Nonlinear Adaptive Control
Professor Srikant Sukumar
Systems and Control
Indian Institute of Technology, Bombay
Week 11
Lecture No: 64
Initial Excitation in Adaptive Control (Part 4)

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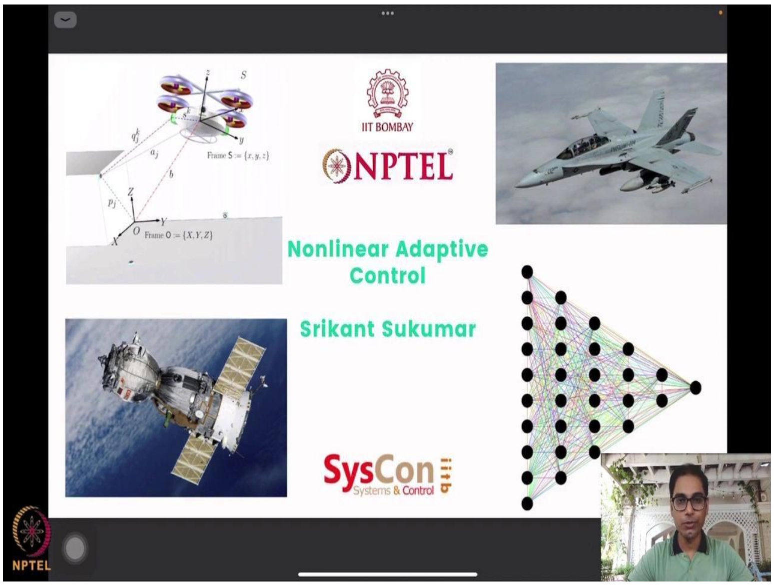

Hello, everyone, welcome to yet another session of our NPTEL on nonlinear and adaptive control. I am Srikant Sukumar from systems and control, IIT Bombay. So, we are into the last legs of this course. And we are in the middle of week number 11. So, well, I mean, in the background, what you see now is a bunch of different kinds of applications that adaptive control has found using these are things like, drones, fighter aircraft, space crafts, neural nets.

So, this is sort of the applications or at least to one set of applications that I am hoping that the algorithms that you will learned in this course will help you work with. I am of course, always fascinated by newer and newer applications, I have seen quite a bit of automobile system applications also. And I am always open ears to know what you are working on. So, please ping me and let me know what sort of things that you are using these adaptive control ideas for. And, I mean, I can say for sure, it will be very interesting for me to learn.

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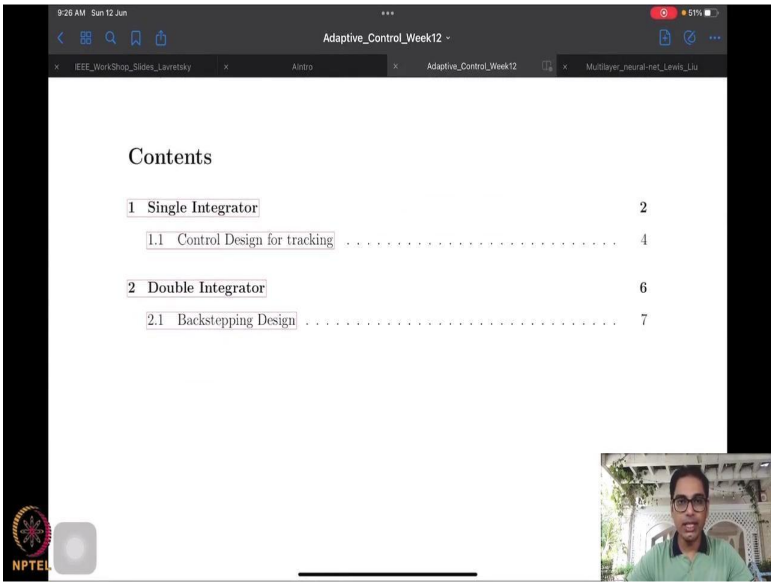

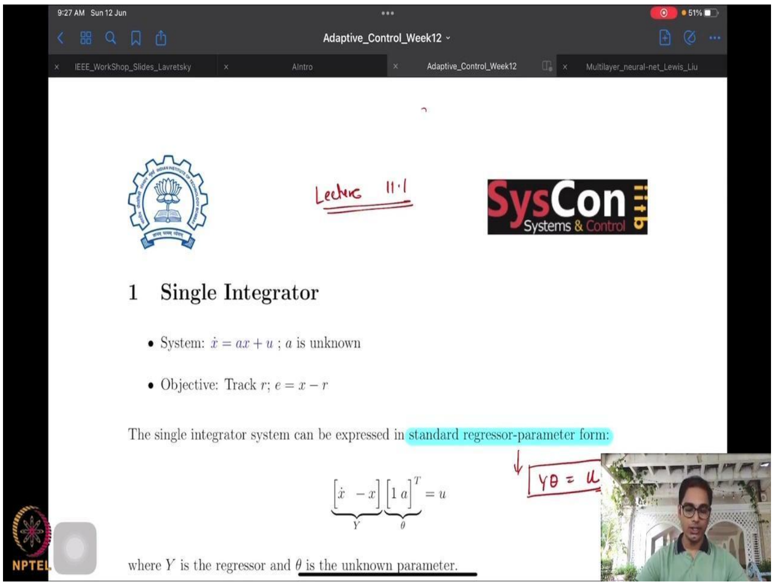

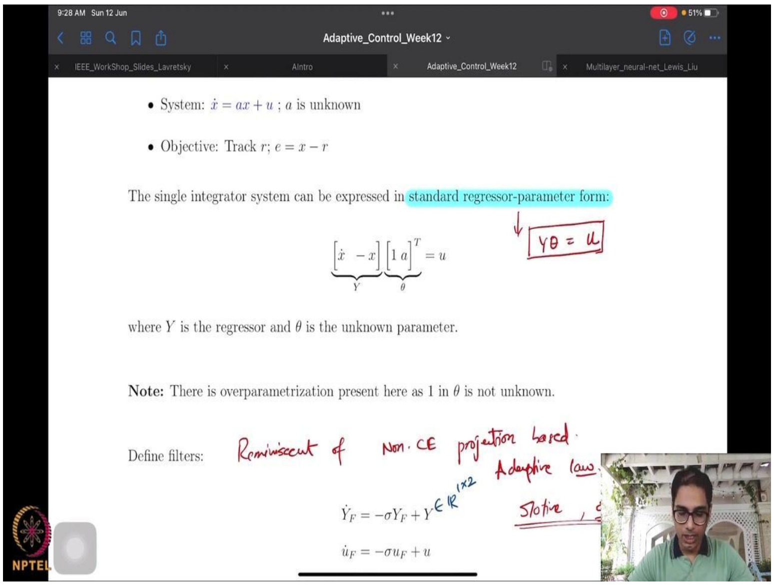

Now, what we are doing, since the beginning of this week is the initial excitation based adaptive control. So, we already know the standard adaptive control for parameter learning requires the knowledge requires persistence of excitation condition, we already know, this is more like an, a condition on identification and not something or, in fact, something that precedes, adaptive control itself, and not something that can be easily gotten rid off.

