_3}{R_3 + R_4} V_s$$

$$V_{CD} = V_{cA} + V_{AD} = V_{cA} - V_{dD}$$

$$V_{CD} = \left[ \frac{R_1}{R_1 + R_2} - \frac{R_3}{R_3 + R_4} \right] V_s$$

$$V_{CD} = 0 \Rightarrow \frac{R_1}{R_1 + R_2} = \frac{R_3}{R_3 + R_4} \Rightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$\Rightarrow R_1 R_4 = R_2 R_3$$

Bridge sensitivity:

$$\Delta V_{CD} = \frac{\partial V_{CD}}{\partial R_1} \Delta R_1 + \frac{\partial V_{CD}}{\partial R_2} \Delta R_2 + \frac{\partial V_{CD}}{\partial R_3} \Delta R_3 + \frac{\partial V_{CD}}{\partial R_4} \Delta R_4$$

$$\frac{\partial V_{CD}}{\partial R_1} = \left[ \frac{(R_3 + R_4) - R_1}{(R_1 + R_2)^2} \right] V_s = \frac{R_2 V_s}{(R_1 + R_2)^2}$$

Another very important part of any such kind of electrical circuit is the measurement of impedance. Impedance refers to resistance inductance or capacitance depending on which kind of circuit you are dealing with. If you are dealing in the pure resistive circuit then the most common way of measuring resistance is in terms of Wheatstone bridge any kind there are several other bridge circuit also, but there is the simplest one and the most popular and most accurate one that is why we generally prefer going by this.

Look at the in the Wheatstone bridge what we have here we generally have four resistors connected in a form just what shown here. So, whenever there is a under balance situation this V out is equal to 0 and the if we connect this terminal C and D through some suitable voltage measuring meter then you are going to get zero reading from there. And, now if any of the resistance changes then the value of V C and V D will not be equal to each other and that can be sensed again using a voltage measuring instrument.

Actually Wheatstone bridge can operate in two ways like several other measuring instruments in deflection mode or null mode. Deflection mode means whenever there is a change in one of the any of the resistor let us say a particular situation there is a small change in value of R 1. So, if there is a change in the value of R 1 then there will be some value in this of V out also it will not remain equal to 0 and accordingly there will be certain kind of deflection sensed by the voltmeter that you are putting there, that is the deflection mode measurement.

The other one the null mode is whenever we are seeing the deflection then we are going to alter the value of any other resistor say  $R_3$ . So, because of the change in  $R_1$  there is some deflection there; if the deflection is over certain scale we can directly get the voltage measurement, but if the deflection is not over in the scale then we are going to modify any other resistance like  $R_3$ . We keep on modifying  $R_3$  till the deflection goes back to its initial null situation; initial zero situation, that is the null mode of measurement.

The deflection mode of measurement is very easy because very easy if you are looking for a dynamic measurement, but the problem is in case of deflection mode the value of the resistance or value of the voltage generated that is  $V_C$  minus  $V_D$  that is the function of this  $V_S$  whereas, in case. So, if there is change in the value of  $V_S$  we have to recalibrate the voltmeter accordingly.

However, if you are going for a null mode then the final reading is 0 and therefore, we do not have to depend on the value of  $V_S$  just we have to sense the change in this value of  $R_3$  that will give you measurement about the change in  $R_1$ . Several measuring tools like the strain gauges which we are going to talk about in the next week; there are several temperature sensing devices etcetera which makes use of Wheatstone bridges and gets the actual output signal converted to certain kind of electrical signal through this.

Now, how to calculate the relation for a balanced bridge or to be more specific the relation between these resistances? As shown here  $I_1$  is the current that is flowing through presently we considering this terminal to be open that is C and D are not connected with each other. So,  $I_1$  is the current that is flowing through resistor  $R_1$  and  $R_2$  and  $I_2$  is flowing through between  $R_3$  and  $R_4$ . As the voltage difference between point A and point B is equal to this  $V_S$  only. Therefore, we can write  $I_1$  should be equal to this  $V_S$  divided by  $R_1$  plus  $R_2$  similarly  $I_2$  should be equal to corresponding voltage  $V_S$  divided by net resistor in the circuit that is  $R_3$  plus  $R_4$ .

