Nonlinear Adaptive Control
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Week 11
Lecture No: 63
Initial Excitation in Adaptive Control (Part 3)

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Hello, everyone, welcome to yet another session of our NPTEL on nonlinear and adaptive control. I am Srikant Sukumar from Systems and Control, IIT Bombay. So we are well into the 11th week of our lectures on nonlinear and adaptive control. And by now we have learned quite a bit of adaptive control, we have covered a large breadth of the theory of adaptive control design and analysis. And I really hope all of you have learned enough material by now to be able to apply in design algorithms to drive autonomous systems such as the Space-X satellite that you see in our background.

Now, in this week, we have started to look at a new paradigm in adaptive control until the week before this, we have always been looking at persistence of excitation for learning of parameters in adaptive control. We never really promised parameter convergence in our typical analysis, you always more or less concerned with only tracking of the errors sorry tracking of the system states to the to the desired states, but of course, parameter learning is also a key aspect and we did spend a decent bit of time looking at persistence of excitation, uniform complete observability, integration, lemmas and things like that, which sort of are what help us to prove convergence in the persistence of excitation domain.

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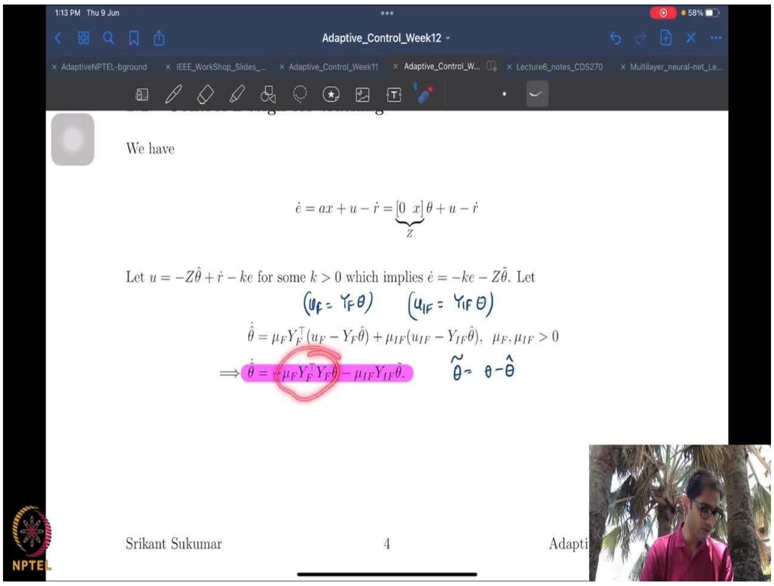

But now, starting this week, we have been looking at initial excitation based adaptive controller where you have essentially 2 layers of filters on your regressor and your control and because of this somehow not somehow, but interestingly, I would say the update law design becomes simpler and also has nice negative terms in theta tilde itself, which is not usually present in typical PE based adaptive control, typical certainty equivalence adaptive control does not contain any theta tilde term.

Of course, there is a theta hat term in sigma epsilon modified adaptive controls, but that those have deteriorated performance even in absence of disturbances. So this is rather nice, rather powerful, and you also of course, show that V dot become negative definite in the presence of initial excitation. Of course, if there is no initial excitation, there is no persistent excitation then you will end up in the same kind of trouble in the presence of disturbance. So I wanted to make a few comments on the results we have.

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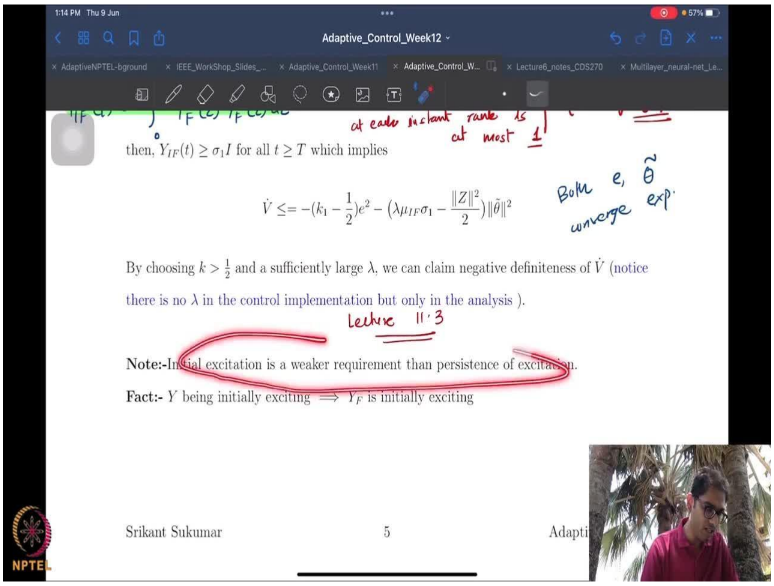

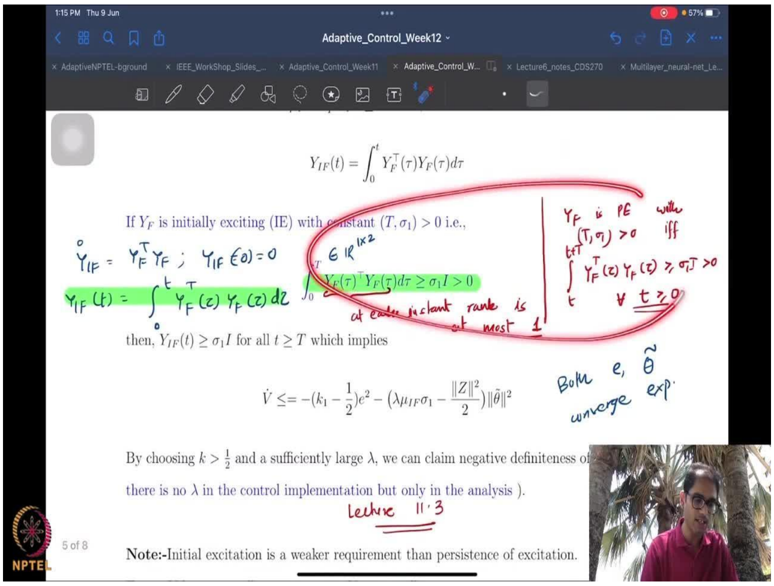

So anyway, so this is where we start, I will mark this is lecture 11.3 and what I want to do is make of course, a few comments, several comments. The first thing it is written here is of course, that the initial excitation is a weaker condition then persistence excitation that should be obvious from here itself, we spoke about it in the previous lecture, that here you need excitation to happen on all sliding windows time, and here the excitation is required only at initial time, so that is would be obvious. The second thing is that the original regressor Y initially exciting is sufficient for Y F to be initially exciting. So therefore, you do not need to verify initial excitation property on the filtered signal, but on the original signal itself is sufficient, this is rather useful.