So, in order to make it a little bit more realistic, several researchers worked on trying to relax this condition. And one such relaxation is what we are looking at right now. And this is based on using only initial excitation instead of persistent excitation. So, the idea being that only for an initial phase of time, you have sufficient excitation and beyond that, you do not. And so the question is, then can you still, achieve your parameter convergence and parameter learning in those cases. So, this is the general idea. Now, we started with looking at a single integrated system. And this was, was rather interesting, I hope, we do not write it as a differential equation, but we write it in this regressive parameter form.

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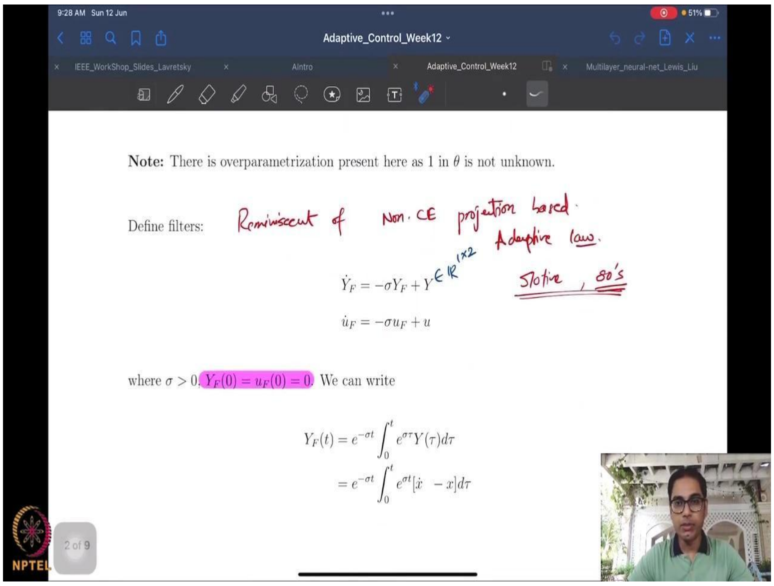

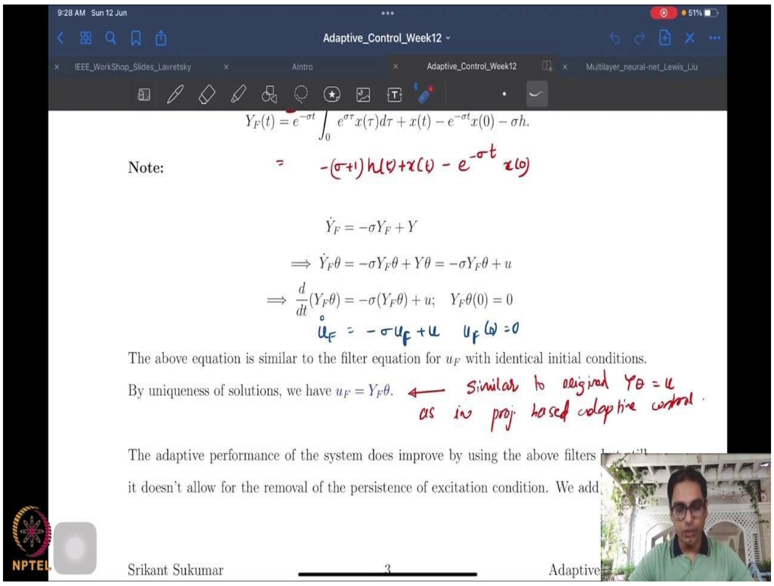

And then we go on to look at development of these multiple layers of filters and so, we have 2 filter layers here. And we have one filter layer, which is the YF, we talked about implementability of this filter, and then connect the filtered variables also, just like we have a connection between the standard, the original variables, Y and u.

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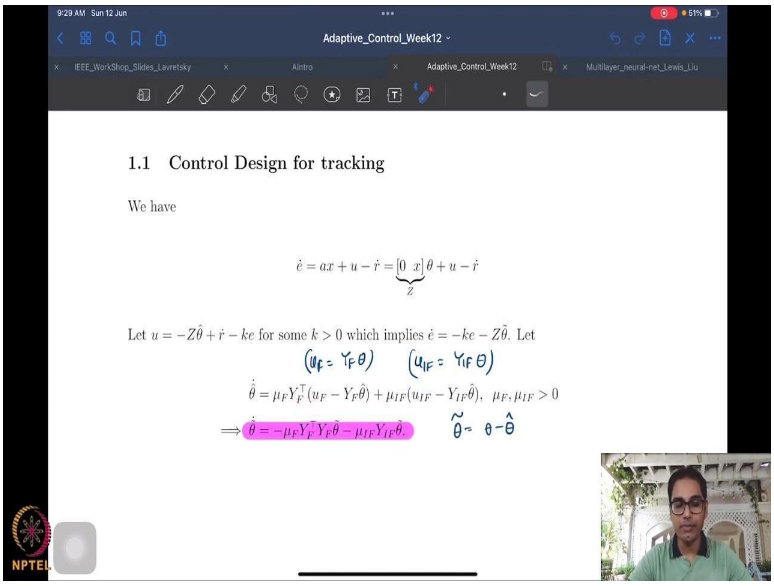

And, of course, then finally, we design a second filter layer. Once we did the second filter layer design, we then started to address the tracking problem. Until that point, we did not even really care about the tracking question itself. So, we, of course, design this parameter update without any Lyapunov analysis at this point, because we are rather comfortable with the fact that there is nice negative terms appearing here. Which is unlike what you get in standard adaptive control laws.

