Nonlinear Adaptive Control
Professor Srikant Sukumar
Systems and Control
Indian Institute of Technology, Bombay
Week 11
Lecture No: 61
Initial Excitation in Adaptive Control (Part 1)

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Hello, welcome to yet another session of our NPTEL on nonlinear and adaptive control. I am Srikant Sukumar from systems and control, IIT Bombay. I warmly welcome you to the week number 11 of our course. And I hope, I really hope that we have had a very good journey together. We have already learned several methods, several algorithms that will help design robust adaptive nonlinear laws that drive autonomous systems such as the Space-X satellite that you see in our background.

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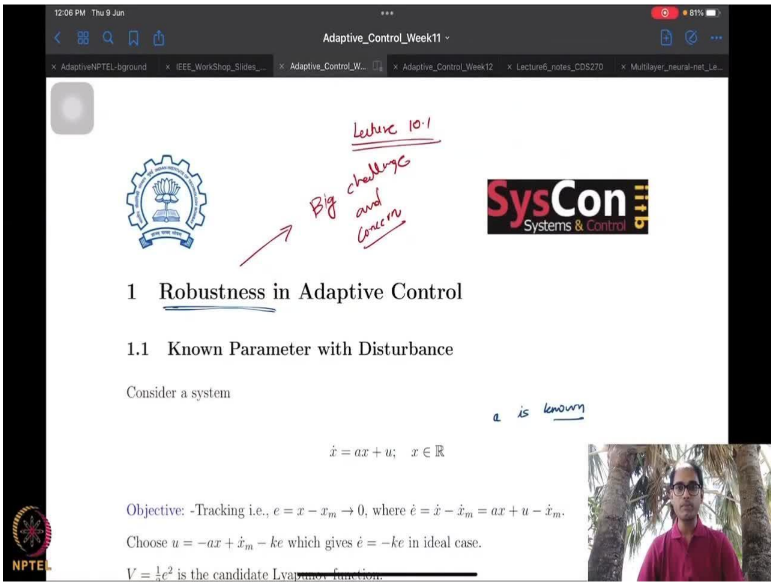

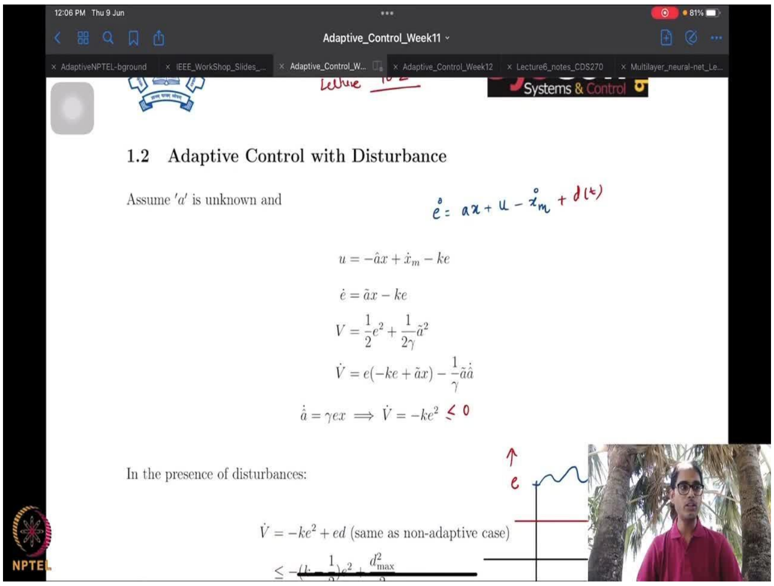

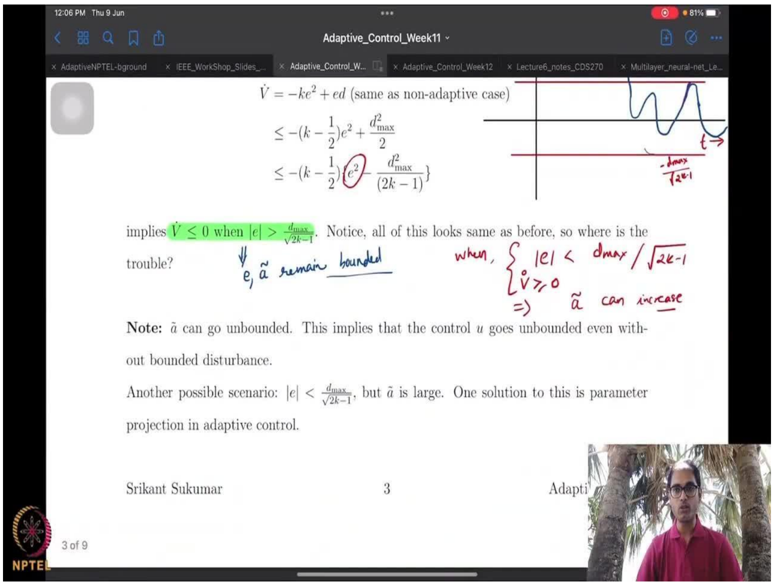

With this sort of an introduction, I want to talk a little bit go back a little bit to what we were doing in the previous week. So we actually started off talking about what is robustness and what is the need for robustness in adaptive control. And we got a pretty good idea that in the presence of disturbance, a disturbance that would not really trouble a non-adaptive laws, what happens in adaptive control setting is that this disturbance can result in our parameters growing unbounded and subsequently the control also becoming unbounded in order to maintain a nice bound on the tracking error.

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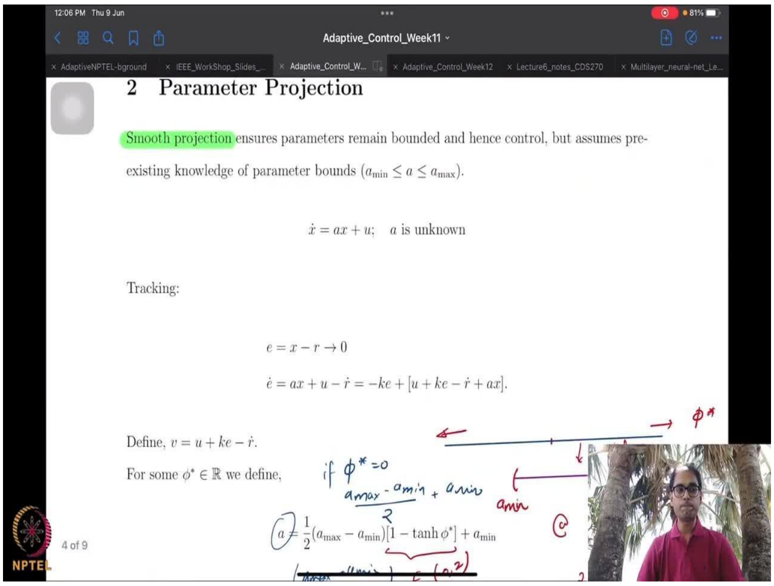

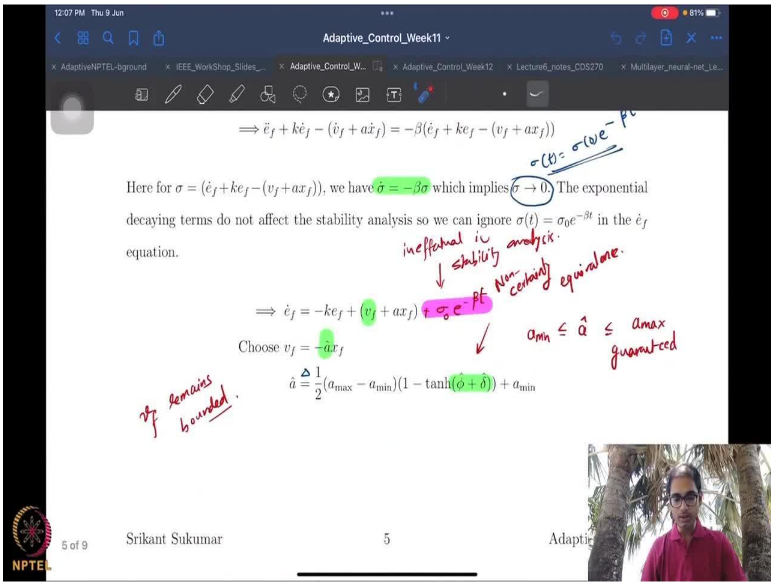

This was of course rather undesirable and we first talked about tackling this using the parameter projection method, in which we assumed some prior knowledge of bounds on the parameter and this prior knowledge of bounds on the parameter, we design adaptive laws using projection functions like tan hyperbolic functions, which will allow you to keep the parameter values within this given bounds.

