Nonlinear Adaptive Control
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Week 11
Lecture No: 62
Initial Excitation in Adaptive Control (Part 2)

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Hello and welcome to yet another session of our NPTEL on nonlinear adaptive control. I am Srikant Sukumar from Systems and Control, IIT Bombay. So we just started our 11th week of lectures on nonlinear adaptive control. By now we have covered a large variety of methods to design and analyse algorithms that will drive autonomous systems robustly, such as the Space-X satellite that you see in the background. So I hope you have found the exposition of nonlinear adaptive control interesting and I hope you will continue to be with me until the end of the course.

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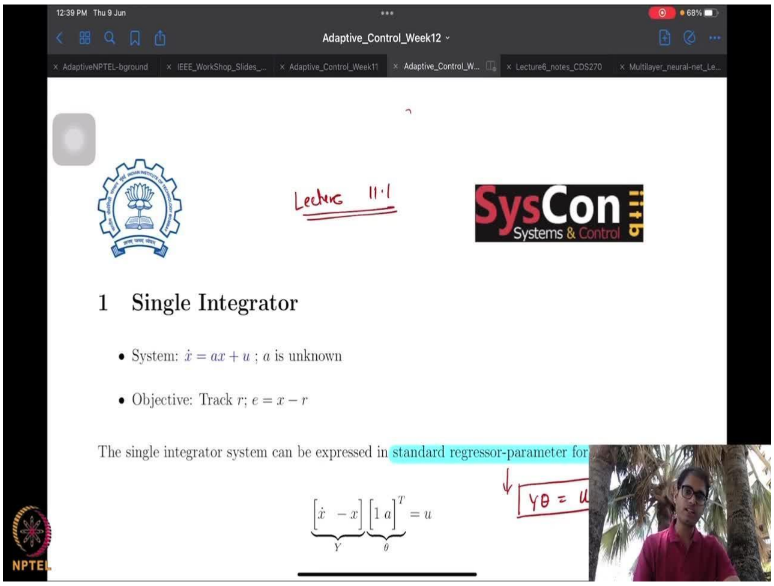

So what we were doing last time is that we started to look at a particular method of adaptive control, which allows us to sort of relax the persistency of excitation condition. So as we sort of discussed a little bit last time persistency of excitation is a rather strong requirement for identification, which well until recently was thought to be sacrosanct. So now what we want to do is to do this kind of real time parameter learning without having persistence of excitation, but with a weaker sort of condition.

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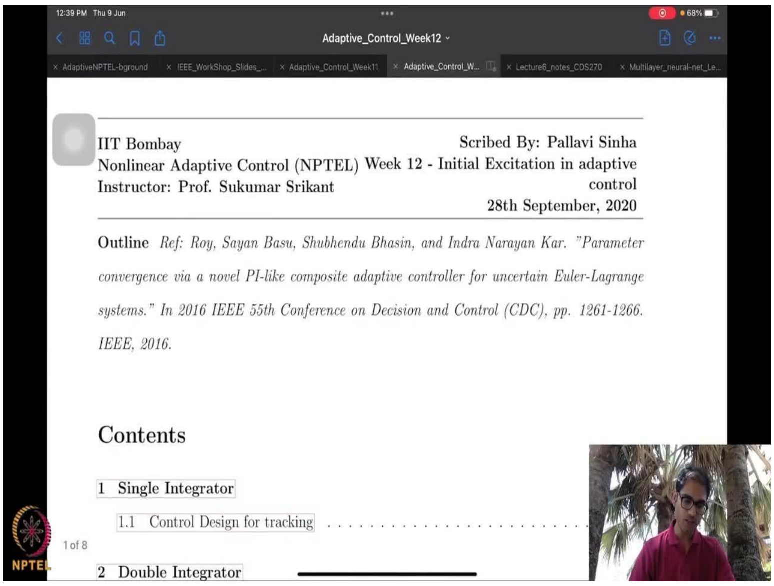

So in order to do that, we in order to understand that, of course, the reference was this work by Sayan Roy, Shubhendu Bhasin and Indra Kar, they of course, have a lot of subsequent work in this direction also which I would strongly urge all of you to look at very interesting stuff. So one of the well, I mean, we started to look at very simple setup of course for illustration, we like to see these basic problems, but we have also seen in the past, and I hope you are convinced by now, that even if I give you a vector problem, things are not going to be significantly different.

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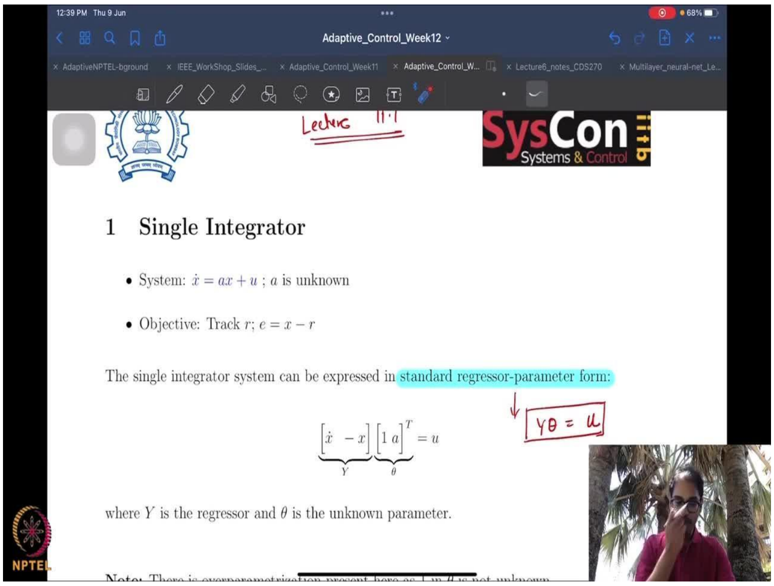

