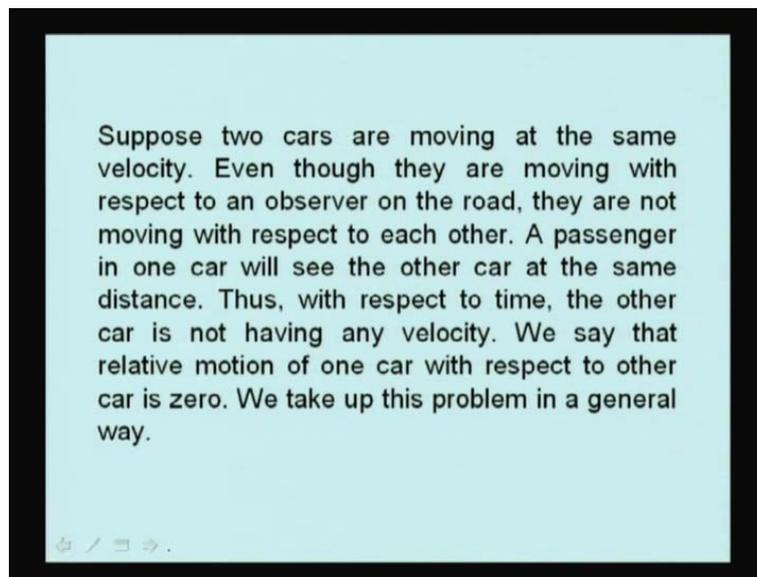


Engineering Mechanics
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Kinematics
Module 10 Lecture 25
Relative motion

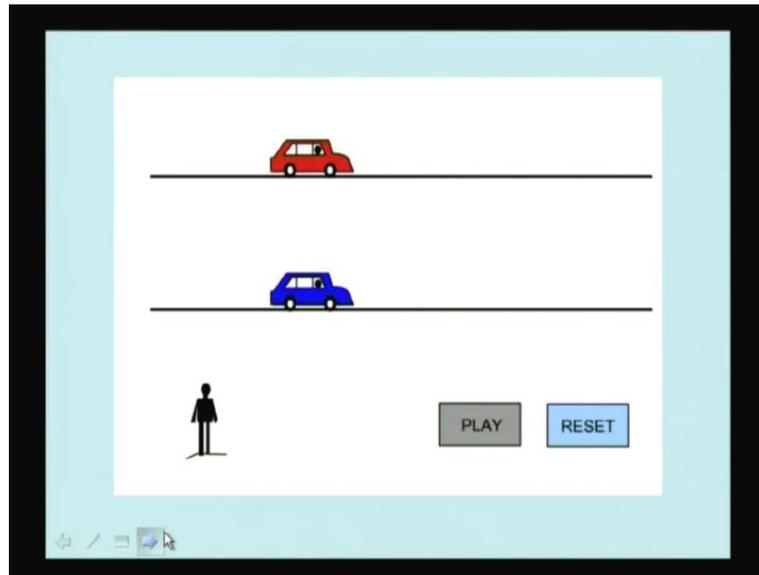
Today, we are going to discuss about the relative motion. Motion of one particle with respect to the other particle. That can be described.

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If you have two cars which are moving at the same velocity, even though they are moving with respect to an observer on the road, they are not moving with respect to each other. A passenger in one car will see the other car at the same distance. Thus, with respect to time, the other car is not having any velocity with respect to passenger in the car. We say that the relative motion of one car with respect to the other car is zero. We will discuss this problem in a more general way, when the cars may not move only in the straight line but they may be undergoing curvilinear motion.

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This is one animation showing the motion of two cars; red car and blue car. The driver in the red car always sees the blue car. Therefore, with respect to the red car, blue car is not having any relative motion.

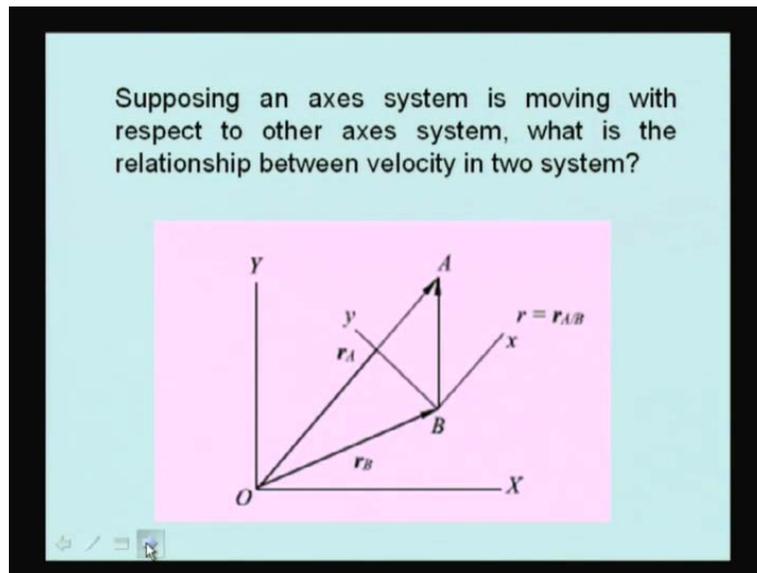
However, drivers will come to know that they are moving at a high speed because they will see the other surrounding objects like, tree and all this. Outside the person is seeing that both the cars are moving with the same velocity. We are familiar with the concept of relative motion. In train also, sometimes we get the illusion that the train is moving because the other adjacent car starts moving, but when we see other objects like poles etc., we get the idea that it is our train which is not moving. In a way, we attach one axis system in our mind.

When we observe the objects moving, we have some reference frame; in that reference frame, we study its motion. Therefore, this lecture can be considered about expressing the displacement, velocity and acceleration with respect to particular coordinate system. Those coordinate systems, itself maybe translating with respect to another coordinate system or it may be rotating. Imagine that a boy is revolving a rod in a horizontal plane in the circular path. Now, if a particle is situated at the middle of the rod and another particle is at the end of the rod, then the particle which is at the middle of the rod will always see the particle at the end of the rod. Therefore, what happens, it may be observed that the particle is not having any relative motion with respect

to this; however, from outside, we observe that both the particles are having different velocities. Particle, which is at the end of the rod moves with a higher velocity compared to the particle which is at the middle. Therefore, there is a relative motion between the two particles.

However, we have taken its rotating coordinate system and that is why the middle particle does not observe the velocity of the particle at the end.

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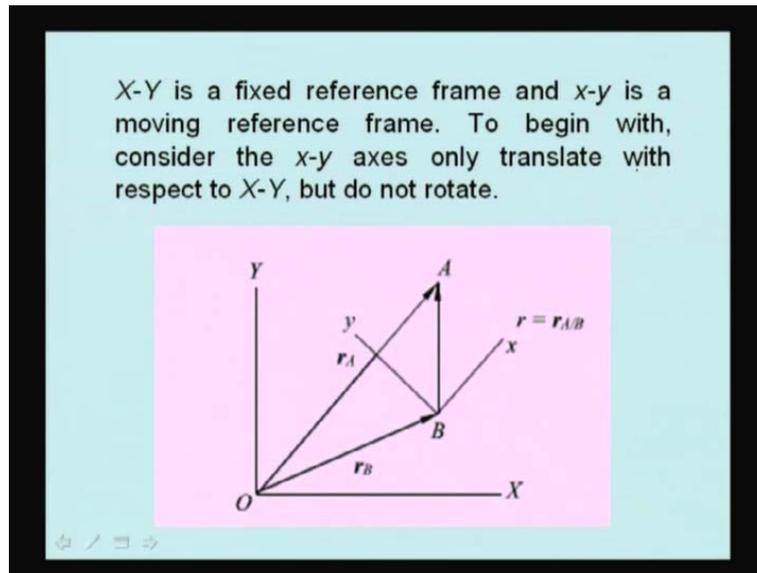


Therefore, it all depends on what type of axes system we have chosen. Here, in this figure, O X and Y are the axes system shown. One particular particle, a point is indicated by B; r_B is the position vector. This is A; another point A. r_A is the position vector. If we fix a coordinate system, you can consider that B may be one particle and A, the other particle. Both may have different velocities.

If we fix up another coordinate system attached to the particle B which is indicated by x and y, in the most general form, it can have this thing. Supposing an axes system is moving with respect to the other axes system; that means x-y is moving with respect to another axes system that is OX and Oy system, X-Y system, then what is the relationship between velocities in two systems?

The difference between these two position vectors r_A and r_B is indicated by another vector that is called r_{AB} , r_B plus r_{AB} gives you r_A ; or, we can write r is equal to r_A with respect to B. This is the position vector of A with respect to B, which is indicated by r .

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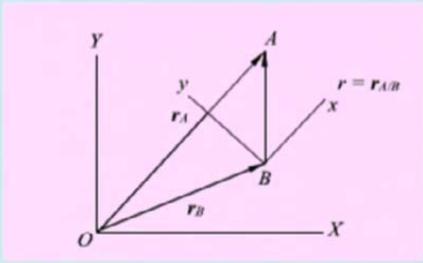
We have already mentioned that X-Y is a fixed reference frame and x-y is a moving reference frame. To begin with, we consider that x-y axes only translate with respect to X-Y, but they do not rotate. In previous examples, one car is translating with respect to the other car, but it is not rotating because they are all moving in a straight line path.

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If A is any particle. The position vector of A as measured relative to the frame $x-y$ is

$$\mathbf{r}_{A/B} = x\hat{i} + y\hat{j}$$

where subscript A/B means "A relative to B" or "A with respect to B".

