Fundamentals of Nuclear Power Generation

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Lecture – 06
Energy and momentum conservation

Hello friends, welcome back to the MOOCs course on the fundamentals of nuclear power generation and today we are looking to finish our second module on the topic of radioactivity and nuclear reactions. We already had 3 lectures on this particular topic and if you are gone through all of them carefully, I am sure you have quite decent knowledge about this phenomenon of radioactivity.

In our first 2 lectures, we discussed mostly about the natural phenomenon on natural radioactivity of certain isotopes, where an isotope can go through spontaneous decay leading to the emission of certain particles and also release of high amount of energy and in certain situations the daughter particles or daughter isotope which is the product of this dissociation or disintegration itself can be radioactive in nature leading to the formation of a long chains or long radioactive chains.

So, such kind of natural radio activities limited to only certain isotopes. However, in the 3rd lecture you are introduced to this very interesting topic of artificial radioactivity or induced radioactivity, where any stable isotope can be forced to represent or display radioactive behaviour by striking with a suitably chosen high energy particle, like we have seen the examples, if we quickly revisit our coverage on our lecture 3.

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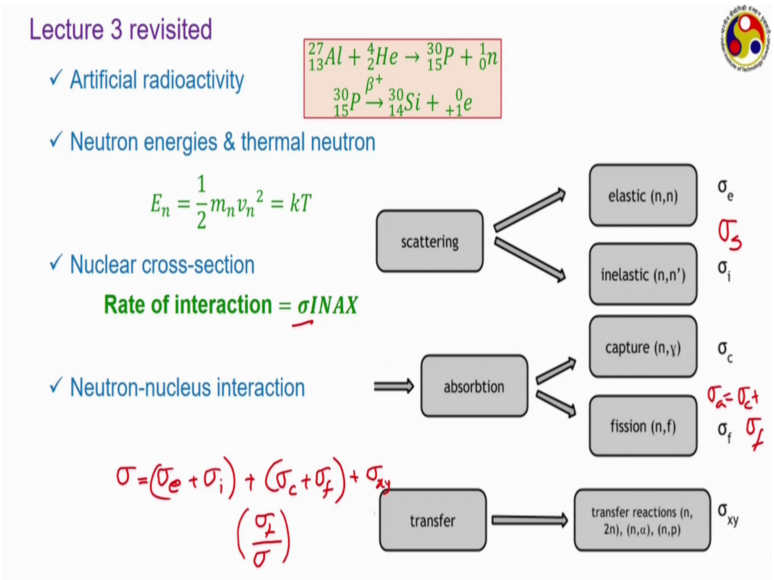

In the topic of artificial radioactivity, which was first introduced by the daughter of Mary and Pairy curie and that is Juliet curie with her husband Frederick or Irrn curie with her husband Frederick Joliet. So, where they took the very stable isotope or aluminium 27, strike that with high velocity alpha particles leading to the formation of phosphorus 30, which was found to be radioactive in nature and that displayed continuous emission of beta positive particles or positrons.

So this way, the way they showed; similarly, lots of other researchers works on that and now it is a proven phenomenon that any stable isotope can be converted to a radioisotope by striking it with a suitably chosen particle. While, the choice of particle can be alpha particle, can be electrons, which is the beta particle or something else. Neutron is generally the most favored 1, the reasons we have already discussed neutron being electrically neutral, it does not face any kind of electrostatic repulsion force from the nucleus and therefore, it can easily approach to the nucleus which is not possible for proton or alpha particles.

It also has a mass of about 1 mu, which is sufficient to cause some kind of damage to a nucleus, if it has high amount of velocity. However, the nature of the interaction a neutron is going to have with a nucleus depends on 2 factors primarily 1 of them is the energy level of the neutron, the kinetic energy that is we have classified the neutrons depending upon their velocity level or energy level into several categories, but out of that primarily 3 terms are very commonly used; slow neutron, intermediate neutron and fast neutron.

Fast neutron generally refers to neutrons having energy greater than 1 MeV, where as slow neutron term is mostly reserve for neutrons having low energy less than 1 electron volt or in certain situations 10 electron volt intermediately something in between, but there is also the concept about thermal neutron, which refers to a neutron which attention thermal equilibrium with it is surrounding and accordingly it is having the lowest possible energy depending upon the temperature of the surrounding; corresponding to 23 degree Celsius the energy corresponding to a thermal neutron is 0.025 electron volt and velocity of about 2200 meter per second, which you that the common values that we associate with thermal neutrons.

Thermal neutrons have certain interesting properties which shall be discussing later on, but for thermal neutrons the value that has just mention they can be just calculated by equating it is kinetic energy which is half m n v n square to the 1 the thermal distribution that is k into T, where k is the Boltzmann constant, accordingly we got that value for 20 degree Celsius, this energy is found to be 0.025 electron volt and the other important factor which governs the nature of neutron nucleus interaction is the nuclear cross section.

Cross section is a concept, which relates to the probability of having a certain interaction happening when a neutron of certain energy strikes a nucleus. The rate of such interaction is been found to be proportional to the neutron flux, which is this I; it is also found to be proportional to the nuclear density or number of nucleus per unit volume of the target material and also it has been found to be proportional to the volume of this. So, NA into X, this 3 terms together represents the total number of nucleus present in the target and I is the characteristics of the neutron beam that is coming to hit this target, but the pro constant of proportionality is this sigma, which we have defined as the nuclear cross section.

It, can be visualized as the amount of area that a nucleus projects or presents to a beam of neutron to cause any particular kind of interaction. We have seen there are several possible interactions and I am producing this particular figure which we discussed earlier. Commonly, we can have 3 types of interactions 1 is first is scattering, where the neutron and nucleus interact with each other, exchanges some energy and momentum then go to different directions.

Scattering can also be of 2 type's elastic and inelastic, elastic scattering refers to a billiard ball or carrom coin types of collision, where they come from different directions and there is a perfect conservation of both momentum and kinetic energy. Just think about what happens in a carrom board, like if we strike a stationary coin with the striker then a part of the momentum and energy carried by the striker is transferred to the coin and then accordingly both of them choose a certain directions and velocity to move on, that is exactly what happens in a elastic collision.

The inelastic collision refers to much higher velocity (Refer Time: 07:15) neutrons, in case of inelastic collision, is are the there is a perfect conservation of momentum, but kinetic energy is not conserved because a part of the kinetic energy is transferred to certain photons, generally gamma photons. Corresponding collision cross sections are represented as a sigma e for elastic scattering and sigma i for inelastic scattering, quite often they are combined together into 1 common terms sigma s, which represents the summation of sigma e and sigma i.

The second kind of interaction that is possible is the absorption, when the neutron is able to penetrate inside the nucleus and after penetration there can be 2 types of possibilities 1 where the neutrons remains inside the nucleus farming, another isotope of the parent which is capture or radioactive capture, other is the neutron enforces enough amount of energy or in imports enough amount of energy to the nucleus, so that it gets split into 2 components 2 daughter nucleus which is fission.

So, both radiative capture and fission are absorption type, sigma c and sigma f are the corresponding cross sections and quite often we use sigma a to represent the absorption cross section, which is nothing but the summation of the sigma c plus sigma f and there is 3rd kind of interaction which is a much rarer compared to the previous 2 which is transferred, that is when a neutron strikes a nucleus it leads to the emission of some more neutrons or certain particles like alpha particles or protons etcetera.

While, this third kind of interaction is mostly associated when certain heavier particles like alpha particles strikes the nucleus, but occasionally it is also can be associated with the neutron collisions. So, all these 5 types of interactions are possible when you have a neutron nucleus interaction and corresponding to each of them there is a certain probability or you can say 1 nucleus present certain amount of area to the neutrons to have each of this kind of interactions. So, the total cross section a neutron can have or I should say a nucleus can offer to a particular neutron is just a summation of all this.

So, total sigma can be summation of sigma e plus sigma i, this 2 corresponding to the scattering part plus capture plus fission corresponding to the absorption part plus the transferred part which may be relevant in certain situations, but mostly it is negligible. So, this is the total cross section a nucleus can offer to a particular in a neutron of particular energy level and hence if our interest is to identify exactly what is the probability of say fission occurring, I must mention which I have already done earlier also.

The value of each of this cross section for a given nucleus depends upon the velocity level of the neutron itself. Now, if I provide you a certain nucleus and a neutron of a certain velocity then all this 5 numbers are given, so we know all this cross sections. Now, if you want to know exactly what is the probability of fission occurring then that should be the fission cross section divided by the total cross section that is available to this particular neutron nucleus here.

So, this is the probability of fission that can occur in that particular situation and this way we can identify the probability of all other kinds of interactions as well. So, with this concepts of artificial radioactivity, today we shall be checking about the conservation of energy and momentum and then we shall be solving some numerical problems to give you some more idea about how to tackle this kind of situations.