Now, if suppose  $V_{CA}$  refers to the voltage drop or voltage rise on here moving from C to A how much will be that? that voltage rise should be equal to  $I_1 R_1$  that is  $R_1$  upon  $R_1$  plus  $R_2$  into  $V_S$ . Similarly  $V_{DA}$  that is voltage rise on here moving from D to A that should be equal to  $I_2 R_3$ ; that is  $R_3$  divided by  $R_3$  plus  $R_4$  into  $V_S$ . So,  $V_{CD}$  net voltage difference when voltage change when here moving from C to D that should

be equal to  $V_{CA} + V_{AD}$  that is  $V_{CA} - V_{DA}$  that is equal to  $R_1$  upon or  $1 + R_2 - R_3$  upon  $R_3 + R_4$  whole into  $V_S$ .

Now, under balanced situation we know that  $V_{CD}$  is equal to 0. So, what we can write for this  $R_1$  by  $R_1 + R_2$  is equal to  $R_3$  by  $R_3 + R_4$ . So, if we apply suitable modification modifying principles on this, then we can always convert this particular principle suitable mathematical operations, this will become  $R_1$  upon  $R_2$  is equal to  $R_3$  upon  $R_4$  or generally it is written in this way that is  $R_1 R_4$  is equal to  $R_2 R_3$ . Let me write with some more space  $R_1 R_4$  is equal to  $R_2 R_3$ .

Look at  $R_1$  and  $R_4$  opposite to each other this is  $R_1$  this is  $R_4$  whereas,  $R_2$  and  $R_3$  are opposite to each other. So, the products of the opposite resistances will be equal to each other to have the balanced condition and what about the sensitivity of this bridge? So, we know that  $V_{CD}$  is given by this relation. Therefore, the sensitivity we will be governed by change in any one of them that is whenever there is a change in the value of any one of the resistances there will be change in the value of  $V_{CD}$  also.

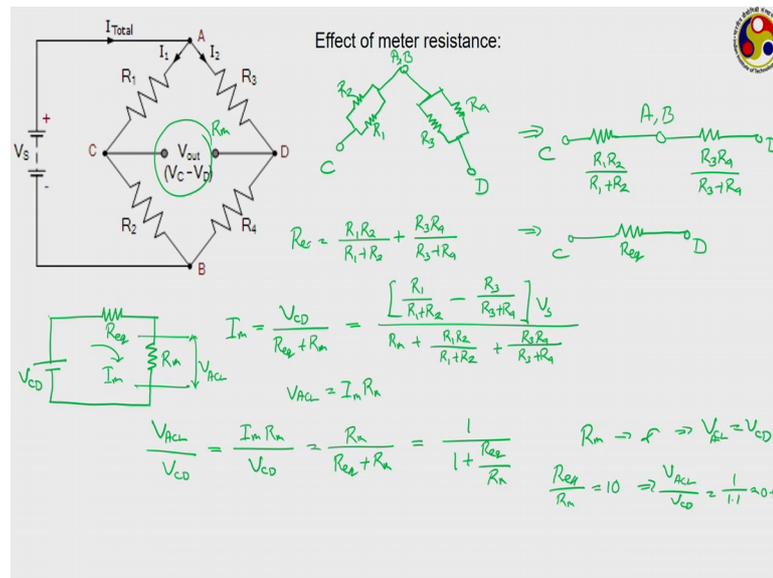
Let us say all the resistances are changing by small amount like  $R_1$  is changing by  $\Delta R_1$  that is to become  $R_1 + \Delta R_1$  this becomes  $R_3 + \Delta R_3$ , this becomes  $R_2 + \Delta R_2$  and this becomes  $R_4 + \Delta R_4$ . Then  $\Delta V_{CD}$  remember  $V_{CD} + \Delta V_{CD}$  that is the modified value of  $V_{CD}$  what actually should not consider this because this is initially equal to 0. It should be equal to  $\Delta V_{CD} = \frac{V_{CD}}{R_1} \Delta R_1 + \frac{V_{CD}}{R_2} \Delta R_2 + \frac{V_{CD}}{R_3} \Delta R_3 + \frac{V_{CD}}{R_4} \Delta R_4$ .

So, this way you can now put the values; like this is simple differentiation like I can show you one example. So,  $\Delta V_{CD} / \Delta R_1$  will be what? This will be equal to  $R_1 + R_2$  into  $1 - R_1$  divided by  $(R_1 + R_2)^2$  into  $V_S$  is equal to  $\frac{R_2 V_S}{(R_1 + R_2)^2}$ . This way we can calculate other four components and get an expression for the sensitivity. Commonly we do not have simultaneous change in all the four resistances in devices like strain gauge etcetera we generally have only one of the resistance acting as the gauge and other three remaining constant.

