Theory & Practice of Rotor Dynamics
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Module - 5
Torsional Vibrations
Lecture - 15
Three Disc Rotor System

So, we will be considering three degree freedom system in this present analysis.

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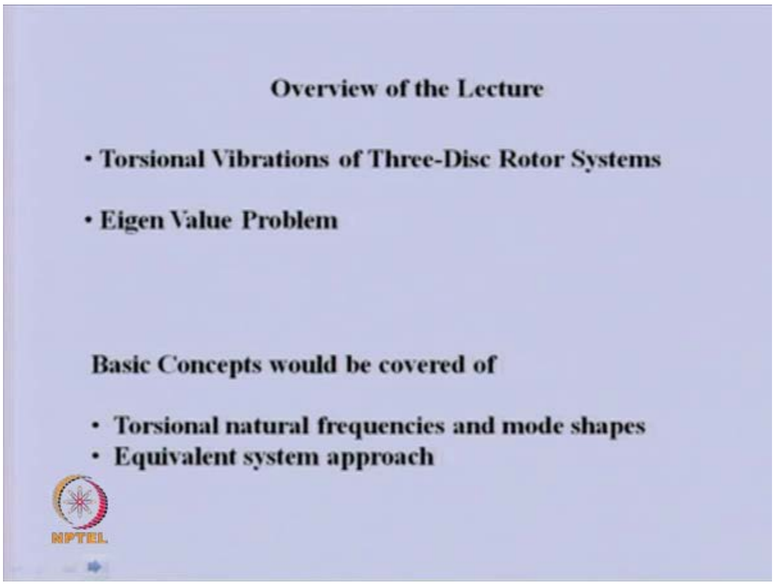

So over all, overview of the lecture is torsional vibration of three disc rotor system and we will, I will be introducing the Eigen value problem and basic concepts will be covered of the torsional natural frequencies, mode shapes and equivalent system approach. So, this will be the overall concept which will be introducing in the lecture.

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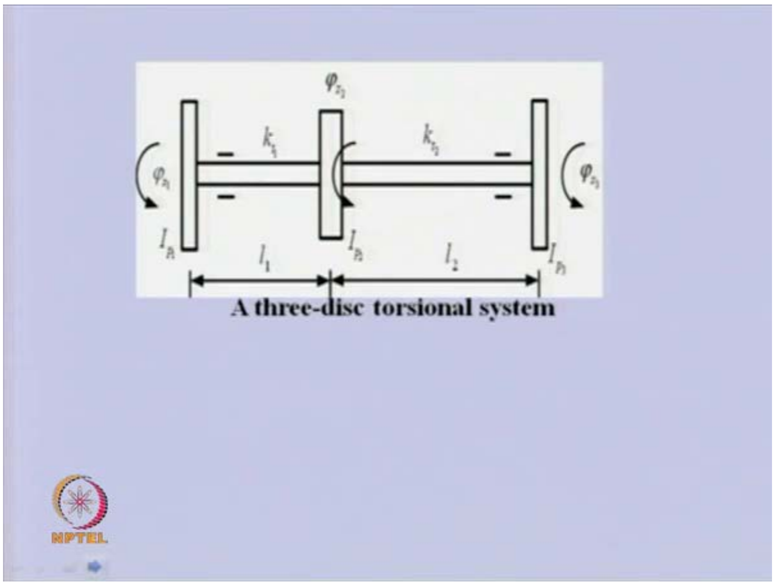

So, let us take one rotor systems in which have three discs. This discs are rigid, they have high polar moment of inertia, this shafts are flexible in torsional, but they do not have a such inertia effect and here you can able to see that during torsional oscillation, these disc have relative torsional twist. The first disc is having relative twist, the absolute; that is absolute displacement phi z 1, the second disc is having phi z 2 and the third is having phi z 3 and this rotor system is mounted on friction as bearing.

So, during motion as such these supports are not preventing its torsional motion. So, this particular case is a also similar to their free free condition. The shaft which is between disc and one and two is having torsional stiffness k t 1 and the shaft which is between this two and three is having stiffness k t 2. Dimensions are given here. The disc are having polar moment of inertia I p 1 I p 2 and I p 3 of disc one, two, three respectively. Now, to obtain the equation motion we will obtain the free body diagram of each of the disc.

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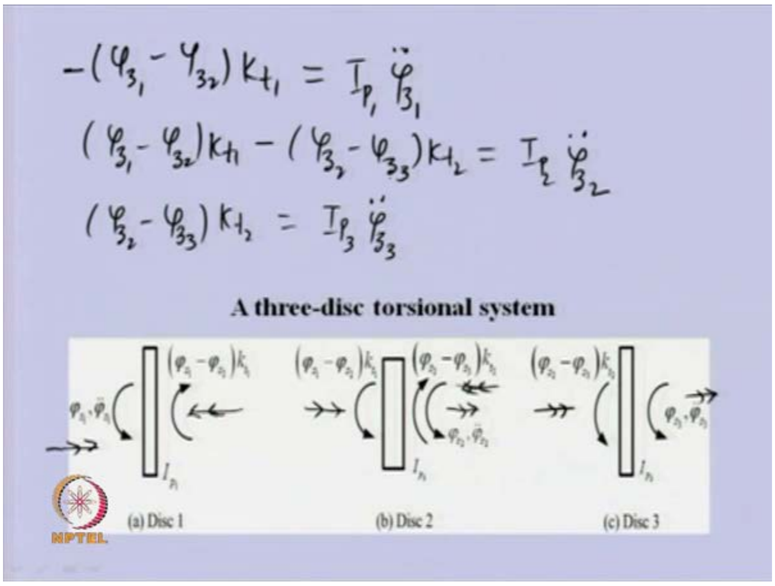

So, this is the free body diagram of three discs. You can able to see that this is the free body diagram of disc one and in this particular case. We have taken the positive convention that if we are looking from the right hand side on to the disc the counter clockwise motion is positive direction. So, we can able to see this I have drawn in counter clockwise direction, which is the displacement of the disc which I have taken positive in the vector form.

This will be the positive direction according to the right hand rule. This is the reactive torque root from the shaft one. So, you can able to see this is phi z 1 minus phi z 2. This is the relative twist of the shaft and this is the torsional stiffness of the shaft one. The direction of this is opposite to the displacement. So, you can able to see this according to the right hand rule, the vector direction will be opposite to the displacement. So, this is the free body diagram of the disc one.

We can able to take the torque balance. So, I am taking, so this is the external torque out of the disc. So, we can able to right phi z 1 and phi z 2 k t 1, this is the minus sign because is opposite to the displacement direction be equal to I d 1, I p 1 polar moment of inertia of the disc and the angular acceleration of the disc. So, this the equation of motion of the first disc from the free body diagram, second disc is connected by shaft one left side and shaft two in the right side.

So, you can able to see that the angular displacement is counter clockwise direction and we are looking the disc from the right side. So, basically this is the positive direction for angular displacement. This is the torque which is coming from the shaft one. So, because this torque is should be opposite to this one because there acting at the opposite and at the same shaft. So, you can able to see the direction of this is opposite to the direction of the, this particular torque.

Another torque from the shaft two is this one in which this is the phi z 2 minus phi z 3. This is the relative twist into the torsional stiffness and that is opposite to the displacement of the disc. So, this particular torque direction I can able to show in the opposite direction to the displacement. So, basically in this particular second disc we have, this is one torque and this is another torque which is acting. These are external torque. So, we can able to obtain the equation of motion for second disc like this. This from the, this particular moment, which is and the direction of the displacement, so positive, this is negative direction. k t 2 should be equal to rotor inertia of the disc.