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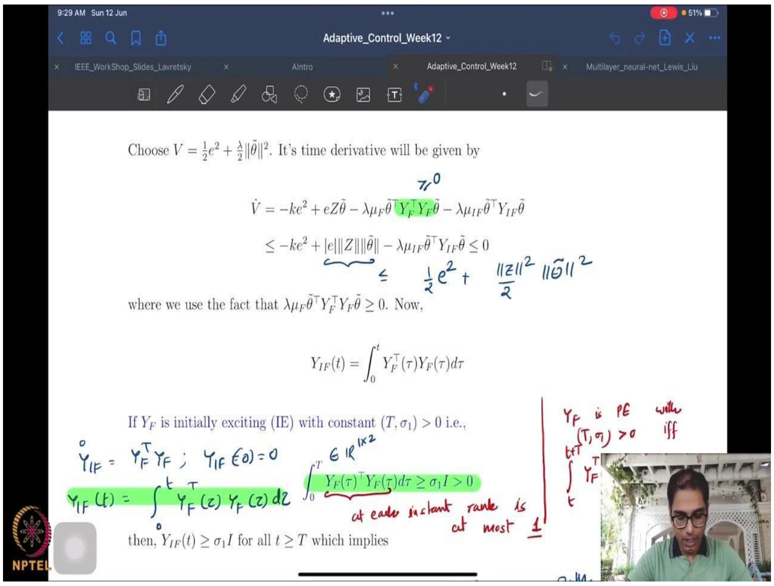

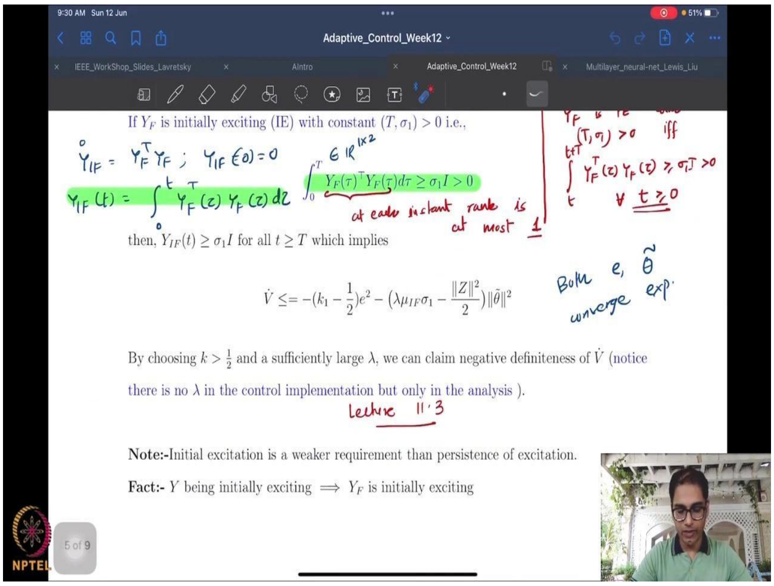

Now, once you have, this sort of a situation, we then choose our control also. So, the controller is also already chosen. And so, all we are left with now is to do the analysis. This of course, we remember is reminiscent of the other non-certainty equivalence method we looked at which was in connection with parameter projection. So, I hope you can connect these dots. So, we do the analysis, we, we of course, have this dummy variable lambda, which we use to dominate things, again, similar to what we had earlier.

And finally, we said that if there is this initial excitation condition, which was of course, much weaker than the persistent excitation condition, we can show negative definiteness. So, we had this rather nice property. I mean, and whenever you see something nice, I hope just like I do, I try to critique it, and I try to figure out where the issues are, what is good, what is bad not just that, I mean, of course, there is always, there is a very, very powerful method.

I have advocated it, myself and I strongly recommend that you look at these authors other works, because although there are other composite adaptive controllers, which use similar ideas, in my opinion, these authors seem to have done a rather simple, intuitive way. So, of course, we said that you do not have to verify the initial expectation for YF itself, it is sufficient verified it for the original regressive Y.

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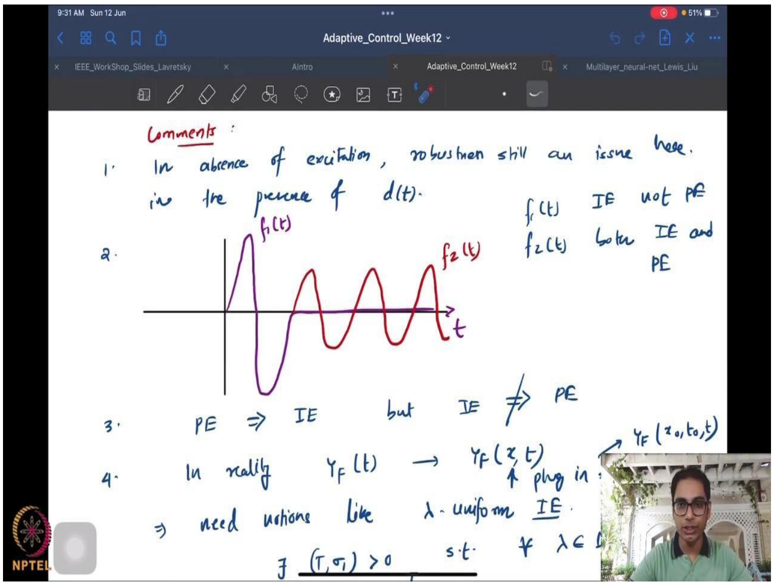

Now, few comments were in order. So, it is also what we saw first is that there is no excitation robustness is still an issue will not handle the robustness issue, for that you still need to do projection or sigma modifications and things like that. Second, of course, we showed how the IE condition is significantly weaker requirement.

And of course, we also talked about the fact that the initial excitation condition is not uniform. In fact, the condition because the regressors typically, almost always depend on initial conditions, because it depends on the state. So, of course, it depends on the initial conditions, you need parameter dependent version of this YF, we looked at this in the integral lemma discussion, a while ago, we talked about persistent excitation. But in this context, also, the same idea holds.

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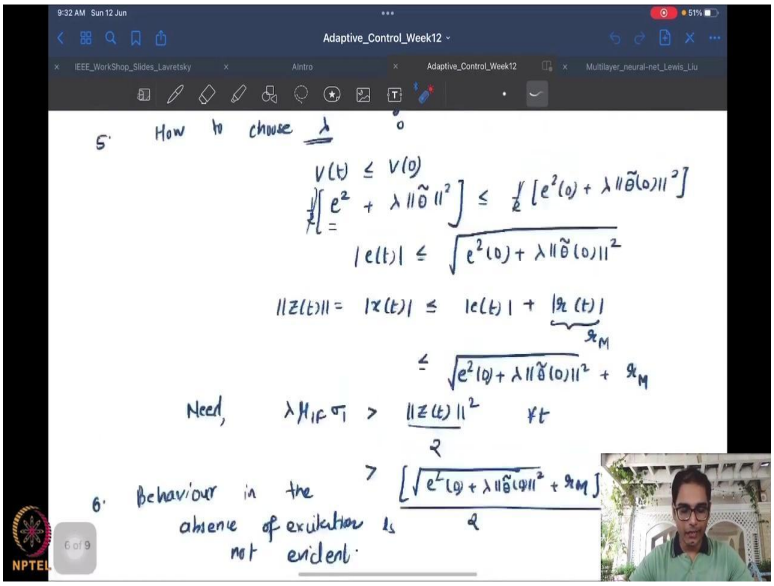

And finally, not finally, well, we also looked at how, or what would be a possible way of choosing lambdas. And we saw that this is not super easy. It is sort of a nonlinear question. The good thing for us is that such a lambda existence of such a lambda is significantly easier to show. And we do not care about actually choosing lambda because it does not appear in the control. The final thing we sort of mentioned and we discussed a little bit is that behavior in the absence of excitation is not very evident. So, this was one of the more key points, I would say.