So, suppose in a vertical situation  $R_1$  is the one that is actually your sensor. So, the resistance of  $R_1$  will keep on changing and the remaining three will remain constant. So, you have to consider only the sensitivity with respect to one of the terms. This is just the

way several devices like strain gauge which is used for measurement of deflection or measurement of strain, a resistance thermometer which is used for measurement of temperatures they all work following this kind of Wheatstone bridge. But, this analysis was done considering an open circuit.

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Whenever we are connecting or we are closing the circle using some voltmeter then there will be certain effect on this and then what will be the effect on this? Now, instead of having this V CD as an open we have connected one voltmeter here which is having a resistor  $R_m$ . Then, how can we see this as the effect of this  $R_m$ ? Generally using the Thevenin's principle there we have to modify the circuit. So, to modify the circuit just look at this A, B, C, D points, there we can generally draw this one. So, this is your point C, this is your point D and this is your resistor  $R_1$  and  $R_2$  these are resistor  $R_3$  and  $R_4$ . So, this is a modified version of the same circuit.

Then, you can modify this one as a is just a straight line kind of structure. So, you can have basically reduce this one to two resistors. This is your point A and B superimposed on each other. So, this is the point A or B this is D, this is C. So, what will be this resistance we have  $R_1$  and  $R_2$  parallel to each other. So, this one will be  $R_1 R_2$  divided by  $R_1$  plus  $R_2$ . Similarly the other one is the combination of  $R_3$  and  $R_4$  in parallel. So, it is  $R_3 R_4$  divided by  $R_3$  plus  $R_4$  and this particular circuit now can be viewed just as one combined resistance between this point C and D which is R

equivalent.

So, what will be your  $R_{\text{equivalent}}$ ?  $R_{\text{equivalent}}$  will be these two resistor in series, that is,  $R_1 R_2$  by  $R_1$  plus  $R_2$  plus  $R_3 R_4$  by  $R_3$  plus  $R_4$  and now, when we have the meter in series it effectively reduces to the circuit somewhat like this. This is your equivalent resistor and this is the meter resistor. So, your  $R_{\text{equivalent}}$  this is  $R_m$  and this is your  $V_{CD}$  this how we can reduce the same circuit when you are having this  $R_m$  coming to picture.

So, what is the net current if  $I_m$  is the net current that is flowing through this circuit then offering to the meter I should say then  $I_m$  can be when there was meter was not present there was no current flowing through between flowing through this line CD. Now, as the  $R_m$  comes into picture, so, there will be current flowing resistor this. So, this  $I_m$  will be equal to this  $V_{CD}$  divide by the net resistance of the circuit which is  $R_{\text{equivalent}}$  plus  $R_m$  and the expression for  $V_{CD}$  was developed in the previous slide which was  $R_1$  by  $R_1$  plus  $R_2$  minus  $R_3$  upon  $R_3$  plus  $R_4$  into  $V_S$  divided by  $R_m$  plus  $R_{\text{equivalent}}$  which is  $R_1 R_2$  by  $R_1$  plus  $R_2$  plus  $R_3 R_4$  by  $R_3$  plus  $R_4$ .

While  $V_{CD}$  is supposed to be the actual voltage that you are trying to measure, but because of the presence of this  $R_m$ ; this resistance  $R_m$  then there will be certain change this means what we are what I am trying to say that the voltage that you are trying to measure is  $V_{CD}$  and so, across your meter the actual voltage drop should be equal to  $V_{CD}$ , but practical because of the presence of this  $R_{\text{equivalent}}$  or  $R_m$  combination the actual voltage drop that is happening across this resistor let us say is  $V_{cl}$  or  $V_{Acl}$  which refers to actual.