So, this is the equation motion of the second disc. Similarly, third disc is connected in the left hand side by the shaft two and this particular torque will be equal and opposite to this one. So, the torque direction will be opposite to the previous one sorry this will be opposite to this one which is left side, node is right side and this we always we are taking in the positive direction, angular displacement direction. So, from here you can able to obtain the equation of motion for the third disc, that is phi z 2 and phi z 3, k t 2 should be equal to inertia of the third disc. So, you can able to see that we obtain equation of motion of all three discs and we can able to rearrange these in matrix form that will be more convenient to handle.

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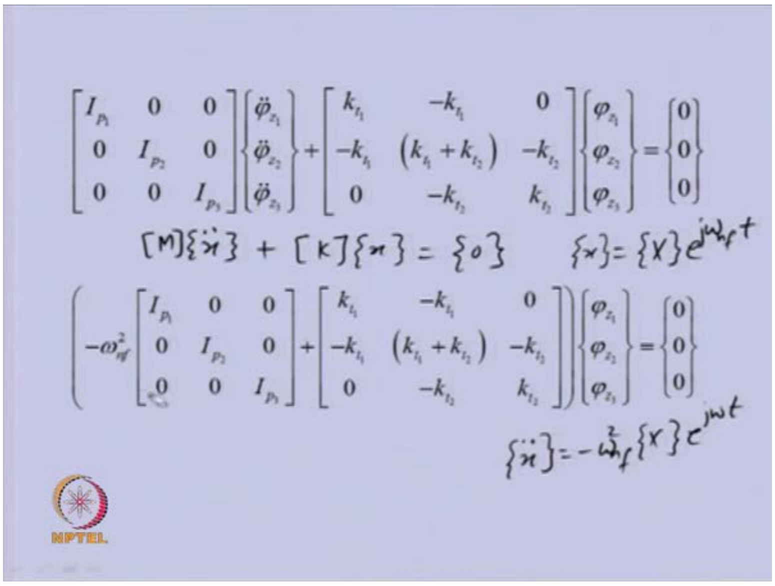

So, in the next slide… So, this is the equation of motion which we derived in the previous slide. I hope, put them in a matrix form. So, you can able to see this, this is nothing but the mass matrix of the rotor system. This is the inertia, this is a angular accelerations, these are the displacements and all this stiffness terms are here. This stiffness matrix you can able to see this is a symmetric matrix. In this particular case the mass matrix is diagonal and the form of this equation is similar to standard form of the multi degree of freedom system, vibration system.

So, where you can see M is here this matrix, this vector is this vector, stiffness matrix is this one and this is the displacement vector. Now, for free vibration because we are dealing with the free vibration we expect the displacement will be having some amplitude and harmonic component that will be the natural frequency of the system. So, if we substitute this in the equation of motion this one, we can able to see that we need to derive or take derivative of this with respect to x with respect to t and because this is the constant. So, we will get omega square for that particular case.

So, if we substitute we can derivate this twice. So, basically I will be getting on this expression and this expression if we substitute these two in the equation of motion, I can get this equation in which, I can able to see omega square minus has come here and I have taken the displacement vector which is amplitude this capital X outside. Now, you can able to see this is a homogeneous equation and the one solution is that all the displacements are 0. So, in that particular case motion is not taking place. So, we are not interested in such analysis. So, if for non zero value of this displacement the determinant of this should be 0.

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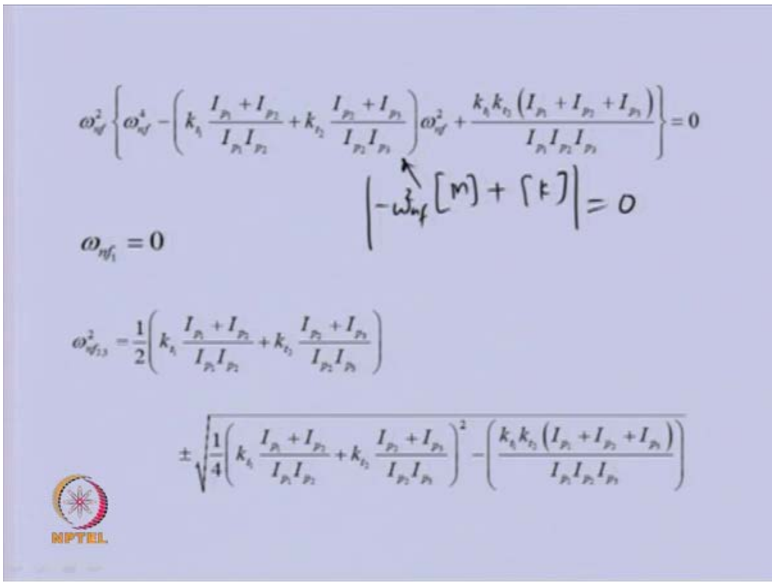

So, that will give a polynomial. So, you can able to see that if we take the determinant of those matrices to 0. So, that is something. So, this is nothing but and plus k minus omega n f square determinant equal to 0. So, we will get a polynomial like this. So, we can able to see that the one solution of this is when this natural frequency itself is equal to 0 and second is when these terms are 0. So, from first one we got 0 natural frequency, second term we will be getting two more roots, positive roots. So, that expression expanded here. So, if we know the polar moment of inertia of various discs, the torsional stiffness of the shafts, we can able to obtain the natural frequency of the system.

So, here we got three natural frequency; out of that one is having 0 value. So, we have a seen how to obtain the natural frequency of a three disc rotor system. Now, when we are having multi degree of freedom system then we need to obtain the relative twist or relative angular displacement of the disc. That means the mode shape of the disc. Now, we will see how we can able to obtain the mode shape from the, our equation of motions. So, for that we will take this particular equation. So, in this equation we can able to expand any two of the equation. So, like first equation and second equation we can take.

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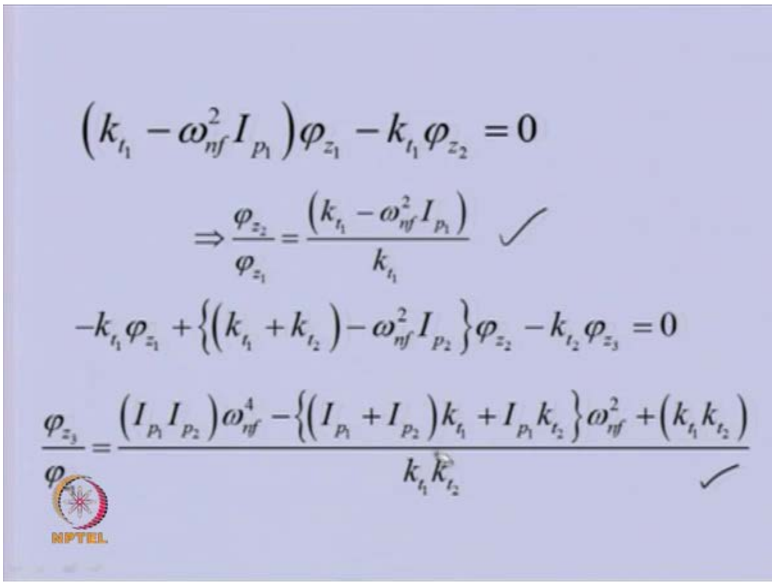

In the subsequent slide we have… So, this is the first equation from that equation and from this you can able to see that we can able to obtain the ratio between the two displacements, angular displacements like displacement of the disc two divided by displacement of disc one. That will be in terms of various stiffness, mass moment of the inertia and the natural frequency at the system. Similarly, from the second equation of that matrix equation we will get these expression and from here you can able to see that we can able to obtain the ratio of the displacement 3 with respect to 1, this 2, this particular displacement can be eliminated from the here, because we have already relation between that the 2 and 1, so that can be eliminated here.