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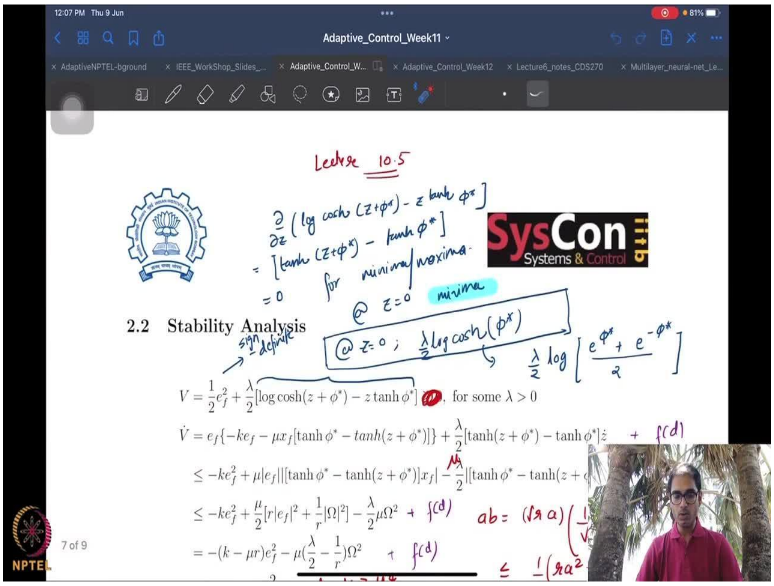

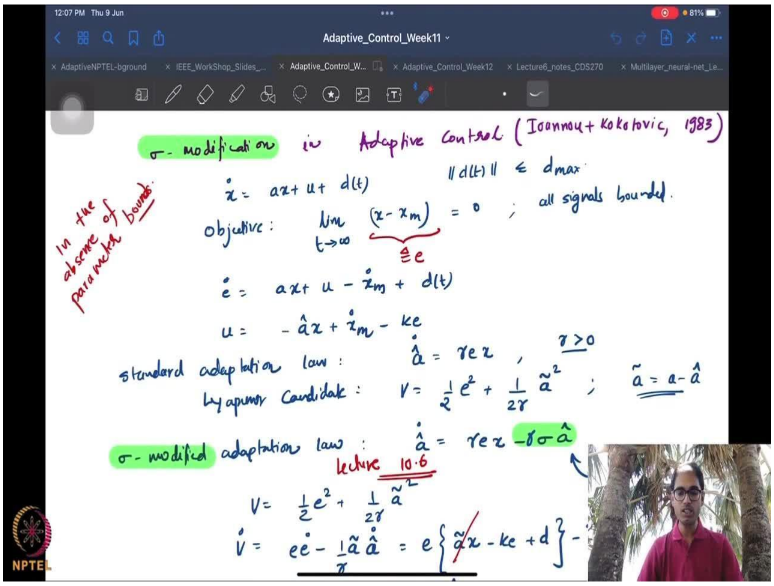
Adaptive_Control_Week11
σ- modification in Adaptive Control (Ioannou + Kokotovic, 1983)
$\|d(t)\| \le d_{max}$
$\dot x = ax + u + d(t)$
objective: $\lim_{t\to\infty} \underbrace{(x - x_m)}_{\triangleq e} = 0$; all signals bounded.
In the absence of parameter bound.
$\dot e = ax + u - \dot x_m + d(t)$
$u = -\hat a x + \dot x_m - ke$
standard adaptation law: $\dot{\hat a} = \gamma e x$, $\gamma > 0$
Lyapunov Candidate: $V = \frac{1}{2}e^2 + \frac{1}{2\gamma}\tilde a^2$; $\tilde a = a - \hat a$
σ- modified adaptation law: $\dot{\hat a} = \gamma e x - \gamma\sigma\hat a$
lecture 10.6
$V = \frac{1}{2}e^2 + \frac{1}{2\gamma}\tilde a^2$
$\dot V = e\dot e - \frac{1}{\gamma}\tilde a\dot{\hat a} = e\{\cancel{\tilde a}x - ke + d\} -$
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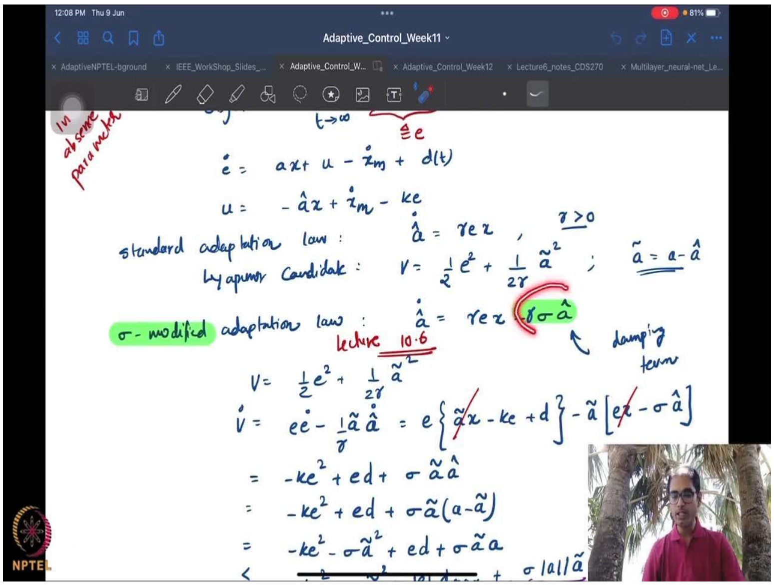
Adaptive_Control_Week11
$\dot e = ax + u - \dot x_m + d(t)$
$u = -\hat a x + \dot x_m - ke$
standard adaptation law: $\dot{\hat a} = \gamma e x$, $\gamma > 0$
Lyapunov Candidate: $V = \frac{1}{2}e^2 + \frac{1}{2\gamma}\tilde a^2$; $\tilde a = a - \hat a$
σ- modified adaptation law: $\dot{\hat a} = \gamma e x - \gamma\sigma\hat a$
damping term
lecture 10.6
$V = \frac{1}{2}e^2 + \frac{1}{2\gamma}\tilde a^2$
$\dot V = e\dot e - \frac{1}{\gamma}\tilde a\dot{\hat a} = e\{\cancel{\tilde a}x - ke + d\} - \tilde a[\cancel{ex} - \sigma\hat a]$
$= -ke^2 + ed + \sigma\tilde a\hat a$
$= -ke^2 + ed + \sigma\tilde a(a - \tilde a)$
$= -ke^2 - \sigma\tilde a^2 + ed + \sigma\tilde a a$
$\le \quad + \sigma|a||\tilde a|$
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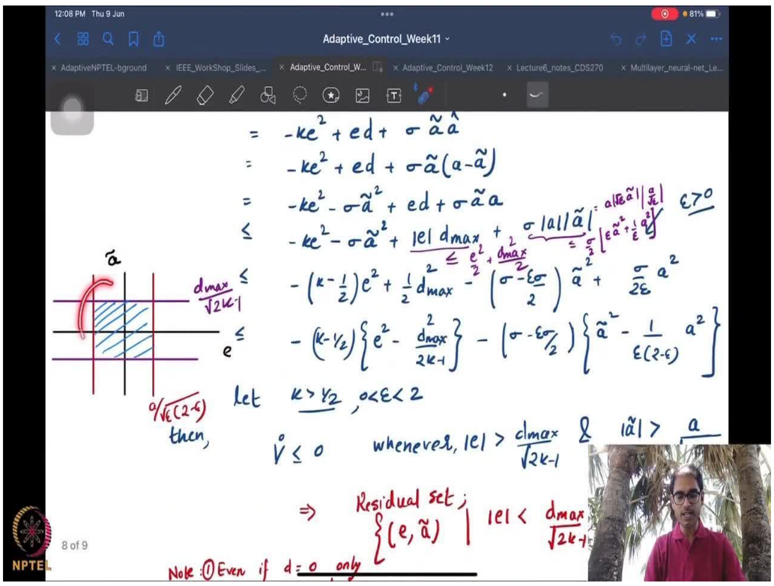
Adaptive_Control_Week11
$= -ke^2 + ed + \sigma\tilde a\hat a$
$= -ke^2 + ed + \sigma\tilde a(a - \tilde a)$
$= -ke^2 - \sigma\tilde a^2 + ed + \sigma\tilde a a$
$\le -ke^2 - \sigma\tilde a^2 + |e|d_{max} + \sigma|a||\tilde a|$
$|e|d_{max} \le \frac{e^2}{2} + \frac{d_{max}^2}{2}$
$\sigma|a||\tilde a| : a\sqrt{\epsilon}|\tilde a|\frac{|a|}{\sqrt{\epsilon}} \le \frac{\sigma}{2}[\epsilon\tilde a^2 + \frac{1}{\epsilon}a^2]$; $\epsilon > 0$
$\le -(k - \frac{1}{2})e^2 + \frac{1}{2}d_{max}^2 - (\sigma - \frac{\epsilon\sigma}{2})\tilde a^2 + \frac{\sigma}{2\epsilon}a^2$
$\le -(k - 1/2)\{e^2 - \frac{d_{max}^2}{2k-1}\} - (\sigma - \epsilon\sigma/2)\{\tilde a^2 - \frac{1}{\epsilon(2-\epsilon)}a^2\}$
$\tilde a$
$\frac{d_{max}}{\sqrt{2k-1}}$
e
$a/\sqrt{\epsilon(2-\epsilon)}$
let $k > 1/2$, $0 < \epsilon < 2$
then, $\dot V \le 0$ whenever, $|e| > \frac{d_{max}}{\sqrt{2k-1}}$ & $|\tilde a| > a$
$\Rightarrow$ Residual set; $\{(e, \tilde a) \mid |e| < \frac{d_{max}}{\sqrt{2k-1}}$
Note: ① Even if $d = 0$ only
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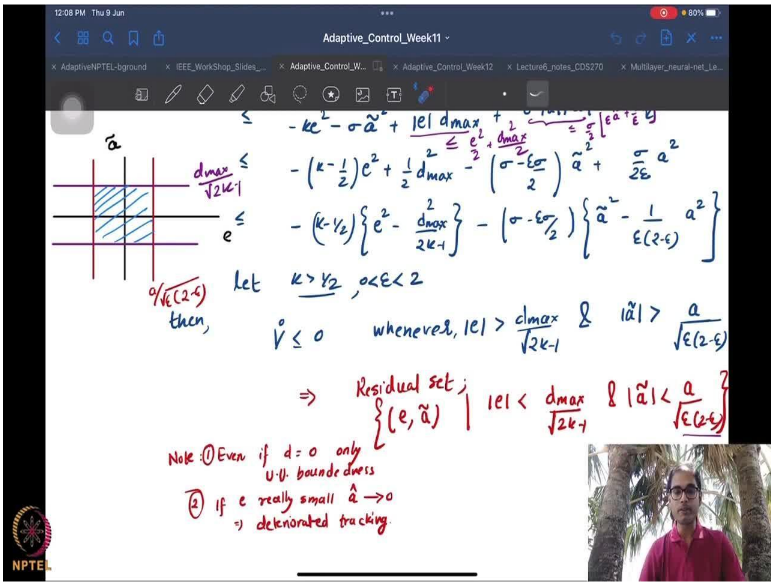

