Theory of Rectangular Plates - Part 1
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Lecture - 04
Tutorial: Transformation of Tensors

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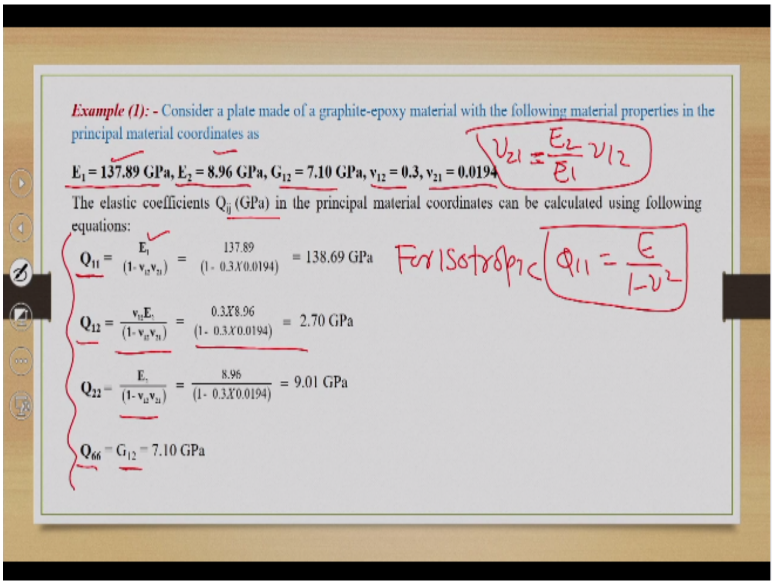

Now I am going to solve some simple problems like if somebody gives you Young's modulus E1, E2, G12, mu 12 and mu 21. Even before that I may ask that E2/E1 and mu 12=mu 21. So using those relations, we can find out mu 21. If E2 is known, E1 is known, mu 12 is known then you can find out mu 21. Why it is required? You see that elastic coefficient using these materials.

When we are going to develop a plate theory, we have to find out Q11, Q12 numerically. So E1 1-mu 12 mu 21. For isotropic Q11 will be E/1-mu square okay. So in the assignments we may ask that how to evaluate a Q11 for a given material property, similarly how to evaluate Q12, mu 12, E2. These are the very standard relations or I would like to say the basics of structural mechanics if we are not comfortable with this, we cannot proceed further. We must know that how to evaluate Q11, Q12, Q22 and Q66 is nothing but G12.

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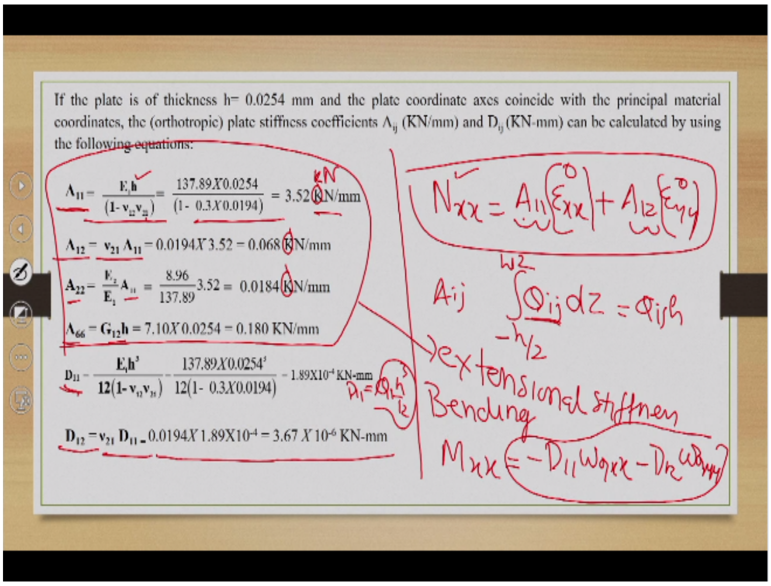

If you remember exactly, there I have said Nxx, what is that? A11 of epsilon xx of 0+A12 of epsilon yy of 0 that is the plate constitutive relations. So for that we know how to evaluate A11, A12. So A11 the definition is Qij dz –h/2 h/2 where Qij is a constant. So basically Qij*h times will be Aij. Here ij goes from 1 to 2. So A11 will be nothing but E/1-1 to 21*thickness of that plate. So we can evaluate A11.