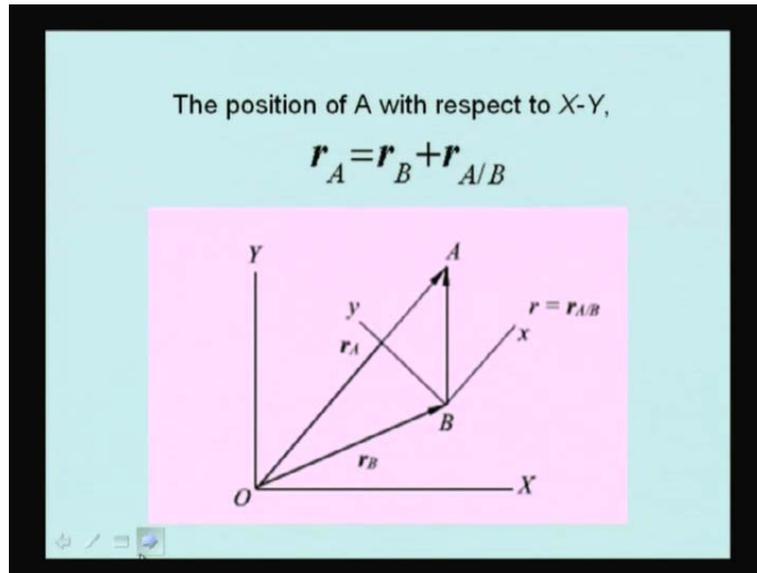


The diagram shows a coordinate system with origin O and axes X and Y . Two particles, A and B , are shown. Particle B is at position vector \mathbf{r}_B from O . Particle A is at position vector \mathbf{r}_A from O . The position vector of A relative to B is $\mathbf{r} = \mathbf{r}_{A/B}$, which is the vector from B to A . The components of $\mathbf{r}_{A/B}$ are x and y , shown as projections onto the X and Y axes respectively.

This case becomes quite simple; if A is any particle, the position vector of A as measured relative to the frame $x-y$ is $r_{A/B}$ is equal to $x\hat{i} + y\hat{j}$, where subscript A slash B means A relative to B or A with respect to B . X Y are the components of the position vector and they have been expressed here in terms of the unit vectors in the frame X - Y .

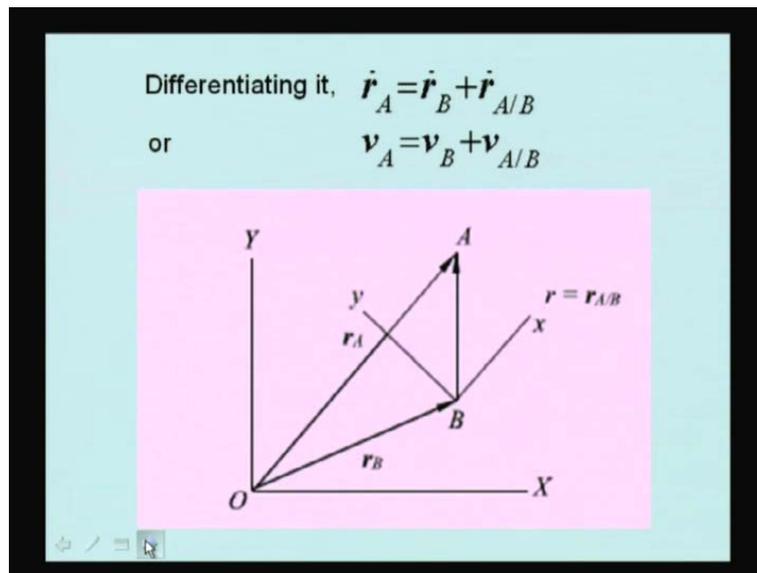
We may have adapted unit vectors in the fixed reference frame, that means we could have such that \hat{i} and \hat{j} . However, it is known that if the axes are parallel to each other, if x is parallel to y , in that case \hat{i} will be equal to \hat{i} . In this case, we have expressed it $x\hat{i} + y\hat{j}$.

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The position vector of A with respect to X-Y is given by \mathbf{r}_A is equal to \mathbf{r}_B plus $\mathbf{r}_{A/B}$, A with respect to B. With this, we can differentiate this position vector with respect to time because we know, we have some idea about vector calculus; vectors can be differentiated.

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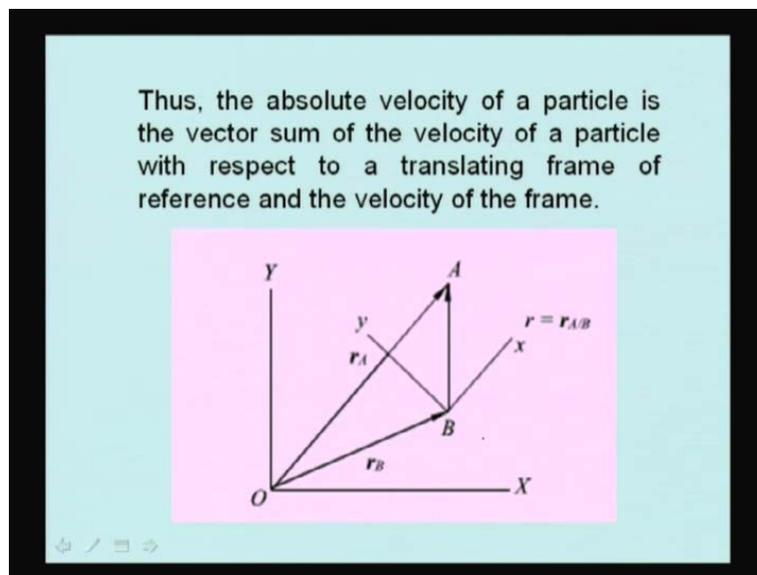


Differentiating it with respect to time and indicating the differentiation with respect to time by dot, we get $\dot{\mathbf{r}}_A$ equal to $\dot{\mathbf{r}}_B$ plus $\dot{\mathbf{r}}_{A/B}$ or $\dot{\mathbf{r}}_A$ is nothing but the velocity of A change

in the position vector of A because r_A is the position vector with respect to OXY frame, X Y frame. Therefore, $r \dot{A}$ gives the absolute velocity of point A; that is indicated by v_A .

Similarly, r_B is the position vector of B in X-Y reference frame. Therefore, $r \dot{B}$ indicates the velocity of point B that is absolute velocity B plus v_A with respect to B because $r_{A/B}$ is the position vector of A with respect to B. Therefore, $r \dot{AB}$ will be velocity of A with respect to B. Therefore, we get a simple relation that v_A is equal to v_B plus $v_{A/B}$; that means velocity of particle A is equal to velocity of particle B plus velocity of particle A with respect to B.

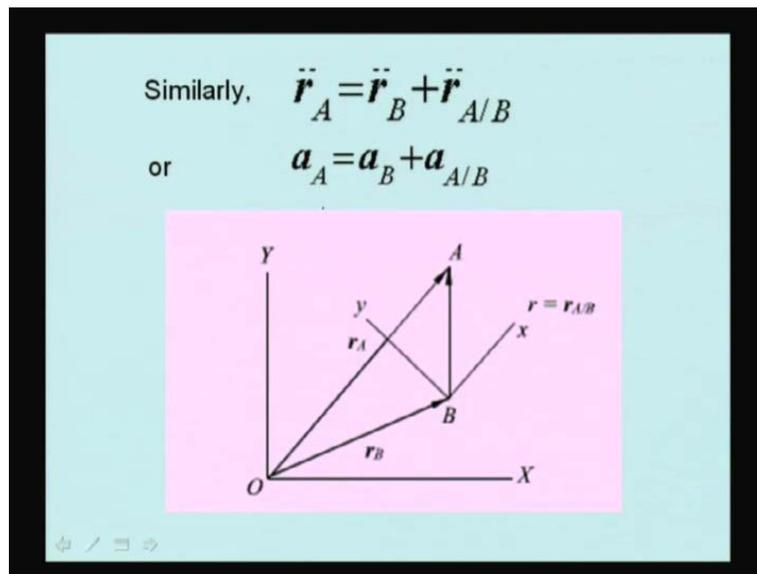
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Therefore, absolute velocity of a particle is the vector sum of the velocity of a particle with respect to a translating frame of reference and the velocity of the frame itself. If we say velocity of the frame because frame is attached at B, v_B can be called as the velocity of the frame and v_{AB} can be called velocity of the particle A with respect to this translating frame which is attached at particle B.

We need not mention again about that there is a particle B. Instead, we mention that there is a particle A, whose velocity is measured with respect to a reference frame which is moving. Reference frame origin is indicated by B; B may be physical particle or may not be particle.