So, what will be your  $V_{Acl}$ ? Actual voltage drop across the meter will be  $I_m$  into  $R_m$ . So, whatever expression you have you can convert this to a much simpler form as for doing some manipulation. But, generally we are more interested to know the relation between this actually measured voltage because of the presence of the meter resistance and the true one  $V_{CD}$  which we are trying to measure

So, if we measure this  $V_{Acl}$  by  $V_{CD}$  then what we have your  $V_{cl}$  is  $I_m R_m$  by  $V_{CD}$  and  $I_m$  upon  $V_{CD}$  was  $R_m$  divided by  $R_{\text{equivalent}}$  plus  $R_m$  or if we write this way it is  $1$  by  $1$  plus  $R_{\text{equivalent}}$  upon  $R_m$  that is your meter resistance is actually dependent on the ratio of this two resistors  $R_{\text{equivalent}}$  and  $R_m$ . In a situation if your  $R_m$  tends to

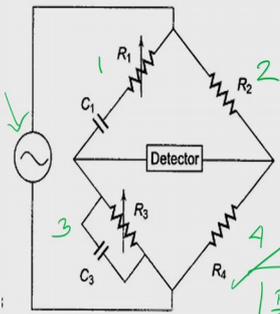
infinity then you have the denominator reduces to one that is your actual measured voltage  $A_{cl}$  is equal to  $V_{CD}$ . So, whatever you are going to measure using a voltmeter there is a true value. But, practical cases  $R_m$  is always having some finite value. It is never possible to have a voltmeter with infinite resistances. So, we have to reduce the value of  $R_m$  to certain limit and that depends upon this particular ratio of  $R$  equivalent to  $R_m$ .

Let us assume suppose a situation you have this is equal to 10 then your voltage ratio  $V_{Acl}$  by  $V_{CD}$  is equal to  $1$  by  $1.1$  which is roughly equal to  $0.91$  that is there will be about 1 percent loss in the sorry 9 percent loss in the final value what output your voltmeter is going to show that will be only 91 percent of the true output. Therefore, we always have to go for a value of  $R_m$  which is sufficiently high particularly if we have some idea about the equivalent resistor resistance of this Wheatstone bridge then we have to choose the voltmeter such that this rms significantly high to get a proper reading from this.

So, this is the way we can analyse any Wheatstone bridge circuit. This kind of analysis for Wheatstone bridge stone circuit will repeatedly come back, particularly the sensitivity related issue we shall be discussing repeatedly in case of strain gauge and other devices. But, let us move to now AC resistance bridges or AC impedance bridges.

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Impedance meters: Wien bridge (AC)



$Z_1 Z_4 = Z_2 Z_3$        $Z_2 = R_2$   
 $Z_4 = R_4$   
 $Z_1 = R_1 - \frac{j}{\omega C_1}$   
 $Y_3 = \frac{1}{Z_3} = \frac{1}{R_3} + j\omega C_3$   
 $Z_2 = Z_1 Z_3 Y_3 \Rightarrow R_2 = \left(R_1 - \frac{j}{\omega C_1}\right) R_4 \left(\frac{1}{R_3} + j\omega C_3\right)$   
 $\Rightarrow \frac{R_2}{R_4} = \left(\frac{R_1}{R_3} + \frac{C_3}{C_1}\right) - j\left(\frac{1}{\omega R_3 C_1} - \omega R_4 C_3\right)$   
 $\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1}$   
 $\frac{1}{\omega R_3 C_1} - \omega R_4 C_3 = 0 \Rightarrow \frac{1}{\omega R_3 C_1} = \omega R_4 C_3 \Rightarrow \omega = \frac{1}{\sqrt{R_1 R_3 C_1 C_3}} = 2\pi f$   
 $R_1 = R_3 = R$        $\frac{R_2}{R_4} = 2$   
 $C_1 = C_3 = C$        $f = \frac{1}{2\pi RC}$   
 $\Rightarrow f = \frac{1}{2\pi} (R_1 R_3 C_1 C_3)^{-1/2}$

AC bridges or AC impedance meters are required when you are looking to measure the

value of some capacitance or inductance like this is the first example which is Wien bridge. Wien bridge is a meter which is generally used to measure the capacitance and resistors of certain capacitor. Let us say for this particular case our objective may be to measure the value of this  $C_1$  or the resistor  $R_1$ .

So, just look at this. This is just like Wheatstone bridge circuit only, but they with a change here  $R_2$  and  $R_4$  resistors are ok, but this is the one that we are trying to measure which is a capacitor having a capacitance  $C_1$  and resistance  $R_1$  and here we have another resistance a what resistor rheostat kind of thing where we can modify the value of the resistance  $R_3$  with  $C_3$  capacitance in parallel.