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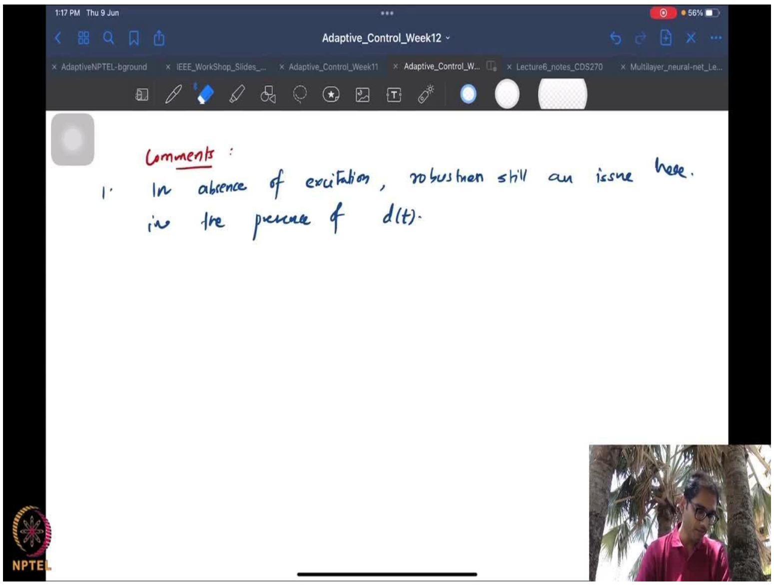

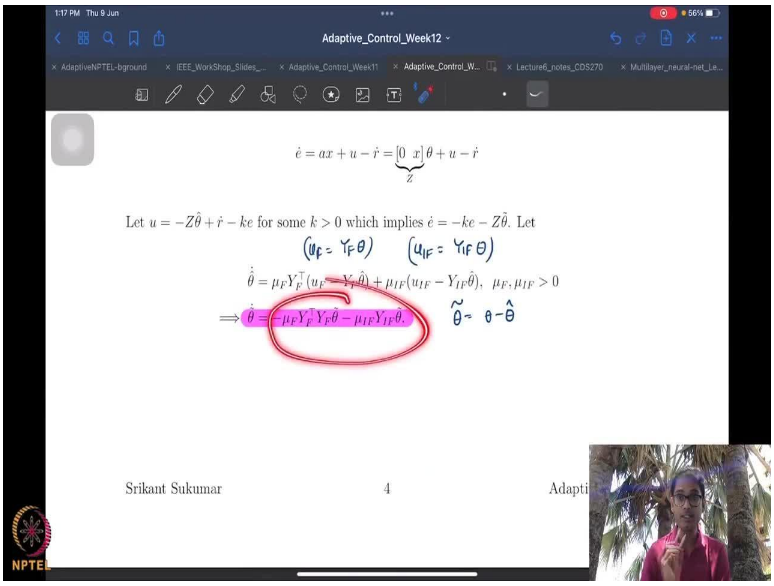

Few other comments like I said, into add a page, few other comments. 1 so comments on what we have obtained one like I said in the absence of persistence or in the absence of excitation of anything of excitation, robustness still an issue here. So please do not think that just because I did some new method I got rid of persistence, I have initial excitation, yes, initial excitation is easier to obtain than persistent excitation. Like say if you have some excitation initially it is okay, more than okay. But if you do not have even that, then you still have robustness issue in the presence of disturbance, robustness still an issue here in the presence of disturbance. So you will have to resort to the same kind of methods. I mean, you will not have a big advantage here.

So remember that, remember what happens, I mean, if you go back here, I mean, this becomes more evident, if you look at this. Yes, there is theta tilde terms in this equation, which is nice, just like sigma epsilon modification, but these terms are scaled by Y I F and Y I F transpose Y F. And these are only semi definite at best, if you do not have excitation, if you do not have any excitation, these are only semi definite not definite, therefore, these terms may not contribute to giving you robustness. So this is, again, a problem.

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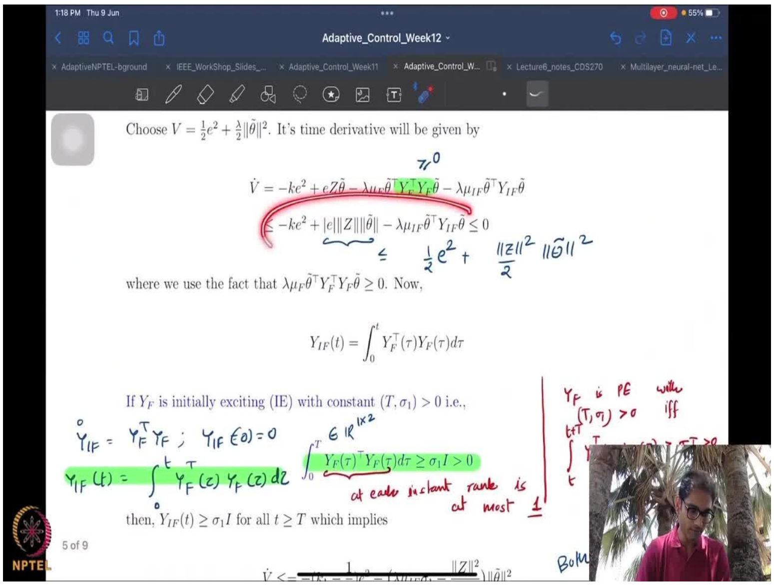

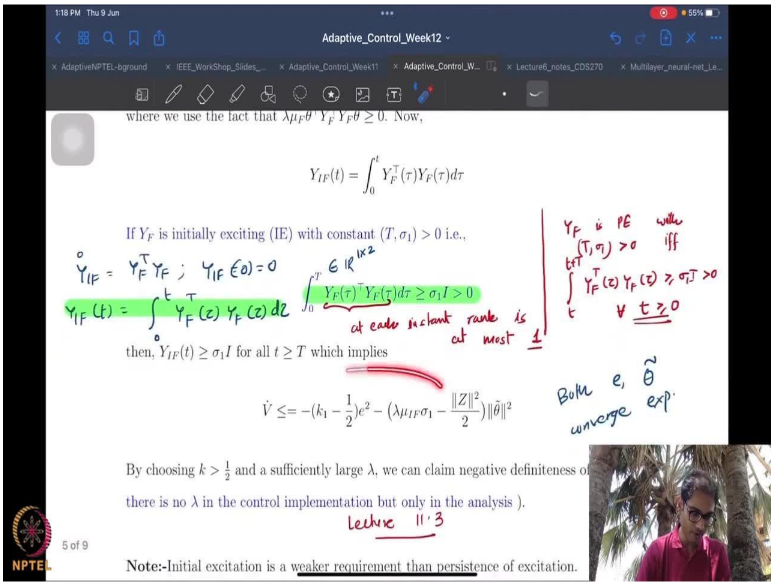

So I mean, if you look at the Lyapunov analysis, essentially, you have this term, this is your actual term, this you get this term only after you have excitation. If you do not have excitation, you are left with this term. And then you have some serious issues still, I mean, because this term is not necessarily this is not necessarily positive definite, it is only a negative definite, this is only semi definite. So you still have this non-strict kind of Lyapunov function, which therefore means that you still have robustness issue in the presence of disturbance. So you have not resolved robustness per say. Great. So I hope you understand that.