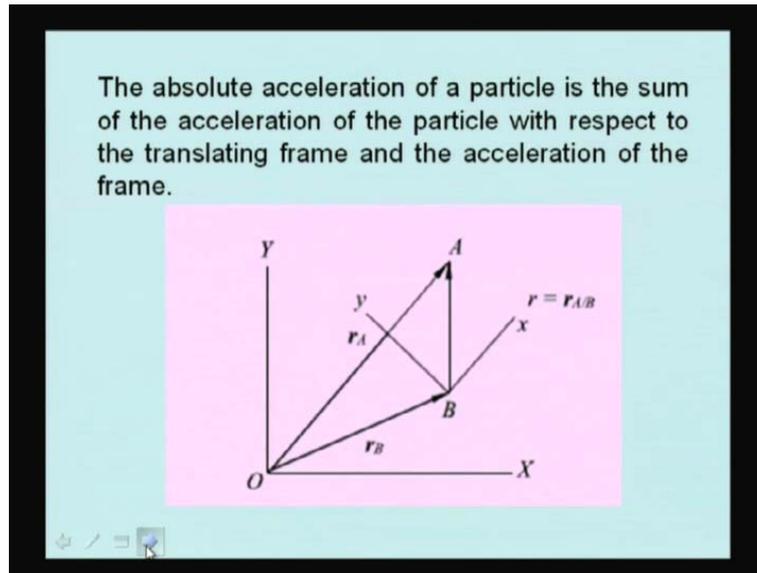
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In the same way, we can find out accelerations. We take the double derivative, which we indicate by double dot or the vectors. $\ddot{\mathbf{r}}_A$ is equal to $\ddot{\mathbf{r}}_B$ plus $\ddot{\mathbf{r}}_{A/B}$ or \mathbf{a}_A is equal to \mathbf{a}_B plus $\mathbf{a}_{A/B}$.

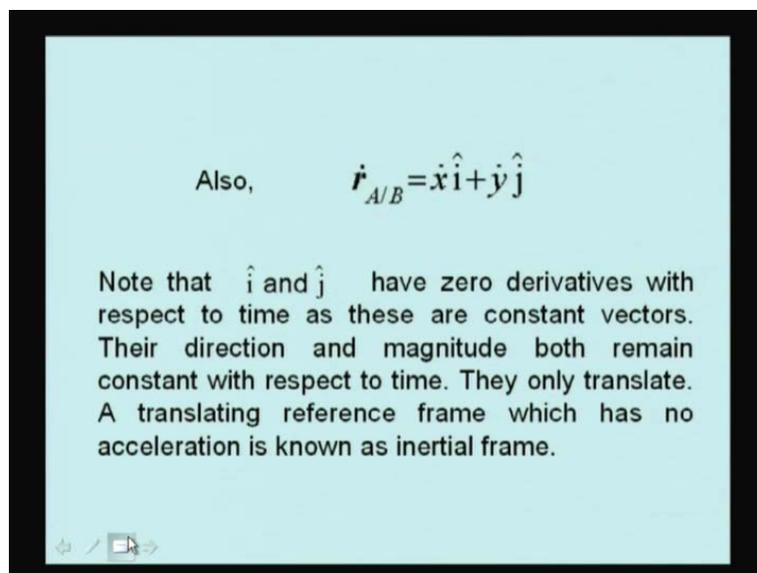
Here, in this case, \mathbf{a}_A is the absolute velocity absolute acceleration of point A, \mathbf{a}_B is the absolute acceleration of point B and $\mathbf{a}_{A/B}$ is the acceleration of point A with respect to point B in the reference frame which is moving.

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Thus, the absolute acceleration of a particle is the sum of the acceleration of the particle with respect to the translating frame and the acceleration of the frame itself. Because we can see that a_B is the acceleration of the frame; that means, a_B is the acceleration of the origin of the moving reference frame.

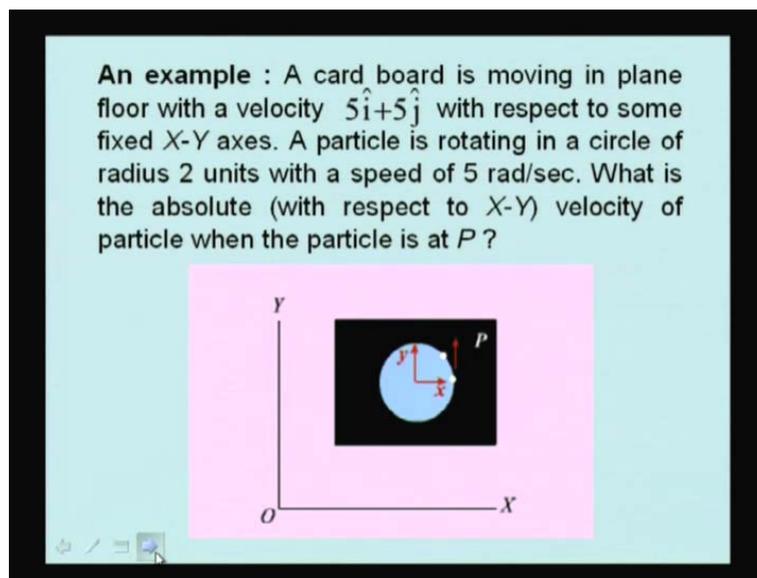
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Also, in the component form, $\dot{r}_{A/B}$ which is the velocity of A with respect to moving reference frame; that can be written as $\dot{x} \hat{i} + \dot{y} \hat{j}$. Note that \hat{i} and \hat{j} have 0 derivatives with respect to time, as these are constant vectors. In translating motion, \hat{i} and \hat{j} does not change with time. Their direction and magnitude both remain constant with respect to time.

A translating reference frame which has no acceleration is known as inertial frame. If a translating reference frame is having acceleration, it is known as non-inertial frame. A rotating reference frame cannot be an inertial frame. An inertial frame is that reference frame which is translating with respect to the other reference frame with constant velocity.

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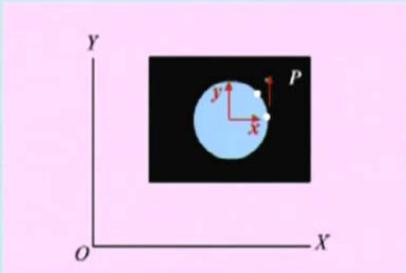
We present one example of using these relations. In this figure X-Y system is shown; that is fixed reference frame. In this frame, a cardboard is moving in this plane floor with a velocity $5 \hat{i}$ plus $5 \hat{j}$ with respect to fixed X-Y axes.

In this cardboard which is black cardboard, a particle is rotating in a circle of radius 2 units with an angular speed of 5 radian per second. Question is what is the absolute velocity of particle with respect to X-Y frame, when the particle is at point P? This is the velocity of the particle P and that has been shown here.

We can attach an axes system $x-y$ which is translating with respect to $X-Y$, this moving axis system has been indicated by red lines.

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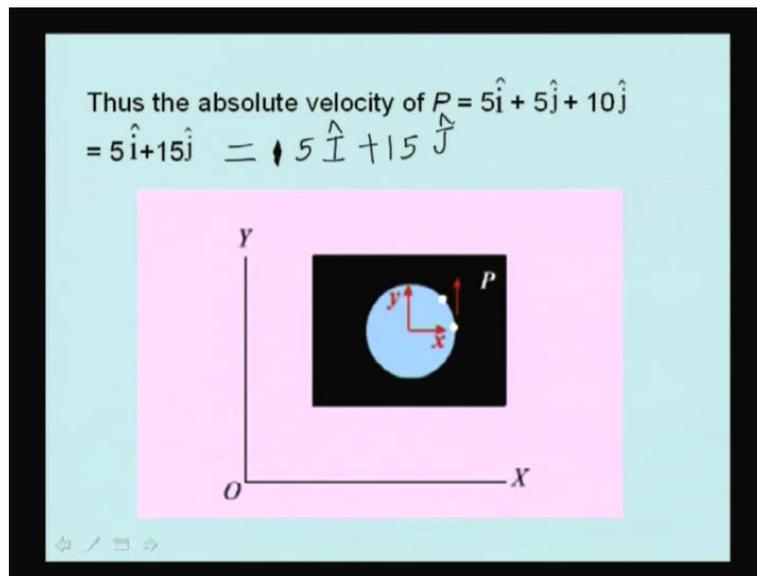
Solution :
Let us fix a $x-y$ axis system at the center of the circle. With respect to $x-y$ system velocity of $P = \omega \times r = 2(5)\hat{j} = 10\hat{j}$



Fixing the $x-y$ axis system at the center of circle with respect to $X-Y$ system, velocity of point P is given by $\omega \times r$; that means ω is mentioned, that is r is 2 units and ω is 5 radian per second. This is $2 \times 2 \times 5\hat{j}$; that means it comes out to be $10\hat{j}$ unit, $10\hat{j}$.

We have to notice that this small j is the unit vector in the moving reference frame. This small j is in the moving reference frame. We could have said capital J also; of course in this particular case capital J will be same as small j . One has to always pay attention that in which system is he expressing the unit vector. So, with respect to $X-Y$ system velocity of point B has been specified as $10\hat{j}$.

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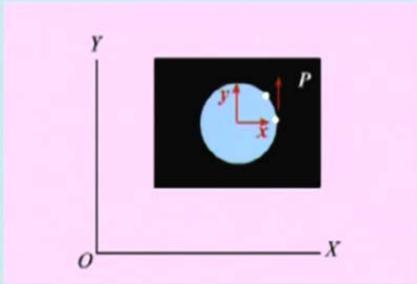
Thus, the absolute velocity of P is equal to the velocity of the reference frame itself plus the velocity of the particle in that reference frame. It has been already mentioned that the cardboard is moving in a plane floor with a velocity $5\hat{i}$ plus $5\hat{j}$; that means the reference frame which is attached with the cardboard is having the velocity $5\hat{i}$ plus $5\hat{j}$. Therefore, absolute velocity of P is $5\hat{i}$ plus $5\hat{j}$ plus $10\hat{j}$ that is equal to $5\hat{i}$ plus $15\hat{j}$.

One has to notice that although I am talking about the absolute velocity of particle B, however the answer has been expressed in terms of the unit vector in the moving reference frame. If one can write final answer in this manner also which is also very correct. $5\hat{I}$ plus $15\hat{J}$, where I and J may indicate the unit vectors in the absolute reference frame.