Whenever we are going for this kind of circuit we just alter the Wheatstone bridge relation slightly. Let us say this is our  $R_1$ , this is 2, this is 3, this is 4. Like in case of Wheatstone bridge we have seen that the product of resistances in the opposite arms are equal to each other. Similarly here instead of resistance we write the generalized relation in terms of impedance which is  $Z_1 Z_4$  is equal to  $Z_2 Z_3$ ,  $Z$  refers to the impedance of the corresponding arm.

Now, here  $Z_2$  is equal to  $R_2$  because there is only one resistor there similarly  $Z_4$  is equal to  $R_4$  again there is only one resistor here. Now, what about the other two arms? For the arm  $Z_1$  we are having a resistance and a capacitance in series. So, its impedance can be written as  $R_1 + j\omega C_1$  which is the standard way of writing this. I hope you know this from your basic electrical knowledge. You can review any basic electrical books for this where  $j$  is the  $j$  refers to root over minus one and  $\omega$  is the angular frequency of the AC voltage that we are imposing on this, that is the characteristic of this particular AC source.

And, what about 3 then? Instead  $Z_3$  we generally write  $Y_3$  the admittance which is just opposite of  $Z_3$  which is easier to write it will be equal to  $1/(R_3 + j\omega C_3)$  it is easier to write. So, with this principle now we know that we have  $Z_1 Z_4 = Z_2 Z_3$  or we modify this as  $Z_2 = Z_1 Z_4 Y_3$ . So, putting the values  $Z_2 = R_2 = Z_1 Z_4 Y_3$  that is  $R_1 + j\omega C_1$  into  $Z_4 = R_4$  and  $Y_3 = 1/(R_3 + j\omega C_3)$  and that is  $R_2/R_4 = (R_1 + j\omega C_1)/(R_3 + j\omega C_3)$  and that is  $R_2/R_4 = (R_1 + j\omega C_1)/(R_3 + j\omega C_3)$  and if we simplify this or take a product of this it is  $R_1 R_3 + j\omega C_3 R_1 = R_2 R_3 + j\omega C_3 R_2$  minus  $\omega R_1 C_3$ .

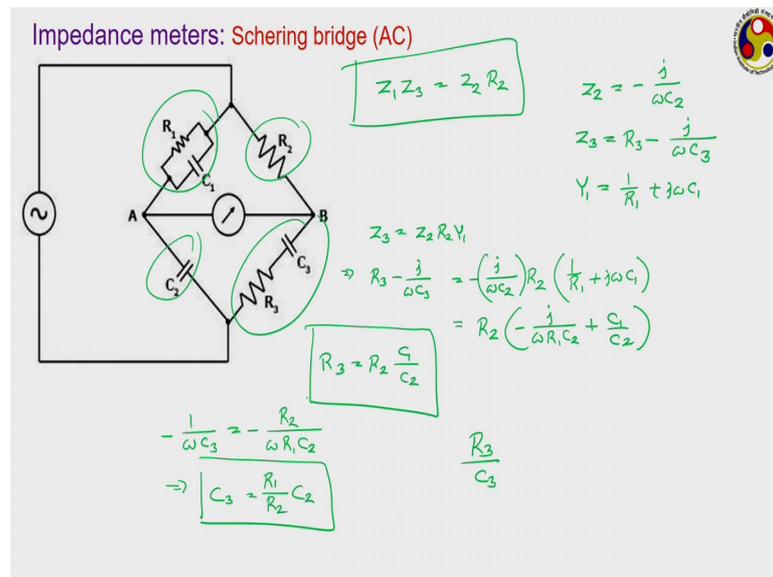
So, if we compare the right hand side and left hand side or I should say the real and imaginary parts of the right hand or left hand side comparing the real part we have  $R_2$  upon  $R_4$  from the left hand side is equal to  $R_1$  upon  $R_3$  plus  $C_3$  upon  $C_1$  from the right hand side that is giving relation between the resistance and the capacitor that we are putting.

Similarly, equating the imaginary part from the left hand side we have  $1$  upon  $\omega R_3 C_1$  minus  $\omega R_1 C_3$  is equal to 0, because there is no imaginary part on the left hand side. So,  $1$  upon  $\omega R_3 C_1$  is equal to  $\omega R_1 C_3$  that is  $\omega$  is equal to  $1$  upon  $\sqrt{R_1 R_3 C_1 C_3}$  and sometimes we would like to represent  $\omega$  in terms of frequency that is equal to  $2\pi f$ . So, your  $f$  becomes  $1$  upon  $2\pi$  into  $\sqrt{R_1 R_3 C_1 C_3}$  whole to the power minus half.