If we know A11, if we know A12 and through some experiments is there able to at any point the strain epsilon xx 0, epsilon yy 0 we can evaluate Nxx. A12 is mu 21*A1, A22 is E2/E1 A11, then A66 up to here this is your extensional stiffness. These formulas are important, we must remember these things so that we can evaluate or we can calculate during the assignments or exams or some point of time.

Now we are talking about the bending stiffness. If you want to evaluate movement Mxx, what is that? It is –D11 of w, xx-D12 of w, yy. So this is your movement, so you can evaluate D11 is E1 h cube 12- or in another terms I can say that Q*h cube/12, in other sense D11 is this thing. Somehow you know that Q11 you can calculate the D11 h cube/12 or you know the D11 you can calculate the Q11.

Similarly, D12 one can calculate, so units should be small k, there is some mistake kilonewton.

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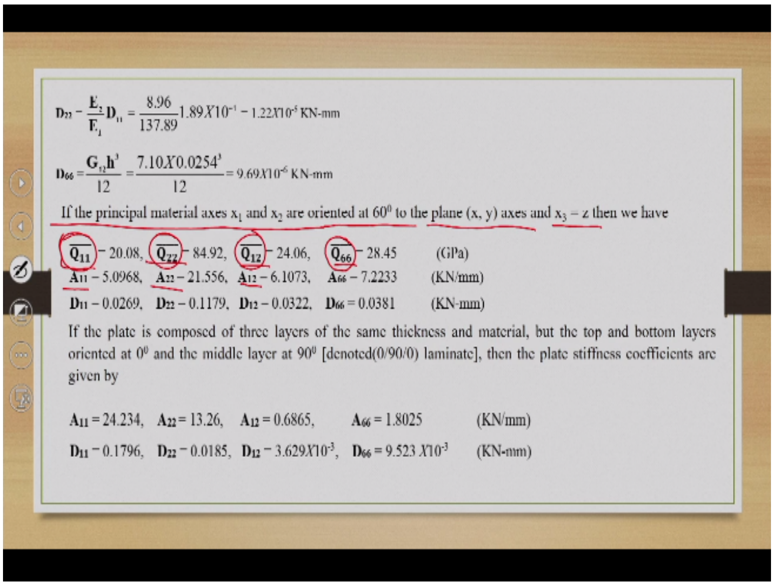

Then D22, D66, now the question that material axes are oriented at 60 degree and x3=z then you have to evaluate bar transformed one which will be a cos 4 and so on. Then based on this effective transform reduced stiffness A11, A22 can be evaluated okay.

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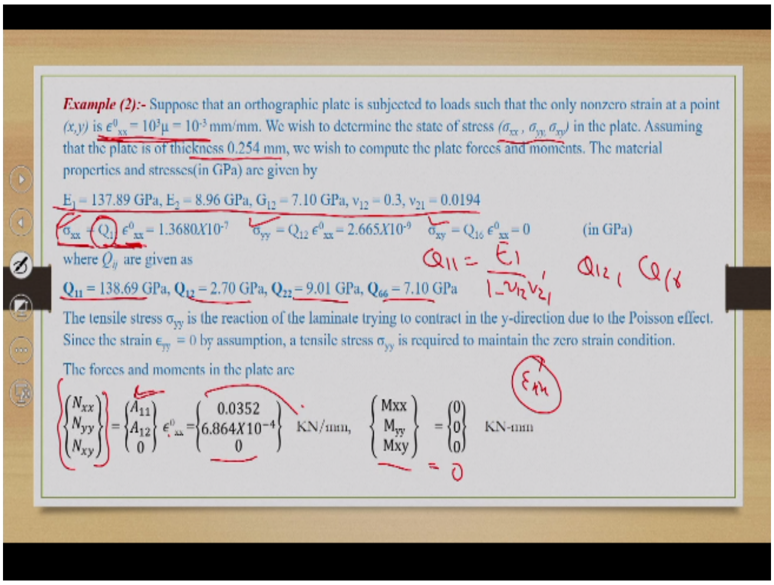

I have already told you that if your E1, E2, E3 all things are given and then at any some point of time epsilon xx 0 is given, so it is we are going to read the statement that an orthotropic plate is subjected to only nonzero strain is this and we have to be interested to find out that state of stress at that point and thickness of the plate is given, material properties are given so you can evaluate sigma x axis Q11 of epsilon 0.