So, the ratio of displacement of 3 with respect 1, this expression we obtain. After simplification this will be this form. You can able to see this also contains various polar moment of inertia, torsional stiffness of this, the shaft and natural frequency. So, these two equations, this equation and this equation can be used to obtain the relative twist between disc two and one and three and one for corresponding to various natural frequency. Like first one was the 0 natural frequency.

So, we need to substitute 0 here. So, you can able to see that the repeating the last sentence. So, these two equations can be used to obtain the relative displacements and for various natural frequency like for first natural frequency which is having 0 value you can able to substitute omega n f as 0. So, that will be the relative twist between two disc, disc two and one. Similarly, for 0 natural frequency this term and this term will vanish.

So, the remaining term will give the relative displacement between the disc three and one. Same exercise we can able to do for the second natural frequency that is your non zero value. So, if we substitute that here we will get the relative displacements corresponding to the second natural frequency or here for corresponding to the second natural frequency we will get the relative twist between this three and one. And lastly because we have the three natural frequency, so if we put the third natural frequency corresponding to that much of frequency we will be getting the relative between this 2 and 1 from here and 3 and 1 from this expression. So, these two can be used for obtaining the relative twist between various discs corresponding to different natural frequencies. Now, through simple example we will see how we can able get the natural frequency and mode shape for three disc rotor system.

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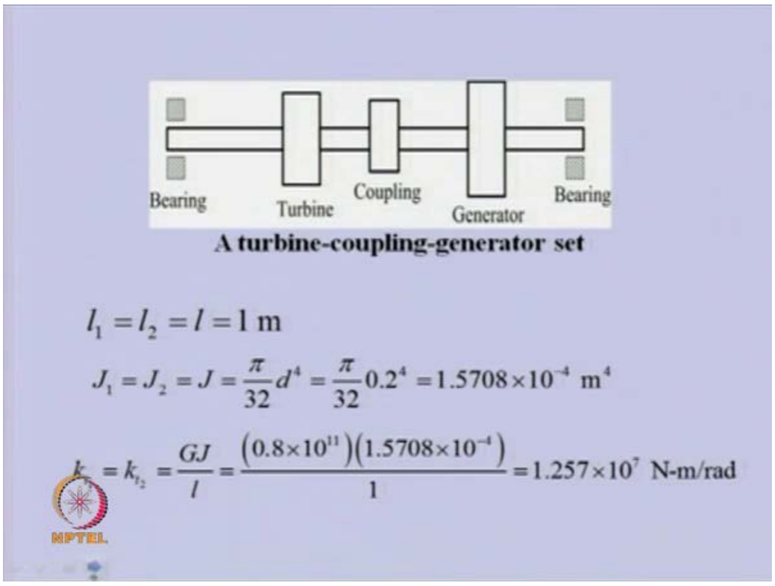

So, here we have turbine coupling generator system and such these bearings are frictionless and as such they are not imparting any role in the torsional vibrations of this system. So, basically we want to obtain the torsional natural frequency at this system. Various geometries are given. So, l 1 l 2 are nothing but the distance between the turbine and coupling and coupling to generator. So, these are 1 meter. Various property of the shafts are given like 0.2 is the diameter. 0.2 meter is the diameter of the shaft and we have torsional stiffness of the shaft.

In this particular reason because you can able to see that as such we are not considering the mass of the shaft. So, this segment and this segment of the shaft will not play any role in the torsional oscillation. Only, these two will be playing role because there is no mass attached at these ends. So, the torsional stiffness of this segment and these segments are same, because that dimensions are same and it is given here.

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$$\omega_{nf_1} = 0$$

$$\omega^2_{nf_{2,3}} = \frac{1}{2}\left(k_{t_1}\frac{I_{p_1}+I_{p_2}}{I_{p_1}I_{p_2}} + k_{t_2}\frac{I_{p_2}+I_{p_3}}{I_{p_2}I_{p_3}}\right) \pm \sqrt{\frac{1}{4}\left(k_{t_1}\frac{I_{p_1}+I_{p_2}}{I_{p_1}I_{p_2}} + k_{t_2}\frac{I_{p_2}+I_{p_3}}{I_{p_2}I_{p_3}}\right)^2 - \left(\frac{k_{t_1}k_{t_2}\left(I_{p_1}+I_{p_2}+I_{p_3}\right)}{I_{p_1}I_{p_2}I_{p_3}}\right)}$$

$$\omega_{nf_2} = 611.56 \text{ rad/s}$$

$$\omega_{nf_3} = 2325.55 \text{ rad/s}$$

Now, this expression which we know for natural frequency of the system, so we can able to substitute and here you can able to get first as 0 natural frequency and 2 flexible mode natural frequency is 611.56 radians per second and 2325.55 radians per second. Now, corresponding to this, if we want to obtain the natural mode shapes.

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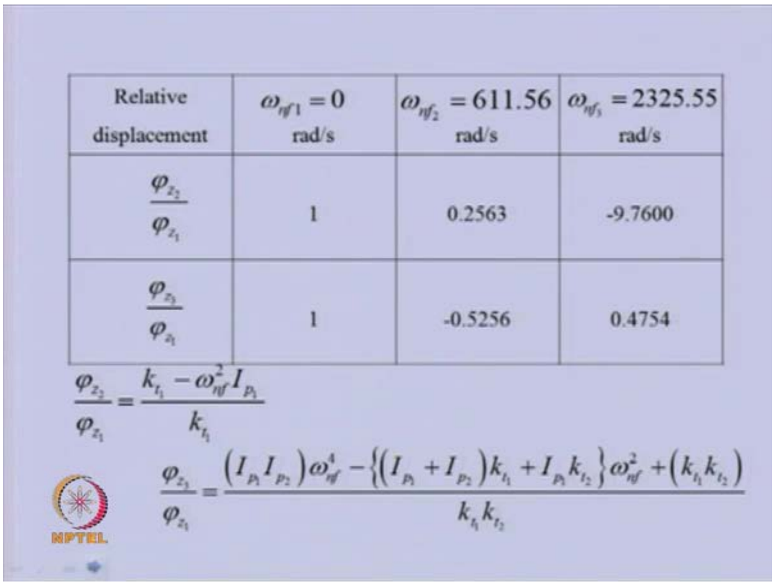

We can able to use these expressions and this ratio which we obtained earlier is of this form and the in this particular case you can see when the natural frequency is 0 both the disc two and three related to disc one is having same motion. That is something like all the disc are having same amount of displacement. So, there will not be any relative twist between them. So, that we call it as a rigid body mode. Corresponding to the second natural frequency if we substitute the values in these two expressions, we will get these ratios. So, you can able to see that one is positive and another is negative and if we want to plot these relative two displacements, you can able to plot it.