So, this is the frequency of the applied voltage signal that we have got if we know the applied value of the  $f$  then using this relation and this relation we can calculate the value of  $R_1$  and  $C_1$ . It is possible to write them as exclusive relation between  $R_1$  and  $C_1$  like from this relation maybe we can get a relation between  $R_1$  equal to something and put that back in the second relation to get something some value of  $C_1$  in terms of  $f$ , but generally that is not required for a this, this is the way we generally analyse an Wien bridge oscillator.

Quite commonly we generally go for  $R_1$  equal to  $R_3$  equal to  $R$  and  $C_1$  equal to  $C_3$  is equal to  $C$ ; particularly when you are looking to measure the value of the frequency and you have a control on this resistance and capacitance  $R_1$  and  $C_1$ . In that case what we have? We have  $R_2$  upon  $R_4$  is equal to 2 and the value of  $f$  is equal to  $1$  by  $2\pi R$  into  $C$ . So, the value of the frequency can easily be measured in this particular case for an unknown AC signal. This is Wien bridge is used to measure both resistance and capacitance or sometimes the frequency of an AC wave.

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Another variation of this which is called the Schering bridge, just look at this quite similar, but here you have just one bridge or one arm with the plane resistor. This arm is having only a single capacitor this is having resistor and capacitor in series and this is having them in parallel.

So, if we follow the same principle for all of them now, here we can write following the in this given  $Z_1 Z_3$  is equal to let us say  $Z_2 R_2$ . Here  $Z_2$  refers to this particular one. So, your  $Z_2$  will be equal to minus  $j$  upon  $\omega C_2$ ,  $Z_3$  resistors and capacitors is 3. So, it will be  $R_3$  minus  $j$  upon  $\omega C_3$  and for the arm number 1, we are writing the admittance such there in parallel like we did in the previous case. So,  $Y_1$  will be equal to one upon  $R_1$  plus  $j \omega C_1$ .

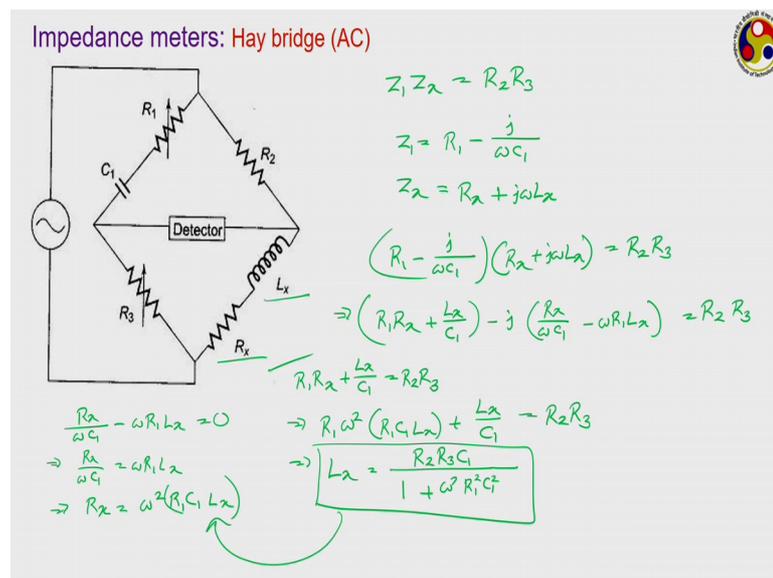
So, if we put all of them together then writing  $Z_3$  is equal to  $Z_2 R_2 Y_1$ . We have  $R_3$  minus  $j$  upon  $\omega C_3$  is equal to  $\omega C_2$  into  $R_2$  sorry this should be should have been  $R_2$  and  $Y_1$  is  $1$  upon  $R_1$  plus  $j \omega C_1$ . Writing this is combining all of them we have so, minus  $j$  upon  $\omega R_1 C_2$  minus or I should say plus we have  $C_1$  upon  $C_2$ .