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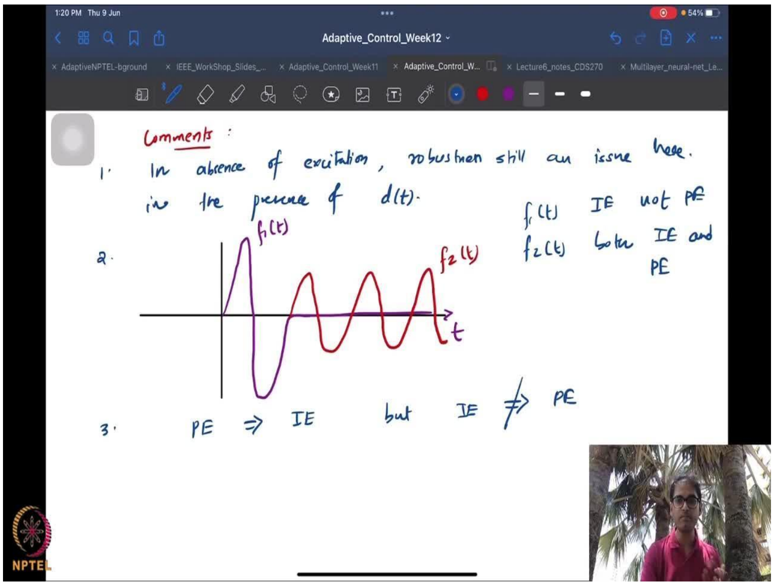

Now the second thing is, of course, that it is an easier condition. So if you have for example, so if you if I try to draw some axis, and what I am going to do is try to draw 2 signals. So if this is a signal for example, so this is a signal, say, I call it f 1 of t and then I have a signal which is say the same, but then after this is still continuous, and this signal is f 2 of t. So it should be obvious to you that. So it should be obvious to you that f 1 of t is initially exciting, not persistently exciting, and f 2 of t both I E and PE.

So again, of course, it should also be evident to you therefore, I mean, so of course larger class of signals are initially excited. So it should be evident to you that persistence excited persistent excitation implies IE, but IE initial excitation does not imply persistent excitation, so this is important. So therefore, this indicates that initial excitation is a weaker requirement. So therefore easier to satisfy, we just need some excitation at some initial time and you are done and you it is enough to identify things.

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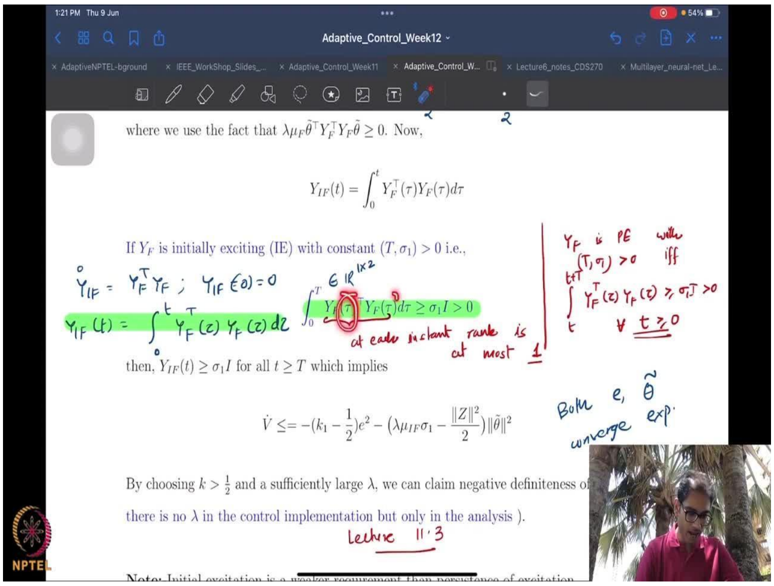