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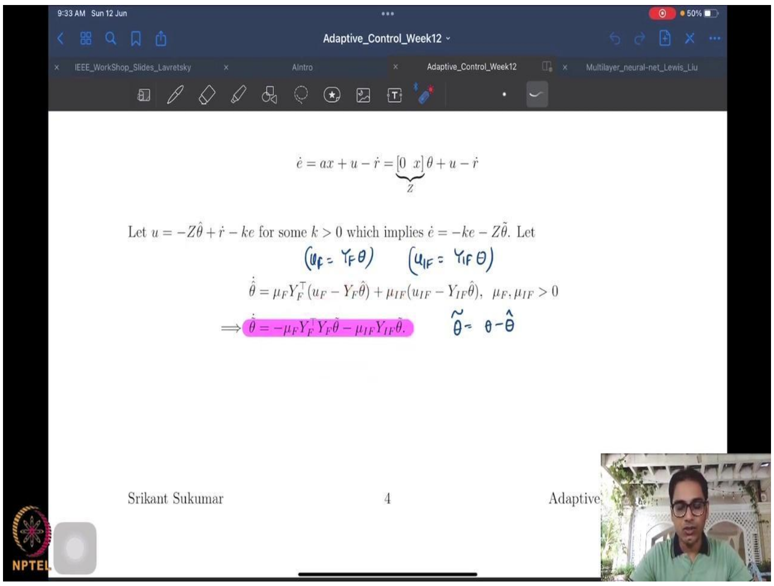

Because if you look at the update law, when you see that both these terms are very excitations dependent. The first term sort of depends on the YF transpose YF it is only semi definite, so the definiteness of this, of course, is not guaranteed. And the definiteness of this is guarantee with the residual excitations. But if that is not, then there is also not guaranteed to be definite, none of these are actually negative definite. And so, how will it perform.

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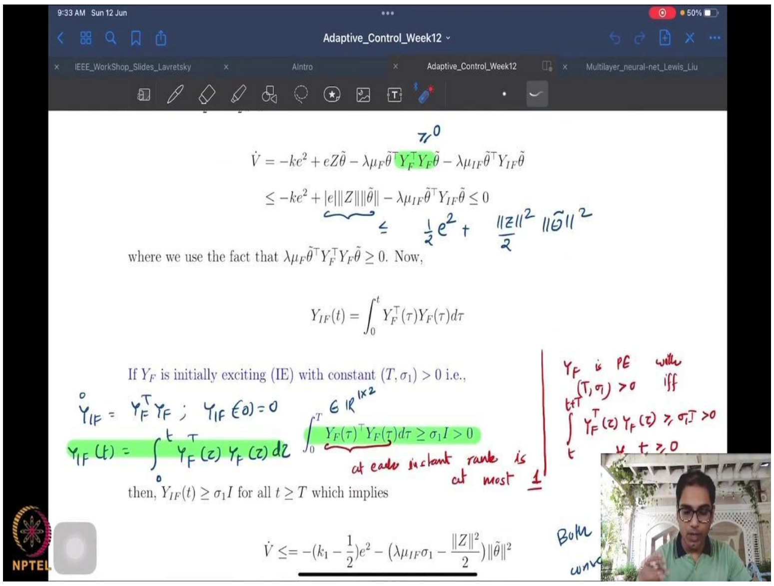