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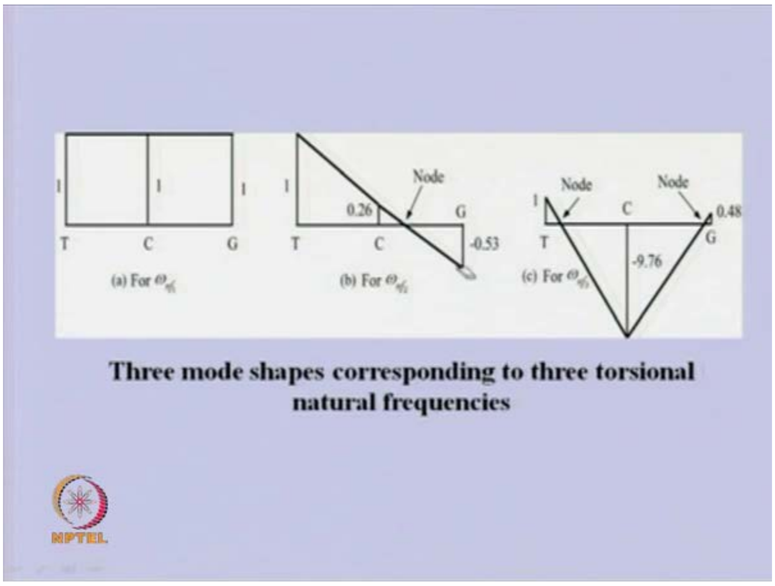

So, this is the plot of the mode shape. The first one was having all disc same displacement. So, this is a turbine, coupling, generator. These are the displacement, three displacements. For the second natural frequency you can able to see that the, this is for turbine, we can take this as 1, corresponding to that second disc will having 0.26 displacement and third will be having negative. So, I have shown in the negative direction.

Corresponding to the third natural frequency here if we substitute this in these two ratios we will see that one becomes negative, another is positive. So, the mode shape will take this shape. The disc one if we take value 1, the disc two will be having a large negative value and disc three will be having small positive value. So, here you can able to see that in second natural frequency there are two nodes appearing where there will not be any torsional oscillation.

In this first natural frequency there will be one node between coupling and the generator. And these nodes are very important because high level of stresses we expect at these nodes. Now, the same problem we will solve using a direct approach. In this particular approach we will not be obtaining the equation of motion from free body diagram, but we will try to obtain the natural frequency of this system by qualitative nature of the mode shapes of the system. So, in this because we know there are three discs and there are three possible modes and those three possible modes from that we will try to obtain the natural frequency at the system. So, let us see how this can be obtained.

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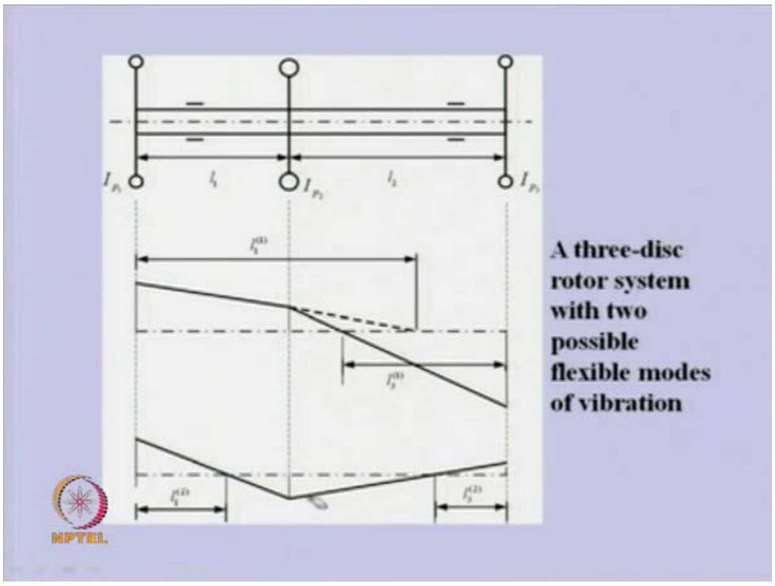

So, this is the three rotor system. We are interested in obtaining the torsional vibration of torsional natural frequency of this. Because the supports are something like free free, corresponding to the torsional vibration. So, in this particular case we will, we expect that the one natural frequency will be 0, because it is very clear that when we have free free vibration condition, boundary conditions we have one of the natural frequency 0 and correspondingly the mode shapes is we call it as rigid body mode in which all the disc are having same displacement.

So, such this particular mode is having no practical relevance. So, we will not be considering that particular natural frequency determination, but we are interested in obtaining the two other natural frequency which are corresponding to flexible mode. So, for this flexible mode you can able to see, we can have, this is the possible mode shape in which you can able to see that this is the, one of the mode shape in which this two disc are having displacement in synchronous with each other. But this particular disc is having displacement in the anti synchronous with this.

So, that means in one of the mode we expect that may be this and this disc will be having same direction of motion. Like let us say clockwise and this will be having displacement in the other direction and anti clockwise. So, this has been shown here. So, you can see this is the 0 axis and we have positive direction displacement of this two disc, but this third disc is having negative direction. So, this is one of the possible natural mode. This particular disc rotor system can have.

Other possibility is the may be outer to this are having same direction of motion maybe clockwise or anti clockwise, but the central one is having motion opposite to the other two. So, here you can able to see that this is having positive displacement. This is also having positive displacement, but the middle one is having opposite to this two. So, this is another possible mode. In the second case we have taken this two is having same displacement.

It may happen that may be the second and third is having same displacement as compared to the first one. So, other case is possible, but that will determine by the relative mass moment of inertia of the disc as well as the stiffness. So, in this particular case we assume a specific case and depending upon the relative values of the natural frequency and the torsional stiffness of the shaft we can have a other possible mode shape in which the disc 2 and 3 is having same displacement as compared to the wall. But for the present case let us consider other disc 1 and 2 are having same displacement as compared to the third one.

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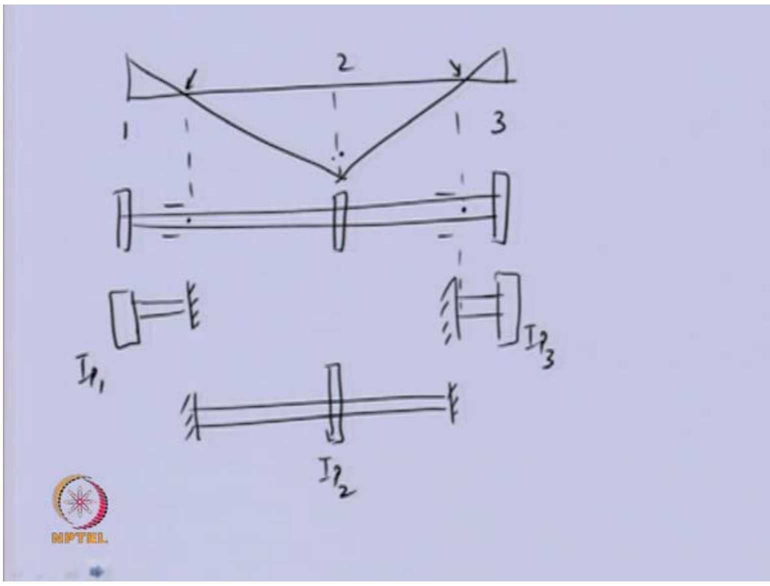

Now, if we consider this particular system, let us say the third one case in which we have the two outer disc having positive displacement and the central disc is having negative displacement. So, this is the disc one location and this is disc two location and this three disc location. So, in this particular case you can able to see these are the node position where there is no displacement. So, as such we have the rotor system like this. This mounted on frictionless bearing. So, you can able to see that the node position is here and here. So, we can able to break a three rotor system into three single rotor system like this. So, I am taking one of the rotor system from the node position as this one.