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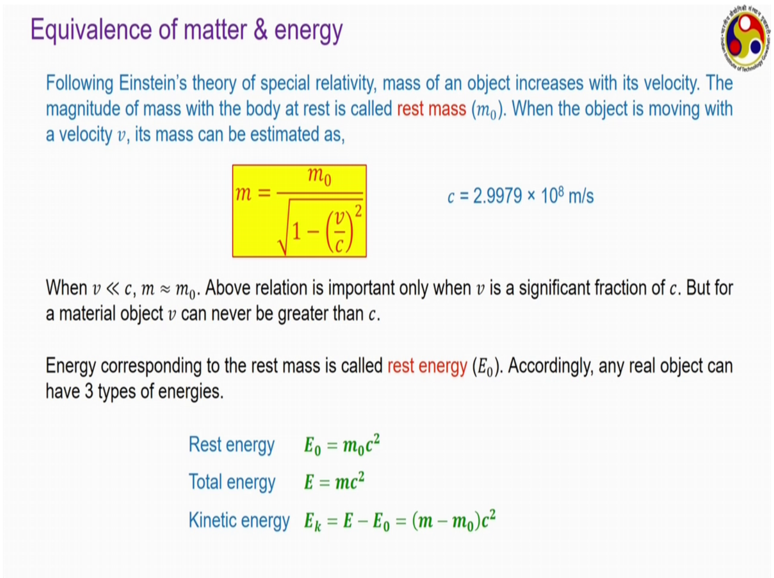

But, before that a particular concept I have to mention which I could have done during the first model itself, but still thought about putting it better here, the concept of a rest mass. While, in common physics point of view, we keep mass and energy as 2 separate entities, but following Einstein's theory of special relativity both are interchangeable that is mass can be converted to energy and energy can also be converted into mass and hence the special theory of relativity says that or theory of special relativity says that the mass of a body is not constant rather it keeps on changing with the velocity, but when the velocity of the particle or body is 0, then corresponding mass is called the rest mass, which is commonly denoted by m dot or m 0.

All the values of mass for proton, electron, neutron, whatever we have discussing so far or the mass of any particular nucleus all of this are actually a rest mass; that means, rest mass for a proton, if someone asks you, you have to say that, that is 1.007825 amu, because those are the values that we have already mentioned those are all for rest mass, but when the particle is moving with certain velocity then corresponding mass can be given by this relation no proof required for this, but can be a directly take it as m equal to m dot m naught divided by root over 1 minus v by c whole square, c will being the velocity of the light in vacuum that is 2.9979, you can give an 8 meter per second.

Now, you can clearly see c being such a very large number, we need to have quite large value of this v to have any contribution on the value of m. If the v is small compared to c, then the denominator basically reduces to 1 and m remains quite similar to m naught, but this particular relation is important only when the velocity is a good portion of this c, that is this v needs to be a good 10, 20 percent of this c, then only the value of m will be substantially different from the rest mass value.

Otherwise, there is no point considering this concept of changing velocity with kinetic energy we can always operate with the rest mass, but another thing we can learn from here, if v becomes greater than c then what happens, then the denominator becomes imaginary, which is definitely not possible which signifies that no material object can have velocity greater than v, the largest it can have is equal to the velocity of the light in that case also it is mass becomes infinite, but energy accordingly, we can define 3 different energies for a particle. The energy corresponding to this rest mass is called the rest energy, that is if the value of this rest mass m naught is completely converted to a energy then the amount of energy that we are going to get that is equal to this rest energy.

Similarly, when a particle is moving it is mass is expected to be different from the rest energy, rest mass and hence it is energy level also will be different from the rest energy. So, this rest energy is just m naught c square, that is if rest mass is completely converted to energy this is the amount of energy we are going to get and that is what we are calling as rest energy, but when it is moving then total energy is of course, m into c square that is all.

Then, what is the difference between the 2, the difference has to be the velocity with which it is moving, so that is what it offers the kinetic energy, which is the difference between the total energy and the rest energy or can be represented as a m minus m naught into c square. So, in a (Refer Time: 14:58) whenever particle is stationary, we can continue to use the rest mass and with rest energy, but whenever particle is moving then the velocity of the particle will have some kind of contribution to it is energy level and we must encounter that in kind in calculations related to nuclear energy in particular, where we have conversion of mass to energy or energy to mass.

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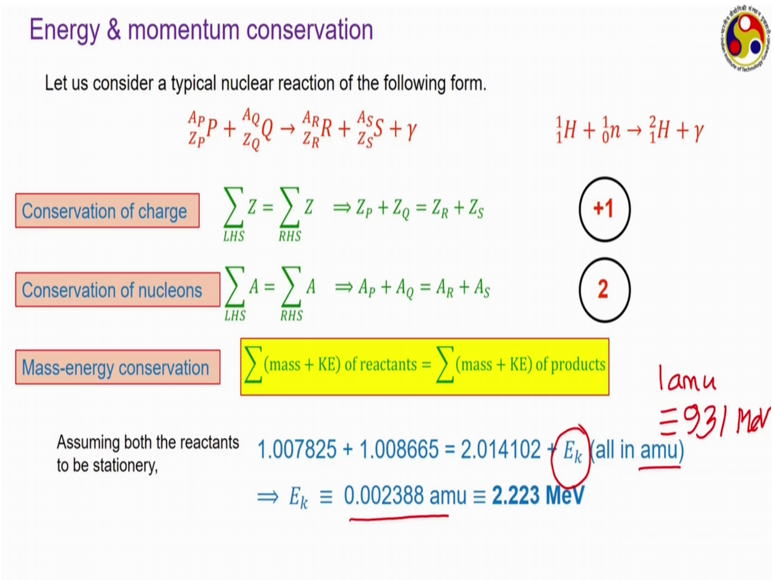

Now, let us take a simple or typical nuclear reaction, where P and Q are the reactants, R and s are the products and also gamma photon can be found on the right hand side as a product, then there are 3 different conservations or balance that we have to ensure first is the conservation of the charge, that is total charge on the left hand side must be equal to the total number of charge on the right hand side or as per the equations shown here Z P plus Z Q is the total number of charge present on the left hand side. So, that should be equal to ZR plus ZS, gamma definitely does not have any charge because that is a photon or you can also visualize that as the electromagnetic wave.

Second balance is for the number of nucleons, total number of nucleons also must be balanced the parent nucleus present here P and Q that is the reactance, they can exchange their charge on nucleons to form the products, but total number of charge and total number of nucleons must be conserved. So, summation of A on the left hand side should be equal to summation of A on the right hand side or as per the given equation AP plus AQ should be equal to A R plus AS.

Third, the most important 1; mass energy conservation, while in typical thermodynamics or heat transfer we are bothered about conservation of mass and conservation of energy separately, because there we neglect any kind of conversion of mass to energy or energy to mass but we cannot do that while we are dealing with nuclear reactions because here there is significant portion of mass can get converted to energy or I should say small amount of energy can be converted into mass can lead to huge amount of energy generation.

So, mass and energy should be considered together and hence we have to see this conservation equation as mass energy together. So, that is the summation of mass plus kinetic energy of the reactant should be equal to mass plus kinetic energy of the products. Here, 1 important thing we shall be seeing examples later on, here generally we use some kind of unit for mass and some other kind of unit for this kinetic energy, but when we are doing this balance we have to stick to 1 particular unit, that is either the kinetic energy need to be presented in terms of mass unit or the mass needs to be represented in terms of the energy units.

Let us take 1 example, we have taken example of a very simple equation, where the proton or a common hydrogen nucleus is being struck by a neutron and leading to the production of deuterium and gamma energies. So, first we have to conserve charge, how many charge we have on the left hand side? That is just 1, because neutron does not have any charge.

So, to and whatever the right hand side, again there is total just 1 number of charge because gamma is neutral again. So, conservation of charge we have plus 1 amount of charge on both sides, so charge is conserved. Now, how many number of nucleons we have, let us check out the number of nucleons, on the left hand side we have 1 contributed by this proton or the hydrogen nucleus and 1 contributed by this neutron. So, total we have 2 nucleus from the left hand side and on the right hand side again we have 2 nucleons here, related to this deuterium.

So, the total number of nucleons that is also conserved, which is 2 in this situation. Now, come to this mass energy conservation. So, the I am the directly writing the values of mass in amu here, but the mass values you need to take from some kind of table or data book. So, total mass of this hydrogen is 1.007825, this 1 mass of neutron is 1.008665, 0 Standard and actually I would suggest you try to remember the mass values of proton and neutron because that will be repeatedly used in different parts of this course.

The third 1 of course, I do not want you to remember or I do not expect you to remember which is for deuterium, it is provided from some data source which is equal to 2.014102 amu and last is the kinetic energy, gamma photon is mass less. So, it is rest mass is 0, therefore, we do not need to write the mask for that, but EK represents the total kinetic energy available with the products here.

Here ,1 assumption that we have you have taken is that both the reactants are stationary, if the reactants are also moving then there should have been a kinetic energy term added here as well, because then the left hand side also is having some kinetic energy contribution, but important thing is there, here this energy terms that we are adding like only 1 in the left hand side in this equation, but possibly can be something on the right hand side as well, that 1 energy transit EK that also needs to be represented in the same unit in terms of which we have represented the mass, that means, it has to be in terms of amu.