However, it is not so important to always express the absolute velocity in the absolute coordinate system. Any coordinate system can be adapted. Only thing that one has to understand is that in which coordinate system the components have been expressed.

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With respect to x - y system, acceleration of the particle $= -\omega^2 r = -25 \times 2 = -50$ units (towards centre). The acceleration of x - y with respect to X - Y system is zero. Hence the absolute acceleration is $-50\hat{i}$.

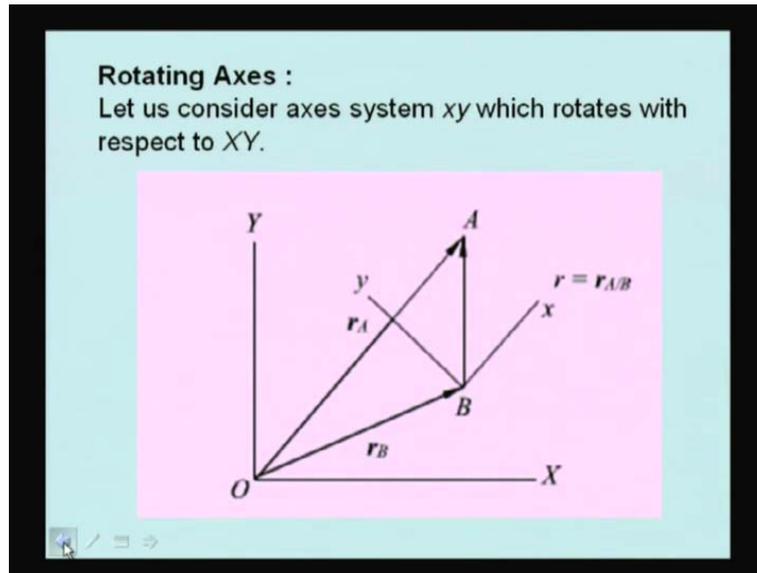


The diagram illustrates a particle P moving in a circular path within a rotating coordinate system (x, y) . The particle's acceleration is directed towards the center of the circle. The fixed coordinate system (X, Y) is also shown, with the origin O .

Now, we discuss about the acceleration of the particle with respect to x - y system; that means with respect to moving coordinate system. Acceleration of the particle is equal to minus omega square r . We have used minus term. It is directed at the momentum and it is directed towards the negative x direction. Therefore, this is equal to minus 25 into 2 omega is equal to 5 radian per second, r is equal to 2 units; this becomes minus 50 units towards center.

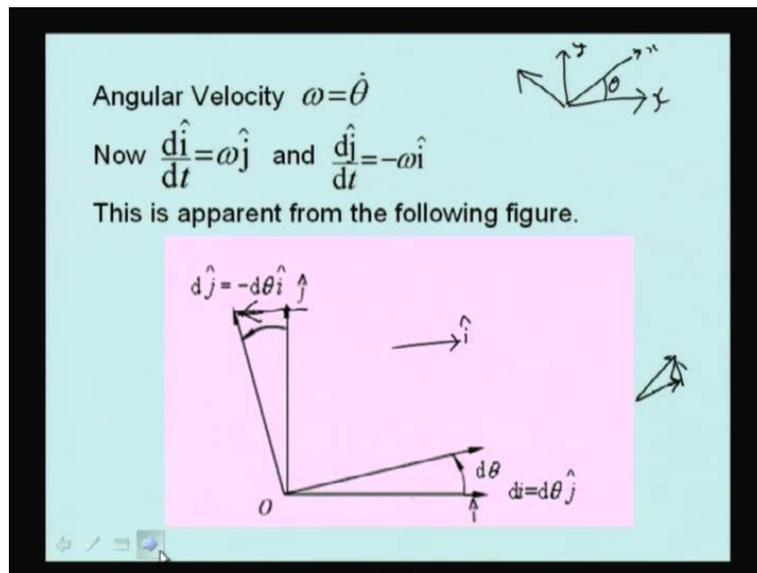
The acceleration of x - y that means 50 units towards center that is why minus 50 sign. The acceleration of x - y with respect to X - Y system is 0 because the cardboard is moving with a constant velocity. Hence, the absolute acceleration is minus **5i** itself.

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We discuss the rotating axes system. Let us consider axes system xy which rotates with respect to XY system. Again the same figure; here also, the same relation is valid that r_A is equal to r_B plus r_{AB} . That relation remains same whether this coordinate system is moving, translating or rotating.

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We also define that the angular velocity of the rotating axis is ω , where ω is equal to $\dot{\theta}$; that means $d\theta/dt$. We have the axis $x; y$ rotates by an amount θ . Because it is rotating, this θ can be differentiated with respect to time.

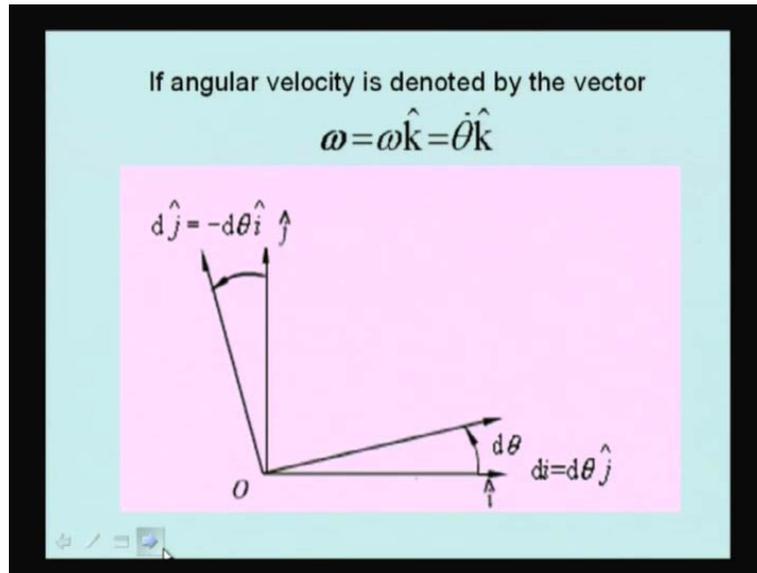
We can easily see that if the unit vectors i and j are expressed in the moving coordinate system; that means rotating coordinate system, then di/dt will be ω times j and dj/dt is equal to minus ω times i .

In the translating reference frame, di/dt was 0; however, in the rotating reference frame, di/dt will not be 0 because the vector is although it is maintaining the same magnitude, it is rotating; that means it is changing the direction. This is apparent from this figure.

Here, the unit vector i has rotated by an amount $d\theta$. Therefore, there is a change in the position of the vector. If this is the vector, after $d\theta$ time this vector before their difference is indicated by another vector and this magnitude will be equal to $d\theta$. Because these are this one. It will be in the j direction because in the unit $d\theta$ tends to 0, di approaches the ji direction. Therefore, if we differentiate di with time, we get $d\theta/dt$ that is ω times j .

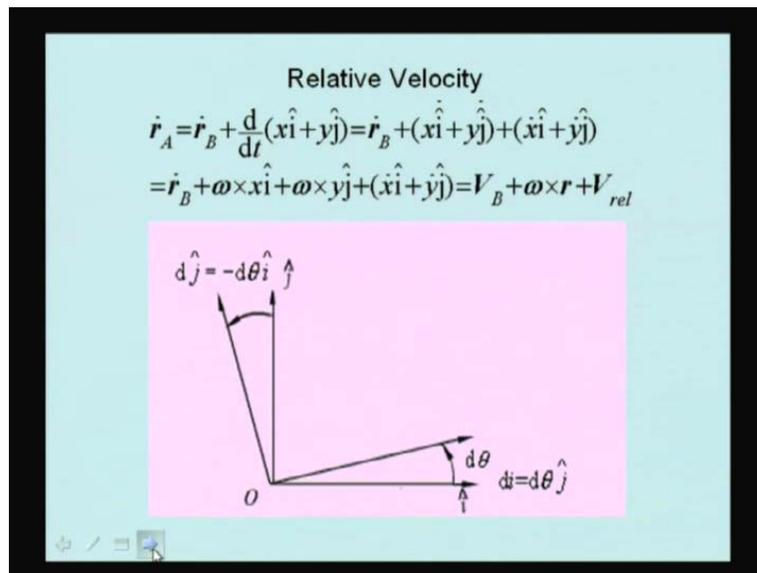
Similarly, the vector j has rotated to a new position. Therefore, dj will be directed towards this side. This is dj and its magnitude is naturally a unit vector, it is $d\theta$ dj , but it is in the minus direction of i because positive i is this direction. Therefore, it is minus $d\theta$ into j , dj is equal to minus $d\theta$ into i .

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If angular velocity is denoted by the vector, that is omega is equal to omega k hat, that is d theta by dt or theta dot multiplied by k hat, k hat is the unit vector and it is along the z direction that is normal to the plane.

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Then we can also write that omega di by dt is omega cross i; like that dj by dt is also omega cross j. Relative velocity of the point A with respect to moving frame x-y can be found by

differentiating the basic relation r_A is equal to r_B plus r_{AB} ; differentiating it with respect to time, we get \dot{r}_A is equal to \dot{r}_B plus $\frac{d}{dt}(x\hat{i} + y\hat{j})$ is equal to \dot{r}_B . Differentiating $x\hat{i} + y\hat{j}$ with respect to time, we get one term $\dot{x}\hat{i} + x\dot{\hat{i}} + \dot{y}\hat{j} + y\dot{\hat{j}}$. Then, we get $\dot{x}\hat{i} + \dot{y}\hat{j} + x\dot{\hat{i}} + y\dot{\hat{j}}$.