So, this particular rotor system I am taking as separately. Similarly, here because this is node position, so 0 displacement, so this rotor system is another, I am taking separately and in between these two, I am taking without system like this. So, you can able to see that we had three disc rotor system earlier. This was single system based on the information of the mode shape we have broken this single system to three systems, one two three. This is a cantilever kind of boundary condition, this also cantilever kind of boundary condition.

This we know is I p 3, this is I p 1, this is having I p 2 out of inertia and these lengths, if we know at present they are unknown, the node position is unknown. So, if we know that we can able to obtain the natural frequency of this system, this system and this system and because we have broken them from an original system, natural frequency of all these three system should be same and should be equal to the original system. So, this is the basic idea by which we can able to obtain the natural frequency of the original system.

So, here the main aim is to obtain what are these segments, these lengths of the shaft segment. If we can obtain these line segment length then we can able to obtain the natural frequency of either of these natural system which and all are belonging to the same system. So, there natural frequency should be same as the original system. Now, we will be writing the natural frequency of these systems for unknown length of the shafts and from there we will try to obtain the length of these shaft segments.

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$$\omega_{nf_A}^{(2)} = \sqrt{\frac{k_{t_1}^{(2)}}{I_{p_1}}} = \sqrt{\frac{GJ}{l_1^{(2)}} \frac{1}{I_{p_1}}}$$

$$l_1^{(2)} = \frac{I_{p_3}}{I_{p_1}} l_3^{(2)}$$

$$\omega_{nf_C}^{(2)} = \sqrt{\frac{GJ}{l_3^{(2)}} \frac{1}{I_{p_3}}}$$

$$\omega_{nf_B}^{(2)} = \sqrt{\frac{k_{t_2}^{(2)}}{I_{p_2}}} \qquad k_{t_2}^{(2)} = \frac{GJ}{l_1 - l_1^{(2)}} + \frac{GJ}{l_2 - l_3^{(2)}} = GJ \frac{l_1 + l_2 - l_1^{(2)} - l_3^{(2)}}{\left(l_1 - l_1^{(2)}\right)\left(l_2 - l_3^{(2)}\right)}$$

$$\omega_{nf_B}^{(2)} = \sqrt{\frac{GJ}{I_{p_2}} \frac{\left(l_1 + l_2 - l_1^{(2)} - l_3^{(2)}\right)}{\left(l_1 - l_1^{(2)}\right)\left(l_2 - l_3^{(2)}\right)}}$$

$$\frac{1}{l_1^{(2)} I_{p_1}} = \frac{1}{I_{p_2}} \frac{\left(l_1 + l_2 - l_1^{(2)} - l_3^{(2)}\right)}{\left(l_1 - l_1^{(2)}\right)\left(l_2 - l_3^{(2)}\right)}$$

So, this is the natural frequency of the cantilever, first one which is left side, left hand, left side of the figure. So, this is the I p 1 and this is the torsional stiffness of the shaft segment of the cantilever thing and in this, this length is unknown. This l 1 is unknown to us, but other properties we know. So, this is the natural frequency expression for the left hand side cantilever beam rotor system, single disc rotor system; this is for the right side, the extreme right side corresponding to the disc three.

So, I p 3 and you can able to see this particular length is the shaft segment for the that particular cantilever beam in the right side. This also unknown, then the third natural frequency that is for the middle rotor system, which was having boundary condition of fixed fixed condition and the disc in between. If we do not have the idea about the length because these lengths are unknown, so this natural frequency, we are acting as the torsional stiffness of the shaft and the polar moment of inertia at the disc two. So, for fix fix case this torsional stiffness of the shaft in terms of unknown lengths, we can able to obtain like this. We can able to see that l 1 is the total length of the left hand side of the shaft segment.

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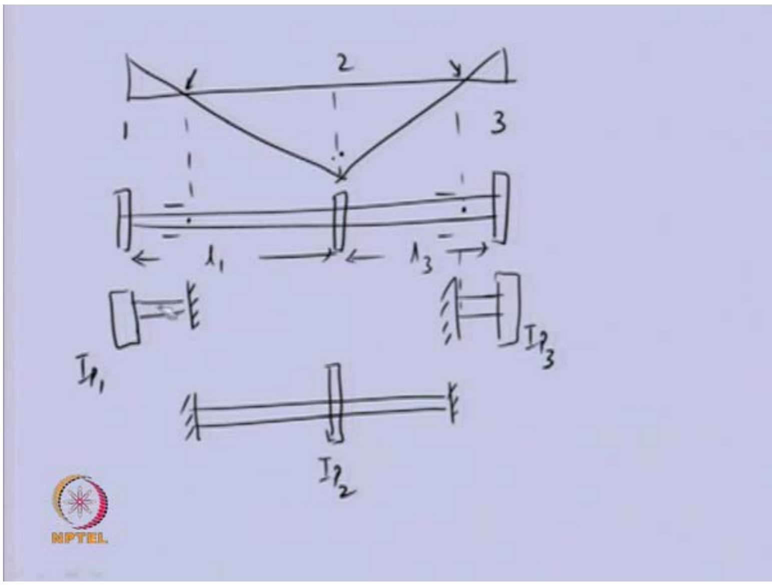

l 1 is from here to here and this is l 3 and if we know this we can able to get this by subtracting l 1 minus this length. So, the same thing we have done here. So, this is the torsional stiffness of the shaft and this we can able to substitute here to get the natural frequency. So, here we can able to see that l 1 of subscript 2, l 3 subscript 2 is unknown and basically if you see overall here one is this natural, this equation, the second equation and this is the third equation. So, we have three equations.

They are natural frequency expression and they belong to the same natural frequency. So, that means we can able to equate the equation one and two. If we equate this two you can able to see that you will get this relationship between the l 1 subscript 2 and l 3 subscript 2.

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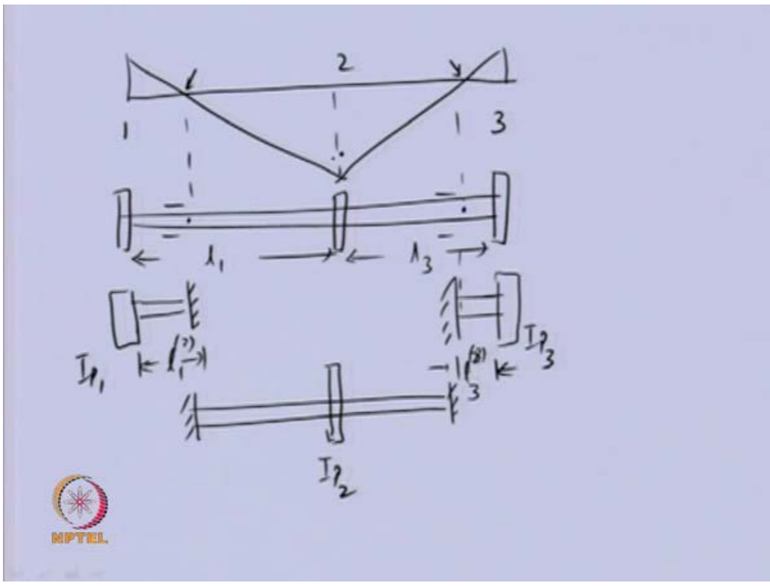

So, these are the shaft segment lengths. So, this is l 1 subscript 2 and this is l 3 subscript 2. So, we got one relationship between them. Then if we equate the first one and the third one we will get this expression. So, this is another expression which is relating the l 1 subscript 2 and l 3 subscript 2 and using this in this equation we can able to eliminate one of the unknown that is l 1 subscript 2. So, you can able to see we can able to substitute here in other places. So, we will left with only l 3 subscript 2 in this expression.