Now one of the, of course, we did the stability analysis, and we figured out some nice properties for such projection based adaptive control laws. One of the sorts of issues are well, I mean, one of the sort of concerns that may be raised on in such solutions is that you require knowledge of the bounds and parameters though it is a very reasonable assumption, but if you do not have such a knowledge, then there also exist solutions, they are called the sigma modification and the epsilon modification. The basic idea being that the standard certainty equivalence adaptive law is augmented with a damping term, which was not present in the typical certainty equivalence law. And what this damping term does is it creates a residual set in both a tilde that is the parameter error and the tracking error.

Now one of the issues, well, a couple of issues one of the issues is that even if the disturbance is 0, we only get bounded performance with such kind of analysis, which may more or less be okay for most practicing engineers, but may not be okay with some theoreticians. They would say that there is no external disturbance, why am I still getting sort of poorer performance, not ideal performance.

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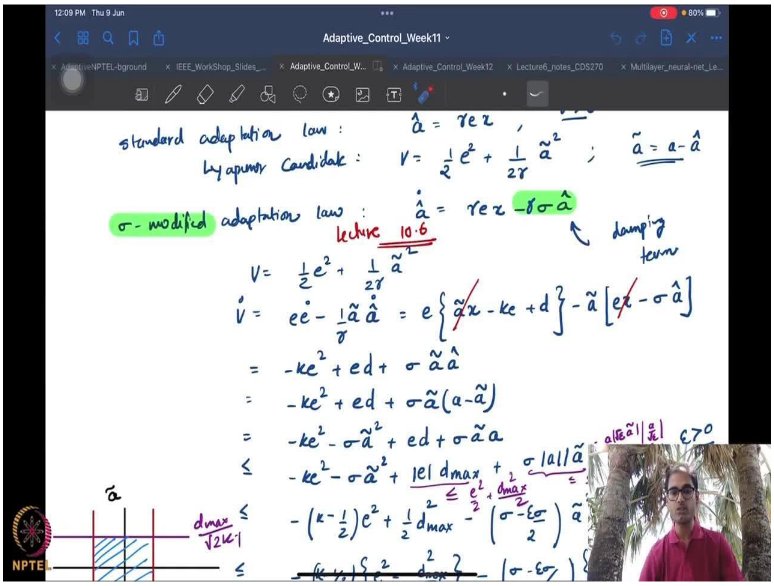

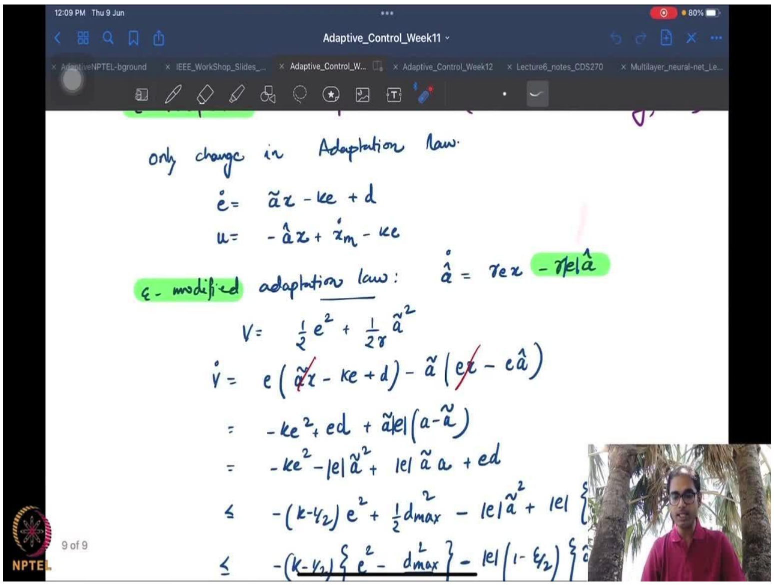

The other issue is that, because of the structure of a hat dot, when e goes to 0, that is, e becomes very close to 0, you see that this term starts to dominate significantly and whatever parameters value has been learned a hat, suppose a hat has gotten rather close to the true value a, that gets unlearned because this term will push every a hat to 0, actually, you unlearn the true parameter value and this can of course, cause a deterioration of the tracking error performance also. And this is where the epsilon modification came in, where instead of constant gain sigma, the gain is now in absolute value of the error itself. Therefore, when the error goes to 0, then this term also goes to 0, and whatever parameter value you have learned until that point remain as it is, there is no change. So that is sort of what you would do the best you can do in some sense.