In this case, $\dot{\hat{i}}$ is not equal to 0; instead $\dot{\hat{i}}$ is given by $\omega \times \hat{i}$. Therefore we get, \dot{r}_B plus $\omega \times x\hat{i} + \omega \times y\hat{j} + \dot{x}\hat{i} + \dot{y}\hat{j}$. $\omega \times \omega$ can be taken common and it will become $\omega \times (x\hat{i} + y\hat{j})$ which gives V_B plus $\omega \times r$ plus $\dot{x}\hat{i} + \dot{y}\hat{j}$ can be called as relative. Because this is the velocity of the point A in the frame of reference, moving reference frame, that is the relative velocity here; because position vector is xy , that means relative.

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Thus
$$\underline{V}_A = \underline{V}_B + (\underline{\omega} \times \underline{r}) + \underline{V}_{rel}$$

The relative acceleration equation may be obtained by differentiating the relative velocity relation.

$$\underline{a}_A = \underline{a}_B + \underline{\dot{\omega}} \times \underline{r} + \underline{\omega} \times \underline{\dot{r}} + \underline{\dot{V}}_{rel}$$

The diagram shows a coordinate system with origin O. The unit vectors \hat{i} and \hat{j} are shown. A small angle $d\theta$ is indicated between the original and rotated unit vectors. The differentials are given as $d\hat{j} = -d\theta \hat{i}$ and $d\hat{i} = d\theta \hat{j}$.

Thus, we get a relation that is V_A is equal to V_B plus $\omega \times r$ plus V_{rel} . We get one extra component; in this case it is not equal to $V_A - V_B$. Instead, we get one additional term which is important, that is $\omega \times r$. If the reference frame is only translating, in that case ω becomes 0 or a translating reference frame ω is 0. Therefore, V_A is equal to V_B plus V_{rel} . Likewise, we can find out the expressions for relative acceleration.

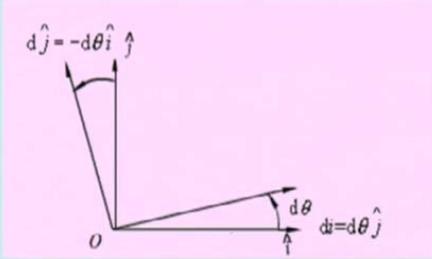
The relative acceleration equation may be obtained by differentiating these relative velocity relations. Relative velocity relation is given here; that is V_A is equal to V_B plus ω cross r plus V_{rel} . Differentiate V_A with respect to time, you get a_A ; differentiate V_B with respect to time you get a_B . Differentiate ωr with respect to time, you get two terms because it is a cross product of two vectors. One is given by ω dot into cross product r plus ω cross product r dot. Differentiate V_{rel} with respect to time, you get \dot{V}_{rel} , that is differentiation of V_{rel} with respect to time.

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Now $\dot{r} = \frac{d}{dt}(x\hat{i} + y\hat{j}) = \dot{x}\hat{i} + \dot{y}\hat{j} + (x\dot{\hat{i}} + y\dot{\hat{j}}) = \omega \times r + V_{rel}$
 $\omega \times \dot{r} = \omega \times (\omega \times r + V_{rel}) = \omega \times (\omega \times r) + \omega \times V_{rel}$

It is known that r dot, in this previous expression we got a term - r dot. Let us see, what is r dot? r dot is d by dt $x\hat{i} + y\hat{j}$ which can be written as $\dot{x}\hat{i} + \dot{y}\hat{j} + x\dot{\hat{i}} + y\dot{\hat{j}}$ plus x dot \hat{i} plus y dot \hat{j} . x can be written as ω cross r plus V_{rel} - this relation we have derived. Therefore, ω cross r dot can be written as ω cross bracket ω cross r plus V_{rel} which is equal to ω cross bracket ω cross r bracket close plus ω cross V_{rel} .

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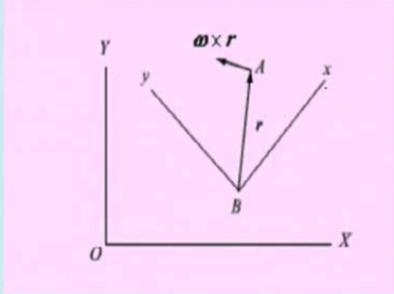
$$\begin{aligned}
 \dot{V}_{rel} &= \frac{d}{dt}(\dot{x}\hat{i} + \dot{y}\hat{j}) \\
 &= (\ddot{x}\hat{i} + \ddot{y}\hat{j}) + (\dot{x}\dot{\hat{i}} + \dot{y}\dot{\hat{j}}) \\
 &= \omega \times (\dot{x}\hat{i} + \dot{y}\hat{j}) + (\ddot{x}\hat{i} + \ddot{y}\hat{j}) \\
 &= \omega \times V_{rel} + a_{rel}
 \end{aligned}$$


Now, \dot{V}_{rel} is basically $\frac{d}{dt}(\dot{x}\hat{i} + \dot{y}\hat{j})$, which can be written as $\ddot{x}\hat{i} + \ddot{y}\hat{j} + \dot{x}\dot{\hat{i}} + \dot{y}\dot{\hat{j}}$; which is equal to $\omega \times (\dot{x}\hat{i} + \dot{y}\hat{j}) + \ddot{x}\hat{i} + \ddot{y}\hat{j}$. It can be written as $\omega \times V_{rel} + a_{rel}$.

Here, again we observe that \dot{V}_{rel} is really not equal to a_{rel} . Relative velocity, if you differentiate with respect to time, you do not get relative acceleration. We get one additional term as well, which is $\omega \times V_{rel}$. In the translating reference frame ω becomes 0. Therefore, \dot{V}_{rel} is equal to a_{rel} ; that means in the translating reference frame, if you differentiate the relative velocity, you get the relative acceleration; but, the same thing is not to have the rotating reference frame.

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Thus, $\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{V}_{rel} + \mathbf{a}_{rel}$
Let us identify each term.
 $\dot{\boldsymbol{\omega}} \times \mathbf{r}$ is the tangential acceleration, since it is perpendicular to unit vector \hat{k} and r .

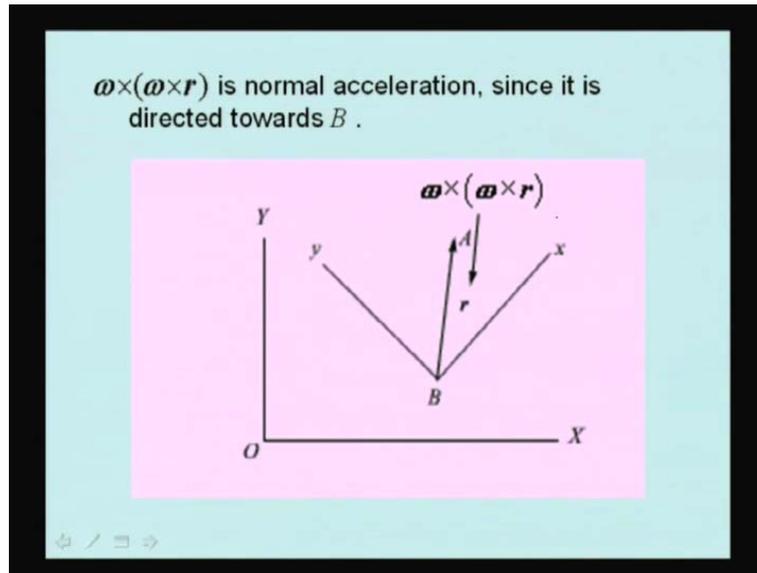


The diagram shows a 2D coordinate system with a vertical Y-axis and a horizontal X-axis. The origin is labeled O. A point B is located in the first quadrant. From B, two axes are drawn: a vertical y-axis and a diagonal x-axis. A vector r points from B to a point A. A vector labeled ω × r is shown perpendicular to r, pointing in the direction of tangential acceleration.

Thus a_A is equal to we get this big expression a_B plus $\omega \dot{r}$ plus $\omega \times \omega \times r$ plus $2\omega \times V_{rel}$ plus a_{rel} .

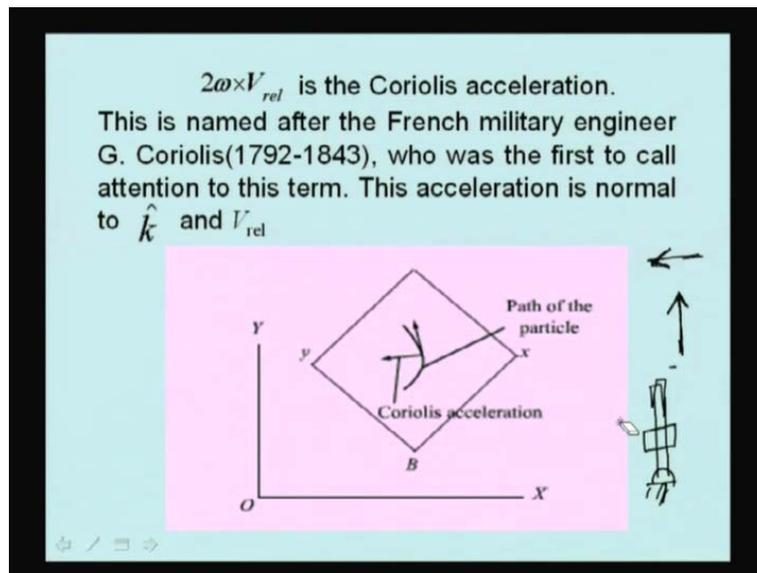
Let us identify each term in it. $\omega \times r$ is the tangential acceleration. Since it is perpendicular to unit vector \hat{k} and r , ω is having the direction \hat{k} . Therefore, $\omega \dot{r}$ is also having the direction \hat{k} . $\omega \times r$ is the vector which is in the tangential direction. This is shown here.