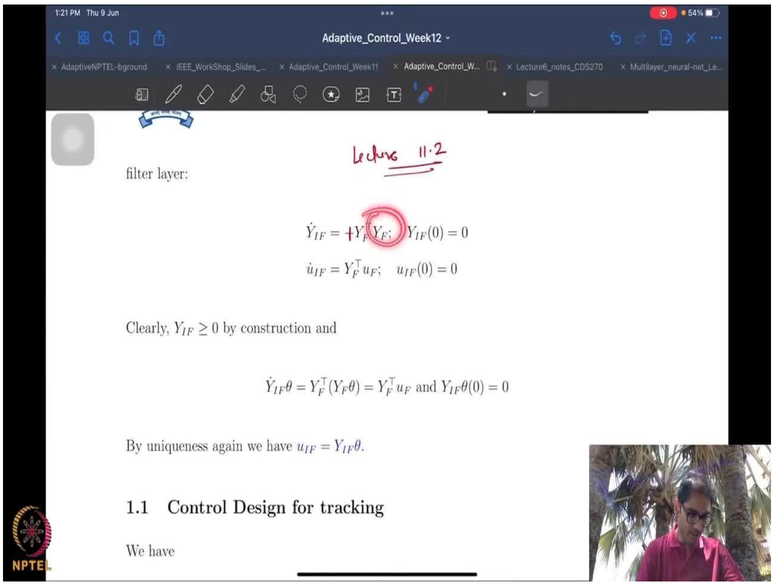

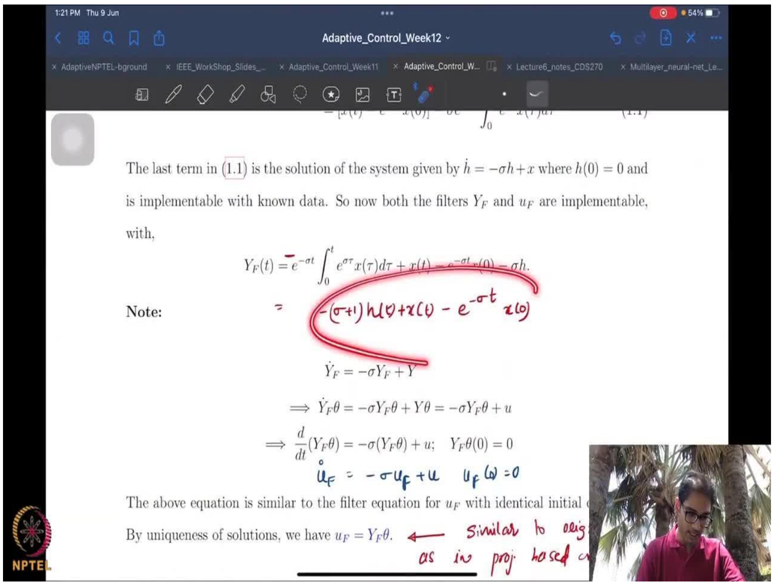

Few more things, if you look at this condition for excitation, initial excitation, this is sort of true even for persistent excitation. Although these conditions are written as a function of time, these are written as a function of time notice, but in reality your Y F is your Y F is obtained how your Y F is obtained by integrating Y or filtering Y and Y contains the states. So Y F in fact, I even written the expression. So the expression for Y F is something like this, where h is also depending on the state.

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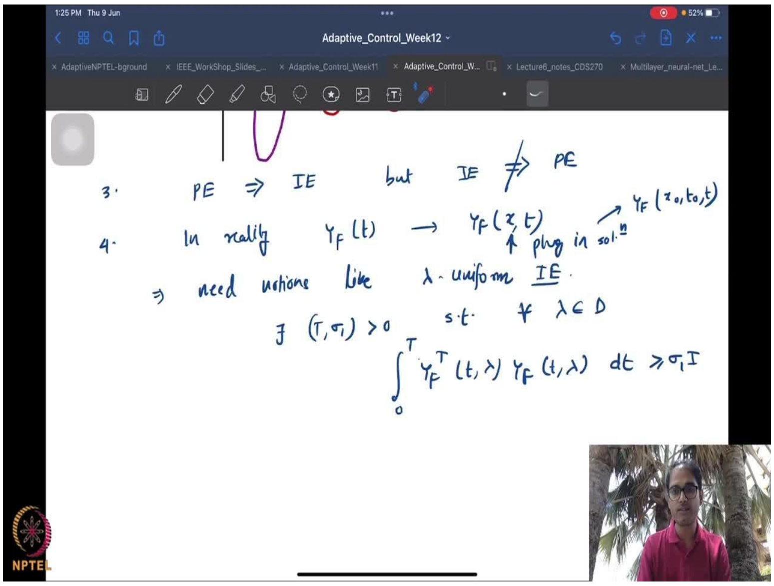

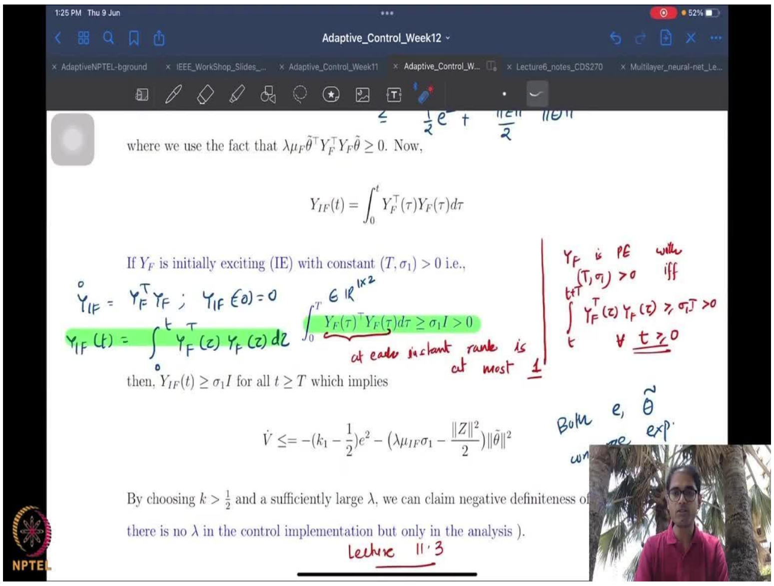

So in reality and this is the case for most problems that you will ever consider, in reality Y F t is actually Y F x, x dot t depends on the states and its derivative or if you do not want to put the derivative because we got rid of its view in whatever some integration by parts it is a function of state and time. And if you see the definition here just like in case of the persistence also is only a function of time. So then it begs the question what happens if Y F is a function of the state?

So we come back to the same issue that we talked about when we discuss the integral lemma, we have to talk about notions now need notions like lambda uniform initial excitation. So what was lambda uniform initial excitation? It essentially says that there exist t, sigma 1 positive such that for all lambda in some domain integral 0 to T, Y F transpose t lambda Y F t lambda dt is greater than equal to sigma 1 I. So basically we need lambda agnostic initial excitation that condition.

Why can we talk about a parameter and not a state? Because if you see if I plug in the solution here, So this becomes a function of actually plug in solution, if I plug in the closed loop solution there is actually a function of I get Y F x 0, t 0, t, is a function of just the initial time state and time, initial time, initial state, and time and therefore, these are parameters because these are constant values. So until now, when we whatever we talked about is sort of non-uniform results because we have to plug in a particular x 0, t 0, then we have to verify the integral condition like this, like this in fact sorry like this and then we have negative definiteness of Y F and all Lyapunov analysis goes through and all that, but that is not very nice.

The way to go would be to define these notions of lambda uniform initial excitation, which would be something like this. In fact which is very identical to when we define lambda uniform persistence of excitation okay and integral lemma also it is very similar to that. If you have lambda uniform it means that for a particular set of initial time conditions, you will get you have this kind of condition satisfied and then you will have your nice parameter convergence also along with tracking results. So this is very-very important point remember, what we did here until here is not uniform with respect to initial conditions, time.

And if you want to have uniformity with respect to initial conditions and time, you have to define the initial excitation differently, you have to define it as lambda uniform initial excitation. Because in this case and in almost every case you will ever believe it with regressor invariably as dependence on states and this is almost unavoidable. So this is again another, very-very important point.