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$$l_1^{(2)} = \frac{I_{p_3}}{I_{p_1}} l_3^{(2)}$$

$$\frac{1}{l_1^{(2)} I_{p_1}} = \frac{1}{I_{p_2}} \frac{\left(l_1 + l_2 - l_1^{(2)} - l_3^{(2)}\right)}{\left(l_1 - l_1^{(2)}\right)\left(l_2 - l_3^{(2)}\right)}$$

$$\left\{\frac{I_{p_2} I_{p_3}}{I_{p_1}} + \frac{I_{p_3}^2}{I_{p_1}} + I_{p_3}\right\}\left(l_3^{(2)}\right)^2 - \left\{\frac{I_{p_3} I_{p_2} l_2}{I_{p_1}} + I_{p_2} l_1 + I_{p_3}\left(l_1 + l_2\right)\right\} l_3^{(2)} + \left\{I_{p_2} l_1 l_2\right\} = 0$$

So, if we do that we will get. So, this is the summary of… So, from first two equation we got this relation, from first and three we got there relation and if we eliminate l 1 subscript 2 form this using this, we will get a polynomial in terms of the l 3 subscript 2. So, we can able to see that, we can able to solve this polynomial for this segment. If we get this segment l 3 subscript 2 using this we can able to get the this also. Once we know these two value we can come back to the figure; you can able to see that if you know this and this or we can able to get this expression, this shaft length and this shaft length.

So, we will be basically, we have, we have obtained the node positions now and using these we can able to get the natural frequency of the system. So, any of these we can able to use it to obtain the natural frequency. All these will give the same natural frequency. Now, through one simple example we will try to see this three disc rotor system, how it can be analyzed.

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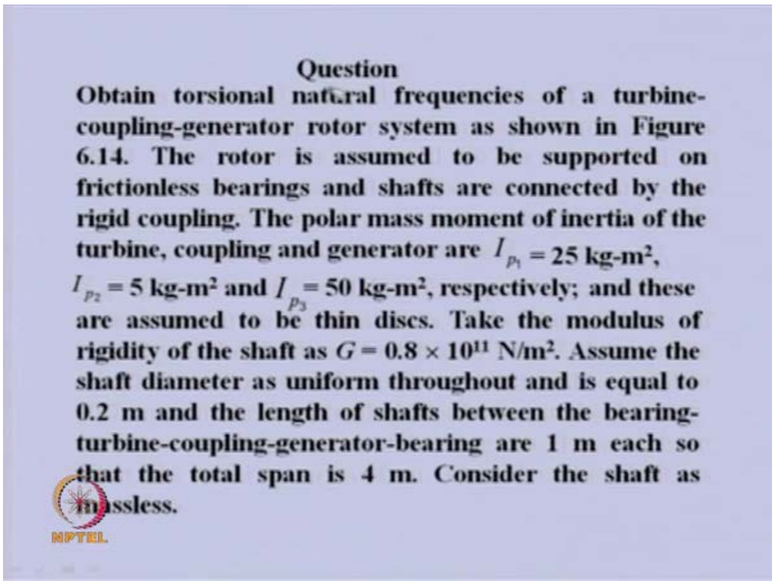

We have, we are interested in the torsional natural frequency of a turbine coupling generator rotor system. This is shown here.

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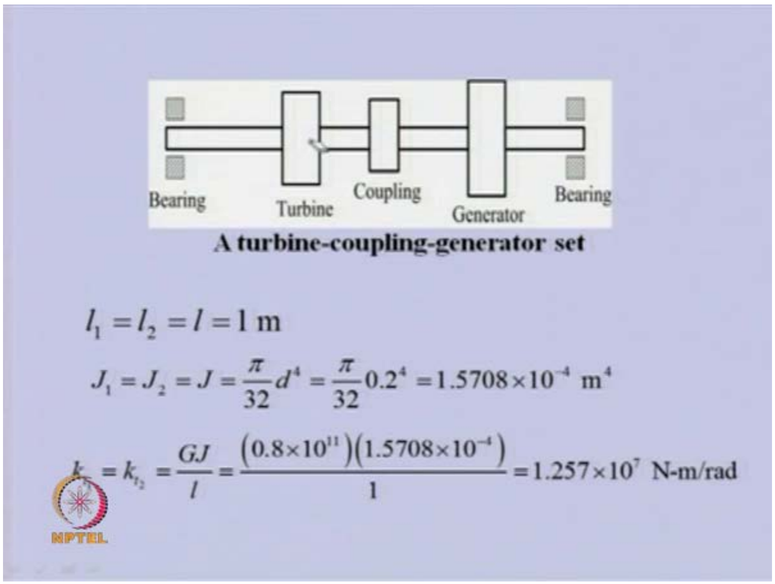

This is a turbine, this is a coupling generator as such is having free bound support. So, as such they are not imparting any torsional vibration or torsional torque, external torque to the rotor system. Various property of the three major mass of the turbine and the coupling and the generator is given here. The shaft property is also given here. We are assuming that the shaft is uniform and as such they do not have inertia and bearing is having no frictional torque on to the rotor and the length of the, this particular shaft segment between the turbine and the coupling and between coupling and the generator is 1 meter.

And as such for the method in direct method which we have studied in which from the possible flexible modes of the system directly we try to obtain the node location and from node location we could able to obtain the natural frequency at the system. So, let us see through an example how it can be done. The example is the same as we, how done previously.

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$$\left[\frac{I_{p_2} I_{p_3}}{I_{p_1}} + \frac{I_{p_3}^2}{I_{p_1}} + I_{p_3}\right]\left(l_3^{(2)}\right)^2 - \left[\frac{I_{p_2} I_{p_3} l_2}{I_{p_1}} + I_{p_2} l_1 + I_{p_3}(l_1 + l_2)\right] l_3^{(2)} + I_{p_2} l_1 l_2 = 0$$

$$\left[\frac{5\times 50}{25} + \frac{50^2}{25} + 50\right]\left(l_3^{(2)}\right)^2 - \left[\frac{5\times 50\times 1}{25} + 5\times 1 + 50\times 2\right] l_3^{(2)} + 5\times 1\times 1 = 0$$

$$160\left(l_3^{(2)}\right)^2 - 115 l_3^{(2)} + 5 = 0$$

$$l_3^{(2)} = \textbf{0.6723 m and 0.04648 m.}$$

So, for this particular thing we have seen that we get a polynomial using the indirect method like this and if we substitute various values of the polar moment of inertia and in this expression we will see that this polynomial will be reduced to this expression. Here, from here we will get two roots because this is a quadratic in l 3 2, you will get two roots.

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$$l_1^{(2)} = l_3^{(2)} I_{p_3} / I_{p_1}$$

$$(l_1^{(2)}, l_3^{(2)}) = (0.09297, 0.04648) \text{ m}$$

and

$$(l_1^{(2)}, l_3^{(2)}) = (1.3446, 0.6723) \text{ m}$$

Corresponding to these two roots using this expression we will get two possible values of the l 1 subscript 2. So, we can able to see that we have two combinations of the l 1 subscript 2, l 3 subscript 2. This is one possible combination and this is another possible combination. So, now let us try to see what are these values, they represent in the rotor system.