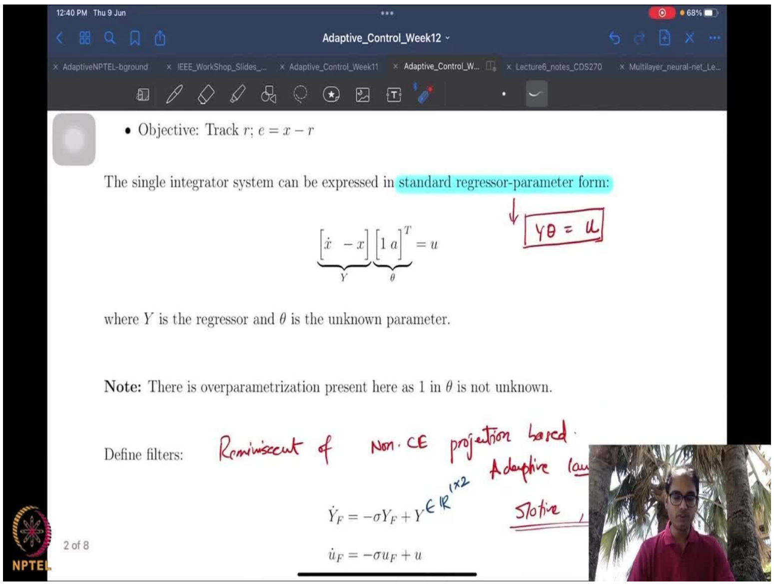

So we started looking at a single integrator system, and a single integrator tracking problem, honestly speaking we did not even go to the tracking aspect of things until now. The first thing we did was we wrote everything in a standard regressor parameter form that is y theta equal to u, and because we wanted to write it in this form, we had to resort some to over parameterization, so we had to introduce 1 also as part of the unknown parameter vector and there was also x dot in the regressor.

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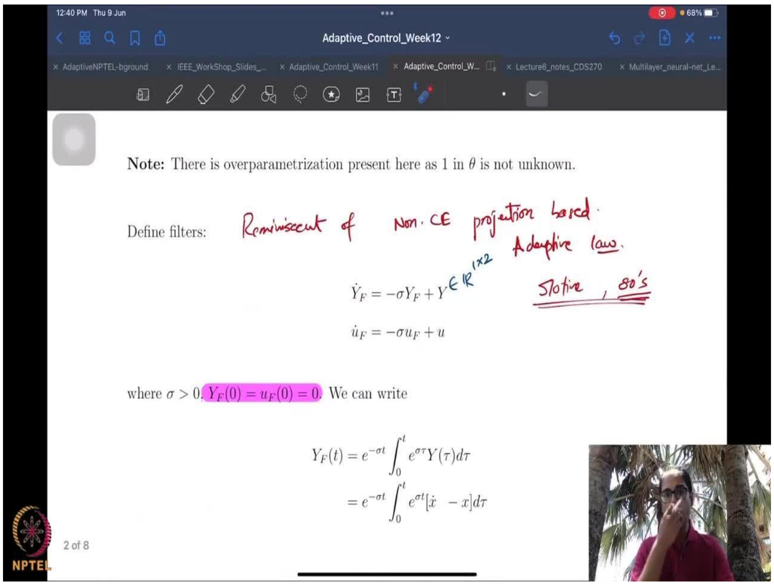

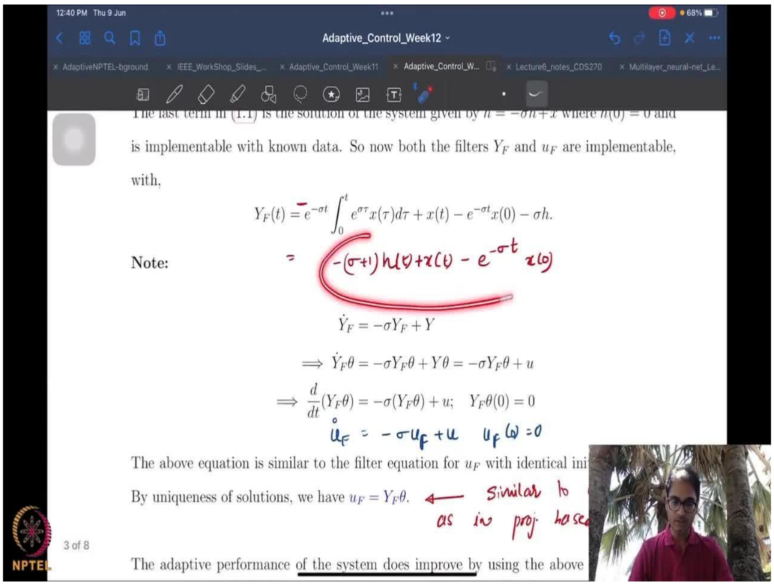

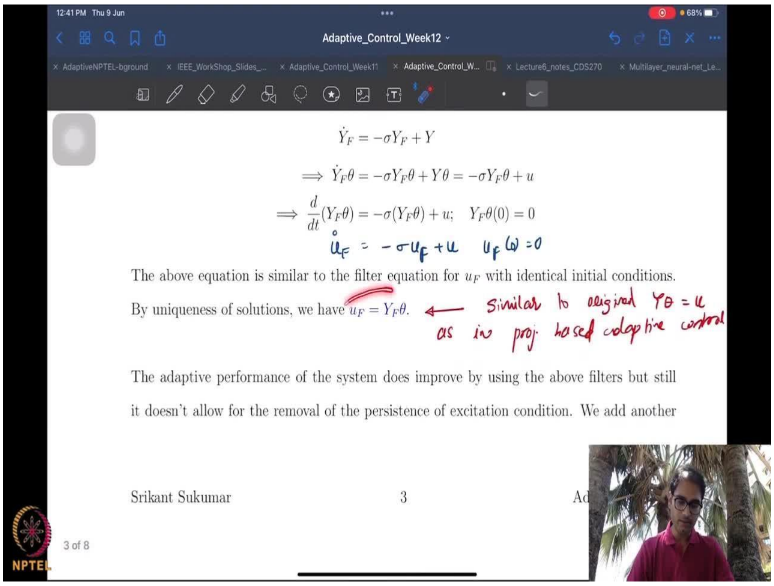

Now what essentially this x dot signifies is the derivative of the state which is typically not assumed to be measured. But then we also showed that this filter itself is implementable by using a simple integration by parts type of scheme. And we even wrote out a nice formula for this, this integration by parts kind of a scheme. Then very reminiscent to what we were doing in projection based adaptive controllers, all of these were motivated by Slotine's work in the 80's is that we, we sort of tried to connect the filtered variables, so we sort of wanted to write the equation in terms of the filtered variables. And again, very reminiscent of our projection based adaptive control scheme the filter equation turns out to be very similar to the original equation and this is also looks like a standard regressor parameter form, which is that the Y and the u have been replaced by the Y F and the u F.