So, by changing the size of this term, we are getting EK is equivalent to 0.002388 amu and we are earlier seen from module 1, that is 1 amu is equivalent to 931 MeV of energy, that is also again coming from this theory of special relativity. So, this amount of mass that is 0.002388 amu, if we multiply that with the 931, then we can see that the corresponding amount of kinetic energy associated with the products is 2.223 MeV. So, while we have done the entire calculation in terms of amu, we can finally, convert the output in terms of energy or mass whichever is convenient to us.

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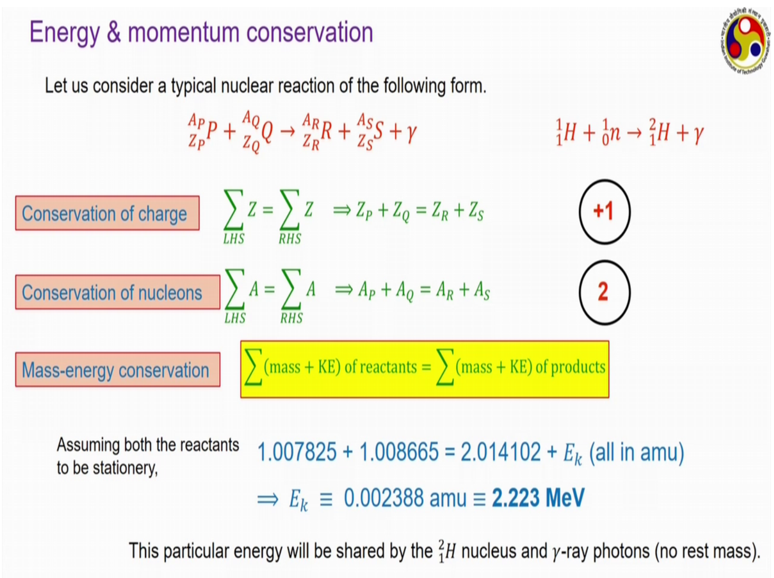

So, this total amount of energy 2.223 MeV that will be shared by both the products, that is the gamma ray photons and also the deuterium, but you have to remember the deuterium is having some amount of rest mass, where as the gamma photon is not having any kind of rest mass.

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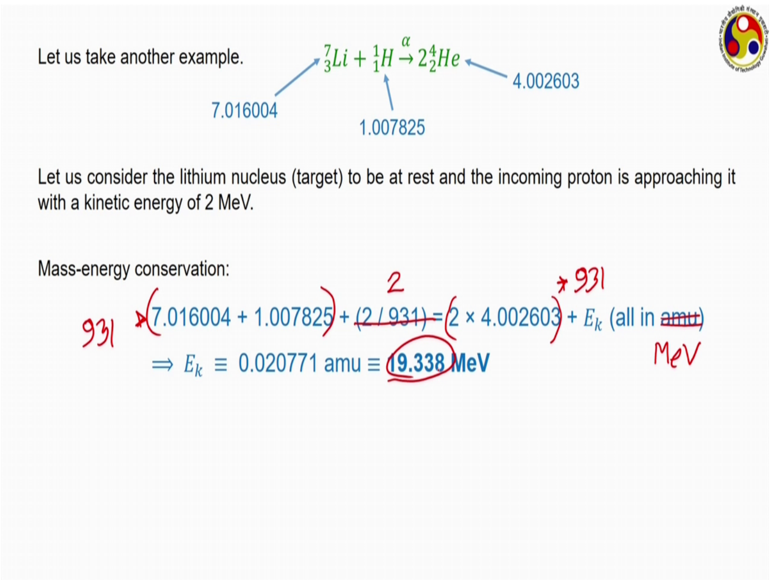

Let us take another example, here we are having lithium 7 isotope being struck by a proton. So, that undergoes alpha decay to produce 2 helium isotopes, mass for all the 3 are given here in no Gama photon involve. So, we are having only 3 isotopes or 4 isotopes rather all are having certain amount of rest mass and another consideration here, we are considering the problem as lithium isotope to be stationary the target and the proton is approaching this lithium isotope with their kinetic energy of 2 MeV.

So, let us write the same equation again, mass energy conservation can be written as here the mass plus kinetic energy of the reactants should be equal to mass plus kinetic energy of the product the product. So, we can choose again I repeat we can choose either mass unit or energy unit let us check with mass unit here. So, on the left hand side total mass corresponds to a mass of lithium 7.016004 plus mass of photon 1.007825 plus we have the kinetic energy of 2 MeV, but we can we cannot write 2 MeV directly here because we need to convert this unit to amu.

So, 2 by 931 is the mass equivalent of this kinetic energy that should be equal to the mass of the 2 helium isotopes plus the kinetic energy carried by this hili 2 helium isotopes all represented in terms of the mass unit which is amu. So, by doing the calculation we can find that corresponding amount of kinetic energy of the product is equivalent to 0.020771 amu, which is actually 19.3 MeV. So, this way we can perform a mass energy balance combined either using in terms of mass or in terms of energy generally it is convenient to use in terms of mass, because all the isotopes that is involved in the reaction most of them will be having some kind of rest mass.

So, it is generally convenient to do in terms of mass, but you can also do all this calculation in terms of energies, like this particular situation if you want to do in terms of energy then what we need to do, the rest mass then you have to write the total energy equivalent for the reactant should be equal to total energy equivalent for the product, in that case the rest mass for both the reactant lithium and proton this thing should be converted to mass, that means, you need to multiply this with 931.

So, once you multiply this with 931, that gives you corresponding energy equivalent, then we do not need to write this thus 2 Mev directly we can write and similarly on the right hand side this is the rest mass of the product and you need to multiply this with 931to get the energy equivalent and EK can stay in terms of it is energy unit in that case everything is represented in mega electron volt.

So, the answer should remain the same, you should get this particular; you should get this particular number directly. So, we can do such mass energy equivalent calculation either in terms of mass or in terms of energy. Also, the concept of binding energy that we have identified earlier or mass defect you can clearly see in this particular example, the mass of the product is 0.020771 amu less than the reactants and therefore, this amount of mass is getting converted to energy.

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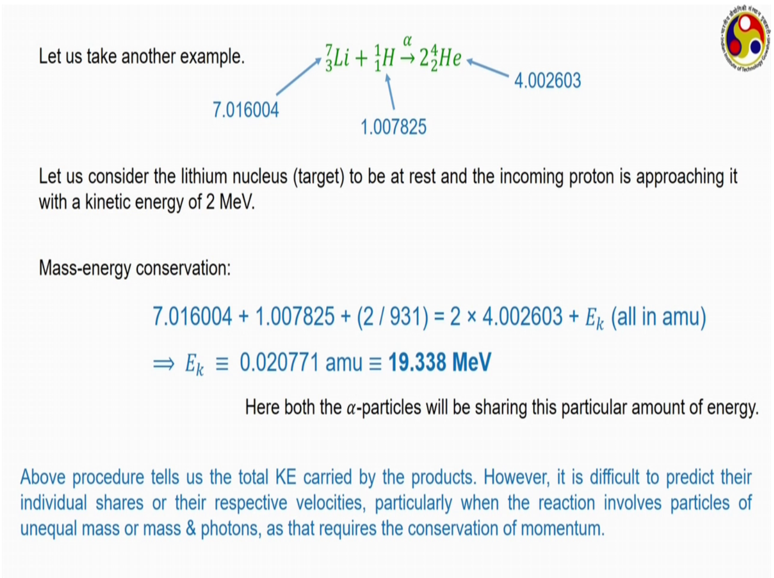

Now, one important point is we know the total amount of kinetic energy available with the products, which is 2 helium isotopes in this case. This 19.3 amu will be carried by both the helium isotopes, but how can we say how much will be carried by both.

In this case, both the isotopes are having equal amount of mass which is expected that this 19.338 MeV amount of energy will be equally shared by both, but if the product are not of same mass, like you think about the example in the previous slide, where we had deuterium and gamma photon as the 2 products, where deuterium is having certain amount of rest mass, gamma photon is having a rest mass of 0. So, the energy contribution of the energy shared for both of them should not be the same, they will be different, but this mass energy equivalence, the mass energy conservation does not give you the share about both of them rather we have to go to the conservation of momentum that is the 4th criterion that is coming to the frame.

In order to calculate; while we can calculate the total energy available with the product just for mass energy equivalence or mass energy conservation to get their individual share, here we are now going to the conservation of momentum, which conservation of linear momentum you can straight away right as the summation of mass into a velocity, that is a linear momentum on the reactants should be equal to the linear momentum of the products.