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Omega cross omega r is normal acceleration since it is directed towards B. omega cross r will be towards j and j cross k again gives you i. This is towards B and this is called normal acceleration or the same thing as you get centripetal acceleration.

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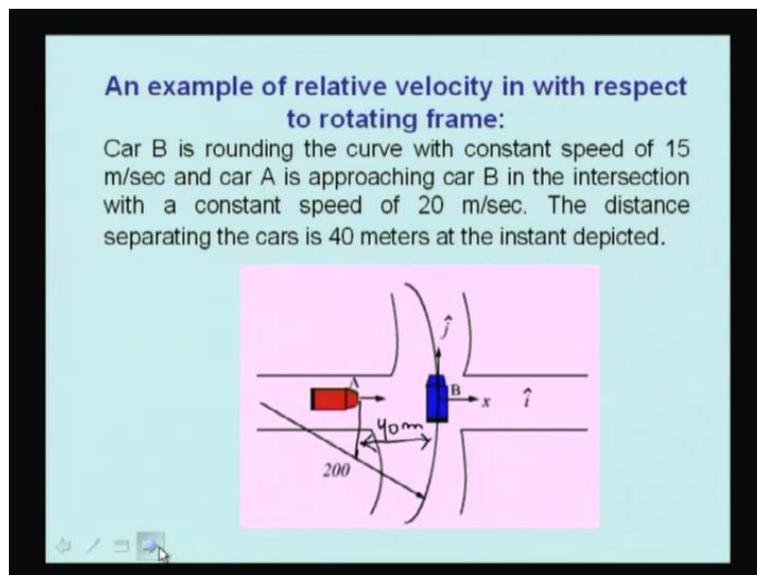
Now, $2\omega \times V_{rel}$ is the other term which we get in the expression for acceleration. $2\omega \times V_{rel}$ is really the Coriolis acceleration. This is named after the French military

engineer G. Coriolis who was born in 1792 and died on 1843 who was the first person to call attention to this term. This acceleration is normal to k and V_{rel} .

If the particle is moving, if the path of the particle is like this and you have the relative velocity of the particle like this, rotate the relative velocity vector in the direction of the rotation; give a 90 degree turn and you get the direction of Coriolis acceleration. That means, if this is the relative velocity, suppose there is a link on which this slider is moving and linkage velocity like this; that means it is moving in the counter clockwise direction. Relative velocity of the slider is shown like this. We have to, because it is rotating in the counter clockwise direction, link itself is rotating in counter clockwise direction. Therefore, rotate the relative velocity in the counter clockwise direction. This is the direction of the Coriolis acceleration.

Similarly, if the rod would link, this would have been rotating in the opposite direction like this. Then, the Coriolis acceleration would have been like this. Coriolis acceleration can be in both the directions. Normal acceleration force is directed towards the center of the curvature.

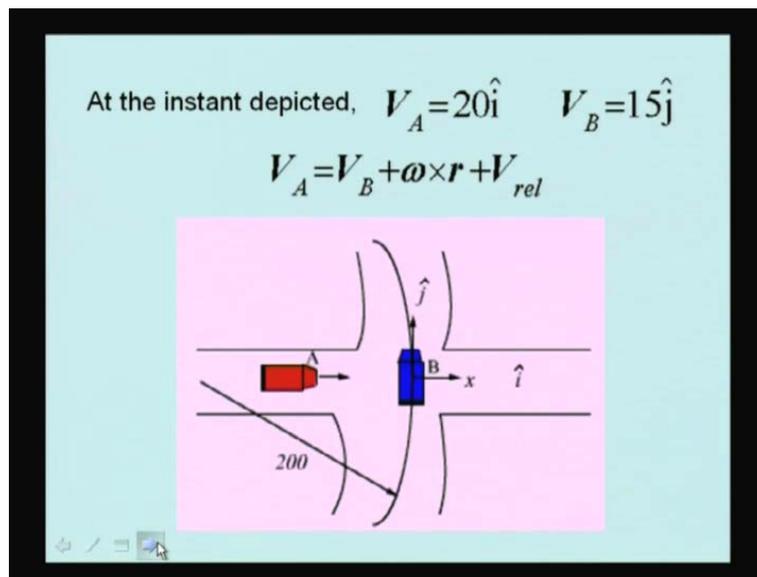
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We will solve one example of relative velocity with respect to rotating frame. Let us consider this problem.

In this figure, there are two cars; blue car is moving on a circular path whose radius is of the curvature 200, the red car A is moving in a straight line and it is approaching B. So, car B is rounding with constant speed of 15 meter per second and car A is approaching car B in the intersection with a constant speed of 20 meter per second. The distance separating the cars is 40 meters at instant depicted. That means this distance is 40 meter. Size of the car is very small compared to the distance 40 meter. So, it can be treated as a particle.

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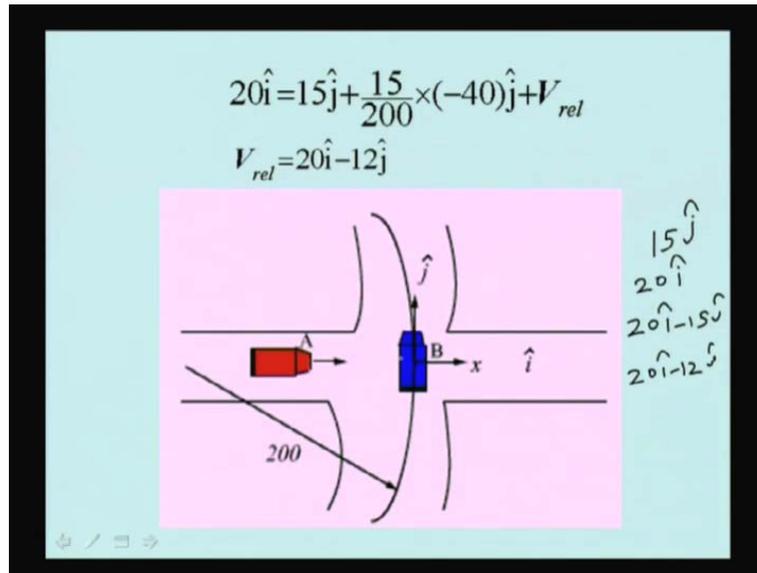


At the instant depicted, V_A is equal to $20i$ because it is mentioned that car A is approaching car B with a constant speed of 20 meter per second. Therefore, V_A comes out to be $20i$.

V_B is moving with 15 meter per second on a circular path, at the instant depicted V_B will be equal to $15j$. Apply the relative velocity formula; V_A is equal to V_B plus ω cross r plus V_{rel} are touching the moving reference frame in car B. Therefore, this is x unit vector is in the i direction that is i then yj. This reference frame is rotating on a circular path.

Therefore, velocity of A is given by velocity of B which is the velocity of the origin of the reference frame and that is same as, the velocity of the car plus ω cross r plus V_{rel} which is the velocity as seen in the moving reference frame or as seen by the driver in car B. Because the driver in car B is moving with respect to that reference frame.

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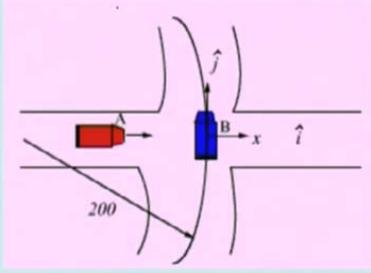


Putting the values, V_A is equal to $20\hat{i}$ is equal to $15\hat{j}$ plus $\frac{15}{200} \times (-40)\hat{j} + V_{rel}$, 15 by 200 is the ω cross minus $40\hat{j}$ because minus plus V_{rel} . Therefore, solving this we get V_{rel} is equal to $20\hat{i}$ minus $12\hat{j}$, this is the velocity of the car A as observed by the person sitting in car B. Remember, car B was having the velocity of $15\hat{j}$ and car A was having the velocity $20\hat{i}$; but, the relative velocity did not come $20\hat{i}$ minus $15\hat{j}$ instead it is coming $20\hat{i}$ minus $12\hat{j}$. Had this car been moving in a straight line, then ω would have been 0 . The velocity, the relative velocity would have come, instead of $20\hat{i}$ minus $12\hat{j}$, it would have come $20\hat{i}$ minus $15\hat{j}$.

Therefore, see the difference a rotating coordinate system makes.

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This is the velocity of car A as seen by the driver (who is treated as a particle along with the car !) in car B. Will the driver in car A will see the velocity of car as

$$V_{rel} = -20\hat{i} + 12\hat{j} ?$$


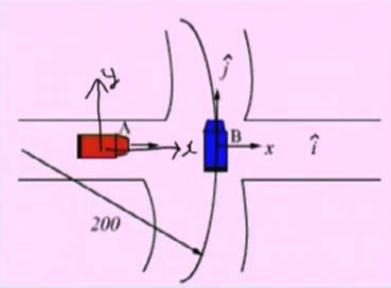
The velocity mentioned there is the velocity of car A as seen by the driver who is treated as a particle along with the car in car B. Will the driver in car A will see the velocity of car as V_{rel} is equal to minus 20i plus 12j? If the velocity of car A with respect to car B is 20i minus 12j then the velocity of car B with respect to velocity of car A is minus 20i plus 12j. No, we cannot say this.