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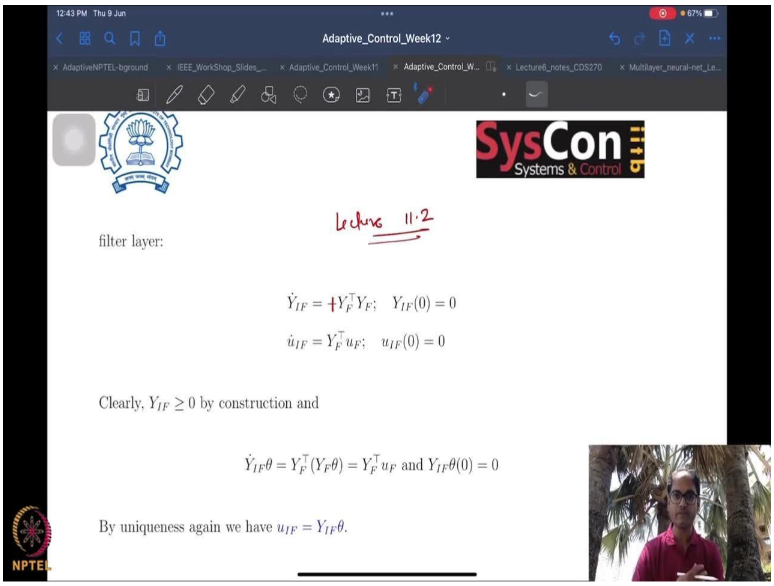

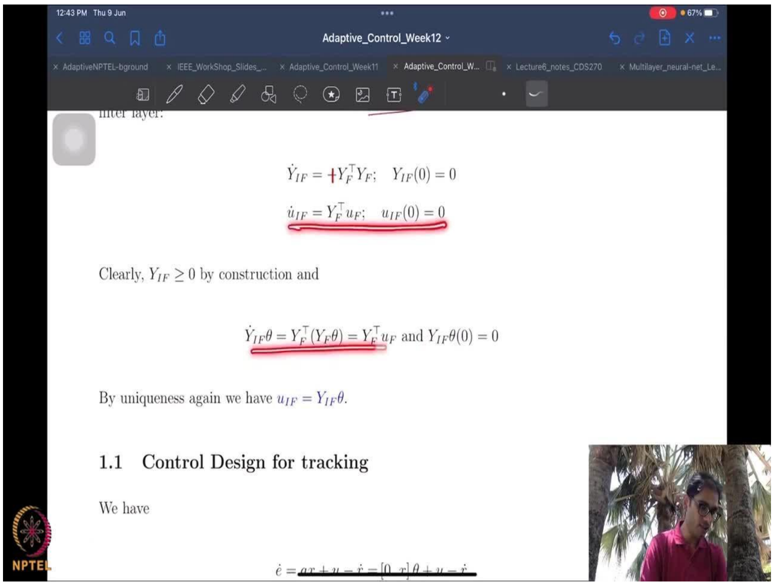

So Slotine already showed that these do improve performance, we also saw the same the creation of an attractive invariant set and we did the projection based adaptive control, but it does not allow us to relax the persistence assumption, which is why these authors who we are referring to now proposed addition of a second layer filter. So this is where we sort of start today. So now the second layer filter, we saw the structure already pretty standard, it is like a Y F dot is governed by minus Y F transpose Y F with 0 initial conditions and u I F dot is governed by Y F transpose what u F with 0 initial conditions again.

So one thing to note is that Y I F is always going to be positive semi definite. I think we have to change, there is a sign issue here, there is actually a plus otherwise, it is not positive semi definite but negative semi definite. By construction, this is positive semi definite, because you start with this zero matrix and then you just keep adding a non-negative definite matrix and non-negative definite symmetric matrix as the derivative and therefore you of course, have positive semi definite Y I F we do a similar exercise, now we try to come up with the equation in terms of the Y I F's and u I F's, so in terms of second layer filtered variables.

And in order to do that I as usual, multiply both sides here by theta. And if you look at this, this Y F theta from our previous analysis is already u F, I substitute the same here, and again, you notice that Y I F theta 0 is 0. And if you see this equation and this equation are exactly the same with same initial conditions, just the variables are differently named. So here you have u I F, here you have Y I F theta.

So therefore, by uniqueness of solutions of ordinary differential equations, the solutions of these 2 also have to be the same, which means that Y I F theta is exactly equal to u I F. So this is again, very similar to the previous equation, in fact as you can imagine, the author's was smart enough to construct this so that such a property does hold, that is the whole idea anyway. So the construction is precisely so in order for this kind of inequality to happen, great.

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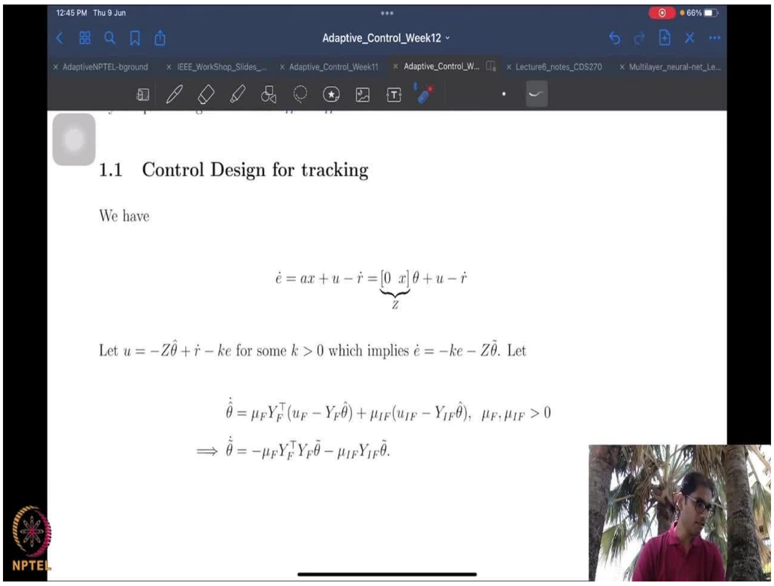