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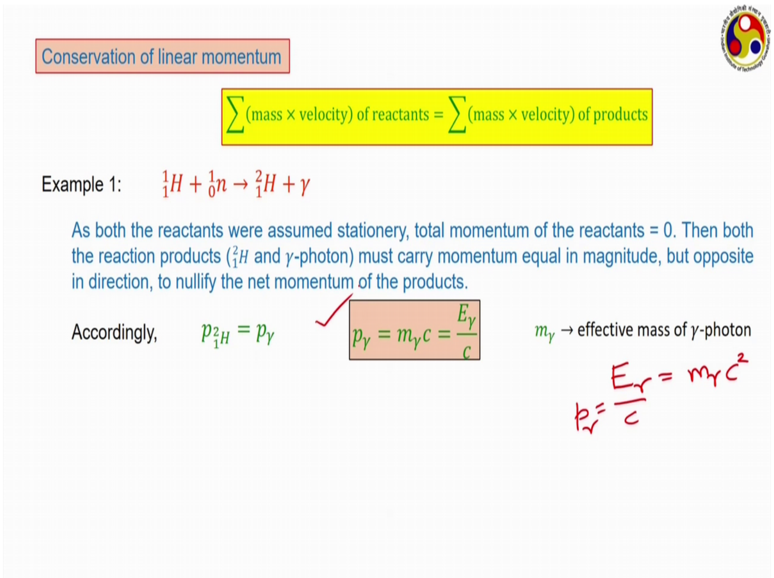

Let us take the first example, which you have discussed. Here, we have deuterium and gamma photon on the right hand side. So, both the product as the both the reactants are assumed to be stationary, then what is the total momentum associated with this left hand side? The left hand side total momentum has to be 0 because both of them stationery or both these reactants are having initial velocity of 0. So, the right hand side momentum also has to be equal to 0, which means the momentum being a vector quantity both the products that is deuterium and gamma or photon should be having equal magnitude of momentum, but exactly opposite in direction, so that net momentum of the product remains to be 0.

Accordingly, we can write the momentum for the deuterium magnitude of the momentum should be equal to the magnitude for the gamma photon, but gamma photon is something that does not have any kind of rest mass. So, we can always equate this momentum of base to the energy, where like if E gamma is the amount of energy carried by gamma photon then that can be represented as E m gamma into c square, m gamma is the effective mass.

Effective rest mass of the gamma photon which is practically 0, I should not say rest mass actually I am sorry, where m gamma represents the effective mass of gamma photon because of it is velocity and being a photon it will move with the velocity of light. So, E gamma is m gamma into c square and hence if you divide this E gamma with c then that should be equal to the momentum of this gamma photon, which is exactly that is written here.

Now, to know the value of or to solve this particular equation, while we know the velocity of this gamma photon you need to know either the energy share of this gamma photon or it is momentum we need to have some information about any 1 of them, then we can calculate the other quantity and accordingly we can calculate the velocity of the deuterium because this is the once you know the momentum of this gamma photon, this magnitude of this 1 also you know and this should be the mass of this deuterium into c square.

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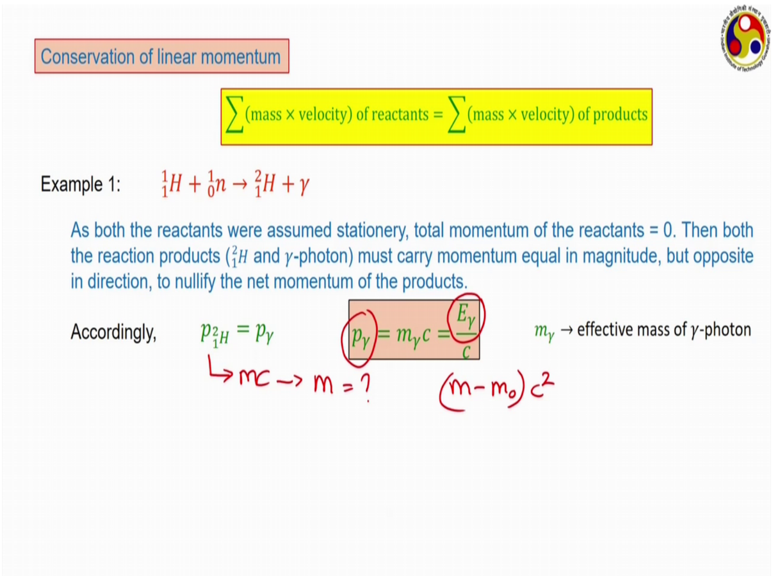

So, we know the mass of that and this mass from this particular once you know the effective mass of this gamma photon, actually this p gamma is equal to m into c, which gives you the value of c, once you know the effective the I am doing a mistake here, sorry I am going back. So, once you know the magnitude of this momentum carried by this deuterium, then we know the value of m into c m referring to the mass of the deuterium. So, as we know the value of c, we now can calculate the effective mass of this deuterium nucleus and the rest mass of this 1 is already known. So, that difference of this, these 2 are the difference m minus m naught multiplied by c square is equal to the kinetic energy of the deuterium nucleus which will give you the velocity of the deuterium.

So, this way by combining this momentum conservation ; conservation of linear momentum with the mass energy conservation we can get complete idea about the energy share of all the products of a nuclear reaction and also complete idea about the exact nature of their velocities.

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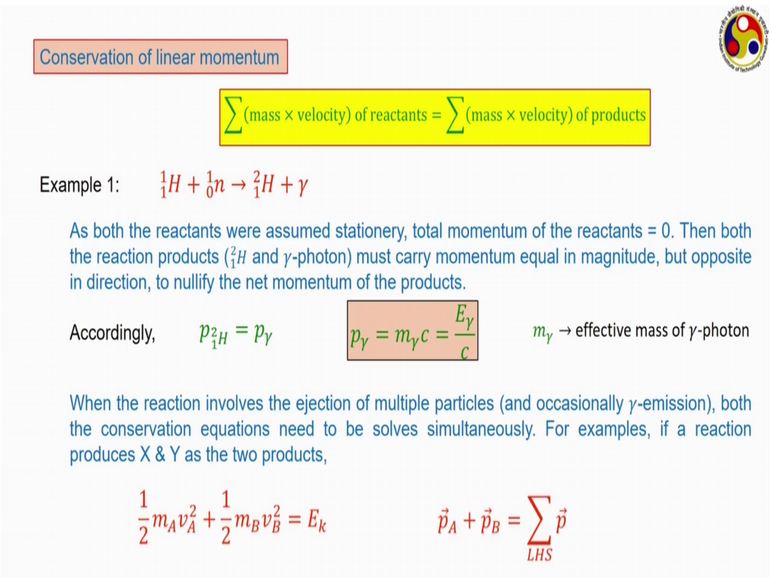

Just, if we take a very simple example say we have a reaction which involves 2 products x and y, where x and y both or any 1 of them can be a particle and other can be a gamma photon, kind of situation then we need to use both conservation of mass energy and conservation of momentum, that is the total kinetic energy will be equal to half m A v A square, actually here I should not use X and Y, I am actually talking about A and B as the 2 products, then half m A v A square plus half m B v B square should be equal to a total kinetic energy, where as p A plus p B should be equal to the total momentum for the left hand side, this being a vector addition I am using this arrows also.

So, in a nutshell there are 4 equations or 4 conservations that we must ensure while dealing with any kind of nuclear reaction number 1 is a conservation of total number of charge, number 2 conservation of total number of nucleons, which will be allow you to discuss the equation completely. Then, you have the conservation of the mass and energy together and number 4 we have the conservation of linear momentum. This 3rd and 4th together will allow you to get the complete idea about energy and velocity distribution of the products and by combining all this 4, we can solve any kind of problems involving radioactivity and nuclear reactions in general.

So, now let us quickly peep into a few numerical examples.

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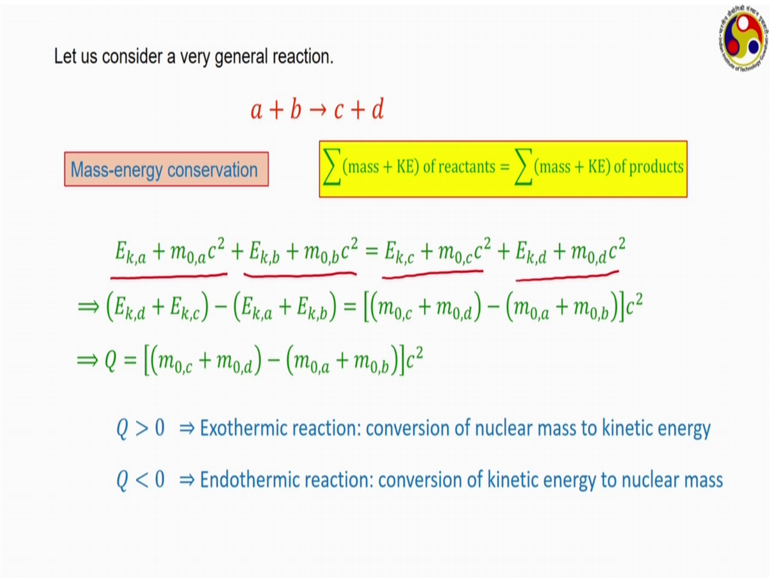

Before that, I finally, would like to summarize we have a very about the energy that we can get from the nuclear reaction, where we have a very general reaction as a plus b producing c plus d where is c and d any 1 of them can be particle and or can be photons, then mass energy conservation says that mass plus kinetic energy reactants should be equal to the same for the products mass plus kinetic energy of the reactants should be equal to the same for the products.