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Suppose the axes system is attached to A,

$$V_A = V_B + V_{rel} \quad V_{rel} = 15\hat{j} - 20\hat{i}$$

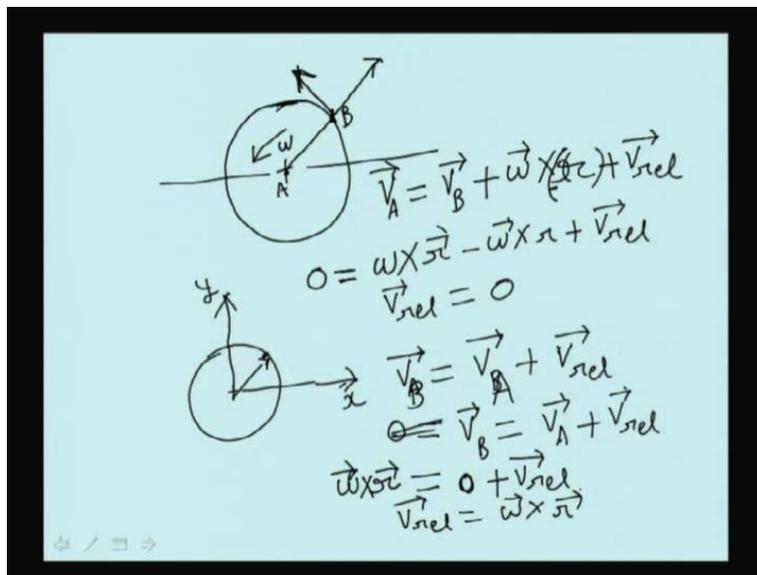
Thus, we see that observations of both drivers are not the negative of each other.



Let us see this thing. Suppose the axes system is attached to A, then V_A is equal to V_B plus V_{rel} . In this case, the axis system is attached to this one. This axis system is not rotating, there is no ω ; ω is 0. Therefore, the expression comes out to be V_A is equal to V_B plus V_{rel} or V_{rel} is equal to $15j$ minus $20i$. Thus, we see that the observations of both drivers are not the negative of each other. That is this thing. Therefore, it is very important to understand these relations in translating as well as using reference frame.

Let us discuss some problems of the concepts developed.

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Now, imagine that a particle is moving. Particle is located at B; that is periphery of this one. The disc is rotating with an angular speed, ω . If we attach a moving reference frame at B, this is x radial direction, other is this one. Then, this frame of reference also rotates with velocity, ω .

In this case, V_B is equal to we get the relation, if we fix the coordinate system at A, V_A is equal to V_B plus ω cross ω cross minus ω cross r plus V_{rel} . If A is a fixed point then V_A is equal to 0.

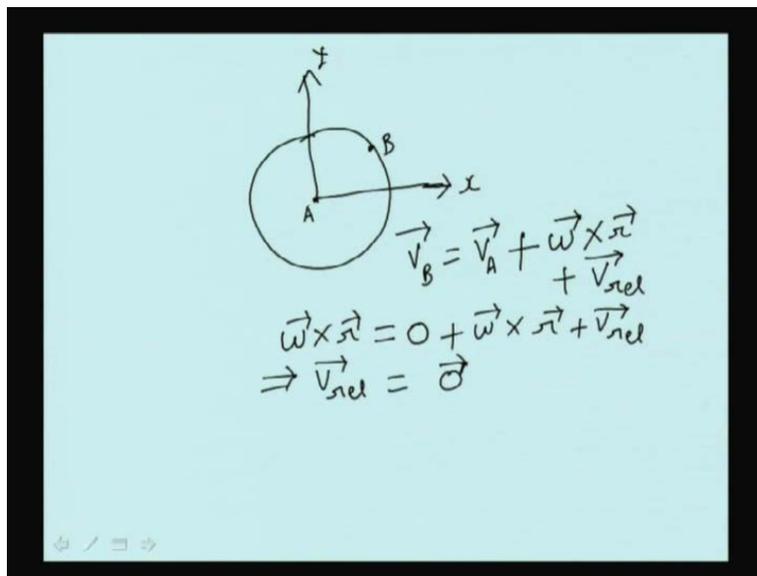
Now, in this case I have fixed an axes system here at ω cross r at this point. Therefore, I have to take this r as minus r , this is minus. Therefore, this is V_B and I can write V_B as ω

cross r where r is the radius of the circle. This becomes $\omega \times r$. This is minus $\omega \times r$ plus V_{rel} which gives us V_{rel} equal to 0; that means if a particle is placed at B and an axis system is attached with that particle, the particle will observe that center is at the same position and therefore you are getting V_{rel} is equal to 0.

However, if we fix up a nonmoving coordinate system here at particle A, then velocity of particle B, V_B are related with velocity of A in this fashion, V_A is equal to V_B plus V_{rel} , as the axis system is not rotating. Therefore, I am not putting ω .

Here, V_A is equal to 0 and this is $V_B = V_A + V_{rel}$. Therefore, V_B is equal to $V_A + V_{rel}$. V_B is known as ωr and V_A is equal to the tangential direction. This velocity is $\omega \times r$. We can write in the vector form; $\omega \times r$ is equal to V_A that is 0 plus V_{rel} . Therefore, the relative velocity comes out to be $\omega \times r$. That means, if you attach an axes system at point A then it will observe particle B to have relative velocity $\omega \times r$.

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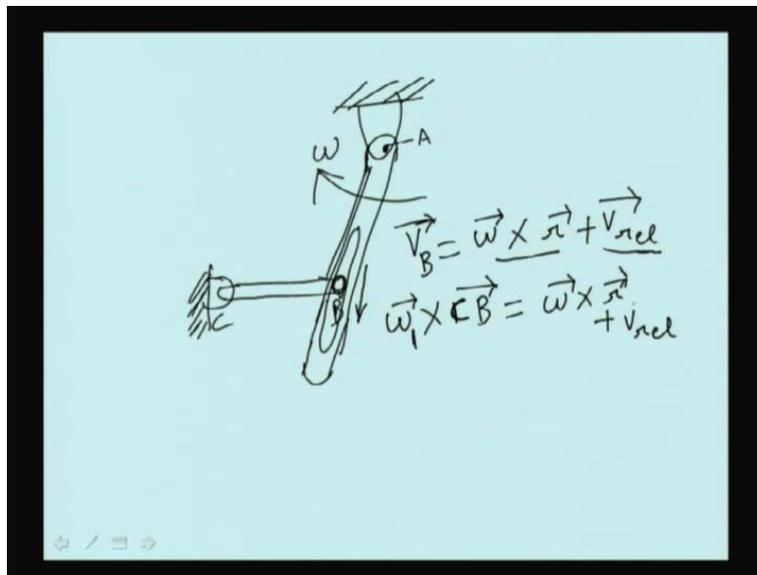


However, what happens, if the axis system is although attached with A, but unlike in the previous case it is not fixed. It is also moving with the disc. In that case, we have axes system which is having the velocity, ω .

Now consider a particle B here. The velocity B is equal to V_A plus ω cross r plus V_{rel} , V_B in this case, is equal to ω cross r because absolute velocity is known to be ω cross r which is equal to 0 velocity plus ω cross r plus V_{rel} , which gives us V_{rel} equal to a 0 vector. Indicating that really it is not that, just particle is not enough. It is important what type of axis system has been chosen.

Same particle A may have the origin. One axis system was constant; not nonmoving. In that case, V_{rel} comes out to be ω cross r . If the axis system is also rotating with the disc, then V_{rel} turns out to be 0 which is inconsistent. As the axis system is rotating, it always follows particle B and thus, it is not able to know the difference in the position. We will discuss one problem concerning this principle.

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Here, this is link; link is there and this link is having a slot. On this slot, a ball is kept which can spin, which can slide and this is another link which is hinged here.

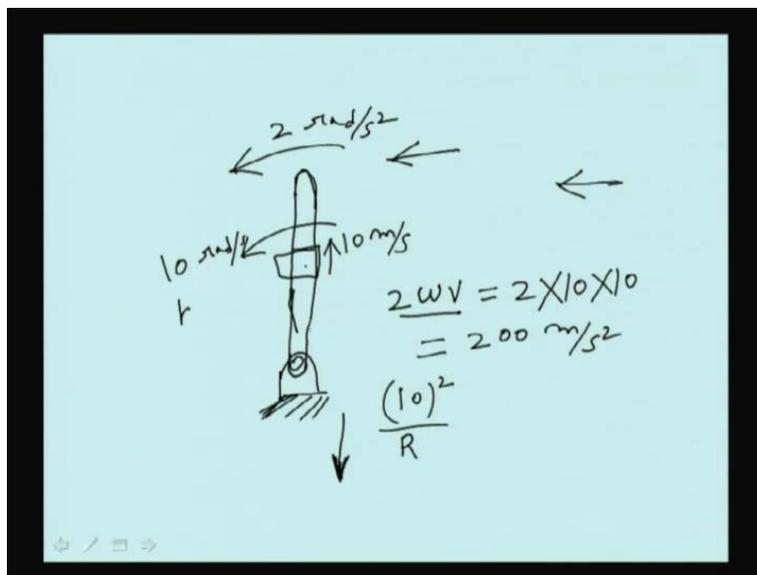
We may be asked to solve this problem. Given that you have the angular velocity ω and this point, that is point A; this point is point B. This is the point then this is point B. We have to find out the velocity of point B with respect to point A and also the angular rotation of this link which can be called as BC. Such type of problems also can be solved using the previous equation.