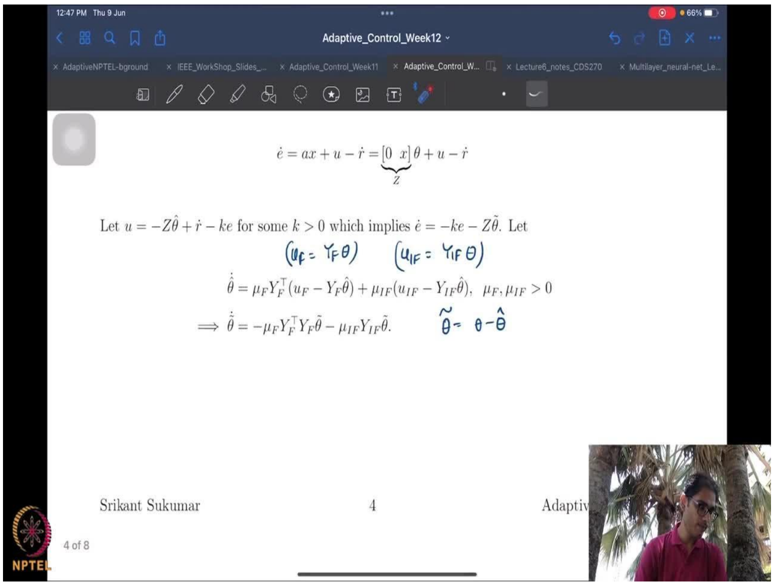

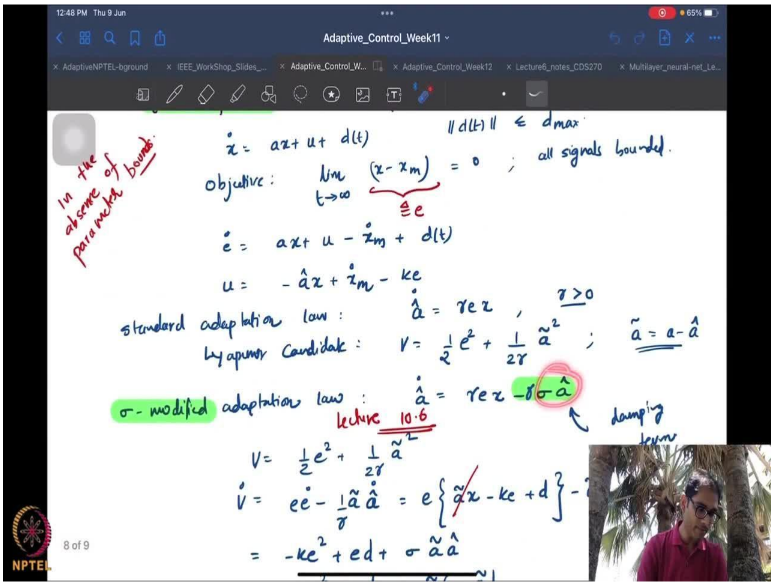

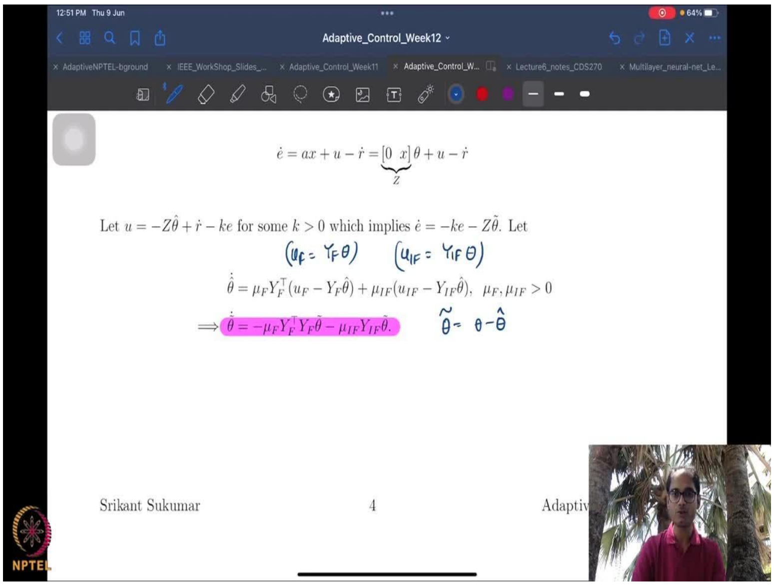

Now, that we have designed these 2 layer filters, we are now going to actually look at the control problem which we sort of neglected until this point. So what is the aerodynamics, it is e dot is a x plus u minus r dot. And of course, because we have new parameters theta, we want to write everything in terms of theta. So we write this as z times theta plus u minus r dot. And now, in accordance with our standard certainty equivalence type methods, we propose a control as minus z theta cap plus r dot and a nice negative term k e with some positive gain k. And with this, what we will get is that e dot is minus k e minus z theta tilde. So as usual, we get this nice error term.

Now, the interesting thing is the way we specify the parameter update law has no connection to Lyapunov analysis. So this is again similar to the projection based method, but there it was motivated in a different way, the choice of the update law, here the choice of the update law was of course motivated in a different way, in fact, it is completely decoupled, we just have 2 terms. So here mu F and mu I F are some positive gains, some positive scalars if you may. And then you have Y F transpose u F minus Y F theta cap and u I F minus Y I F theta cap.

Now if you use the fact that U F is Y F theta and u I F is Y I F theta, in fact the filters were constructed very smartly so that such a thing holds and if you substitute this here, and this guy here, you will get this mu F, Y F transpose Y F theta tilde from here, and mu I F, Y I F theta tilde from here, were of course your theta is sorry theta tilde is theta minus theta cap. So this is a rather cool thing, why? Why is it a cool thing? One thing you already seen is that I did not have to, I mean, even in the sigma epsilon modification we introduced some term in theta cap. So by the way, before I go further, here, we had a theta cap dot and here we have a theta tilde dot, which is minus theta cap dot, so that is why you have a negative sign here, that is why we have a negative sign going from here to here as simple as that.