So, accordingly we can write in terms of either energy unit or mass unit, here we are writing in terms of energy unit because we have already done in terms of mass unit; writing in terms of energy unit we have the kinetic energy of a plus the in a rest energy of a, this is the total energy conservation from a, similarly the total energy conservation from b; kinetic energy plus rest energy that should be equal to total energy contribution from c and total energy associated with the d.

If we rearrange the terms, then we can take all the energy terms on the left hand side and rest mass rest energy related terms on the right hand side and then we get Q, which represents the difference between kinetic energy of the products and kinetic energy of the reactants. Here, Q is the total kinetic energy of the products c and d in this case minus total kinetic energy of the reactance a and b here, should be equal to the difference between their rest masses multiplied by c square, that is the net kinetic energy gained by the product is equal to the difference between the kinetic energy of the reactance and kinetic energy of the products multiplied c square.

If Q is a positive quantity, then that represents an exothermic reaction that is the products will be having more energy compared to the reactants. So, this is an exothermic reaction and mass is getting converted to energy. However, if Q is negative that represents the mass of the product is more compared to the reactants, that means, it is an endothermic reaction and energy has been converted to mass. So, the mass energy conservation which we earlier attain from mass unit point of view, this way we can also attend from energy unit point of view.

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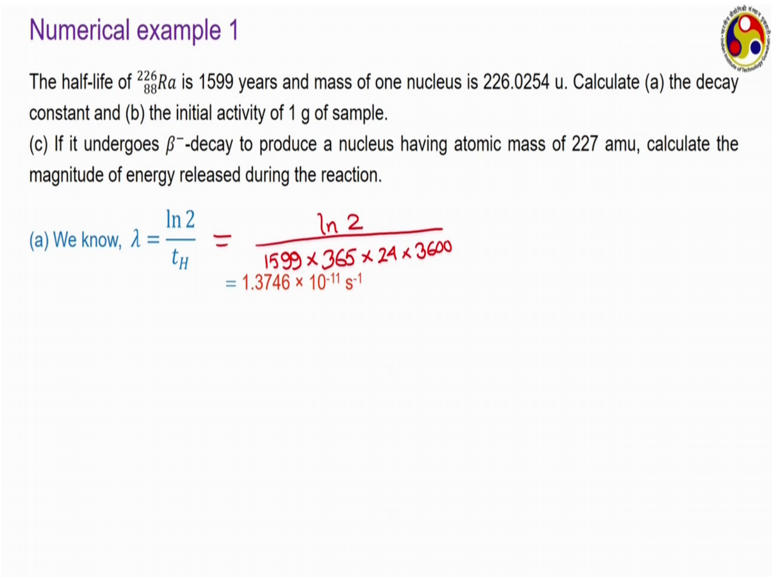

So, let us go to some numerical examples. Our, first example corresponds to the radium isotopes, you just read the problem we have radium 226 isotope which is having a half life of 1599 years and the mass of 1 nucleus is given, you have to calculate the decay constant and the initial activity of 1 gram of the sample, I shall be coming back to the c part afterwards. Now, we know that the decay constant and half life have an inverse relationship and very straight forward 1 lambda is equal to log t by log 2 by t H, you need to very, very careful about the unit of t H. In this case, half life is represented in terms of years, so if you will are going to solve it, then your numerator is straight forward that is log 2, but in the denominator you have t H given in terms of year it is always better to convert that to si unit, that is in second.

Then, what we can write in the numerator? We have 1599 years and in each year we have

365 days, you can neglect the leap year concept and in each day we have 24 hours and in each hour we have 3600 seconds. So, the denominator that is the half life has been converted to seconds and if you put the numbers into calculators, you will be getting this the total value of the decay constant is coming to be something of the order of 10 to the power minus 11 second inverse, the please take a note of this unit of this decay constant, it is just inverse of half lives, so it has to be time inverse. Then, how to calculate the initial activity, I hope you remember the definition of the activity, activity is defined as the rate of decay that is minus dN dt.

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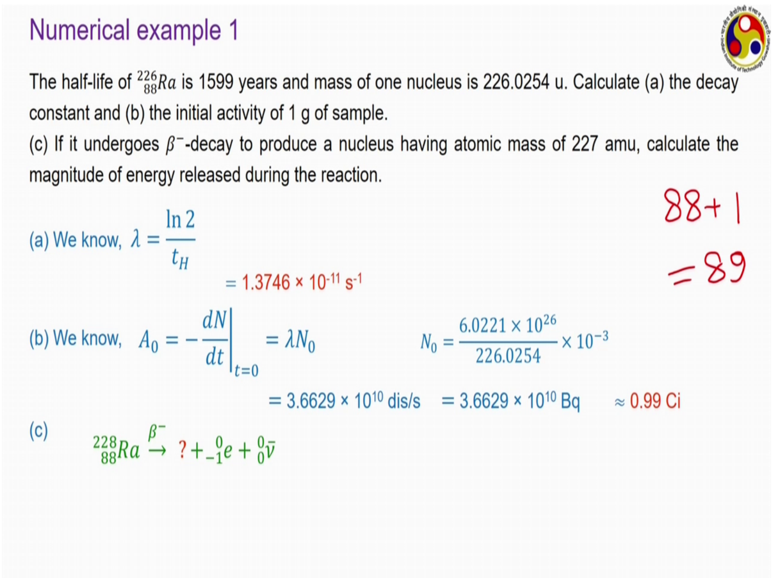

So, initial activity means, at any particular time T equal to 0, the rate of decay that is minus d N d t equal to 0, which can be equate to lambda into N naught, N naught being the number of nucleus, so lambda value you have just calculated. How to get N naught? N naught, we know from that from Avogadro's hypothesis, that 1 kilo mole of any particular isotope will always have 6.0221 into 10 to the power 26 number of molecules inside that, that means, here in this case the molecular weight of radium is given, radium 226 has 226, 0254, that means, 226.0254 kg of radium 226 will comprise of 6.0221 into 10 to the power of 26 number of molecules and from there we are getting the total number of initial isotopes present in the initial sample, multiplying that with lambda we are getting the initial activity and it is coming to be 3.6629 to the power 10 disintegrations per second.

Now, I hope you remember the units of radioactivity which you have discussed earlier in the very first lecture. So, one disintegration per second is equal to 1 Becquerel which is the si unit of radioactivity. So, you can represent this disintegration per second with Becquerel and finally, the classical unit of radioactivity is curie, which is the old unit defined by Marie curie, what is the relation between Becquerel and curie? You remember, 1 curie was equal to 3.7 into 10 to the power 10 Becquerel.

So, dividing this with by 3.7 into 10 to the power 10 we are getting it to approximately 0.99 curie. Does this number 0.99, that is almost 1 curie, does it strike something? Why we are getting 1 curie as a activity of a radium 226? Because, that is how the query was defined; the Q definition of curie was the activity level of radium 226. So, the answer to this problem is expected to be something very close to 1 curie and that is what we are getting.

Now, you are let us go to the part c of this, particular problem. It is mentioned that radium 226 undergoes a beta decay to produce a nucleus is having an atomic mass of 227 amu and we have to calculate the total amount of energy released during the reaction, that means, the concept of mass energy conservation that we have just learned that we have to use here, but before that we have to write the equation.

We know that radium 226 is undergoing beta decay, then what will be the product? Let us try to write the equation first, it is radium that is undergoing beta decay. So, there has to be electron on the right hand side and antineutrino, which we have also learned in the second lecture, but what about the daughter isotope, what to expect to be daughter isotope? So, we must use the other 2 conservations that we have learned, the conservation of charge and the conservation of total number of nucleons.

How many charge we have available on the left hand side? On the left hand side, we have 88 is a total number of charge, on the right hand side we have electron having minus 1 unit of charge, then what about daughter you are going to have? That should be having 88 plus 1 that is equal to 89 as the total number of charge. So, 89 charge if you go to the periodic table that refers to actinium.

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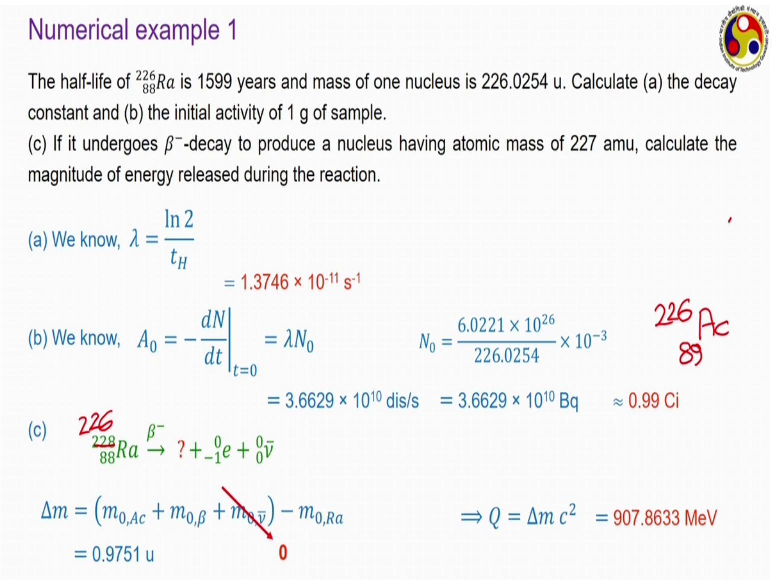

So, the daughter that we are going to get that is going to have 89 a number of charge and from the periodic table that is actinium, which is having 89 number of charge, then total number of nucleons we are having 228 as a total number of nutrions on the left hand side and how many on the right hand side nothing coming from the electron and antineutrino. So, this 1 should also have 228 as the total number of electron, actually 1 mistake that have made while doing this. Here, we are talking about radium 226, so this number should be 226 and here also the actinium product that is 89 Ac 226, that we are referring. So, it is not 22, it should be 26, so we are having actinium 226 as the product, which is having 89 protons and 226 number of nucleons in the product.