So for this purpose, we know how to evaluate Q11, Q11 is what? Q11 is E1/1-mu 12 mu 21 and what is that Q12? And similarly Q16 later on, so they have evaluation and if you substitute it there you can evaluate the sigma xx, sigma yy, sigma xy. Then stress resultant Nxx, Nyy, Nxy, only epsilon xx 0 is nonzero, so only contribution will be A11. So if you multiply with that you can get what will be the movement.

For the case of movement, it should be epsilon xx of 1 so there is no epsilon xx of 1 so movements will be 0. So these types of questions or the numericals when you are going to make a program for a plate, so you must know these formulas that how to evaluate this, you are going to use this, at the end of the day you need some number that deflection should be in some numbers, movements should be in some numbers. So you have to evaluate these things.

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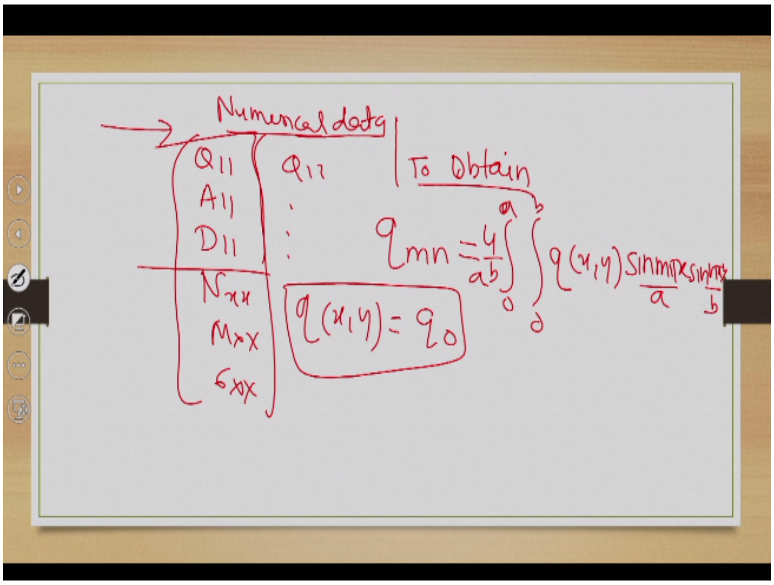

Now I am talking about the now you have known that how to evaluate Q11, how to evaluate A11, how to evaluate D11 and corresponding some Q12 and so on and how to evaluate Nxx and Mxx for a given point or a sigma xx and so on for a given numerical data. The next is to evaluate or obtain if you remember with qmn that qmn definition was that is also required for the solution.

If you want to solve a plate problem, you have to at least up to here all material constants and qmn, so qmn definition wise was 4/ab 0 to a 0 to b and q x, y sin m pi x/a sin m pi y/b. Now I am saying let us say my Q x, y is nothing but just a UDL kind of thing q0, not a function of anything then what will be my qmn? So you have to solve this integration.

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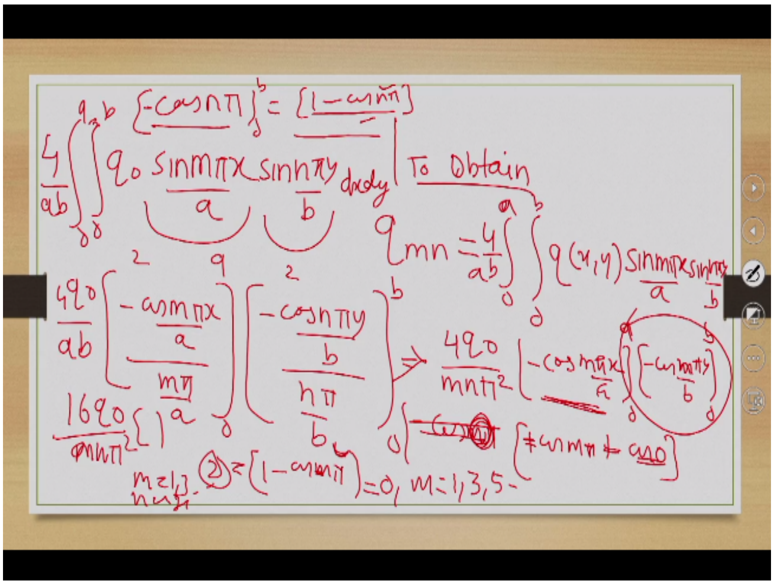

What will be that just put it there. This is very, very much important. Most of the time, we understood the theory that how to represent or how to develop a governing equation for a plate but for making a program for getting the solutions, we must know these basic things that how to evaluate. Program will not give you the final solution. You have to write your own code or your own program that this will be equal to something and that we are going to use that.