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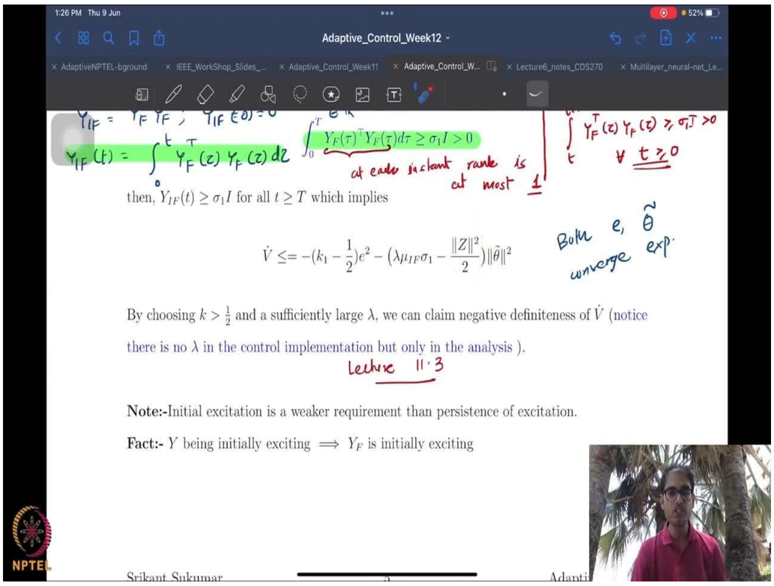

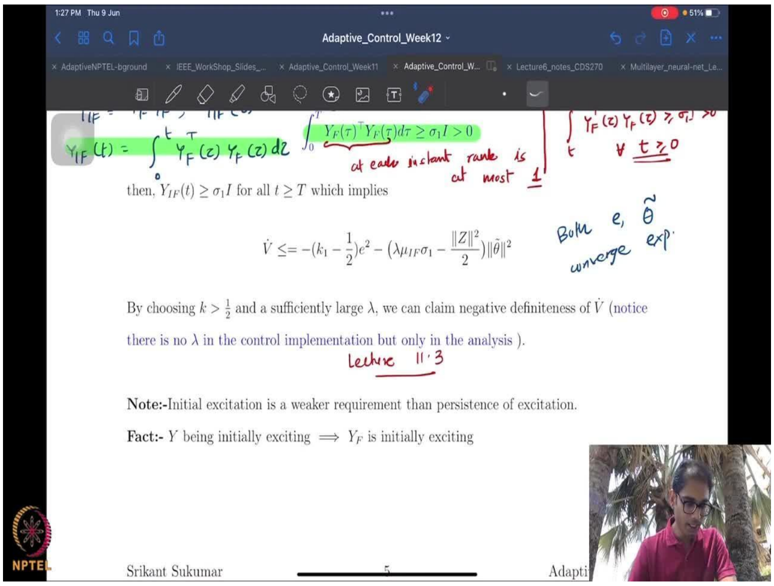

Then, of course, you notice that I mean, we sort of talked about this earlier also but if you notice this there is a state dependent term which did not of course exist before this. So now, how to deal with it is of course an interesting question. And the way the authors have proposed to deal with it is of course, adding this gain lambda, which is only in the analysis and does not appear anywhere else. And so of course, you can sort of dominate this outcome. Now, this is a time for the state varying quantity.

So what does it mean dominated, because how do you talk about boundedness? This is sort of cyclical question, this works fine, it is not a big issue, if you choose a large enough lambda to begin with, then this is a negative term this is a nice negative terms, so V dot is negative semi definite, V is non-increasing, therefore states are bounded. And from that you get some bound on the states and from that, I get some bound on the Z. Using that bound on the Z I will get another value of I will get another value lambda. And from that is how I will choose my lambda. So this is the very interesting point.

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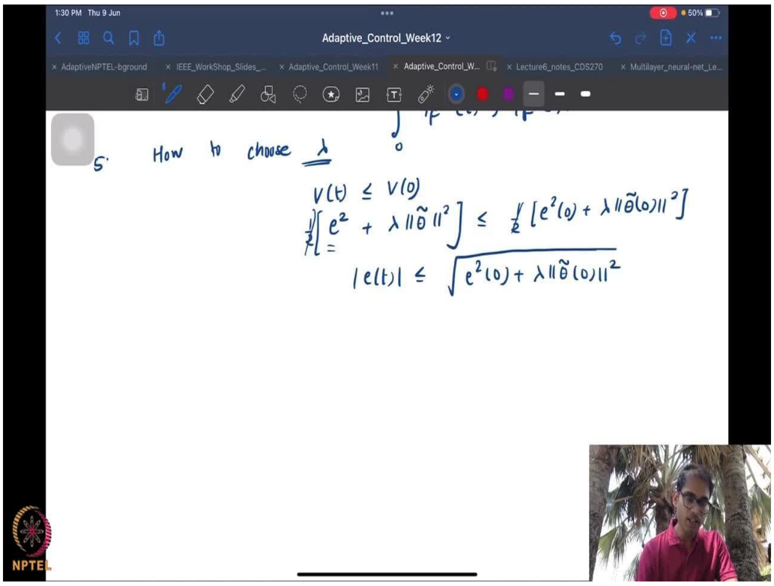