So, we have to write the mass energy conservation now. Writing, a let us first calculate the total mass defect here, let us go by the mass (Refer Time: 41:27) we can take either energy point of view or mass point of view we are taking the mass point of view. So, total mass defect here the mass of the product by the mass of the reactants. So, rest mass of actinium is already given as 227 amu and rest mass of beta which is electron, that we already know that is to be 5.486 10 to the power minus 4 amu and neutrino or antineutrino is mass less mass of radium is given. So, accordingly get the total mass defect 3.9751 amu and corresponding amount of energy is delta m into c square that is 907.86 MeV is the total amount of energy, that will be released during this particular beta decay of radium 226, I repeat you please make this correction this is not 228, this is 226 as per the problem, this is typing error while typing the equation.

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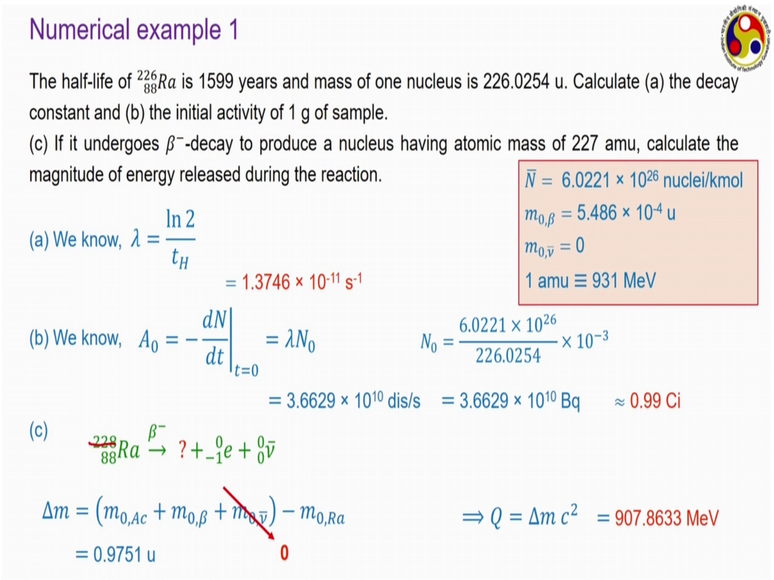

So, certain information that we have used while solving this problem, first is the Avogadro hypothesis that is 1 kilo mole of any particular isotope will be having 6.0221 into 10 to the power 26 number of nuclei, then the mass of electron 5.486 into 10 to the power minus 4 amu that we have used, mass of antineutrino is 0 and 1 amu is equal to or equivalent to 931 MeV, these are the information's that we have used here and while the Avogadro hypothesis probably you already know from school level, this conversion between mass and energy I am sure this 1 you already memorize it by now and I repeat again the mass of the 3 subatomic isotope, the 3 principles of atomic particle that is proton, neutron and electron you please try to remember at least to some of them degree because that will help you only.

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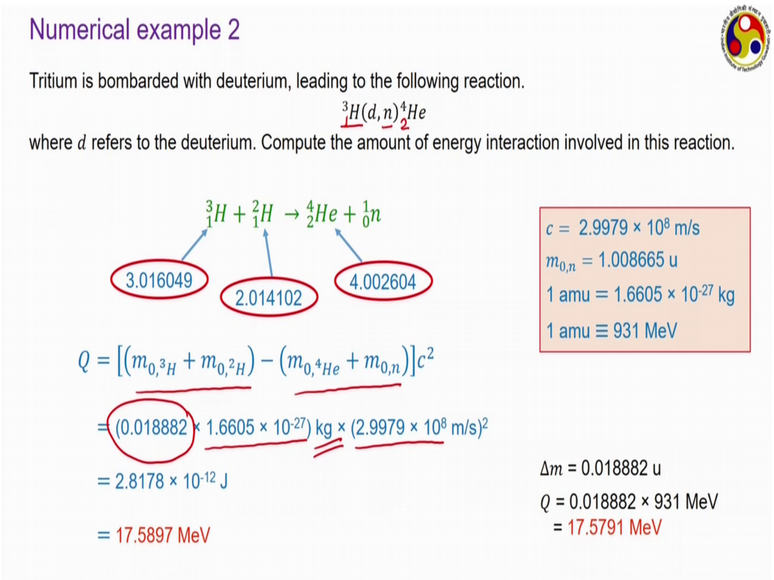

Let us go to a second problem. Here, it is mentioned that tritium is bombarded with deuterium leading to a reaction given as per the shorthand notation that is we have studied earlier; here d refers to the deuterium. Now, we have to calculate the total energy interaction involved in this reaction, first we need to convert this shorthand notation to proper reaction format. What we have as the reactant it is H 3. So, that is tritium or 1 H 3, what we have as the principal product it is helium 4 or 2 He 4, what are the items are there deuterium is the 1 that is reacting with the tritium and on the product we are having this n, which refers to nitrogen. So, accordingly this is a reaction that we are dealing with. So, corresponding mass values have already been provided to you, but mass of neutron I am not providing here because as I mentioned that is something that you are expected to know.

Now, we are writing from the energy point of view that we have studied just 2 slides back. So, the mass of the reactance and mass of the products the balance we are taking here, this is the mass of the reactants, rest mass of tritium and deuterium and this is the summation of rest mass of helium and neutron, the difference here we are multiplying with c square and correspondingly we are getting the answer about 1 important point here, if you calculate the all this masses there they are generally available in terms of amu, you are going to get this particular number, but c is something given in meter per second.

So, this value in amu you must multiply the corresponding conversion factor of 1.6605 into 10 to the power of minus 27 to get into the si unit of kg and then only you will be able to multiply with the velocity of light. So, finally, this much of joule is the energy that you are getting as the output, we can of course, always convert this to MeV using the corresponding conversion factor.

In terms of, here we are gone in terms of rest mass, but we could have also gone from the point of view of the mass defect, that we have used for the previous problem. Like in this case also if you calculate the mass defect, the mass defect is going to be 0.018882 u and you can directly multiply this with 931 MeV. So, you will get this answer which is a less than point 1 percent deviation from the original one.

So, either a approach you can choose and you are invariably expecting to get the same result, these are the information's that you have used while solving the velocity of light in vacuum, the mass of neutron and the conversion between amu and kg, that is a new thing that is coming that is a 1 amu is equal to 1.6605 into 10 to the power minus 27 kg and finally, the EQ, the mass energy equivalence has 1 MeV equivalent to 931 MeV.

Now, while solving this numerical problem we have assumed both the reactant that is deuterium tritium to be stationary, while it is mentioned in the problem that tritium is being bombarded by deuterium, but we are not considering the kinetic energy of the reactants.

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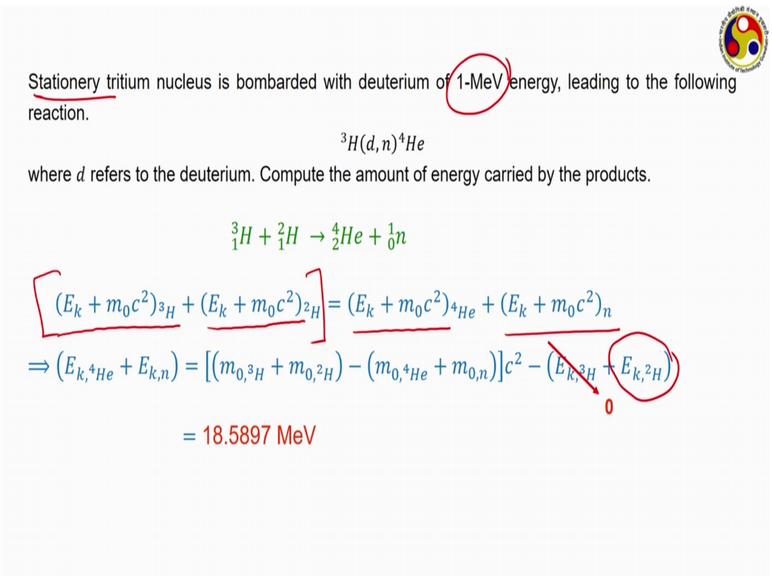

So, let us modify the problems slightly, here the tritium is bombarded with deuterium of 1 MeV energy. So, we have to solve the same problem, but here we have kinetic energy of deuterium also involved in the problem. So, same reactions we have and we are writing the mass energy equivalence now, the equation that we have seen earlier this is the kinetic energy plus the energy corresponding to rest mass that is rest energy for tritium and this is the kinetic energy plus rest energy that is total energy for deuterium. So, this 2 together responds to the total energy content of the reactant and coming to the right hand side, this is the total energy of helium and these are total energy for neutron.