Let us say Q0 sin m pi x/a sin n pi y/b dxdy 0 to a 0 to b, so this is only a function of x, this is only a function of y, so we can integrate. So basically q0 4/ab you put it here and then sin will be – of cos m pi x/a/m pi/a units 0 to a. Then –cos n pi y/b/n pi/b 0 to b. So this you further simplify a and b will get out cancel, so basically 4 q0/mn again pi square will come up here and here –cos m pi x/a 0 to a and –cos n pi y/b 0 to b.

Then, what is that? Put a so it gets cancel out -cos m pi okay and then lower limit- upper limit, you first put it 0, cos m pi only and then this cos 0. So when you put 1 m pi so basically I am going to put here 1 then -, + so cos 0 is 1- of cos m pi. Then if you put pi m=1, cos pi will be -1 then it will become 2. When you put m is 2, then it will be cos 2 pi which is +1 then it becomes 0.

So this series is valid only m=1, 3, and 5 and so on. Similarly, here cos n pi 0 to b – of that so basically 1-cos n pi you can put like that. So if you put 1, it becomes 2. So ultimately 2 from this, 2 from this, so 16 q0/mn pi square and that is valid for m=1, 3 and n=1, 3 and so on. So we can evaluate this function like this.

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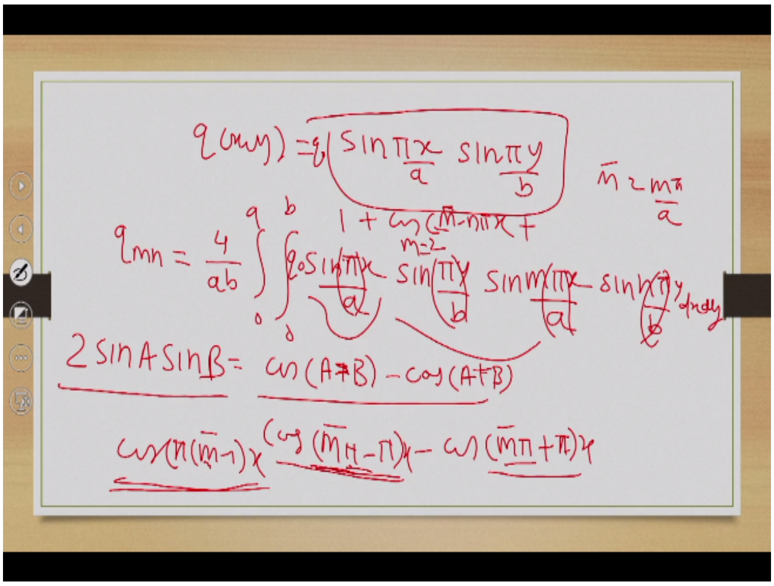

Now come to the second most important function, if your Q x, y is represented sin pi x/a and sin pi y/b sin loading then your qmn will be 4/ab 0 to a 0 to b, put it this thing and you say q0 something value sin pi x/a sin pi y/b and then sin m pi x/a sin n pi y/b dxdy. So here you see that sin x and sin c is basically if you remember sin A and sin B formula which is basically cos A-B-cos of A+B like this some formula.

So if you put it there, so I would like to say let us say m pi is greater so I would like to write like that into this –cos m pi+pi of x and so on. So you can say that cos of pi m-1*x. So this first term when you put m=1 it becomes 0, when you put m=2 okay then series goes like that, so we can say this term can be written as 1+of cos m-n pi x. So basically you will say that m bar where m bar is nothing but m pi/a okay.

Here also pi/a is there, pi/b these things will come out. So here this goes from m=similarly for this case. So if you do that integration of a cos function, it will be sin, you take any value sin pi, sin 2 pi, sin 3 pi, sin 4 pi contribution will be 0.