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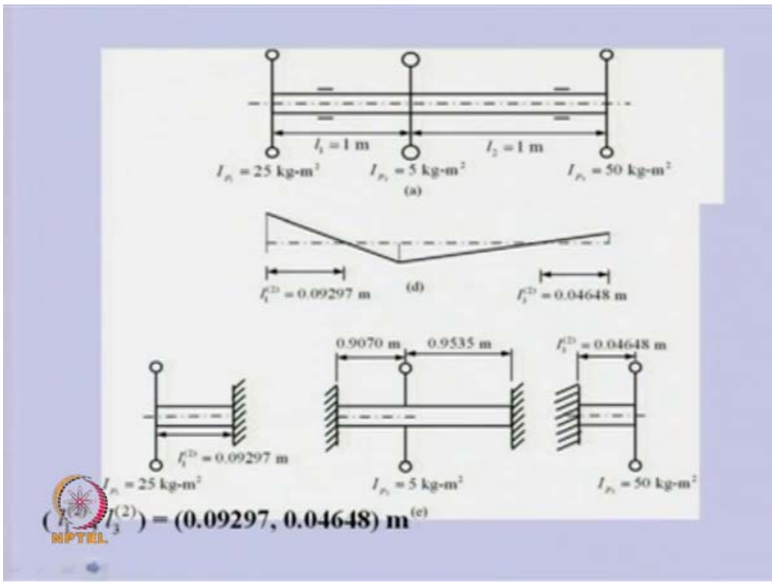

So, for first case that is first possible values l 1 is this one and l 3 subscript 2 is this one. So, basically these are the, this distances this one and this one. So, you can able to see that if node is here and node is here, this system can be broken into three single rotor system. Now because we know the length of this, so natural frequency of this can be obtained or natural frequency of this can be obtained, even this can be obtained and this should be same as the natural frequency of the system.

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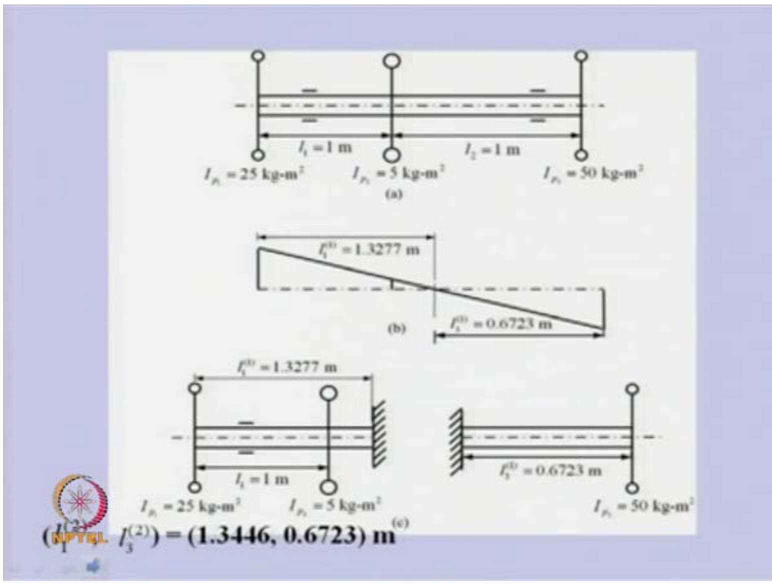

Similarly, for the second communication of the l 1 subscript 2 and l 3 subscript 2 we have these values. For this particular case you can see that this is the node position. In this particular case there are two possible, so 1.3446. So, that is if we take 1.3446 it will. So, this is one of the system and other end this is 0.6723 from the right side. So, this is another node. So, from this we get this particular single system, single degree of freedom.

You can able to see that from 1.3446 basically we are getting the node away from this here. So, that is not possible that we have two nodes in between the two shafts or two discs because we are not considering the shaft mass. So, this particular combination in which, this is visible one because we can have in node here, but another 1.3446 is coming here only. So, it is not possible that we can have two nodes in between these two discs because we are not considering the inertia of the shaft.

So, this is having no relevance, only this is possible. So, if we take that. So, that is this will be cantilever beam of this and we can able to obtain the natural frequency of this corresponding to this length. That will give us the second natural frequency. So, basically from the same solution, we are getting both the flexible natural frequency. One is from this and another from this or even we can able to, if we know the natural frequency expression. For this two degree of freedom system, this will also be having same natural frequency as this one or this one for the second mode in which only single node is possible.

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$$\omega_{nf_3}^{(2)} = \sqrt{\frac{GJ}{l_3^{(2)}} \frac{1}{I_{p_3}}} = \sqrt{\frac{\left(0.8\times10^{11}\right)\times\left(1.5708\times10^{-4}\right)}{0.04648} \frac{1}{50}} = 2325.34 \text{ rad/s}$$

$$\omega_{nf_2}^{(1)} = \sqrt{\frac{GJ}{l_3^{(1)}} \frac{1}{I_{p_3}}} = \sqrt{\frac{\left(0.8\times10^{11}\right)\times\left(1.5708\times10^{-4}\right)}{0.6723} \frac{1}{50}} = 611.42 \text{ rad/s}$$

So, let us obtain. So, this is one of the natural frequency corresponding to two nodes and this corresponding to one node. So, you can able to see that we could able to obtain not only the two node natural frequency as well as the one node frequency from the two possible values of the lengths which we obtained from the quadratic. In the previous lecture we discuss about the step shaft. Now, through one numerical example, let us see how that particular rotor system in which step shafts are there, natural frequency can be obtained?

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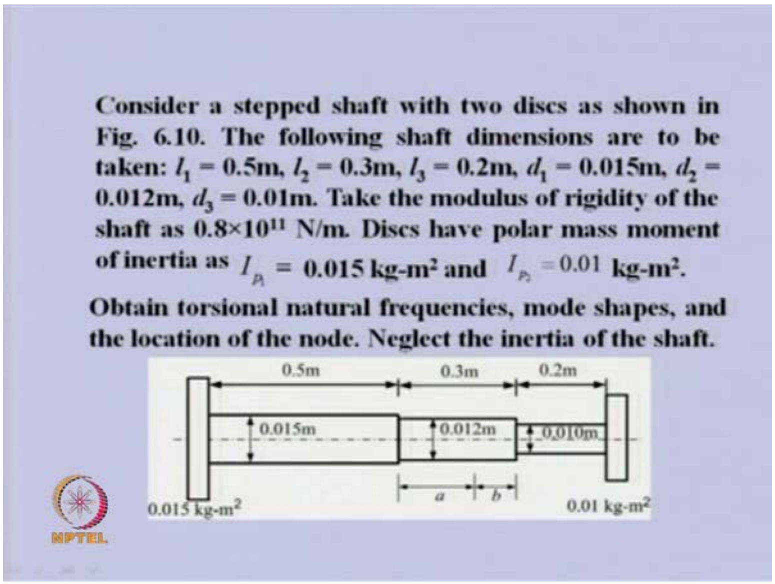

So, in this particular case we have considered a step shaft like this where its dimensions are shown here, length and diameter. We are not considering such inertia of the shaft. Only discs are having inertia and all the dimensions and parameters are as given in this. Now, our aim is to obtain the natural frequency of this two rotor system or system.