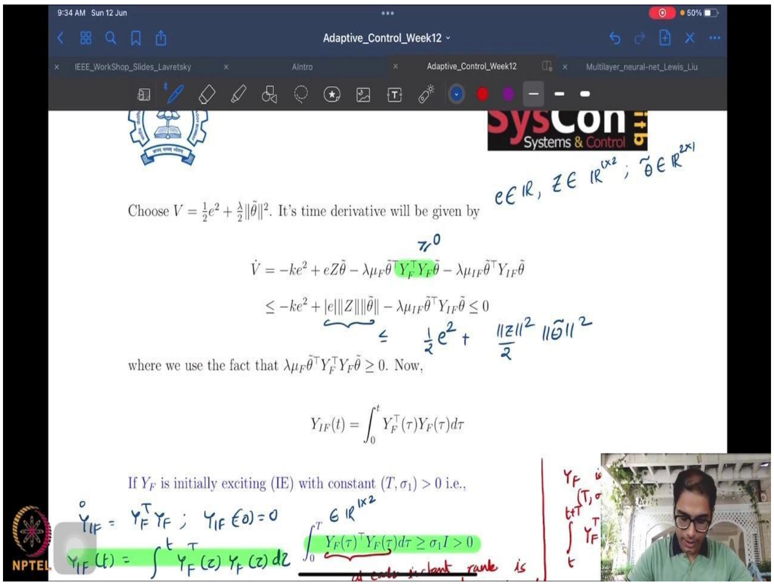

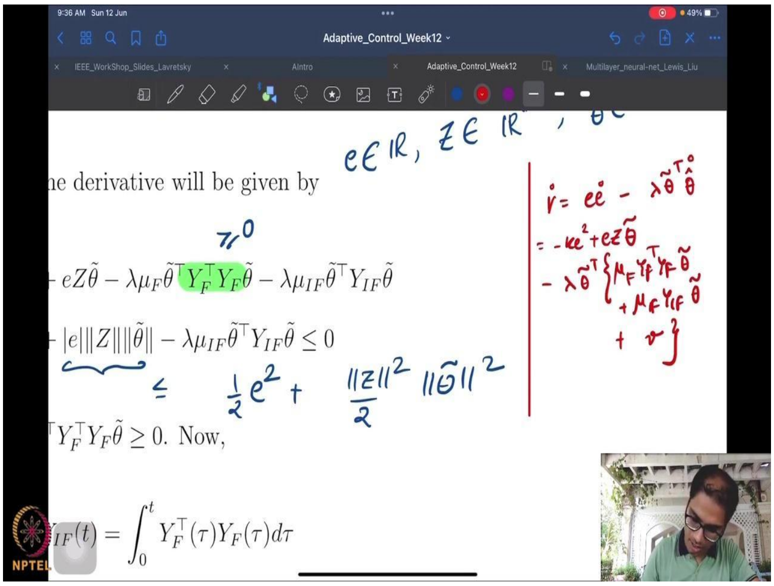

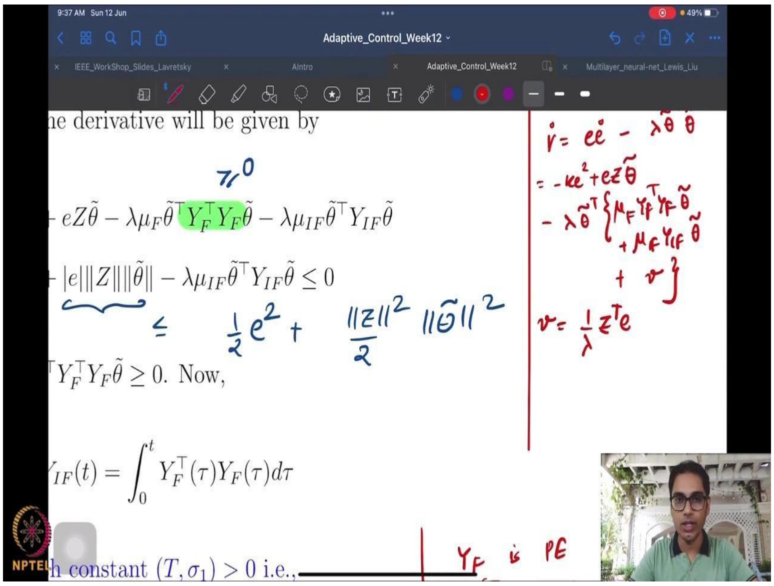

Because if you look at the Lyapunov analysis, this entire Lyapunov argument is based on trying this term trying to dominate this guy. So, you try to break this into squares, and then you use this guy to dominate this. So, our entire argument may be in jeopardy if you do not have initial excitation. Now, what happens how things are different in standard adaptive control? Is that there is neither of these 2 neither is there a negative definite term. So, it is a bit sad, but there is also not a mixed term in the unknown. Because this mix term essentially gets cancelled using your standard adaptive of course one, I mean, I would say once are easy sort of way to deal with it.

One easy sort of way to deal with it. Let me see. I was just wondering about the orders here. So, E of course, so, let me look at the order of things. So, E is here the scalar. Z here belongs to R 1 cross 2, and theta tilde belongs to R 2 cross 1 or R 2. So, now, one of the things that I could have done, even look at the analysis here from in V dot, I would, so I am going to put this as an aside. So, let me draw it a little bit more straight using my zoom I am going to use a bit of an aside. So, if you look at my V dot my V dot is e, e dot minus lambda theta tilde transpose theta hat dot.

So, now, I already know what is e dot so I substituted so, minus Ke square plus eZ theta tilde is what I will get from here. And from here, I get minus lambda theta tilde transpose and theta hat dot was all this mu F YF transpose YF theta tilde plus mu iF YiF theta tilde. And suppose I have plus some additional term mu at my disposal. And if I do have this, what I can show here like what I can show is that if I choose my mu let see choose my mu as 1 over lambda Z transpose e, then I am sort of fine. So, that is the idea.

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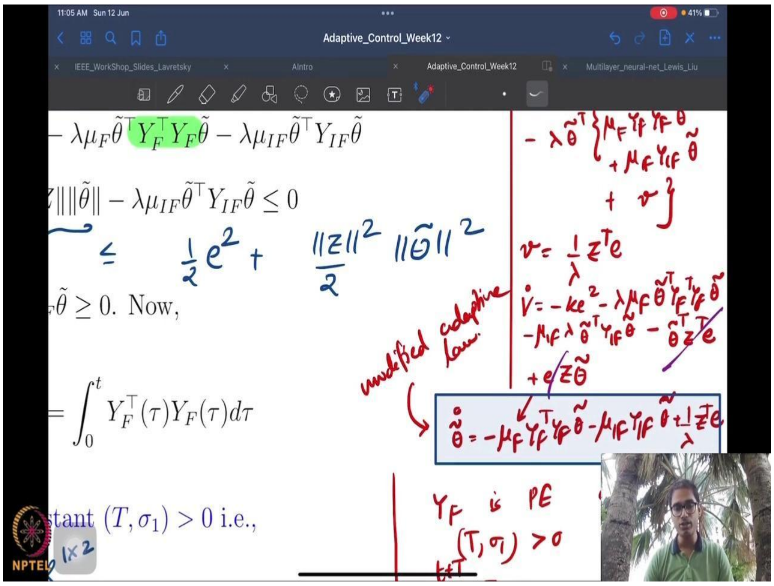

So, what will happen if you make this kind of a choice of V is that if I now write this expression for V dot like this, if you see all have done plug in for the V in here, and if I write this entire expression for V dot, I have this minus Ke squared from here and I have the 2 negative terms from here and subsequently, I will have this guy contributing to a term this one and then this mix term comes back here.

Now, if you notice, we have already mentioned that all terms in V dot are scalars. So, we can take as many derivatives as we solve as many transposes as you want and nothing changes, because the transpose of a scalar is the same scalar so, you can very easily take a transpose of this guy, and E is of course, a scalar. So, e transpose is just E and you will see that this transpose of this exactly matches this term. So, of course, these 2 terms cancel out and what is this nice thing that we obtain, because of this is that there is no mixed term anymore, there are only non-positive terms in the V dot.

Because this is non-positive, this is non-positive and this is non-positive, and so, there is nothing to dominate. So, of course, what is the outcome of this, theta tilde dot now has an additional term. And if you notice, this additional term is essentially the certainty equivalence adaptive update law. So, this is exactly what you would have done if you were not using this initial excitation based law.

So, the initial excitation based law now has terms in addition to what you would do in the certainty equivalence based adaptive law. Now, what happens because of this is that suppose there is no initial excitation either suppose our application is unfortunately such that there is no possibility of initial excitation, forget about persistent excitation. So, what happens is that these terms are then not necessarily definite, so these terms are not necessarily definite.

Now, what that means is that, you cannot really, they do not really contribute to the Lyapunov analysis, so, I might as well ignore them, because they are only negative semi definite at best. So, what this indicates, I mean, you can choose to ignore them or you can keep them and using the Barbalat's lemma but essentially what you will be able to obtain in the absence of excitation is that E goes to 0 because of this term, because, again, we do not repeat all this signal chasing analysis anymore.

We have not done that for a while, because I assumed that all of you have seen enough of the same steps to understand which terms will actually go to 0. So, I can very easily see that this term will go to 0 and the V dot, and you can of course prove that YF theta tilde goes to 0 inside also YiF theta tilde goes to 0, you can prove both of these. So, these may be some kind of attractive set, attractive invariant sets, if you remember, we talked about this attractive invariant set when we discussed this projection based adaptive control also.