So now, in the earlier sigma and epsilon modification type designs if you notice, theta cap dot and also theta tilde dot of course, did contain a term in theta cap, that was the whole idea behind it, sigma modification and epsilon modification that you have this a cap type term. Of course, if I write a tilde dot also I will still have an a cap term. But look at what happened here by virtue of a very-very neat filter constructions. I do not just have a theta cap, I actually have a theta tilde here, theta tilde dot equation contains a theta tilde from this term and also from term. And not just that, I already know Y I F is already positive semi definite, so this is already a non-positive term here, so something really nice.

So unlike Sigma Epsilon modification where I only had a theta hat in theta tilde dot equation here I get a theta tilde in the theta tilde dot equation, which makes this like a very-very nice evolution with a very high chance of being asymptotically stable. And why was this possible? This was possible or this was made possible only because of this kind of relation, if we did not write it in this regressor parameter form, initially, which was Y theta equal to u, we did not do this over parameterization, I would not get U F equals Y F theta, or u I F equals Y I F theta. And because I did that, and notice, the U F is implementable because it is just a one filter, one layer filter of u and u I F is just filtering U F, so obviously, these are all implementable quantity, so I have used exactly implementable quantities, nothing unimplemented.

But because of the regressor parameter form standard structure Y theta equal u, which I started with, I could get theta tilde as u. And now this is looking like a very nice system, it looks like theta tilde dot equals minus k theta tilde, just in a simple case, just anybody I mean, most of you have seen nonlinear control now, for almost 11 weeks more than 11 weeks will understand that this looks like a very promising system, which can be asymptotically stable. Excellent, this is, I mean so that is why I marked this is the magic. This is where the magic has happened, and this magic has happened, because we started with the regressor parameter form and then we constructed smart filters, not we but these students of IIT Delhi constructed some smart filters. So great.

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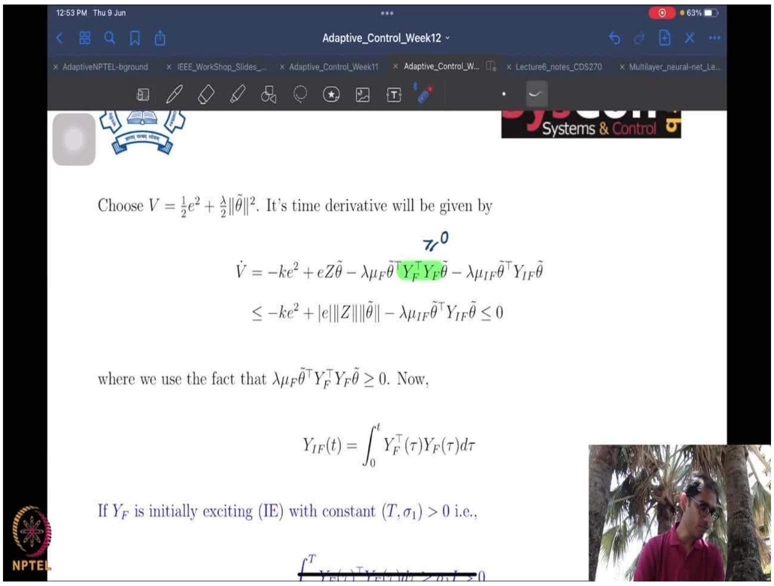

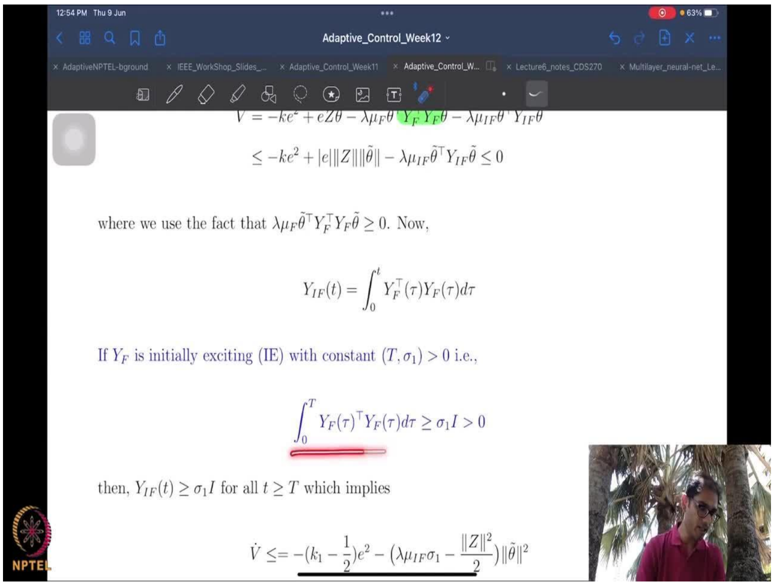

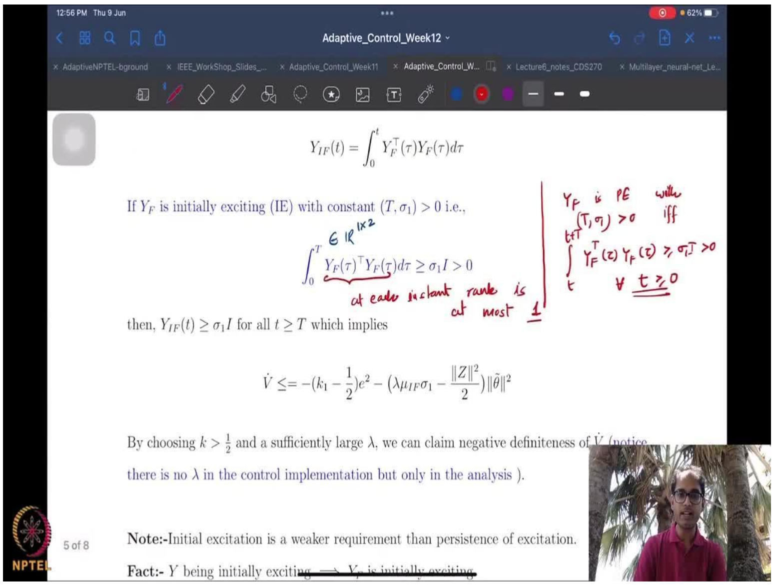

Then we, of course, I mean, do some Lyapunov analysis, and how do we do it? We this is very standard, I take an e square by 2. I have nothing to choose anymore, I just need to do the analysis, nothing remains to be chosen anymore, only the analysis part remains. Great. So I hope you understand. So then I put us put a constant lambda, this again, something that you should remember from your projection based adaptive controller analysis also, I put some arbitrary constant lambda, just for the analysis remember, it is not appearing in the control law, it is not appearing in the update law, because all of that has already been chosen. So there is nothing to be chosen anymore, this lambda is only of use for the purpose of the stability analysis. Excellent.