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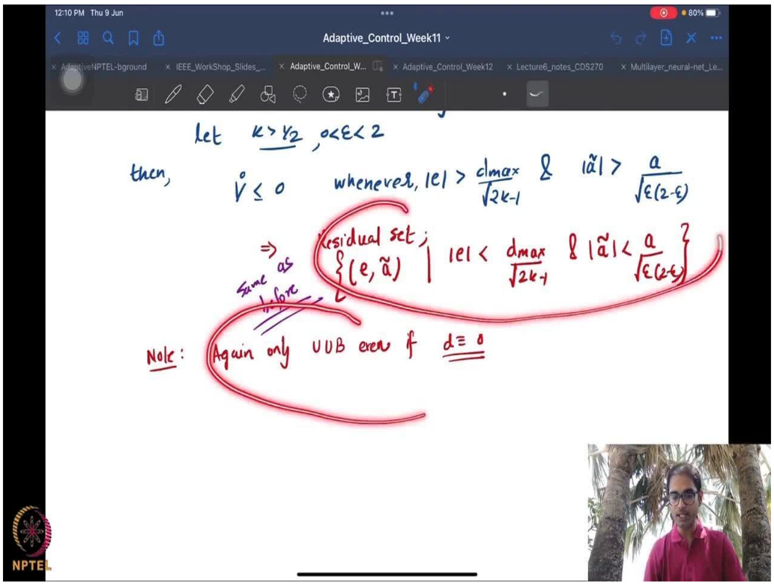

Again of course, this also suffers from the same issue like the residual set expressions are exactly same as before. So even the absence of the disturbance, we will get only uniform ultimate boundedness. But in any case, these were rather useful solutions, I hope, I mean, for realistic applications, these are rather useful solutions, and either projection or sigma epsilon modification are more than sufficient to impart robustness to your adaptive control against unmodeled disturbances or unknown disturbances. So as long as things are the disturbances are bounded you are good to go.

Now we have, by this time seen, covered more or less large breadth of the adaptive control, of course, it is like any other field, it is huge and always more to learn, more to do, and more remains to be taught, but of course, we cannot expect to do everything, but we have covered a very large portion of adaptive control and most importantly all of you have been enabled to do your own design, your own adaptive law designs as you go along.

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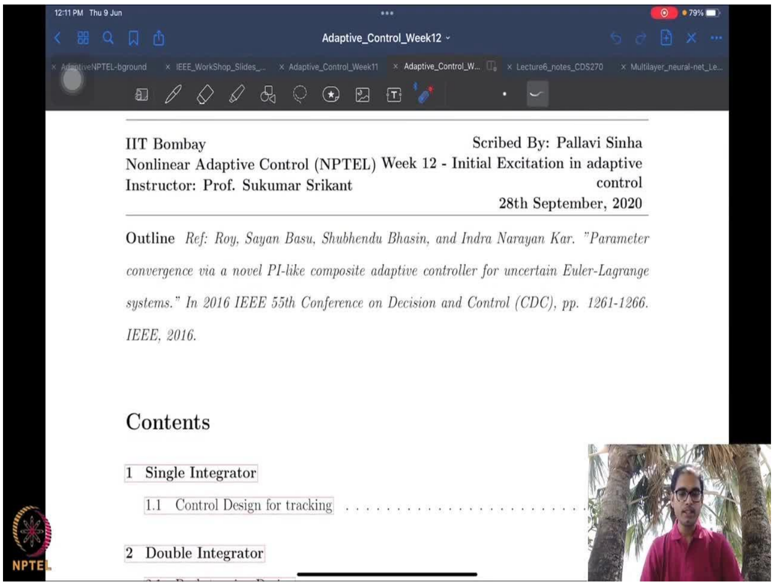

Now what we want to do henceforth is more modern ideas or ideas that are becoming more popular now in adaptive control. One of the first sort of topics that we want to do and this is where we will start today is basically initial excitation based adaptive control. Now as you can see this is not too far pretty recent work. And so one of the sort of issues in adaptive control is parameter convergence or parameter learning as you would say in modern terminology. And this parameter learning requires persistence of excitation, this is well known.

Now persistence of excitation as the word says is persistent, so it has to exist for all time for you to be able to do good parameter identification, and this may not always be possible, this may not especially, when you are sort of close to being done tracking, you want to move on and probably do another tracking problem. So you may not have enough persistence in your signals for you to have learned the parameter value perfectly. So a lot of research went into relaxing this persistence condition and this is what is called initial, well this one of these methods is called initial excitation based adaptive control and the authors here are from IIT Delhi. So very local, may be vocal for local, but very interesting bit of work, interesting sequence of work on relaxing persistence excitation, so this is sort of our reference point.

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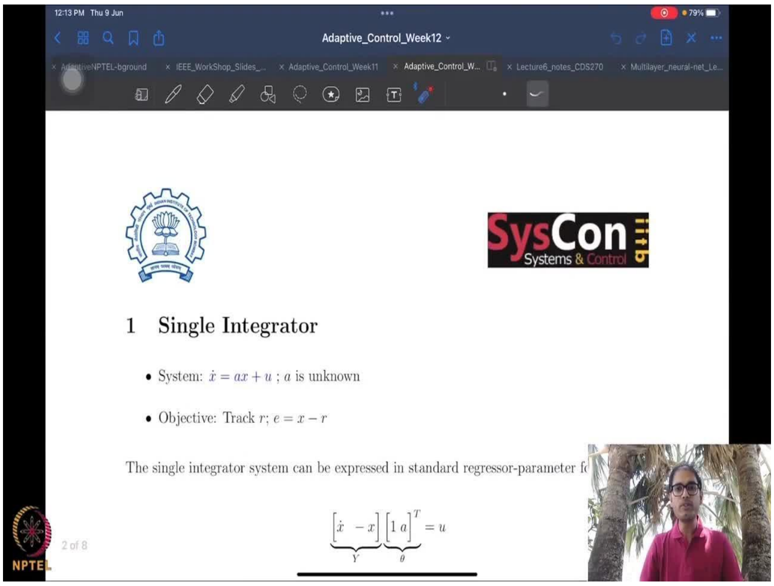

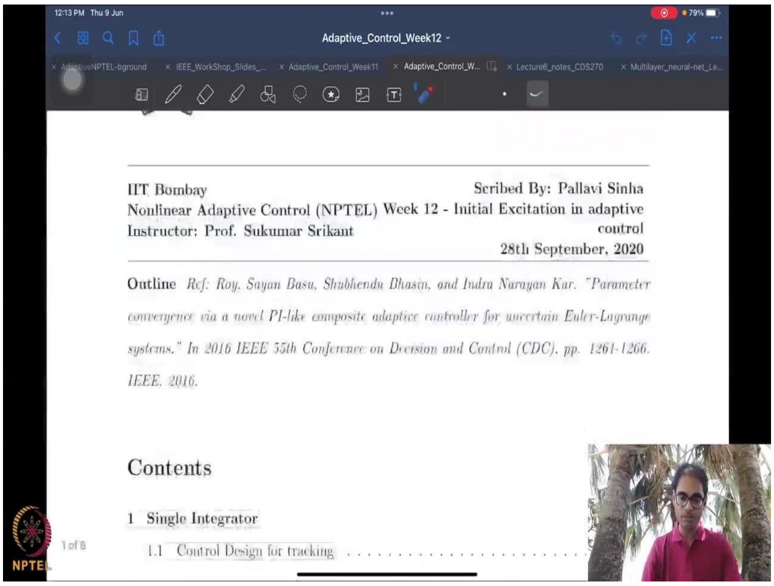

So what we will do is, we will look at a simple version of what they propose, they are proposing all of this for, as you can see Euler-Lagrange system which is essentially the standard model for robot systems. So we will have to look at slightly simpler version of things for classroom illustration purposes, you can very easily extend this to once you understand this, you can very easily extend this to the robot problem or any other control problem.