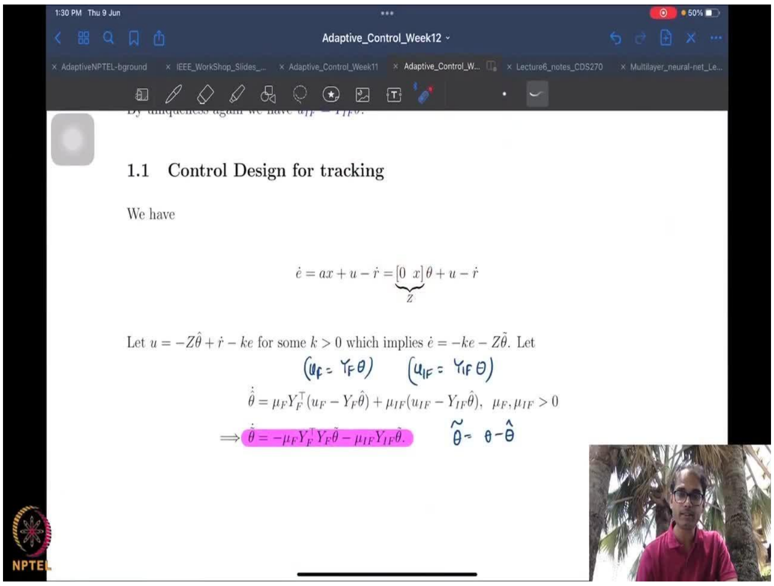

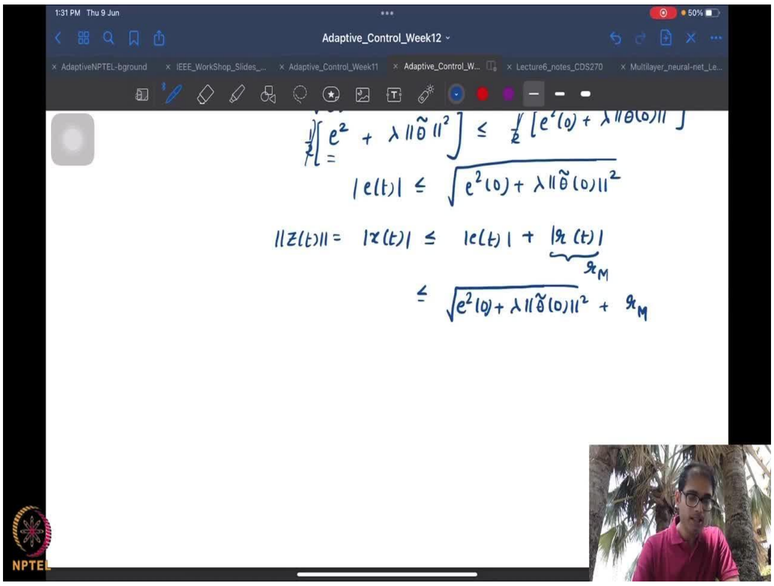

So in fact, I will probably put it more carefully more precisely here how to choose lambda? Again may not be very important for you, but it is important for any theoretician analyst to understand how this choice of lambda is made. So remember, what you end up doing by choosing lambda is proving V t is less than or equal to V 0. So which means that you have your half norm e square plus lambda norm theta tilde square, but in this case e is just scalar, so I am going to just say e square is less than equal to half e square at 0 plus lambda theta tilde square at 0.

And now, with this I of course, want to get a bound on e, I want to get a bound on e. So if I if I look at it conservatively, I can say that e square at t is less in fact I will remove the square root this is e of t is less than equal to square root of e square 0 plus lambda norm theta tilde 0 square. I hope you understand because I know that the sum is less than this, therefore, each term has to be less than this also, I have taken a conservative value because that is the best you will be able to do in this situation, then I have taken a square root on both sides. Great.

Now that I have this bound, what do I know about the Z in this particular case? Z is basically 0 x. So then I will you can say that norm of Z is equal to x in fact, which is less than equal to… Because r is the trajectory that I am trying to track. So this is of course, if I take a bound on the trajectory is bounded. So if I take the upper bound as r m, so this is less than or equal to the square root of e square 0 plus lambda norm theta tilde 0 square plus r m, this is my bound on norm Z.

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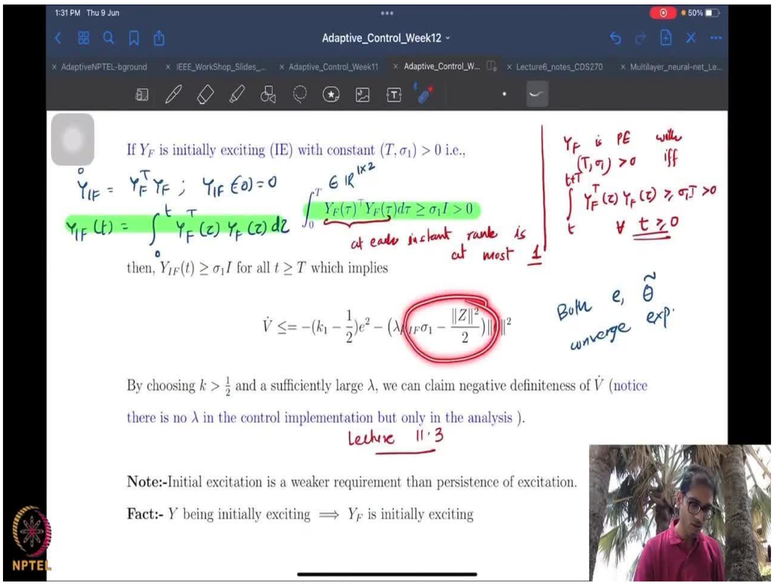

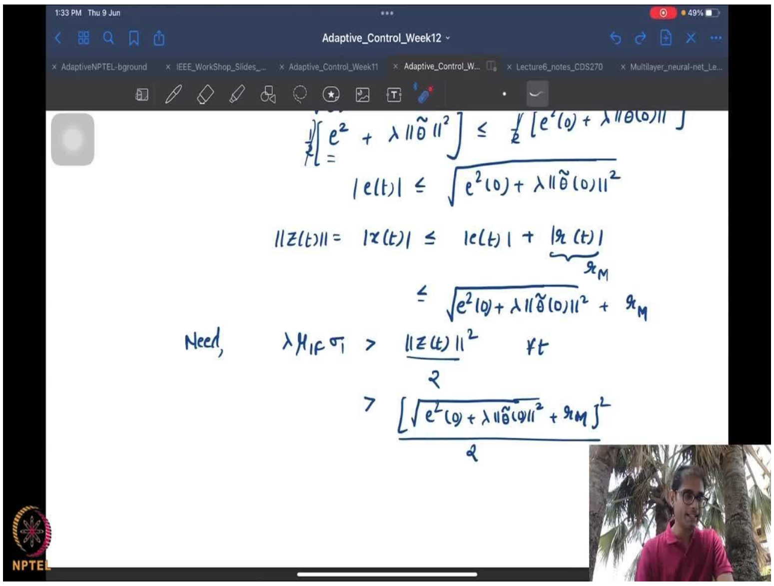

And now what do I need? I need my lambda to dominate this, this is the interesting thing here. And now see this what do I need? I will actually I need that, so what do I need? I need that lambda mu I F sigma 1 should be greater than norm Z t square divided by 2 for all t. So in fact, this is I mean, because this is an upper bound, so it should be able to evident with. So because this does not depend on time, it should be evident that for all t this bound holds the right-hand side does not depend on time. So for all time t, this bound has to hold. So obviously, you want this to be if this is greater than square root of e square 0 plus lambda norm theta tilde 0 square plus r m whole square by 2.

Now, if I want to keep this simple, I mean, actually, I cannot keep this very simple to be honest. There is a little bit of a complication here, I mean, it is not that the solution does not exist here, the solution does exist, I mean, you should be able to find a solution here. The only thing is that there is a lambda on both sides of this equation. So it is like solving a quadratic. So not super obvious how to get this lambda, but there does exist such a lambda. So I hope you remember this sort of a slightly complicated sort of situation that we land ourselves in with this choice of lambda.