So I just substituted for e, so from here I get e dot and I get this guy and then from here, I get lambda times theta tilde transpose theta tilde dot. So which is this, this nice negative sign. And then I know for a fact that this is positive semi definite, why? Because it is a inner product, it is like X transpose X, so Y transpose Y F is positive semi definite. So this is so this is positive semi definite at least, because the symmetric matrix, so real eigenvalues, it is a symmetric matrix constructed out of inner product. So they cannot, so again, non-negative definite.

So we know that so we use this fact to sort of get rid of this stuff, we do not even use this in the analysis. So one might ask why introduced this term, if we did not use it in the analysis? So in this next step, you see that I have dropped this term and only left with this term. So these 2 come in as it is, so this term comes in like this, this term comes in like this, I have replaced this with norm-bounds that is it. But this term I have removed, why we keep this term in the update laws, because it improves the numerical performance, this has been shown and proven. So this definitely improved the… this term in the update law, does improve the numerical performance of the adaptation law and so it makes sense to keep this. Excellent.

So now, if you look at it, I am left with these 3 terms. And now, we talk about the initial excitation property. So we say that this Y F is initially exciting, IE with constants P sigma 1 positive, if this inner product Y F transpose Y F integrated from 0 to T is greater than equal to sigma 1 identity, which is a positive definite matrix. So what does it mean? It means that even if Y F transpose Y F instantaneously is not guaranteed to be positive definite, in fact, impossible.

Because, if you remember, Y F is R 1 cross 2. So if you multiply, so it is at most rank 1. So Y F is at most rank 1. If I multiply and again Y transpose is also at most rank 1, so it should be obvious to you that at each instant, rank is at most 1, because the product rank of the product of matrices is the smallest of the rank of each of the matrix involved. So here, Y F is the only matrix involved, it has rank 1, so the rank of the product can be at most 1.

But what we are claiming is that if I integrate from 0 to T, there is sufficient rotation which is the same thing that we talked about in persistence excitation, but the thing is I here integrate only for a finite from time 0 to cap T. As opposed to this, if I wanted to write Y F persistent excitation Y F is PE with the same constants, if I say with T sigma 1 positive, if it does not look like a Y. If integral t to t plus cap T, Y F transpose tau Y F tau greater than equal to sigma 1 I greater than 0 for all t.

So the difference, so I have sort of made a slightly different kind of definition of persistence than what you would remember from class, because I do not put a lower bound and there its an outer product Y F Y F transpose instead of Y F transpose Y F but it does not matter. First of all, I we discussed this even when we talked about persistence excitation that the lower bound, the upper bound is not so critical, the upper bound is only for the purpose of talking about boundedness of the signals and the lower bound is what you will find in all definitions of persistence for sure, lower bound is the only key thing to be honest.

And then the fact that we use inner product versus outer product does not matter because I can always talk about persistence of the transpose, no problem, as simple as that. So but if you look at the big difference, the big difference here is that you need this condition to hold this positive definiteness condition to hold for all small t. Here, there is no small t at all, no small t. So you wanted to hold only for some initial window, here, you need to hold for all sliding windows, if I if I take a window of time of size t, and I keep sliding it, in every window if I integrate this inner product it has to be positive value so that is a very stringent requirement compared to the initial excitation requirement.

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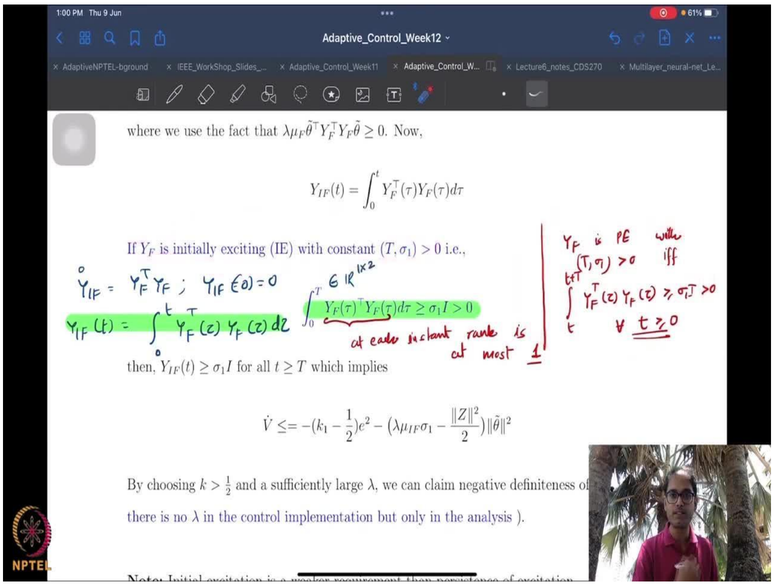