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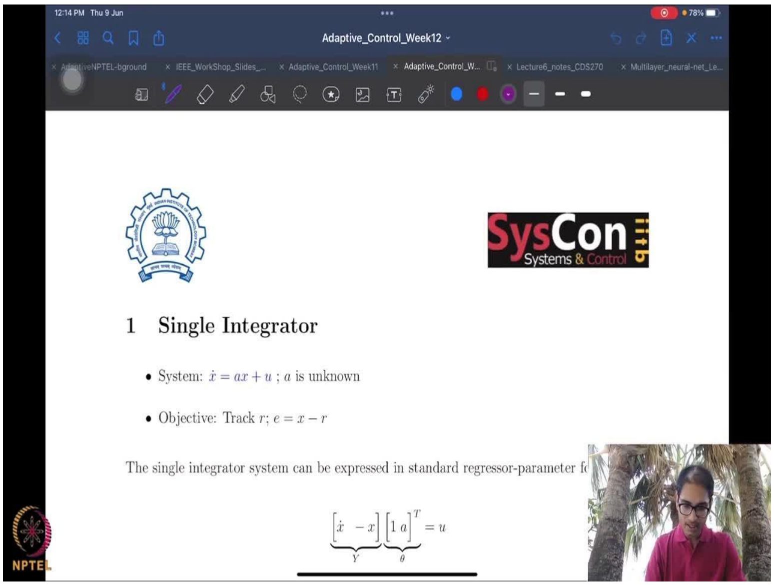

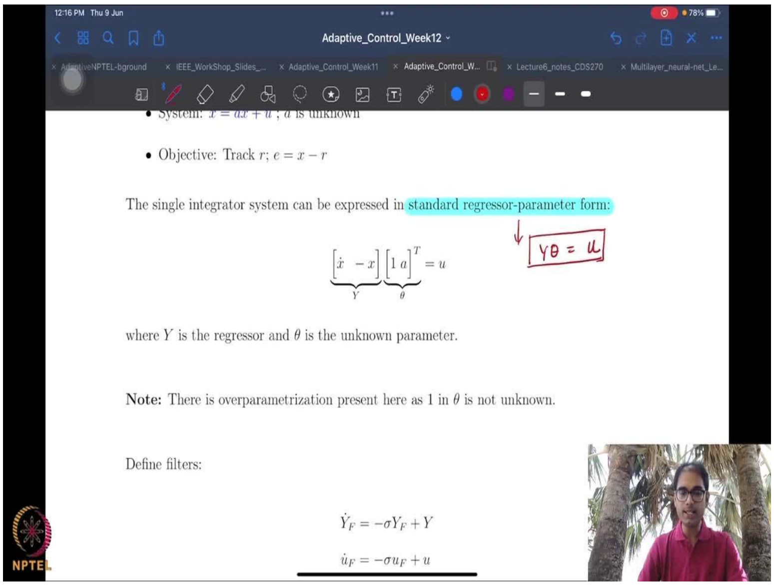

So let us begin this is where we start this week's lectures. Lecture 11.1. So as usual, let us continue with a single integrator setting. So I have a single integrator model, this could have be nonlinear or linear it does not matter, nothing much changes there. As long as you have this kind of a linear parameterization a. So a is unknown, there is a control in a’s, more often than not they would be positive because otherwise this is stable and nothing much to do. And the typical objective is to track a reference signal r and so as always we construct an error signal x minus r.

But before we go on to do any control design, we start to look at the parameter design. So this is going to be similar to the non-certainty equivalence method that we saw in projection algorithm, the projection based adaptive control algorithm. And so this also has a similar feel in the sense that the parameter update is sort of done separately from the Lyapunov analysis. And we will of course talk about it a little bit more later on.

So what we do is we want to express our system, any system for that matter, in this case a single integrator, but we want to express any system in a standard regressor parameter form, and what is the standard regressor parameter form? We want to write our dynamical system as y theta equals u. There is an unknown connected to u, of course that is a separate problem that can be dealt with also, but right now we only deal with the problem where the control gain is not unknown, in fact, it is identity. So we write the system as y theta equal to u this is what we do for all dynamical system, not just for a single integrator.

So in order to do this, for the single integrator, you will notice that I have constructed a regressor and a theta. Now the interesting thing to see here is that because I want to write it as y theta equal to u, I have no choice but to augment the a with a 1 in theta, otherwise this dynamics and this will not match. This I have to do and it is common in this initial excitation based adaptive control. We have to do this, that we have to augment our parameter a with a 1.

So there is a over parameterization, we have we are trying to identify something that we already know, 1 is not unknown, but we will try to identify it. So already you see one, if you name one disadvantage, if you may opt this method. So you have y theta equal to u. So where y is the regressor, which is basically x dot and x, and theta is 1 and a, and u of course, retained as such. So that is what we mentioned here, there is over parameterization as 1 and theta is not unknown.

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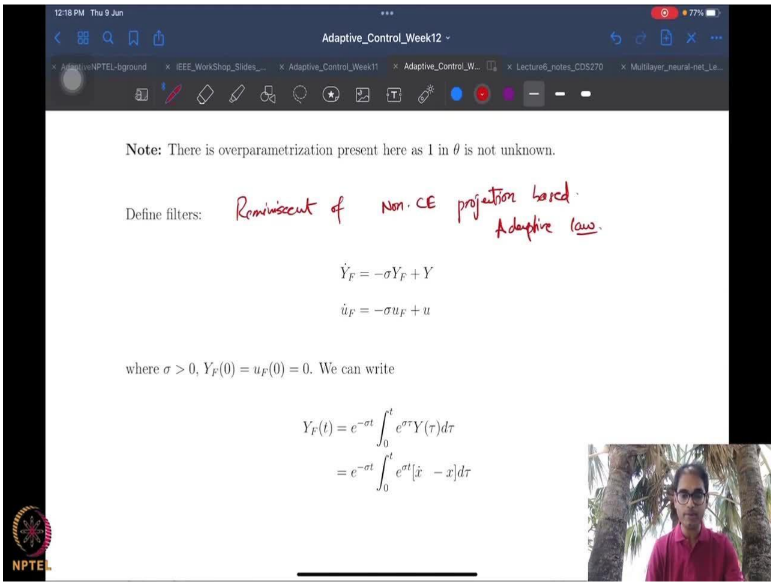