Again, we can just compare the real and imaginary part of both sides. Comparing the real part we have  $R_3$  is equal to  $R_2$  into  $C_1$  upon  $C_2$ . In this case commonly our objective is to identify these values  $R_3$  and  $C_3$  because we generally have control over this particular this three arms. So, we can write this one as so, this gives you an expression

for the R 3 which you are looking for similarly if we equate the imaginary side then minus 1 upon omega C 3 is equal to minus R 2 upon omega R 1 C 2 which gives us C 3 equal to R 1 by R 2 into C 2. A very straight forward simple relation, once we attain the balance situation so, that this particular relation is valid you have to remember. This particular relation is value only on the balance situations. So, once we have the balance then just from a knowledge of the other resistors we can get the value of both R 3 and C 3 to get a value of this capacitance.

Quite often this particular circuit is also used to measure the rate of dissipation of a capacitor. Rate of dissipation is given as just the ratio of the resistors and capacitors like in this case it will be just equal to R 3 upon C 3. You can easily get that from this relation as well.

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Hay bridge another very common AC impedance meter AC impedance meter where we have a an inductance into picture and our objective is to measure this inductance L x. So, following the same principle, but in taking the inductance into consideration. It is interesting that here we have one variable resistor here and one variable resistor and capacitor here which can be adjusted and our objective is to measure this L x and R x.

So, following the same principle we write Z 1 Z x is equal to R 2 R 3. Now, Z 1 we as we have resistors and capacitors we are writing as sorry, we are writing as R 1 minus j upon omega C 1 and Z x. Now, it is not capacitance it is an inductance into picture. So,

how we should write it? I hope you remember or you know this it is  $R_x$  plus just the opposite way we do for capacitance it is plus  $j\omega L_x$  which is the opposite to the capacitance. Capacitors and inductors they are mathematically represent as opposite. So, while in case of  $Z_1$  we have  $C_1$  in the numerator sorry in the denominator here  $L_x$  is in the numerator.

So, now I can easily go back to our relation that is  $R_1 - j$  upon  $\omega C_1$  into  $R_x + j\omega L_x$  is equal to  $R_2 R_3$ . So, we have  $R_1 R_x + L_x$  upon  $C_1 - j$  into  $R_x$  upon  $\omega C_1 - \omega R_1 L_x$  is equal to  $R_2 R_3$ . So, equating the real and imaginary part we have from the real part  $R_1 R_x + L_x$  upon  $C_1$  is equal to  $R_2 R_3$  problem is that like in case of Wien bridge we had the similar situation both  $R_x$  and  $L_x$  are involved.

So, what we commonly do is we separate out  $R_x$  from this relation. So, that we have  $R_x$  is equal to or ok, let us stay with this. Let us do in different way let us equate the imaginary part. So, from the imaginary part we have  $R_x$  by  $\omega C_1 - \omega R_1 L_x$  is equal to 0. So, if you take from there  $R_x$  upon  $\omega C_1$  is equal to  $\omega R_1 L_x$  that is  $\omega^2$  is equal to ok, instead of taking  $\omega$  out let us take  $R_x$  out from this. So, your  $R_x$  will be equal to  $\omega^2 R_1 C_1 L_x$  and let us put it back in the this particular equation.

So, here we are going to have  $R_1$  into  $\omega^2$  in to  $R_1 C_1 L_x + L_x$  upon  $C_1$  is equal to  $R_2 R_3$ . Taking  $L_x$  out from this we have  $R_2 R_3 C_1$  and in the denominator we have  $1 + \omega^2 R_1^2 C_1^2$  and so, that is the expression for your  $L_x$  this can be taken back here to get an expression for  $R_x$ ; thereby providing by complete about the resistance and inductance of the inductor that you are dealing with.

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Impedance meters: Maxwell's bridge (AC)

$Z_1 Z_x = R_2 R_3$   
 $Y_1 = \frac{1}{R_1} + j\omega C_1$   
 $Z_x = R_x + j\omega L_x$   
 $Z_x = R_2 R_3 Y_1 \Rightarrow R_x + j(\omega L_x) = R_2 R_3 \left( \frac{1}{R_1} + j\omega C_1 \right)$

$R_x = \frac{R_2 R_3}{R_1}$   
 $L_x = R_2 R_3 C_1$

$Q = \frac{\omega L_x}{R_x} < 10$

A variation of the Hay bridge or maybe that is an older version is the Maxwell bridge where the calculation is even easier. Quite often this is also called the Wien-Maxwell bridge. Again another circuit where we have just two resistance in this is the inductor that is of our concern and this is a parallel capacitance resistance combination.