In this case, you can attach one axis system on this rod, which is rotating and then you can have V_A V_B is equal to V_A which is 0; that is $\omega \times r$ plus V_{rel} . At this stage, you know the relative velocity direction; that is direction is along this thing, but you do not know the magnitude at this stage. ω , V_A is of course $\omega \times r$; $\omega \times r$ can be easily calculated.

We have fixed up the axis system at point A; r is the radial the position vector of B with respect to A. ω might have been specified to you. So, you will be able to calculate $\omega \times r$ plus and V_{rel} . Only the direction is known. You do not know the magnitude, but then, we know the other equation that if you have the velocity of point B with respect to point C that is also this direction, has been given.

Absolute velocity of point B is $\omega \times BC$ $\omega \times$ vector CB velocity CB, which also has to be given. However, you do not know this is ω . I will indicate it by ω_1 . Although you do not know that the magnitude of ω_1 , you can equate this with this expression, $\omega \times r$ plus V_{rel} and you can find out V_{rel} . We can express these in the component forms and then you can get two equations. There are two unknowns in this problem; one is the ω and other is the magnitude of V_{rel} . That type of problem can be solved.

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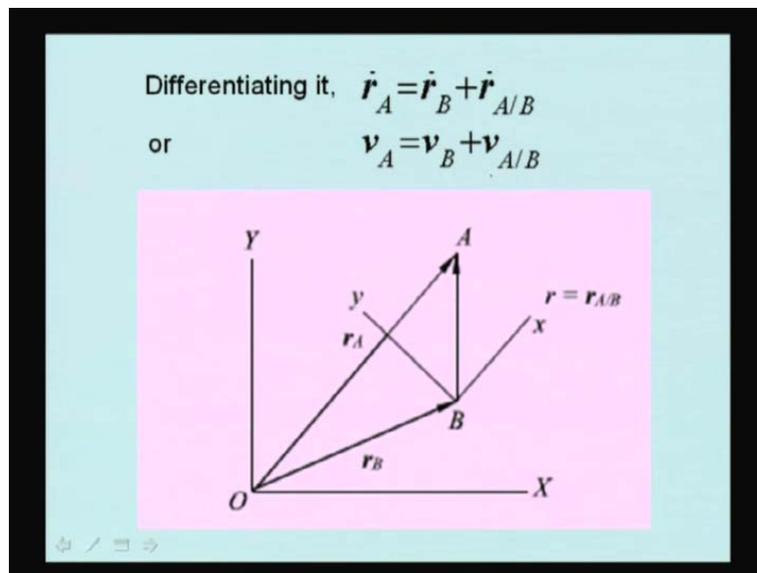


We will solve one simple problem of Coriolis acceleration. If there is a link here, which is rotating at 10 radian per second and a particle is sliding at 10 meter per second, what is the Coriolis acceleration? Coriolis acceleration will be $2\omega \times V$. In this case, magnitude will be $2\omega V$; that means, it will be 2 times 10 times 10, which will be equal to 200 meter per second square. Direction of this Coriolis acceleration will be in this direction. The Coriolis acceleration is not dependent on the position of the slider or the link. It is dependent on the relative velocity; however, there will be another component that is centripetal acceleration of this particle which will depend on the relative position. That is given by $\omega^2 r$, where r is the distance of this. Therefore, this component decreases as the particle is 1.

This component is given in this direction. In this problem, the Coriolis will have the same direction as the tangential acceleration. If the tangential acceleration is provided and it is say 2 radian per Second Square, then the tangential acceleration will be in this direction and the magnitude will be $\alpha \times r$; that means this is 2 radian per second square multiplied by the distance is 1.

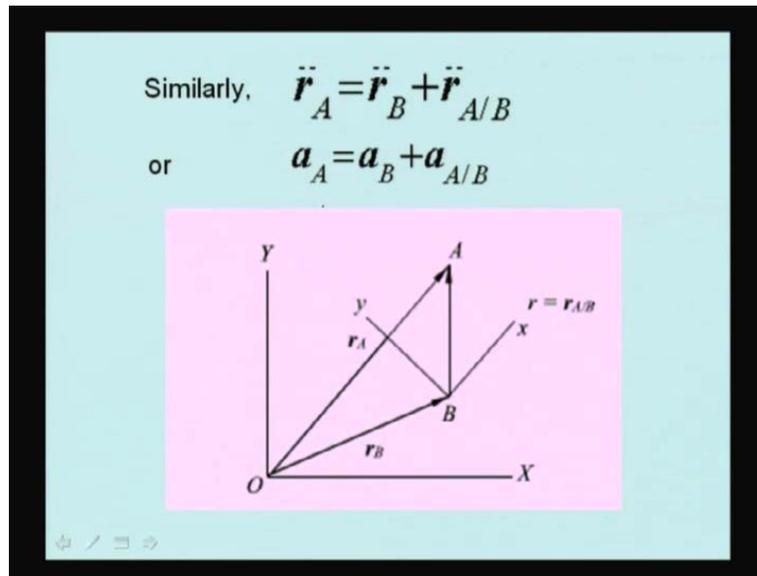
Like that, you can solve this number of problems using the equations developed in this case. Let me quickly summarize, what we have developed.

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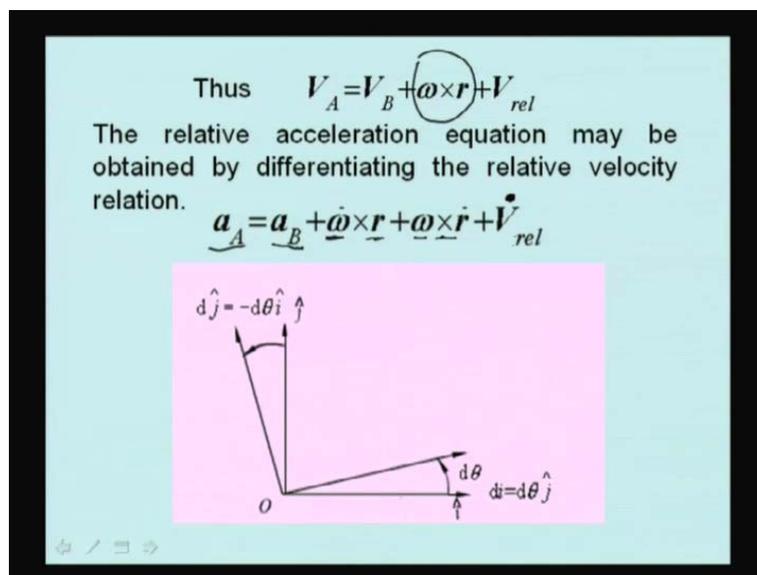
We started with translating reference frame and obtained the simple expression that is V_A is equal to V_B plus $V_{A/B}$.

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a_A is equal a_B plus $a_{A/B}$.

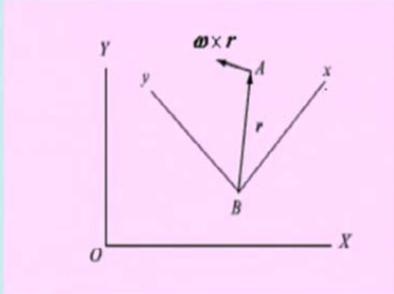
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However, if the coordinate system is rotating, we get the equations V_A is equal to V_B plus ω cross r plus V_{rel} ; remember, there are three components here.

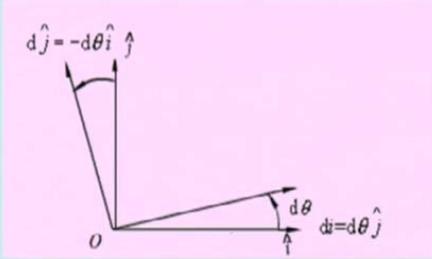
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Thus, $\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{V}_{rel} + \mathbf{a}_{rel}$
 Let us identify each term.
 $\dot{\boldsymbol{\omega}} \times \mathbf{r}$ is the tangential acceleration, since it is perpendicular to unit vector \hat{k} and r .



We get acceleration components, a_A is equal to a_B plus ω cross r plus 2ω cross V_{rel} plus a_{rel} , Here we are getting 5 terms. These two equations are very important. In fact, in this expression, if you put ω is equal to 0, this equation reduces for the equation of the translating reference frame. Therefore, important equations are: one is this.

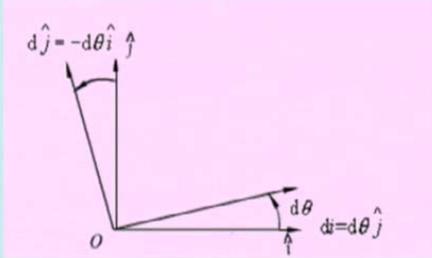
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$$\begin{aligned}\dot{V}_{rel} &= \frac{d}{dt}(\dot{x}\hat{i} + \dot{y}\hat{j}) \\ &= (\dot{x}\hat{i} + \dot{y}\hat{j}) + (\dot{x}\hat{i} + \dot{y}\hat{j}) \\ &= \omega \times (\dot{x}\hat{i} + \dot{y}\hat{j}) + (\dot{x}\hat{i} + \dot{y}\hat{j}) \\ &= \omega \times V_{rel} + a_{rel}\end{aligned}$$


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Thus $V_A = V_B + \omega \times r + V_{rel}$

The relative acceleration equation may be obtained by differentiating the relative velocity relation.

$$\underline{a}_A = \underline{a}_B + \underline{\dot{\omega}} \times \underline{r} + \underline{\omega} \times \underline{\dot{r}} + \dot{V}_{rel}$$


Another equation is this; velocity V_A is equal to V_B plus ω cross r plus V_{rel} .