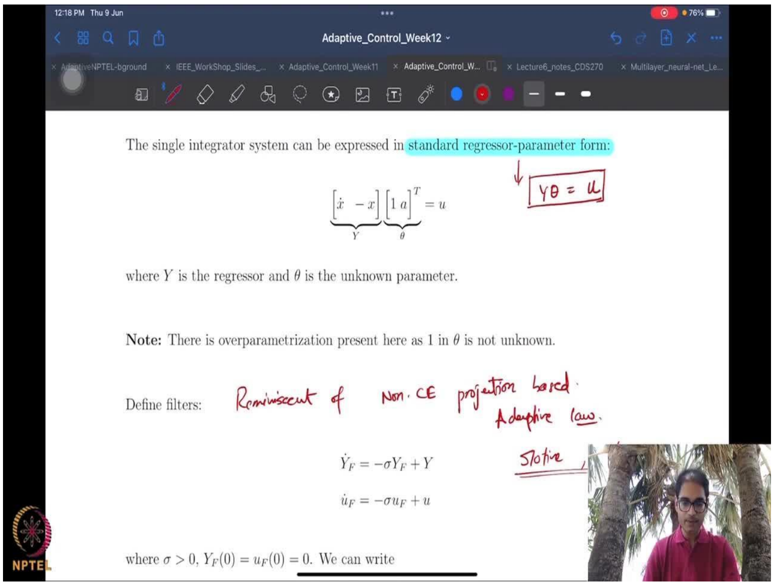

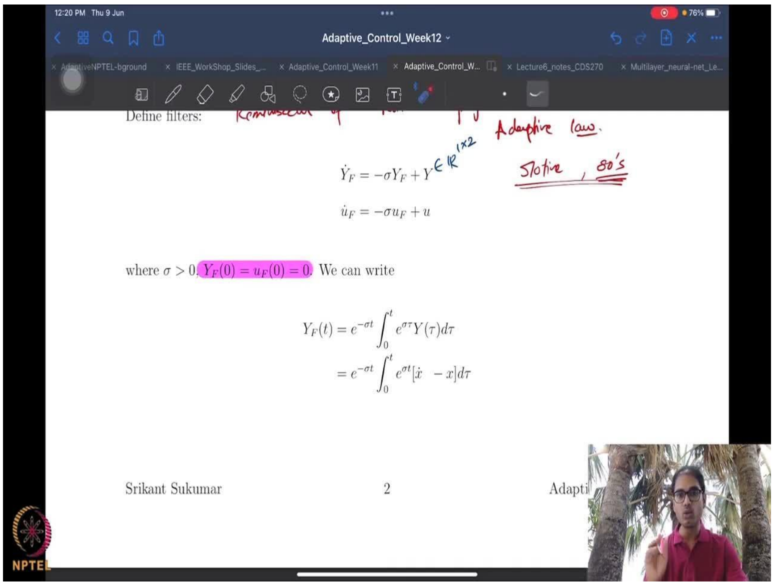

Now we start with the process of defining filters, again, this should be reminiscent of non CE projection based adaptive laws. This is should be reminiscent of non CE projection based adaptive laws, in fact why is it so reminiscent? Because most of these authors have followed an earlier author's work, so slotine had done, had sort of proposed this idea in 80's, now you can look up this reference and you can find this reference. So therefore, a lot of authors are using similar ideas, so even the projection based adaptive control the non CE version, was by Akella. And they also are motivated, of course, by this slotine work in the 80's that is the idea.

Now what is the idea again in pair also if you remember, in this equation, you essentially filtered everything that was known to you, all known quantities got filtered. So what is known, the regressor is known and the control is known. So both of these are passed through a filter, a low pass filter if you may, and with some bandwidth sigma which is positive, of course. And the additional thing here is you asked for what are you required 0 initial conditions. So in our earlier, a sort of result, the initial conditions were not specified, but here we are specifying the initial kind to be 0. These are again filters that are implemented in theory, I mean, in the sense that they are implemented on a computer, they are not real data coming from any real dynamical system, so we do not have to worry about the initial conditions not being exactly 0.

So once we have assumed this sort of filter structure, I apologies, this sort of a filter structure and some initial conditions, I can actually solve for this. So we are sort of trying to verify the implementability of this filter structure filter. So we just integrate this using standard variation of parameters integration. So the solution of this for 0 initial condition is just this e to the power minus sigma t, integral from 0 to t, e to the power sigma tau, y tau, d tau, so that is it, this is the solution. And now I can substitute for y, which is just x dot and minus x. So this is actually if you notice, in this case, y is actually, it is a 2 row vector, row vector of size 2.

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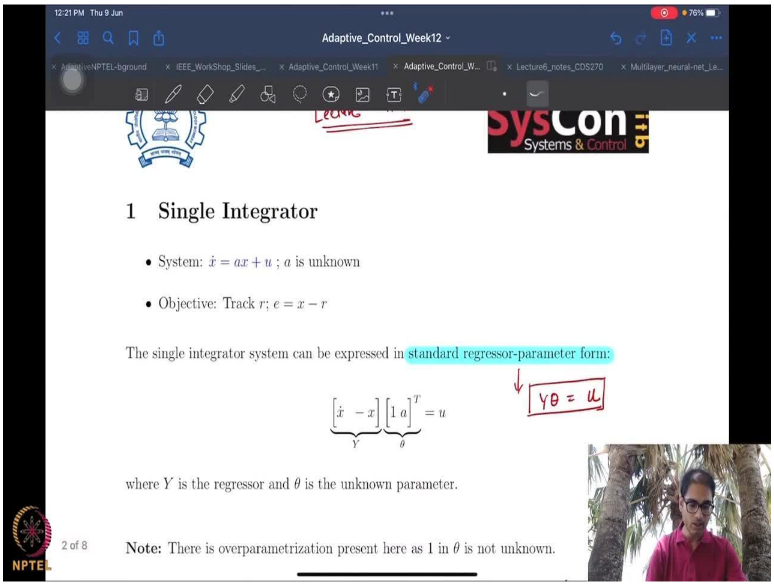

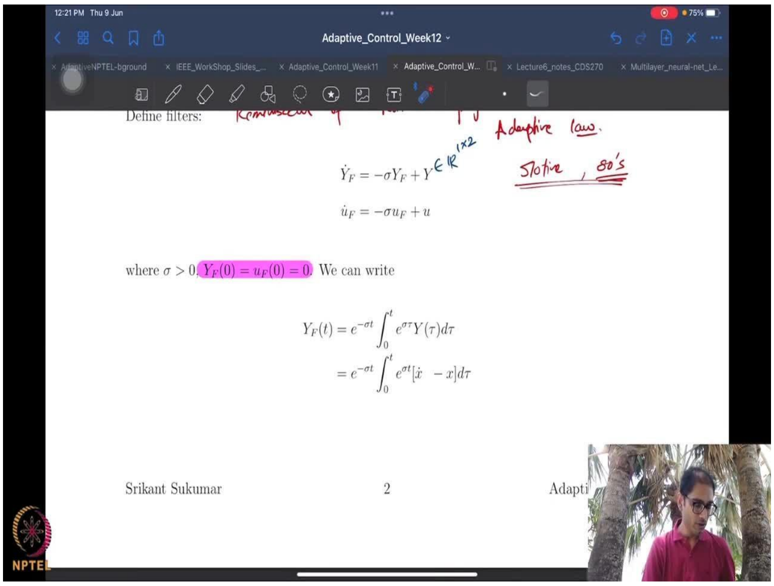

So now one of the if you look at this first term here, there is a little bit of a problem typically in a state space system, x dot is not measured, you measure the state, but not the derivatives of the state. So this is the typical assumption in a state space system. So the question is, can we still implement this kind of a filter? And the answer is yes, otherwise I am assuming the I am sorry, I am sorry for that, otherwise, the authors will not have proposed it I guess. So if you look at this term, you want to see how to deal with this particular term, this term is of course implementable, no problem, because x is available. So all I need to do is integrate this or whatever mean or solve this differential equation with this x piece, no problem.