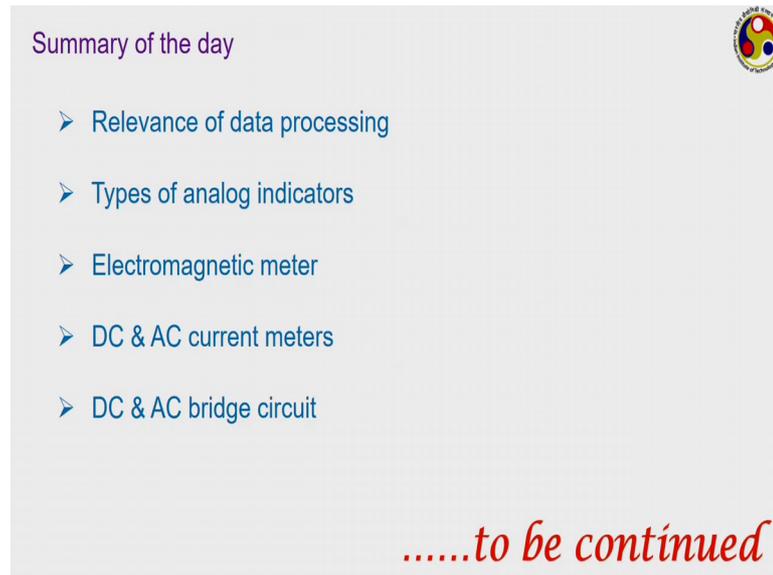
So, again if we write for this one we have  $Z_1 Z_x$  is equal to  $R_2 R_3$ . So,  $Z_1$  we can write directly now because they are they are in parallel. So, we should generally write it is as in the admittance is equal to  $\frac{1}{R_1} + j\omega C_1$  whereas,  $Z_x$  is equal to  $R_x + j\omega L_x$ . So, if you put it in the original equation  $Z_x$  is equal to  $R_2 R_3$  into  $Y_1$  from their  $R_x + j\omega L_x$  is equal to  $R_2 R_3$  into  $\frac{1}{R_1} + j\omega C_1$ . So, sorry, should be  $R_2 R_3$  within bracket  $\frac{1}{R_1} + j\omega C_1$ . So, that is  $Z_x$  and therefore, expanding  $Z_x$  we have  $R_x + j\omega L_x$ , ok.

We now we can directly equate the real and imaginary part equating that we have  $R_x$  is equal to  $\frac{R_2 R_3}{R_1}$  whereas,  $L_x$  is equal to  $R_2 R_3 C_1$ . So, quite straightforward calculation for this from the knowledge of the other three arms we can get the value of the inductance. This is the way to show the values of show how we can calculate or make use of different bridge circuits in case of both DC and AC circuitry to measure the voltage as well as the measure the impedance of circuits.

The one problem the despite being the calculation being such easier one issue is the Maxwell's bridge is that there is a common Q factor that commonly calculate which is

given as  $\omega L \times R$ . This is generally is use this Maxwell's bridge is restricted the value of  $Q$  less than 10 only which is quite restrictive condition. There are when the situation demands a larger value of  $Q$  we have to go for some other bridge that the Maxwell-Wien Wien bridge is not used despite being a much easier circuitry to deal with.

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Summary of the day

- Relevance of data processing
- Types of analog indicators
- Electromagnetic meter
- DC & AC current meters
- DC & AC bridge circuit

*.....to be continued*

So, that takes us to the end of this lecture. There are several things that we have discussed please go through this lecture several lots of method is that we have done, but all are simple mathematics that is a standard pattern in all the calculations. So, if you follow them carefully you will surely able to master them. In the next lecture we shall be solving a couple of numerals to start with where you can make use of this relations.

So, today we have started with the discussion on the relevance of data processing and different components that come under this data processing, then we have discussed about the types analog indicators; particularly electrical based analog indicators. Then discussed a bit more about electromagnetic meters in using the D'Arsonval principle from where we came to the discussion of voltmeters and ammeters.

Then, we talked about DC and AC current meters and exclusively talked about the DC and AC bridge circuit. Under DC circuit we talked only about the Wheatstone bridge, but under AC circuit we have talked about four different circuits. Out of which like Wien bridge which generally is used for measurement of capacitance whereas, we have also

discussed over the Hay bridge and Maxwell's bridge we are used for inductance measurement.

So, that is it for the day. In the next class, we shall be taking it forward discussing a bit about the digital data processing and finish half of this particular chapter. Till then thanks for your attention.