So, similarly, you will also get some kind of an attractive invariant set in the absence of excitation. But of course, we still end up proving that E goes to 0. And this is what we do in standard adaptive control anyway. That we end up proving that the tracking errors go to 0. But of course, we cannot guarantee anything about the parameter estimation error is going to 0. So, learning of the parameter is of course, not guaranteed in the absence of any kind of excitation. But if you have even basic things something as basic as initial excitation, then you are guaranteed to have parameter convergence.

But in the absence of initial excitation, things don't go bad, you still have bounded results, that is your E and theta tilde will be bounded, because V dot is definitely semi definite negative. And on top of it, you will also get the tracking errors to go to 0.

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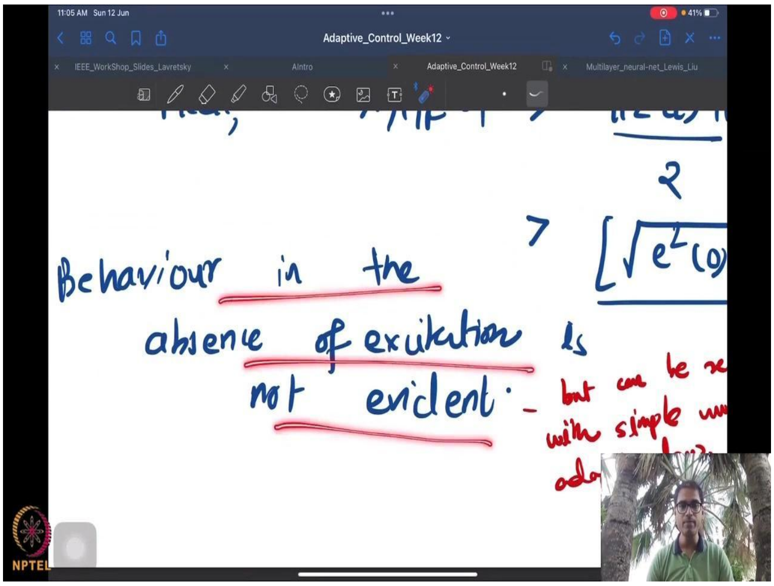

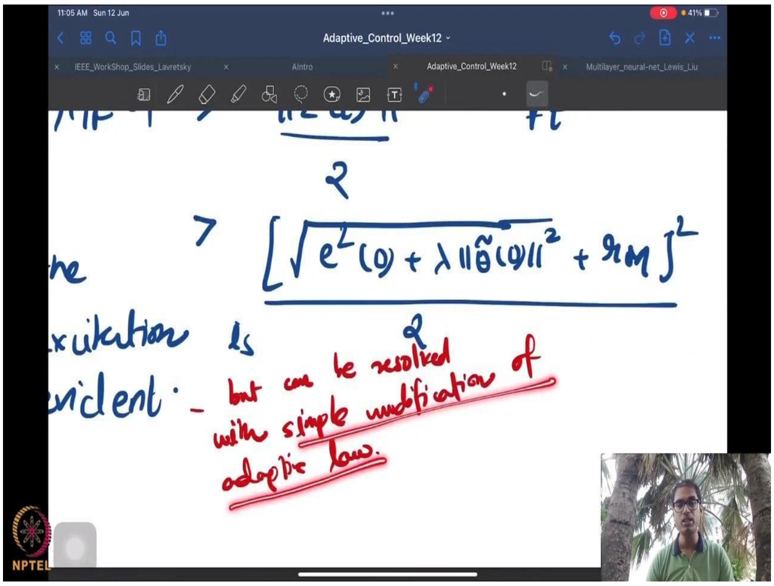

So, this last point that we made last time, that behavior in the absence of excitation is not evident, this can sort of be resolved very easily by making this simple modification of the adaptive law, this new adaptive law, as you notice, contains a mix of your initial excitation based adaptation law and your standard certainty equivalence adaption law.

So, I hope you understand that we can combine the power of these 2 methods that have the initial excitation based law, which is which has these filters and then the standard certainty equivalence adaptive controller to get the best of both worlds in the absence of excitation, I still get nice tracking performance and bounded parameter errors, and in the presence of initial excitation, there is some excitation for some initial time, I get parameter convergence also. So, this sort of last point is something we have this also.

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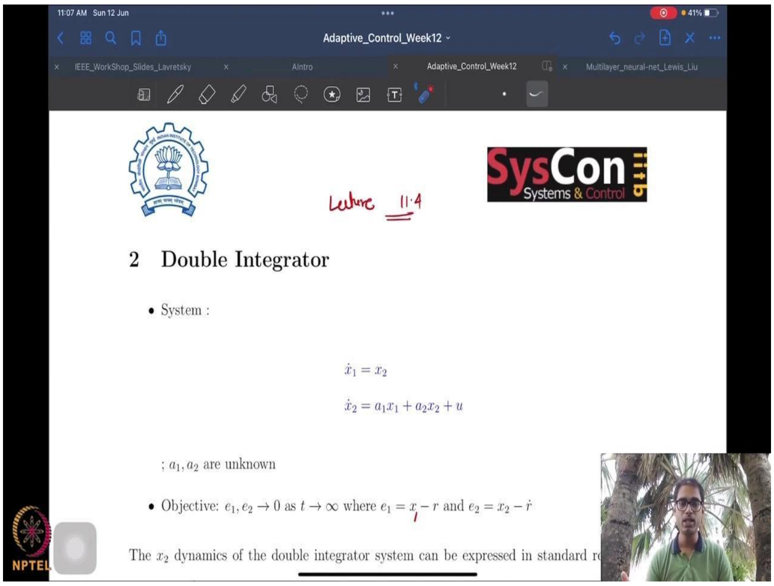

For that long initial, preamble, but it was rather essential to understand better, what initial excitation based adaptive laws are all about. So, whenever there is a new paradigm, it is important to first understand it well to see what are the good things it has and then see what are the limitations also. So, what we want to do now is to look at the combination of this backstepping based ideas that we have seen before with this initial excitation based adaptive law.

Now, I really hope that you remember that this initial excitation based adaptive control design did not really rely on the dynamic so much. So, one would hope that the dynamics itself should not really be impact, the initial excitation based design. So, but let us see how these 2 get combined, that is the backstepping based ideas and the initial excitation based adaptive controllers.

So, the first thing to sort of notice is that we are still going to be looking at the matched uncertainty case only, so we are not considering the unmatched case here. So, here we have the standard double integrator with unknowns, a 1 and a 2 in the control in the same equation. That is the matched case and the objective is, as usual drive these error e1, e2 to 0, where e1 is x1 minus r and e2 is x2 minus r dot.