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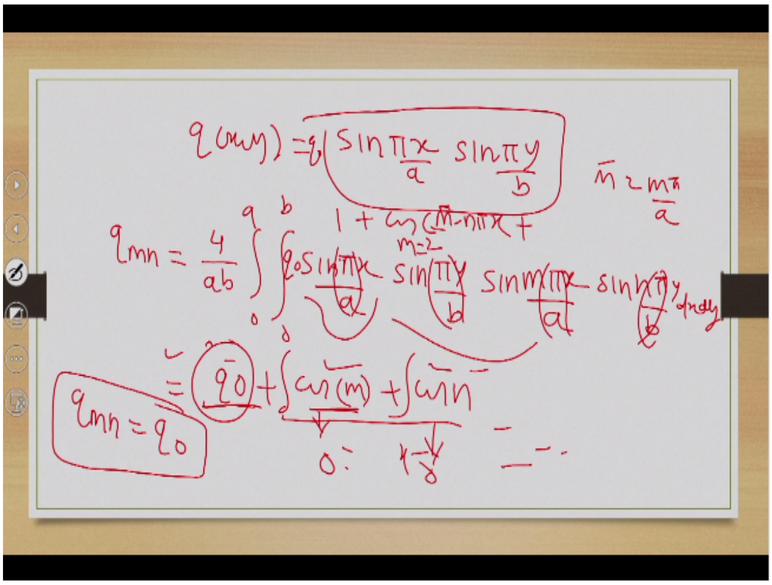

So I would like to say the gist of is this that for such a case it is just becoming q0+cos of something+cos of and something and so on, so there will be integration so 0 contribution, only q0 come up in there, nonzero contribution 1. So qmn a, b they will cancel out and 2 in this and you can take 2 again, so qmn is just q0. So that is the easiest one. When I say that a plate is subjected to its sinusoidal loading then you understood that qmn must be just q0.

For most of the time for simplification or just to expose you that what will be the stresses or moments, we use to assume that our loading is by sinusoidal along x and y axis. So qmn will be q0 and we can solve that deflection, moment and stresses so on.

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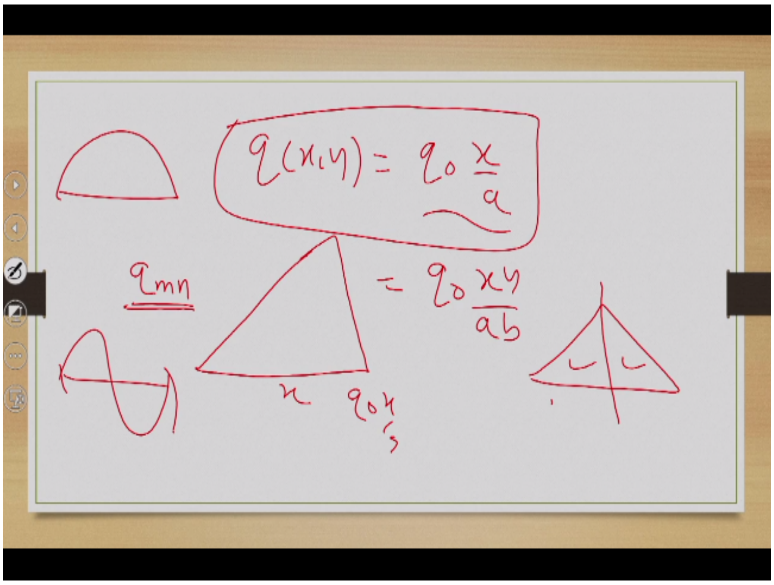

We may give q x, y is like that q0 x/a, a linear function or sometimes q0 xy/ab any kind, so then you are supposed to evaluate that qmn. Even evaluating this qmn is important or I would like to say that sometimes that making a function is a challenging task that if you see in a book that have to represent if this is the only a triangular loading increasing that to say that along x axis sometimes we call it is a hydrostatic loading of q0 x/a.

But if you have a loading like this in pyramid shape then how to make this function, so it will be two series (()) (18:51) one is this, then combining together so one function will be valid up to here and other function will be valid up to here but if you have a sin series 1 or if you have a function like this then you can assume in terms of sin series or some polynomial functions for this case.

So our aim is to find out first how the loading is there and based on that qmn. So evaluating of qmn is a very important task if you are interested to get the deflection and moment of a plate.