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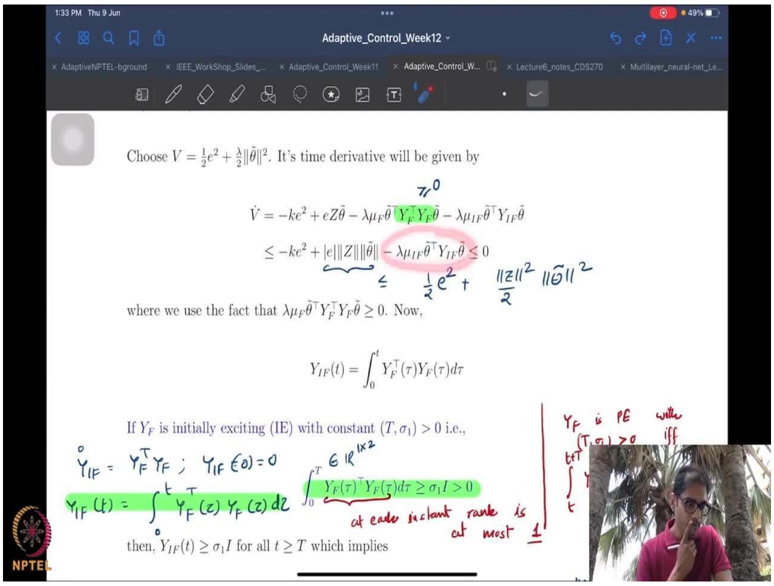

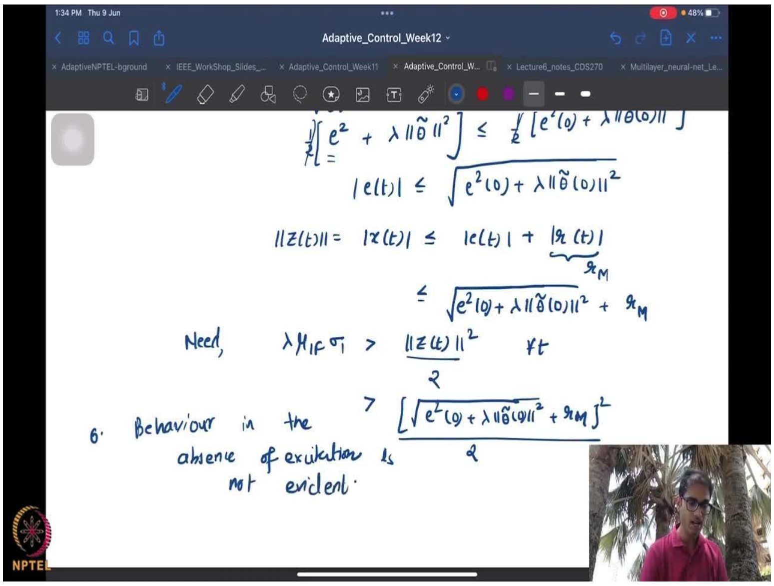

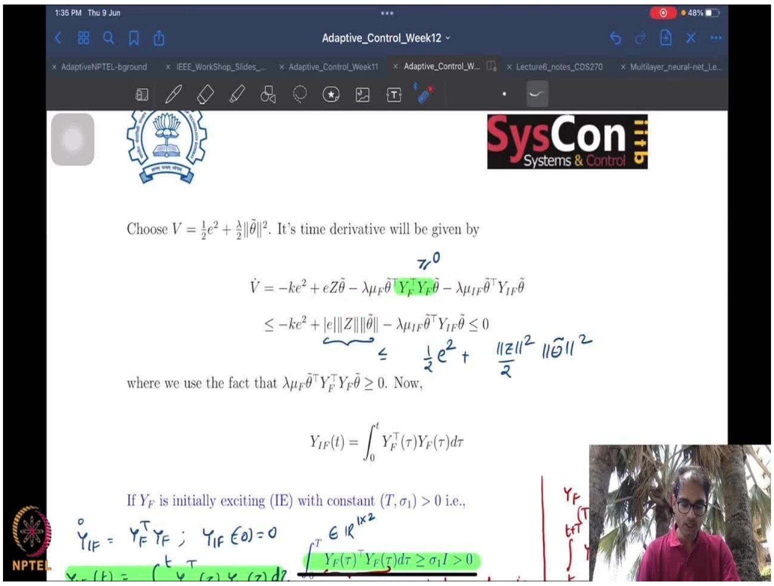

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Adaptive_Control_Week12
SysCon
Systems & Control
Choose $V = \frac{1}{2}e^2 + \frac{\lambda}{2}\|\tilde{\theta}\|^2$. It's time derivative will be given by
$\dot{V} = -ke^2 + eZ\tilde{\theta} - \lambda\mu_F\tilde{\theta}^\top Y_F^\top Y_F\tilde{\theta} - \lambda\mu_{IF}\tilde{\theta}^\top Y_{IF}\tilde{\theta}$
$\le -ke^2 + |e|\|Z\|\|\tilde{\theta}\| - \lambda\mu_{IF}\tilde{\theta}^\top Y_{IF}\tilde{\theta} \le 0$
where we use the fact that $\lambda\mu_F\tilde{\theta}^\top Y_F^\top Y_F\tilde{\theta} \ge 0$. Now,
$Y_{IF}(t) = \int_0^t Y_F^\top(\tau)Y_F(\tau)d\tau$
If Y_F is initially exciting (IE) with constant $(T, \sigma_1) > 0$ i.e.,
$\int_0^T Y_F(\tau)^\top Y_F(\tau)d\tau \ge \sigma_1 I > 0$
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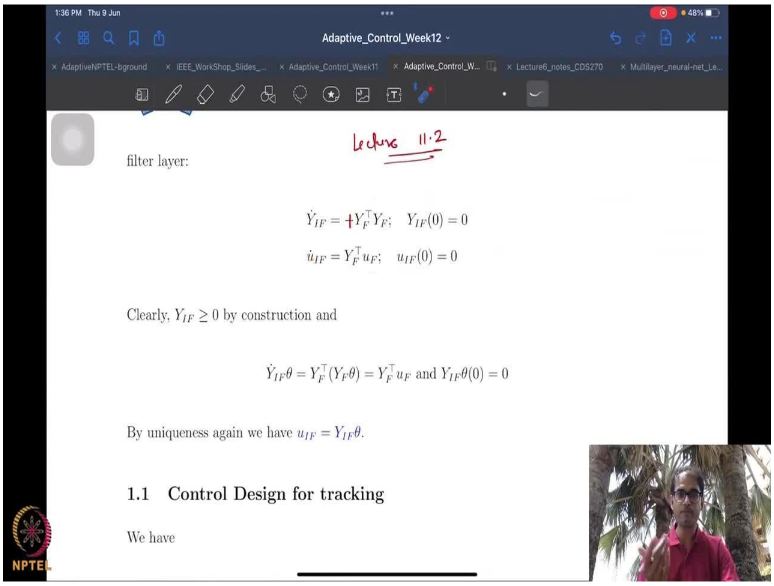