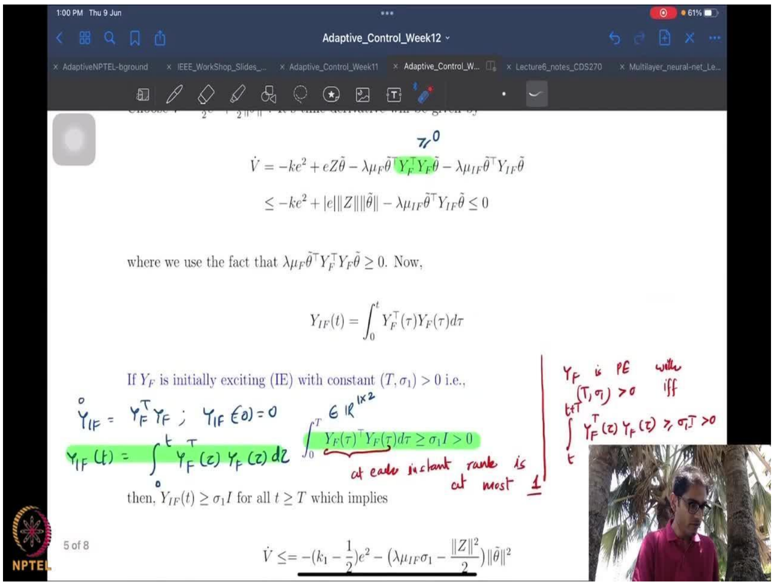

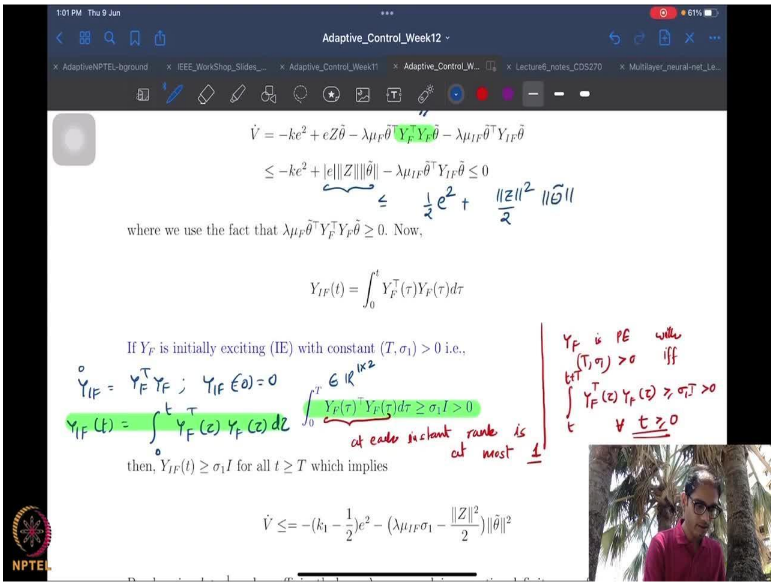

Now what is well known? See, if you have this condition, if you have this initial excitation condition, what do you know, you look at the solution. Now let us look at the very beginning, let us look at the solution of the Y I F equations. So Y I F dot was simply Y F transpose Y F with initial condition at 0 to be equal to 0. So what is Y I F of t? It is actually 0 to t Y I F transpose tau Y F transpose tau Y F tau d tau. And if you look at these 2, you match these 2, they are exactly the same, same integral.

So what does it mean? Means that if I have initial excitation on Y F, then if the small t is greater than equal to cap T, Y I F has to be greater than equal to sigma 1 identity, just by comparing these 2. To compare these 2, you see that if the small t becomes larger than this capital T, then Y I F for that value of small t has to be greater than equal to sigma 1 I, because this integrand is always non-negative always makes a non-negative contribution cannot reduce.

If you have a value of Y I F capital T to be sigma 1 I, which is what we will have from here, then the value of Y F T beyond capital T time will also have to be greater than equal to sigma 1 I. Just by the virtue of how this evolves as simple as that. So because of this initial excitation, which is a significantly less stringent requirement than persistent excitation, that it is evident in the names itself persistent means always exciting, and initial means only initially exciting. So it can be boring later on completely fine.

So for initially exciting sequence, you will have Y I F to be greater than equal to sigma 1 I, when t greater than equal to t. So in this V dot expression, I can have I replace this quantity by sigma 1 I, I replace this quantity by sigma 1 I. And of course, then I also use this fact, now use the sum of squares, so this is less than equal to half e squared plus z squared by 2, theta tilde squared. So of course, I use the sum of squares also, and I get this expression beyond time t greater than equal to t, it does not matter what happens until time equal to t less than t, because until the it is only it is, it is only a finite time, therefore, the system would have expanded only a finite amount, it does not matter how much but a finite amount. And beyond that I have this nice kind of result.

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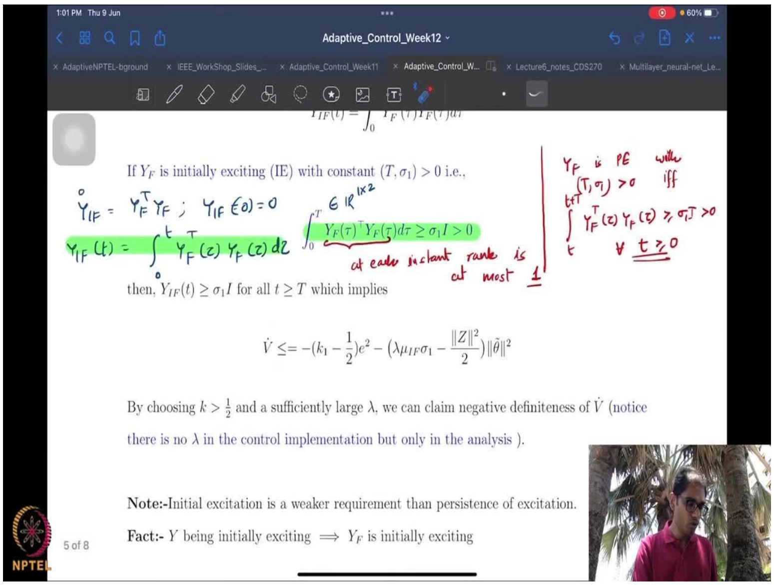