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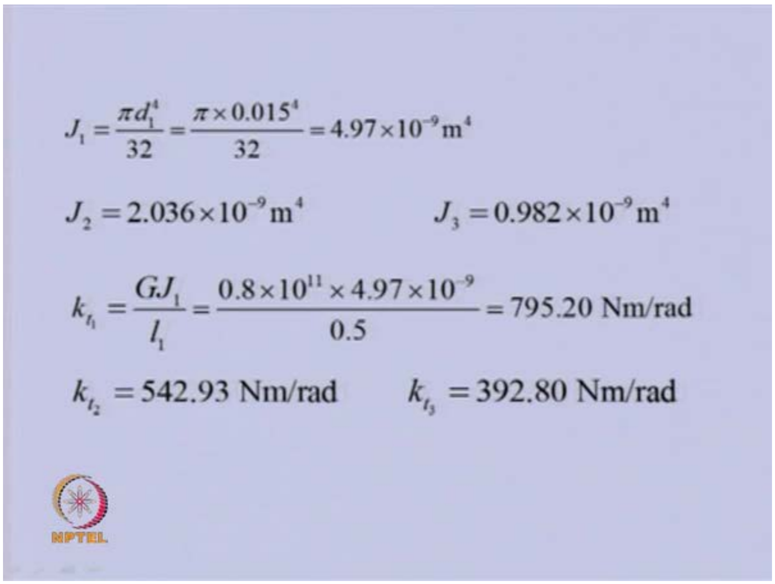

So, various geometrical parameter from the geometry of the shaft can be obtained. Torsion of the first shaft segment, second shaft segment, third shaft segment because they have different steps. So, they can be obtained.

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$$l_e = \frac{l_1}{J_1}J_e + \frac{l_2}{J_2}J_e + \frac{l_3}{J_3}J_e = \frac{0.5}{4.97}0.982 + \frac{0.3}{2.036}0.982 + \frac{0.2}{0.982}0.982$$

$$= 0.0988 + 0.1447 + 0.2 = 0.4435\text{m}$$

$$l_{e_1} = 0.0987$$

$$k_{t_e} = \frac{GJ_e}{l_e} = \frac{0.8\times10^{11}\times0.982\times10^{-9}}{0.4435} = 177.14 \text{ Nm/rad}$$

$$\omega_{nf_2} = \sqrt{\frac{\left(I_{p_1}+I_{p_2}\right)k_{t_e}}{I_{p_1}I_{p_2}}} = \sqrt{\frac{(0.015+0.01)\times177.14}{0.015\times0.01}} = 171.82 \text{ rad/s}$$

So, once we have these we can able to obtain the equivalent length of the shaft corresponding to one of the shaft diameter, which we have taken as the third one. So, corresponding to this, this is the equivalent length of the shaft. Now, we can able to obtain the equivalent stiffness, corresponding to the uniform shaft having diameter same as the third shaft step. So, this is the equivalent stiffness of that and we can able to obtain the natural frequency specially the flexible one because the first is 0 we are not interested in that, we are interested in the flexible natural frequency. So, that is flexible mode natural frequency. So, this is given by this expression in which this is the equivalent stiffness of the uniform shaft. So, this is the natural frequency of the system.

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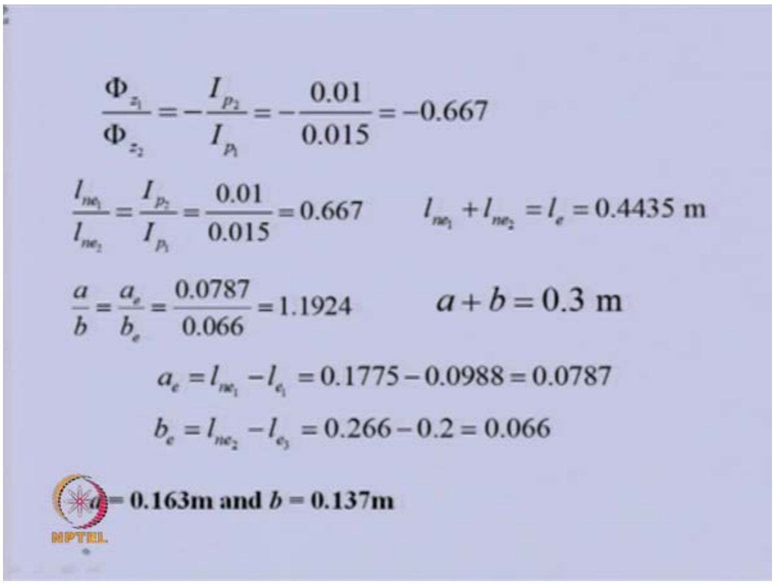

So, once we obtain this the node aim is to have the node location. Where is the node? So, for that we know the relative amplitude is given by this expression. So, this is the ratio of the two amplitudes. The node location ratio is given by this expression and we know that the total length of the either side of the node in the uniform shaft is having value equal to the equivalent length. So, from these two expressions, we can able to obtain the node location.

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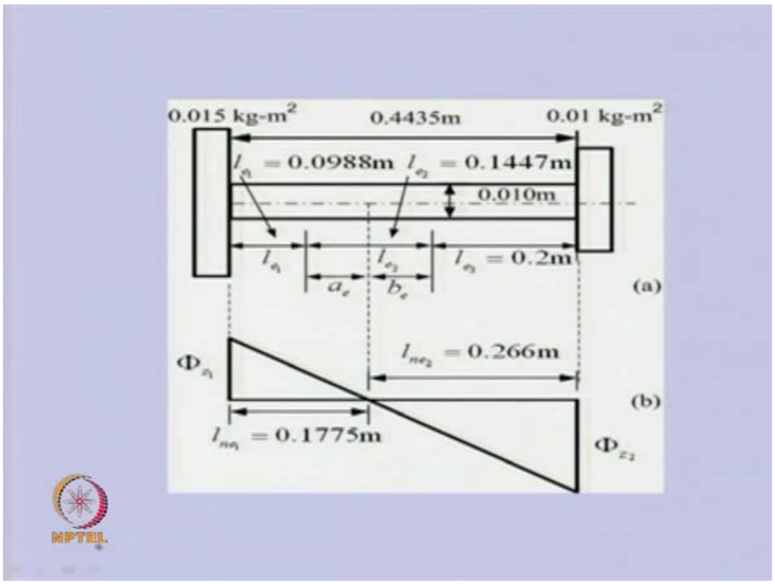

So, if we see the figure, this is the equivalent uniform shaft and we have node location here. Now, we have located the node also. This length we know. So, we know the node location which is for this particular problem coming into the second step position. Now, corresponding to this; actually this is corresponding to the second shaft step and node is here. So, from let us say this end, the position of the node is a e and from this is b. This ratio as we have seen the analysis.

Now, we know these ratio. In original system the node location will be in proportional to this equivalent shaft system. So, from here you can able to see that this ratio is this one and total length of the original step shaft of the second shaft step is 0.3. So, from this we can able to get the A and B and using these two expression we can able to obtain what is the actual position of the node in the step shaft. So, in the present lecture we have mainly analyzed three rotor system.

We consider two approaches, one by obtaining the equation of motion. We obtain the natural frequency and mode shape of the system and by indirect approach in which we use the mode shape information and we try to obtain the node position. From there we obtain the natural frequency of the system. We solve couple of numerical example. So, that the method is more clear and not only for the three disc system, also for two disc system which we covered in the previous lecture that also we are, we are seeing through the mechanical example.

Now, as the number of degree of freedom or the number of disc in the rotor system will be increasing we will face difficulty in solving the bigger polynomial as a of frequency equation or so here we feel that there is a need for more systematic approach by which even if we are handling a bigger system, we should able to solve using less difficulty. So, in the next lecture, we will see more systematic approach. There is a transfer matrix method, which is quite popular in torsional vibration of rotor system, especially when we have bigger systems.