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filter layer:
$\dot{Y}_{IF} = +Y_F^\top Y_F; \quad Y_{IF}(0) = 0$
$\dot{u}_{IF} = Y_F^\top u_F; \quad u_{IF}(0) = 0$
Clearly, $Y_{IF} \ge 0$ by construction and
$\dot{Y}_{IF}\theta = Y_F^\top(Y_F\theta) = Y_F^\top u_F$ and $Y_{IF}\theta(0) = 0$
By uniqueness again we have $u_{IF} = Y_{IF}\theta$.
1.1 Control Design for tracking
We have
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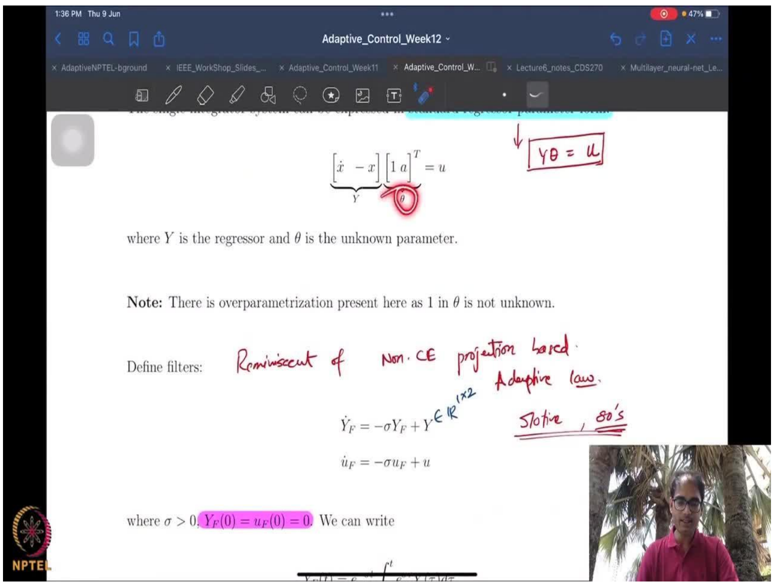

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$\underbrace{\begin{bmatrix}\dot{x} & -x\end{bmatrix}}_{Y}\underbrace{\begin{bmatrix}1 & a\end{bmatrix}^T}_{\theta} = u$
where Y is the regressor and θ is the unknown parameter.
Note: There is overparametrization present here as 1 in θ is not unknown.
Define filters:
$\dot{Y}_F = -\sigma Y_F + Y$
$\dot{u}_F = -\sigma u_F + u$
where $\sigma > 0$, $Y_F(0) = u_F(0) = 0$. We can write
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Now, if its initial excitation is not there, then we are in a rather big soup. Because then we are left with this kind of a term and this is not even positive definite. If there is no initial excitation, then this term could be is not positive definite, let us remember that this term is not positive definite here, then it is not very clear how this kind of a term can be dominated. So to be honest, if you look at this behaviour in the absence of excitation is not evident. I hope this is clear.

If there is no excitation, then I am left with this kind of Lyapunov function derivative, candidate function derivative, and then this term is not positive definite, so I cannot really use this term to dominate anything. And then I am left with this term where this e can be managed here, sure, but then there is still some kind of a term in theta tilde. So at best, I will get somewhere where residual set type of and starts to look like the Sigma Epsilon modification type of situation, unless I can somehow smartly use this term, not very clear, to be honest, that is not very clear whether it will be possible it is obvious that this Y I F is connected to this theta because Y I F is connected to Y, it is not super obvious, it is not super evident how to use this.

Of course, if you retain this term also you still have a Y again a semi definite term again may have some connection to Z, so this may help you dominate this even in the absence of excitation, but it is not very evident, how that would be possible, it is a little bit more of a complicated proposition. So excellent. So as always, when you get something you also give some, I mean, of course, you already gave something in the form of adding more dynamics, I mean, you increase the order of the system, if you, so Y I F is typically, the size of Y I F is more like the size of the regressor, so the size of Y is basically the size of theta. So it is like in this case one parameter, so one additional parameters are 2.

So in this case it became 1 by 2. So therefore, Y F is also 1 by 2 and if Y F is 1 by 2 then Y I F became a 2 by 2 matrix. So you added significant number of states to your system. So you already did ask for something more, but on top of that, you can see that there are some things that may still be a little bit complicated, especially things like the choice of lambda and then what happens in the absence of this initial excitation. In the persistence excitation in the absence of persistence excitation, you still know that you get tracking convergence like 0 tracking errors, in this case, this is not very clear whether such a thing is happening. It may be possible for large values of k and lambda and such, but it is not very clear. So ideally, it would make sense to sort of combine the usual certainty equivalence, PE adaptive law and this law is what would be sort of my recommendation, but of course, I have not done that answer shown it here. I leave it to you folks to try it out. Excellent.

So what we looked at in this session, is that we had completed the analysis for the initial excitation based adaptive controller for a single integrator. In the previous session, we wanted to have a discussion of what the properties are. So we looked at a few rather interesting things interesting features, the positives in the sense that it is a initial excitation is not persistent excitation, and the fact that all these signals are initially exciting and not vice versa. But then we also saw some drawbacks, which is how to choose lambda, what happens in the absence of initial excitation. So there are a few drawbacks also how we do things.

But it is rather important to keep this in mind when we are using the initial excitation based design. In the end, like I said, I propose that you sort of use a combination of the CE adaptation law and IE adaptation law or the PE adaptation law and IE adaptation law. But again, that is not something I have shown here in this discussion. I leave it to you folks to try it out and see how it works. In the upcoming session, we will look at again backstepping extensions for this. So we just want to integrate it with what we have learnt. And I hope you will see that it is not too difficult to do that.

The most important thing to remember is that the in the initial excitation with adaptive control design, the adaptive control law, the update law is designed is completely decoupled from the dynamics, the dynamics almost plays no role, because everything is in terms of the Y, Y F and Y I F, so whatever is inside this Y, Y F and Y I f is what captures the dynamics. The structure of the parameter update law remains the same irrespective of the dynamics therefore, it is very easy to stretch this to any dynamical system. And so it has found use in many different dynamical systems and that is rather nice. So great, I hope you enjoyed this session and I hope you will join me again next time. Thank you.