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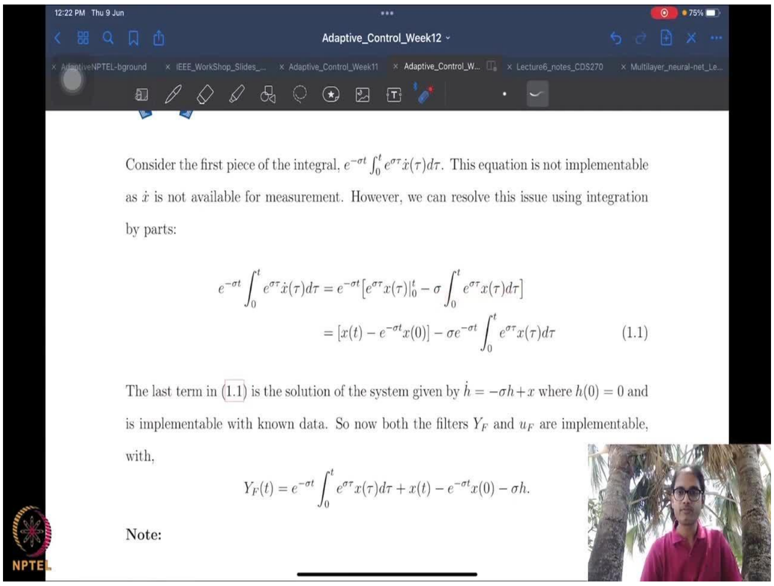

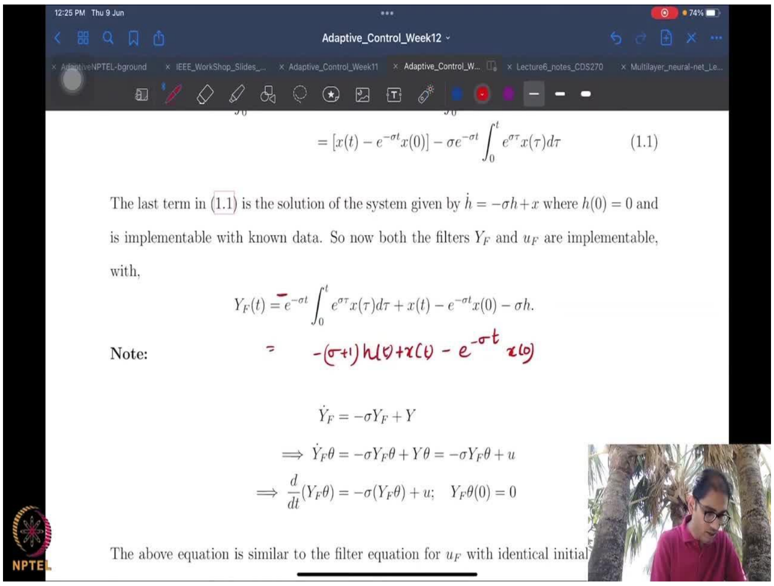

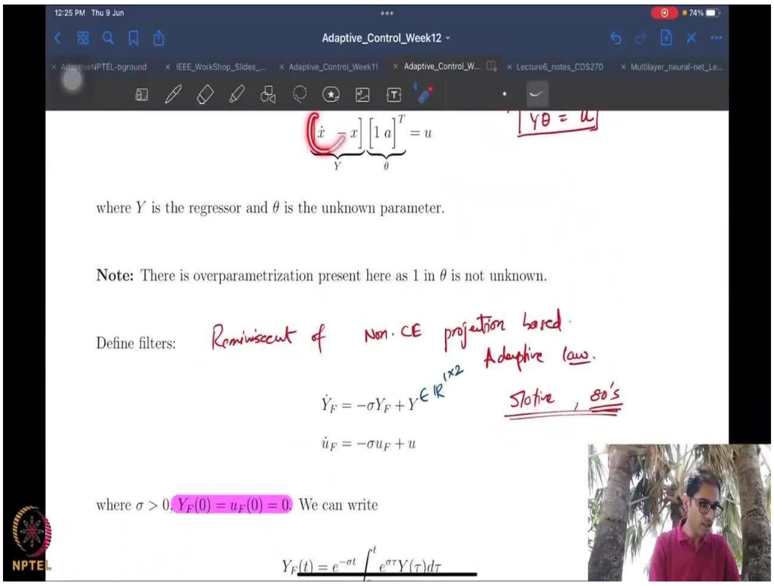

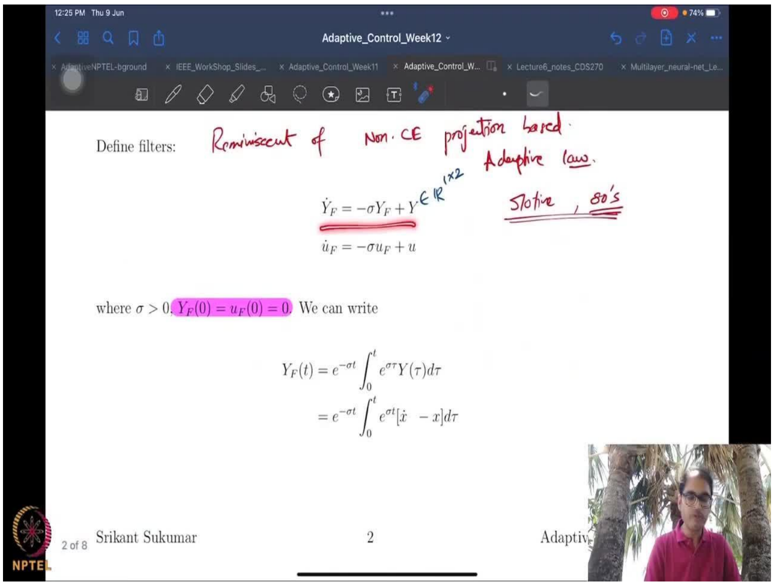

So that is what we do we look at the first piece of the integral here it is this guy, and not implementable, because x dot is typically not available for measurement, this is very standard assumption in state space design. Now how do we resolve this, we just do integration by parts on this guy. So what is integration of parts by parts of this guy, it is just whatever the scaling remains as it is, and then you have first times integral of second and then minus integral of differential of first times integral of second, we are just applying the integral by parts formula.

And now you see something nice has happened, what is this something nice? You see that this thing is now implementable, this x is known, anyway this is implementable, we will anyway look at it further. And this quantity can be evaluated, so this if I evaluate this whole thing right here, you get this, you get x t, x at time t minus e to the power minus sigma t x 0, minus sigma e minus sigma t, 0 to t, e to the power sigma tau, x tau, d tau. So nice, everything looks like an integral or integrable term, everything looks like it is using measurements that are available, so that is good. In fact, if you look at this particular quantity, sorry, this particular quantity, this is actually the solution of the system with 0 initial conditions, very standard very easy to verify. Excellent.

So that is what we do, we just write it in terms of this new variable h just for ease of notation, otherwise, you can directly use this also no problem. So therefore, what do we have? We have Y f is actually equal to e to the power minus sigma t, e to the power sigma tau x tau d tau, this is the second term by the way, this is just the second term here. I hope I got the sign correct. I think this should be the second term is a negative term, so I think this should be a negative sign. This is minus e to the power minus sigma tau 0 to t, e to the power sigma tau x tau d tau, and then you have x t from here, minus e to the power minus sigma t x 0 from here, minus sigma x from here.

And if you now look at this term also this is also in fact equal to h, this is also equal to h in this particular case. So this actually becomes minus sigma plus 1 h plus x t minus e to the power minus sigma t x 0. So the important thing to remember is that, h is implementable because x is known, therefore, Y f is also implementable, pretty straightforward, because x is known and h is known. So in fact, I can be more precise here and say that this is actually h evaluated at t, h evaluated at t, great.

So we have essentially constructed a filter and unusual filter, because you also have the derivative of the states in here that is being filtered. And but the important thing to remember is that this filter is implementable. Excellent, great, because you do not want to design something that is not implementable.