So, before we go forward to doing any control design, just like before we do this, we write this system in this regressor parameter form. And what we do is we only look at this system, because this is the one with the uncertainty. So, we do not really care about writing this in any form. So, we only look at this piece of dynamics, and I am writing this in this Y theta equal to u form. So if I take these guys to the left, I have this Y theta equal to u, as usual, I have this old parameterization here. Because of this additional 1 that I put in here, and that is it. So, and Y is of course, x2 dot and minus x1 and minus x2, slightly that way.

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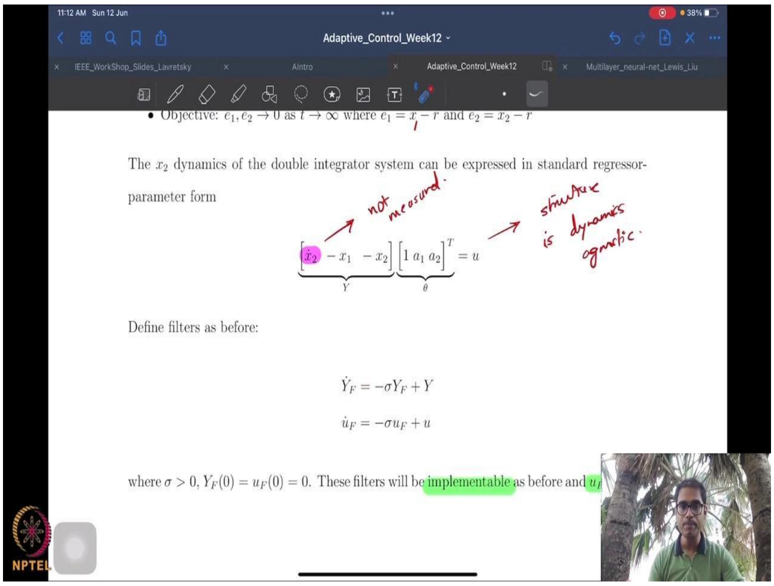

Now remember that, as usual, that this guy should mark it properly. This guy is not measured. So, this guy is not measured, this is pretty standard. Because what is this? If you think of this as a mechanical system, this would be like an acceleration term. And so, usually the assumption is that only the states are measured, and not the derivatives and so on and so forth. That is the usual assumption, of course, you do have mechanical systems where you have accelerometers and acceleration is directly measured, but that is not very common. So, x2 is not measured. Remember that.

So, this is exactly like the single integrator case where you did have one unmeasured quantity. But we did show that even with this unmeasured quantity, you can do this integration by parts and still implement the filter, and So, that is the same here. So, I define 1 layer of filters, just like before, which is the YF dot is minus sigma YF plus Y. And then I also define a filter on control because that is also a known quantity. So, the known quantities in this equation are Y and u that define filters on both.

And of course, the filter gain bandwidth is exactly the same. And we take some, of course, that has some positive gains sigma and initial conditions are assumed to be 0. So the important thing to remember is that, again, we are not showing it here. But these are implementable. The filters are implementable with the same logic as before. So, there are 2 pieces in Y, one piece contains these 2, which are, of course available through measurements. And then there is this guy, which is not available in measurements, so we just do an integration by parts to remove this and that is it. And once we get integration by parts, it becomes implementable.

The other thing to remember is that just like in the previous case, nothing has changed, I mean, only the definition of Y has changed, the structure of the regressor parameter form has not changed. And this is the cool thing. Reason Y this is sort of decoupled from the dynamics itself. So, the dynamics has changed and that change is encoded in the Y itself inside the Y, but the structure Y theta equal to u has not changed. So, that is what I would say that structure is dynamics agnostic.

So, it does not really depend on the dynamics of the system, the structure is Y theta equal to u this is the standard regressor parameter form, and because the structure is not modified in the filtered variables, you will still have the equation uF equals YF theta. So, this still holds nothing has changed here, this is nice. So, those are the important points. Remember, I still write it in this regressor parameter from Y theta equal to u, because of which I have uF is YF theta. And I designed the filters identically, though the dynamics has changed from even the order of the dynamics has changed.

The only thing is there is still, again, similar to the previous case, there is still one unmeasured quantity, but we deal with this using integration by parts. And so, the filter YF is still an implementable filter. Excellent.

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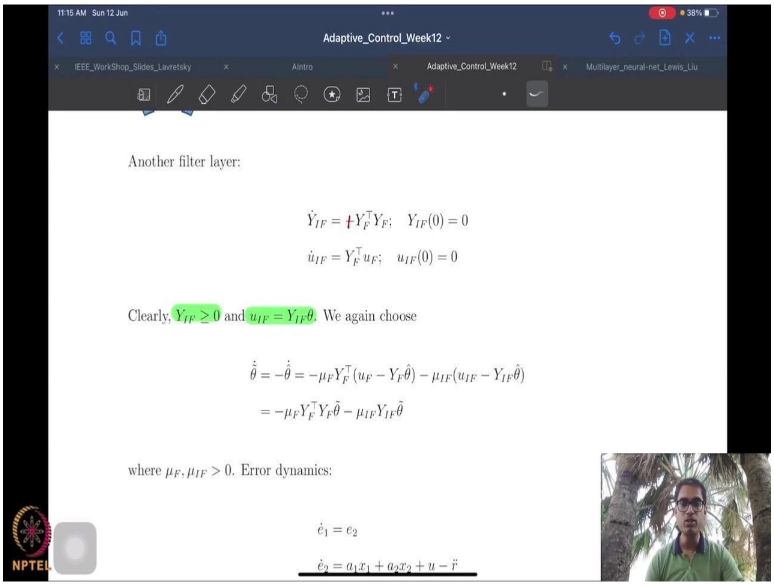