So, we change their sides; that is we keep only the kinetic energy of the products on 1 side and keeping everything else on the other side. Here, it is mentioned that tritium is stationary; look at this particular term tritium is stationery it has mentioned. So, it is kinetic energy is zero, but kinetic energy of deuterium that is given to be 1 MeV. So, we can put the numbers rest mass all are given it is coming to be 18.5897 MeV.

If you compare with the previous problem, where there was no kinetic energy associated with the reactants you will find this is just 1 MeV larger and that is only logical because the rest energy part is remaining the same, but only the 1 MeV additional energy that is coming with the reactance that is being added to the products and hence this is going to be only 1 MeV more than the previous case, but this total amount of energy will be carried by both helium and neutron and unless you solve the momentum conservation we cannot calculate their individual shares.

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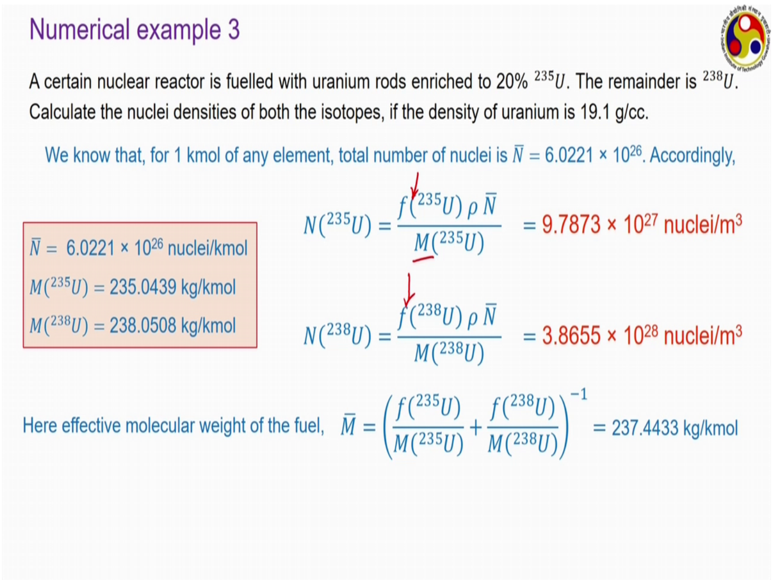

We have a 3rd problem; let us now use the concept of neutron enrichment. Read the problem carefully, it is mentioned that a certain nuclear reactor is filled with uranium rods enrich to 20 percent u 235 and the remainder is u 238. So, we have to calculate the nuclear density of both the isotopes, if the total density of uranium is available to you. Neutron or uranium enriched 20 percent by u 235 refers to 20 percent of the total uranium content is u 235 and reminder is u 238, 1 information.

I have mentioned in the very first lecture or in the very first module, when we introduce the concept of isotopes that natural uranium primary come primarily comprises of uranium 238 and the contribution from uranium 235 is only about 0.7 percent, but by artificial means we can increase the amount of u 235 in the uranium sample and that is precisely what is called enrichment. So, in this problem this, the nuclear reactor is using uranium with 20 percent u 235 enrichment (Refer Time: 50:02).

So, we know that 1 kilo mole of any element will have 6.0221 into 10 into 26 number of nuclei, accordingly number of uranium 235 nuclei should be equal to here this f refers to the fraction of the percentage of uranium 235 in the (Refer Time: 50:22) sample into the density into the Avogadro number divided by the molecular weight M is this, M is the molecular weight, we need to the know the molecular weight for both u 235 and 238 and if you put the numbers this is what we are getting.

Similarly, we can also calculate uranium 238 number nuclei for uranium 238, in this case this f is 20 percent or 0.2 where is for u 238, this is 0.8. So, if a, we at all need to calculate the effective molecular weight of the fuel that this particular nuclear reactor is dealing with, then that also can be calculated as f by M for u 235 plus f by M for u 238 inverse of that and that is coming to our 237.4433 kg per kilo mole. Here, we are again using the Avogadro number and the molecular weight for u 235 or 238, this information of course, you have to take from some data book or will be providing in the problem.

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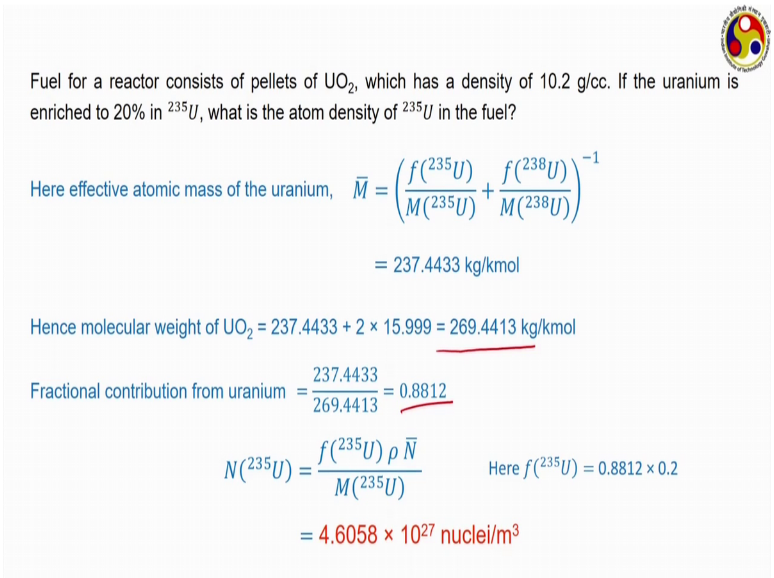

But, the same problem now, we are modifying slightly. It is said that fuel for a reactor consists of pallets of uranium oxide, generally in nuclear reactor we shall be discussing later on, fuel is provided either in the form of rectangular pallets that is blocks having a rectangular cross section or has cylindrical rods. So, and invariably that is generally some kind of compounds of uranium, some oxides or sulfide or hexafluoride's of uranium.

So, in this case you have uranium oxide and it is density is given and the uranium part for that uranium oxide is enriched 20 percent in u 235. So, again you have to calculate the nuclear density of uranium 235 in the fuel. Effective molecular weight you can calculate as was shown in the previous slide for the uranium part. Now, uranium is having a molecular weight of 237 point something, then what about uranium oxide they are the molecular weight of the oxygen, 2 oxygen atom also needs to be added and accordingly this is coming to be the molecular weight of uranium oxide. So, out of this

239, 237 is coming from uranium, so 0.8812 contribution is coming from the uranium in this total molecular weight.

So, using this particular fraction we can easily calculate the total number of nucleus present or nuclear density of uranium 235 present in the sample using the same formula, but you have to remember that this f is here, not just 20 percent; rather it will be 0.2 into this particular fraction, yes. So, accordingly we can calculate total number of uranium 235 nuclei present per unit volume of the fuel and we can also do that same for uranium 238 if required at all. So, this is the corresponding answer after putting the values.

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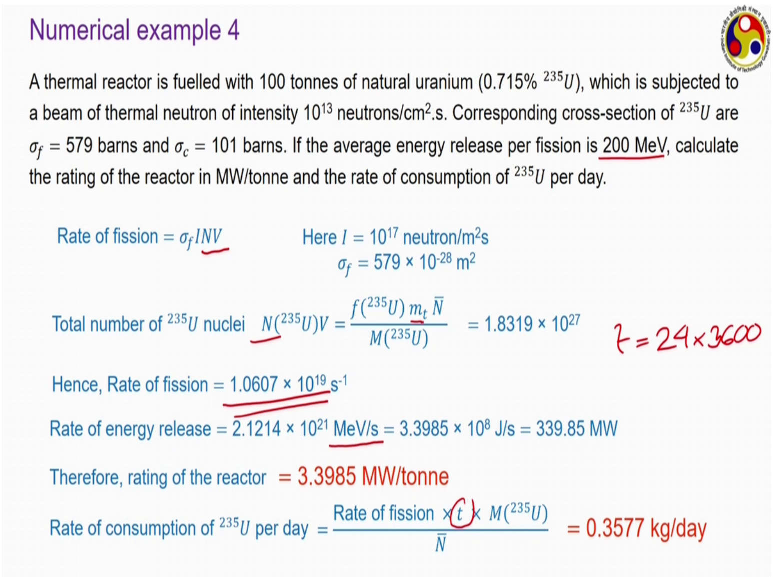

Let us go to the last problem, problem number 4 which is a much more involved problem, which requires the concept of cross section as well. Thermal reactor is filled with 100 tons of natural uranium; natural uranium I mentioned just a while back contains just 0.715 percent uranium 235 and rest you can take as u 238.