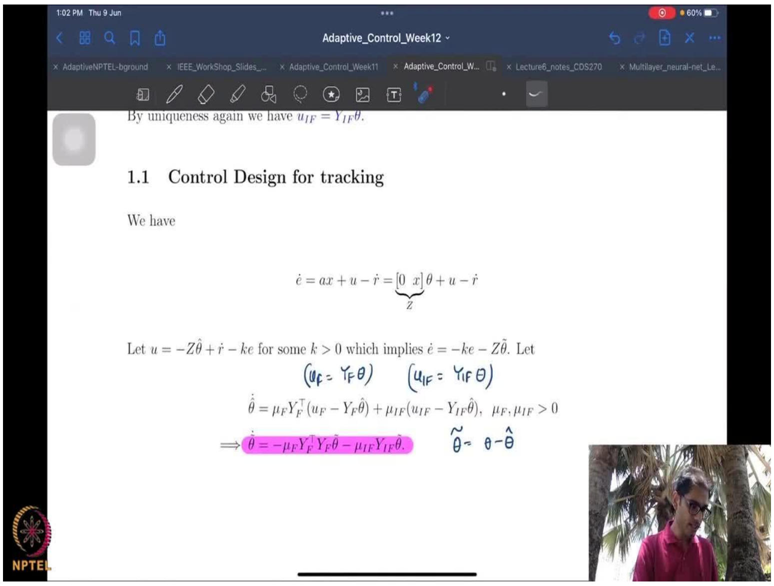

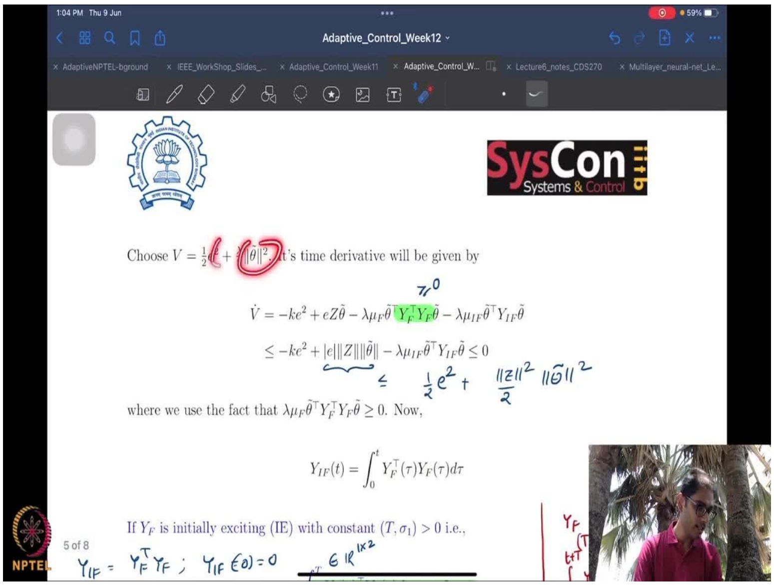

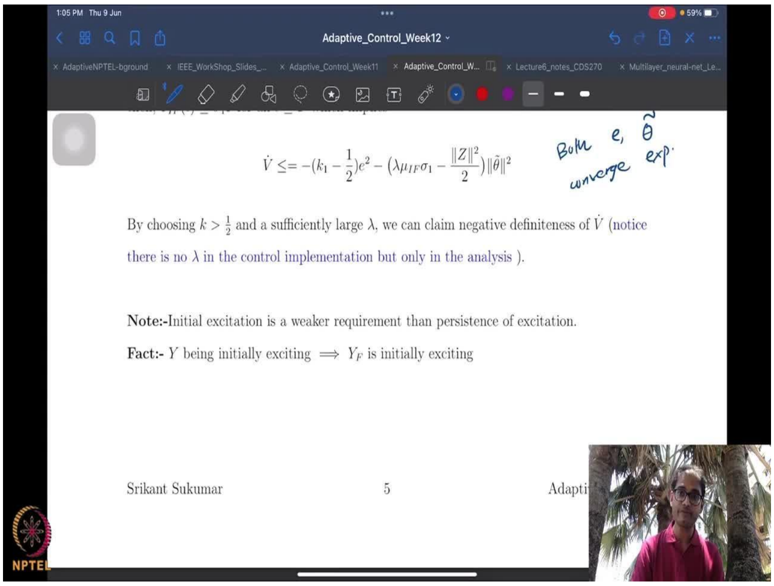

Now, you would as you would sort of think that this is this is time varying and because it contains the state, what is Z? Z contains the part of the regressor that contains this guy that is not a constant. So now, how do we deal with that? That is where the lambda comes in, that is where the lambda comes. So here of course, if k 1 is greater than half, I am done. Here, I just need the lambda to be large enough to dominance because mu I F is, of course, in our hands also you can also choose mu I F to be large, but sigma 1 is not really in our control depends on the initial excitation properties of the signal.

But if you choose lambda large enough suppose you fixed mu I F and fixed sigma 1 is out of control, but if you choose lambda large enough, then you can dominate this. And once you dominate this guy, you know that V dot is less than equal to 0, so V dot is non-increasing, sorry, so V is non-increasing, these non-increasing states are bounded, states are bounded, then this bound continues to hold with some large lambda. And the most important point is this lambda is not required for the implementation is just for the analysis. So this domination is given, it is pretty standard, pretty straightforward.

So once we have this, we have nice negative terms in e and theta tilde, this is under the initial excitation condition, so this is a big difference from our certainty equivalence control, where you never get a term here like this theta tilde. This term happens comes about very in a very straightforward way when you have initial excitation, because of the fact that this update law contains the theta tilde. For the persistence excitation based control, this theta tilde term does not show up very easily there you have to prove results using UCO conditions and are like we already saw that. So it is a little bit more complicated, theta tilde does not show up in the adaptive controller, here it does. And because it does, the Lyapunov analysis also becomes more straightforward.

So this is, of course, nice, you have nice negative definiteness, negative definite V dot. And which immediately means that you have some nice exponential decay in fact, because V was also quadratic in e and theta tilde, and V dot is also negative quadratic in e and theta tilde, so you have a nice exponential decay. So a pretty strong outcome, I would say and therefore, you can have convergence of both e and theta tilde exponentially. So that is what I would say, both e, theta tilde converge exponentially, so pretty strong result here. So that is what it is the certainty equivalence sorry, the initial excitation based adaptive control. Great.

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So what did we look at, we sort of continued our discussion on the initial excitation based adaptive controller, we saw the second filter layer, of course, then we constructed the update law, which interestingly brought about theta tilde terms, nice negative looking terms. And we know that if there is an initial excitation condition, which is significantly less stringent than the persistent excitation condition, we have nice negative definite terms in the theta tilde dot term. And this helps us to prove exponential stability of the system. So we have exponential stability of e theta tilde dynamics, which means that both e and theta tilde are going to converge exponentially in time.

So of course, all this happens for t greater than equal to t. So this is something we have to remember. So in initial time, there may be some finite expansion of the system, which is okay it is still finite. This behaviour can also be governed by choosing gains appropriately and so on. So in the next upcoming session, we will continue our discussion of course on initial excitation based adaptive control, we will look at higher order systems and things like that. And so I hope to see you all there in the subsequent session. Thank you.