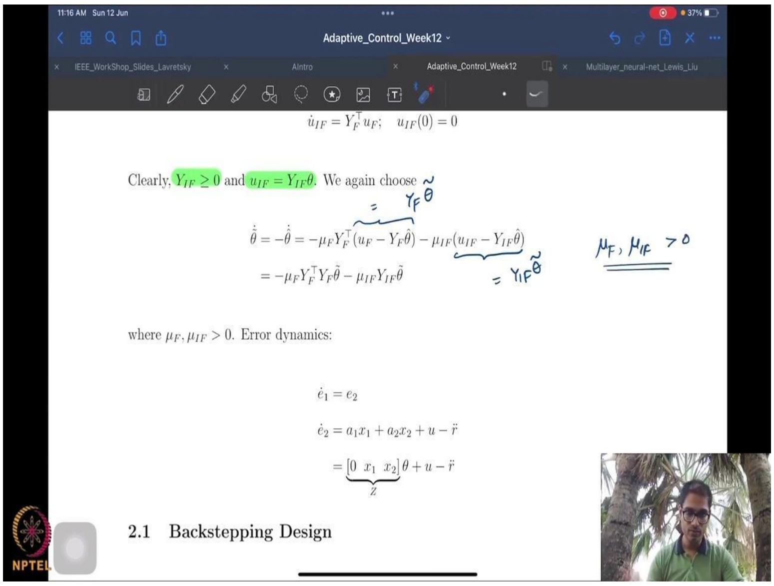

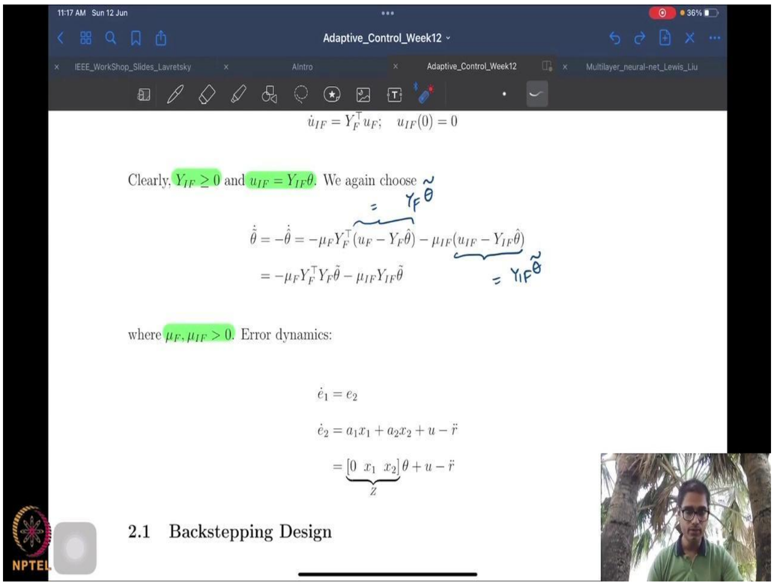

So, now, if you sort of look at what we did in the previous example, in the case of single integrated, we add another filter layer. So, the first filter layer, helps improve performance comes from Slotine's original work in 80s, but it does not really help you alleviate the need for persistence of excitation, So, in order to do that, we need a second layer of filter and that is what we did in the single integrator case and in the double integrator or if you have 100 integrators or if you have a kind of dynamics, you will do the exact same thing nothing changes.

So, the second layer filter is of this form Y IF dot is YF transpose YF. So, I think this is also this plus YF transpose Y IF initial condition 0 and uIF dot is YF transpose uF with the initial condition 0 and of course, as before we have that Y IF is positive semi definite why because it starts at 0 value 0 matrix now, and the derivative is positive semi definite and therefore, YF dot Y IF is now Y IF is positive semi definite the way YF propagates is also becomes a positive semi definite matrix excellent

Now, for uIF also uF transpose uF and the structures are again chosen smartly carefully in what sense in the sense that I get a similar looking filter equation to the first the original regressor parameter form translates to the first layer filter regressor parameter form and then to the second layer filter regressor parameter form. So, the form is not changed at all although the dynamics have changed.

So, as is evident this entire change in dynamics is encapsulated inside this quantity. Now, of course, also this quantity excellent. Now, again we choose our update law that is theta tilde dot is minus theta cap dot as this guy, minus mu FYF transpose uF minus YF theta hat and minus mu iF u iF minus Y iF theta hat the update law is also exactly the same remember, we are not talking about the modification that we proposed here with the certainty equivalence adaptive law added to we are not doing that yet but this is the standard initial excitation based adaptive law.

So, if you do this if you take this in this way you know that this uF minus YF theta cap is going to be equal to YF theta tilde and this guy by a standard regressor parameter forms in the filtered and the double filtered systems and this becomes equal to YiF theta tilde and once you substitute these here you get minus mu YF transpose YF theta tilde minus mu iF Y iF theta tilde and as usual mu F and mu iF are some positive constant. So, again very standard, I am sorry this is already… the fact that mu F and mu iF are positive is already mentioned here, I do not want to repeat it, excellent.

And then we of course, write our usual error dynamics. Which gives me e1 dot is e2 and e2 dot is x2 dot minus r double dot so, this is a1x1 plus a2 x2 plus u minus r double dot. So, then we again write this as Z times theta. So, this is where the Z shows up, you have the control and you have the r double dot. So, this is what you have for your dynamics, excellent.

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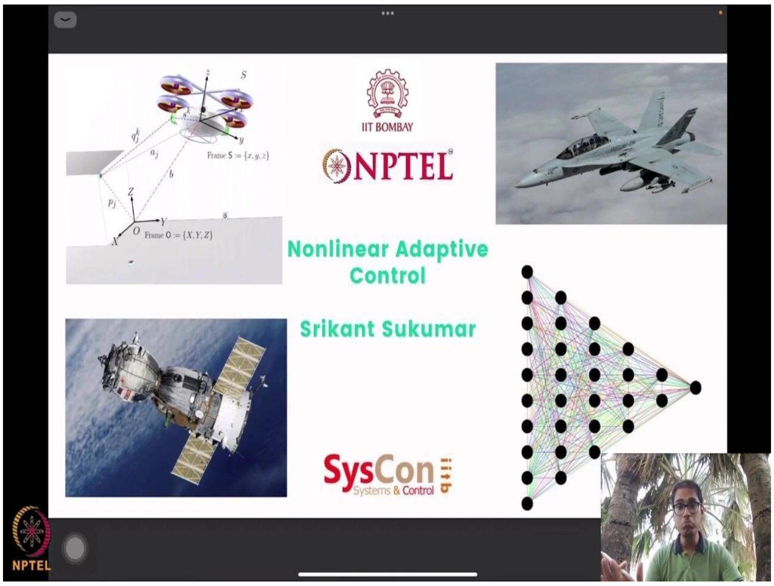

So, what did we look at today, we sort of well, we spent a decent bit of time trying to understand, I mean trying to sort of we go with the implications of initial excitation adaptive control, more specifically, we looked at a small modification to the IE adaptive controller, so, that you add the certainty equivalence adaptive control term to it, which ensures that you are in the absence of excitation the performance of the IE adaptive controller does not deteriorate.

So, that was nice. And after that we started looking at the backstepping based extension for our initial excitation adaptive controller. So, we started looking at that and of course we will continue on the same in the subsequent session. So, I hope you are enjoying our discussion on this initial excitation based adaptive controller and I hope to see you again in the next session, thank you.