It is subjected to a beam of thermal neutron of intensity 10 to the power 13 neutrons per centimeter square second, responding capture and fission cross sections are given, this is the fission cross section, this is capture cross section has no information are provided we can consider other that is scattering cross sections to be equal to 0. If the average energy release per fission is 200 MeV then you have to calculate the rating of the reactor in mega watt per ton and the rate of consumption of uranium 235 per day.

So, here you are talking about fission interaction between the neutron beam that is coming and the uranium nucleus. So, the rate of fission can be sigma f into I into N into V, where I is the intensity of this neutron beam and N into V is the total number of uranium nucleus present in the target. So, here I is given is 10 to the power 7 neutrons per meter square second, of course, the given information the problem is in terms of centimeter square second, but I would always suggest please convert everything to si units (Refer Time: 54:48) much easier to solve.

Similarly, the cross section is given in bonds and we know that 1 bond is 10 to the power minus 28 meter square. So, accordingly sigma f is 579 into 10 to the power minus 28 meter square, but we have to calculate this N into V, which is the total number of uranium 235 nucleus that is present in this particular reactor or where we have 100 tons of total natural uranium fuel.

Now, total number of uranium nuclei we can calculate that is N into V in the previous problem number 3 we are basically calculating this N which is density; nuclear density, but here we are bothered about total number of nuclei that is present. So, N into V that we are to compute if we multiply both side of the previous equation with V then of course, the density is converted to this mass, that is the total mass of the fuel that is present, rest mass is same here again the fraction is just 0.715 percent, Avogadro's number you know and molecular weight of uranium 235 is known, accordingly we are getting this is a total number of uranium 235 nuclei present in the this reactor, where you have 100 tons of fuel. So, then rate of fission just multiplying with this N into V with I and sigma f we are getting this as the rate of fission.

Please, note the unit is second inverse that is this many number of fissions 1.0607 into 10 to the 19 number of sessions are taking place per unit second or per second and in 1 second how much energy is released, that is 200 MeV given in the problems. So, if you multiply this with 200 MeV, then this is the total amount of energy that is getting released in mega electron volt per second and MeV per second can be converted to joule per second or watt using the corresponding conversion factors. So, it is coming to be 339.85 mega watt.

This is the total amount of energy that is being produced or power I should say because it is in megawatt is the power that we are getting from this particular reactor, but objective is to find the rating of the reactor in megawatt per ton, this 339.85 mega watt of energy we are getting from 100 tons of fuel. So, the rating of the reactor is 339.85 mega watt divided by 100 tons or 3.3985 mega watt per ton, this is the reactor rating and if we have to calculate the rate of consumption.

Now, we know that in 1 second I just repeat in 1 second, this many number of fission reaction is happening that is in every second this many number of uranium 235 nucleus is getting consumed, then what will be the total rate of consumption that will be rate of fission, multiplied by the time over which you would like to do this calculation into the molecular weight of uranium 235 divided by Avogadro's number. In this case time refers to 1 day, that is here this time T is equal to you are talking about 24 hours into 3600 seconds.

So, once we put the numbers here it is coming to the 0.3577 kg per day, that means, just spending 0.3577 kg of uranium 235 in 1 day, we are able to produce 339.8 that is about 340 megawatt of energy throughout the day, that is a huge fraction, if you do a corresponding calculation for a say conventional coal based thermal power station the fuel requirement will be in the order of a few tons, a few 100 tons.

But of course, one thing we must consider here, here we are calculating only the rate of consumption of uranium 235, but that is only 0.715 percent of the total fuels. So, if you have to calculate the total rate of consumption of uranium, bring this. That should be 0.3577 divided by 0.00715 this should be the total rate of consumption of uranium to produce this energy level.

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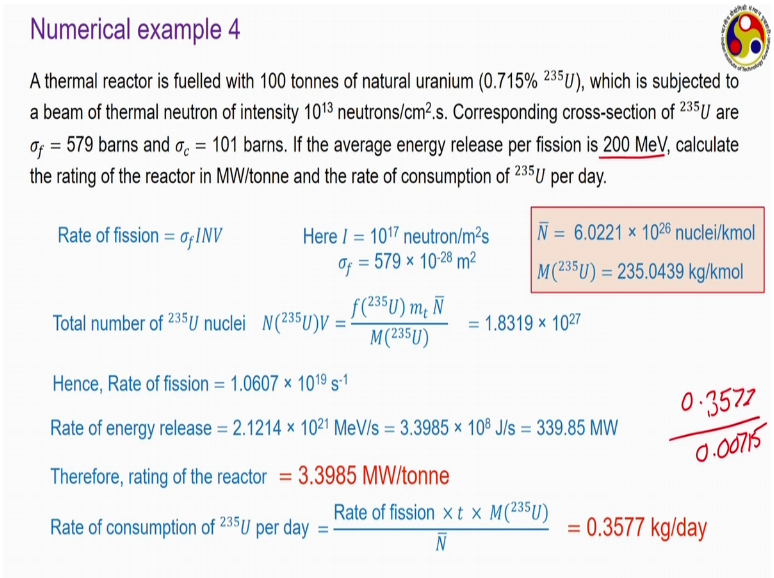

The information that we have used here primarily are the Avogadro's hypothesis and the molecular weight of uranium 235, while solving this problem. So, that is a it for this particular module just following the approach that is described in this for numerical problems you can solve innumerable number of numerical exercises, which I would encourage you to do for any kind of text books and you will be finding quite a few similar numerical problems in the assignment also, I hope you will be enjoying the assignments and before I close off I would like to mention the key points.

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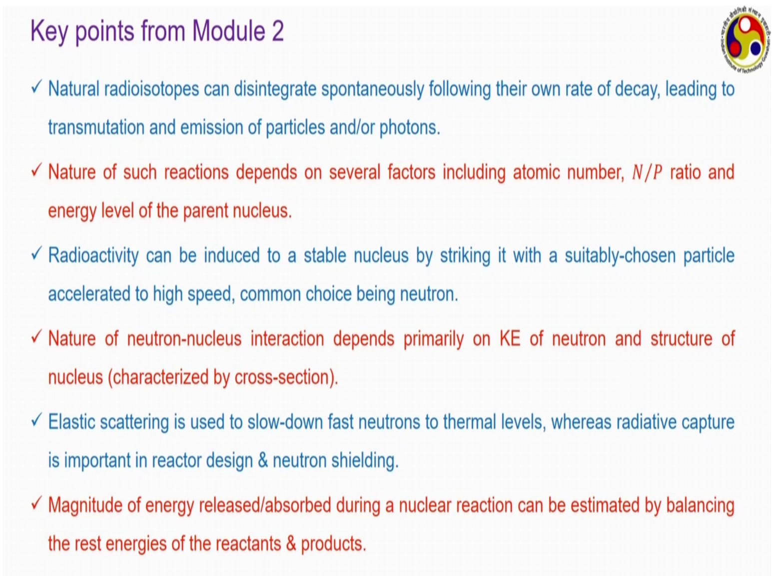

If we summarize the key points from this module, first we have studied about the natural radioactivity phenomenon where we know that natural radio isotopes can disintegrate spontaneously following their own rate of decay and half life and half life being a characteristic of every isotope. So, the rate of decay is you need to any kind of isotope and such kind of decay leads to transportation that is formation of some other isotopes and also emission of particles and or photons, but the nature of such kind of reactions exactly which kind of reaction will take place that depends upon several factors, particularly the total the atomic number that is a number of protons present, the ratio of neutron to proton and also the energy level of the parent nucleus.

This is about the natural radio activity for, but from power generation point of view, we are more bothered about the artificial radioactivity which can be induced to a stable nucleus by striking it with a suitably chosen particle accelerator to high speed. The particle can be alpha particle or proton or something else, but neutron is the most common choice and the entire discussion that we had here and also shall be having in the following modules, will be considering neutron as the soul particle used for inducing this radio activity, but the nature of neutron nucleus interaction also depends on primarily on 2 factors, 1 is the kinetic energy of the neutron and other is the structure of the nucleus which we characterize in terms of the cross sections.

There are different types of interactions possible and each of them has their own use, like the elastic and inelastic scatterings are primarily used to slow down a neutron from the fast neutron level to the thermal neutron level, which be primarily used in the thermal reactors, whereas the radioactive capture is very important in reactor design and neutron shielding or forming the core of a reactor and finally, the magnitude of energy released or observed in the nuclear reaction can be estimated by balancing the rest energy of the reactants and the products and also the if we want to know exactly how much amount of energy is being carried by each of the products, then we need to consider the conservation of linear momentum as well.

So, that is the end of module 2, in the third model we shall be taking it forward from this particular point that is the concept of cross sections we have discussed here. In the third module our topic is fission, we shall be focusing on the fission cross section, we shall be seeing how cross section varies with the energies or energy level of the neutron and how we can utilize that set of phenomenon of moderation, how the elastic scattering can be used in a moderator, this things will be discussed in the 3rd model.

So, I am signing off here. So, have a good day.