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with,

$$Y_F(t) = e^{-\sigma t}\int_0^t e^{\sigma\tau}x(\tau)d\tau + x(t) - e^{-\sigma t}x(0) - \sigma h.$$

$$= -(\sigma+1)h(t) + x(t) - e^{-\sigma t}x(0)$$

Note:

$$\dot{Y}_F = -\sigma Y_F + Y$$

$$\implies \dot{Y}_F\theta = -\sigma Y_F\theta + Y\theta = -\sigma Y_F\theta + u$$

$$\implies \frac{d}{dt}(Y_F\theta) = -\sigma(Y_F\theta) + u; \quad Y_F\theta(0) = 0$$

The above equation is similar to the filter equation for u_F with identical initial conditions.

By uniqueness of solutions, we have $u_F = Y_F\theta$.

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it doesn't allow for the removal of the persistence of excitation condition. We ad

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By uniqueness of solutions, we have $u_F = Y_F\theta$. ← Similar to original $Y\theta = u$ as in proj based adaptive control.

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Srikant Sukumar 3 Adaptiv

Now let us look at some properties of these filtered variables site if you remember even and again, in the projection based productive control design also we wrote, we sort of tried to relate the or write the dynamics in terms of the filter variables also. So we are sort of doing something similar, not exactly the same, but something similar. So if you look at this Y f dot, it is just minus sigma Y f, plus Y. So if I multiply by theta on both sides, I get this. And notice that Y times theta equals to u from our regressor parameter form, I have just substituted here.

And now because theta is a constant, this is possible only because theta is a constant. If I take the derivative, if I take Y f theta as a variable and take the derivative, I get exactly the left hand side and on the right hand side, I have minus sigma again, times Y f theta plus u, and Y f theta at 0, 0, because Y f at 0 is 0. So if you compare it with the u F equation, I will write it here for your reference. To compare it with the u F equation, what do you note? You note that these 2 sorry, these 2 equations are exactly the same. These 2 are exactly the same equations with the same initial conditions, so it is just different notation.

So what does it mean? It means u F is equal to Y f theta. So this is very similar to what we did in projection based adaptive control. And even there, we found an equation in terms of the filtered variables and that is exactly what we are doing here too. Because the original equation was Y theta equals to u in the regressor parameter form and we notice that just like in the projection based adaptive control method, the filtered equation also has a very similar structure, just that the Y is replaced by Y f and the u is replaced by u F. And this is of course, a very important property that gets used subsequently. So remember so that is what, I would say similar to originals Y theta equal to u as in projection based adaptive control.

Again, not a surprise, most of us researchers are always motivated by some past work, and multiple researchers could be motivated by one seminal piece of past work. So this work by Slotine in the 80’s, who also proposed having filters is of course, something that a lot of people refer to. And so of course, you have very similar feel to this as opposed to as sorry, when you compare with projection based adaptive control. Now one thing we know is that the performance of this system does improve by adding filters, how do we know this? Suppose we did not do anything, Slotine did. So they also showed that the performance in improves.

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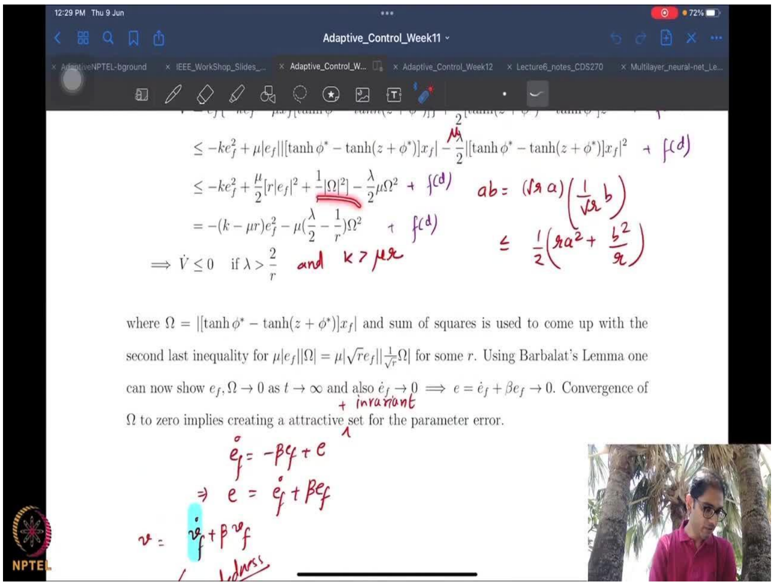
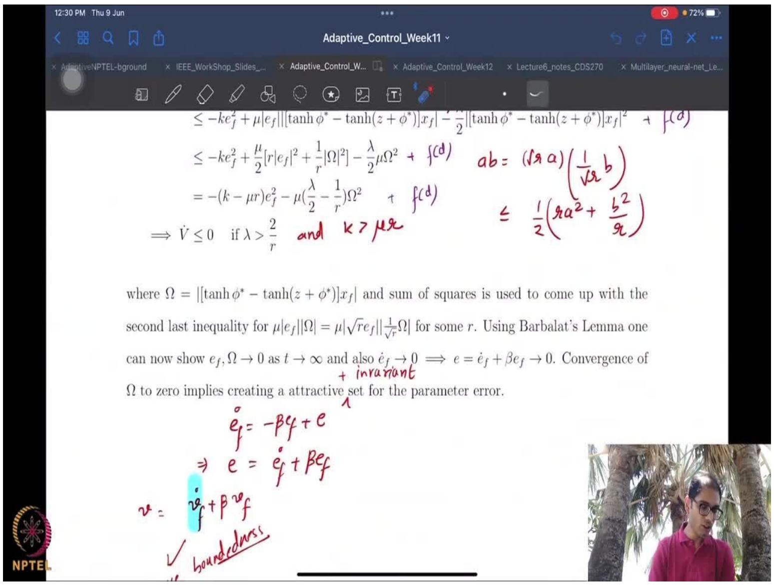

Adaptive_Control_Week11
$\leq -ke_f^2 + \mu|e_f||[\tanh\phi^* - \tanh(z+\phi^*)]x_f| - \frac{\mu\lambda}{2}|[\tanh\phi^* - \tanh(z+\phi^*)]x_f|^2 + f(d)$
$\leq -ke_f^2 + \frac{\mu}{2}[r|e_f|^2 + \frac{1}{r}|\Omega|^2] - \frac{\lambda}{2}\mu\Omega^2 + f(d)$
$= -(k-\mu r)e_f^2 - \mu(\frac{\lambda}{2} - \frac{1}{r})\Omega^2 + f(d)$
$\implies \dot{V} \leq 0$ if $\lambda > \frac{2}{r}$ and $k > \mu r$
$ab = (\sqrt{r}a)(\frac{1}{\sqrt{r}}b) \leq \frac{1}{2}(ra^2 + \frac{b^2}{r})$
where $\Omega = |[\tanh\phi^* - \tanh(z+\phi^*)]x_f|$ and sum of squares is used to come up with the second last inequality for $\mu|e_f||\Omega| = \mu|\sqrt{r}e_f||\frac{1}{\sqrt{r}}\Omega|$ for some r. Using Barbalat's Lemma one can now show $e_f, \Omega \to 0$ as $t \to \infty$ and also $\dot{e}_f \to 0 \implies e = \dot{e}_f + \beta e_f \to 0$. Convergence of Ω to zero implies creating a attractive + invariant set for the parameter error.
$\dot{e}_f = -\beta e_f + e$
$\Rightarrow e = \dot{e}_f + \beta e_f$
$v = \dot{v}_f + \beta v_f$
NPTEL


Adaptive_Control_Week11
$\leq -ke_f^2 + \mu|e_f||[\tanh\phi^* - \tanh(z+\phi^*)]x_f| - \frac{\mu\lambda}{2}|[\tanh\phi^* - \tanh(z+\phi^*)]x_f|^2 + f(d)$
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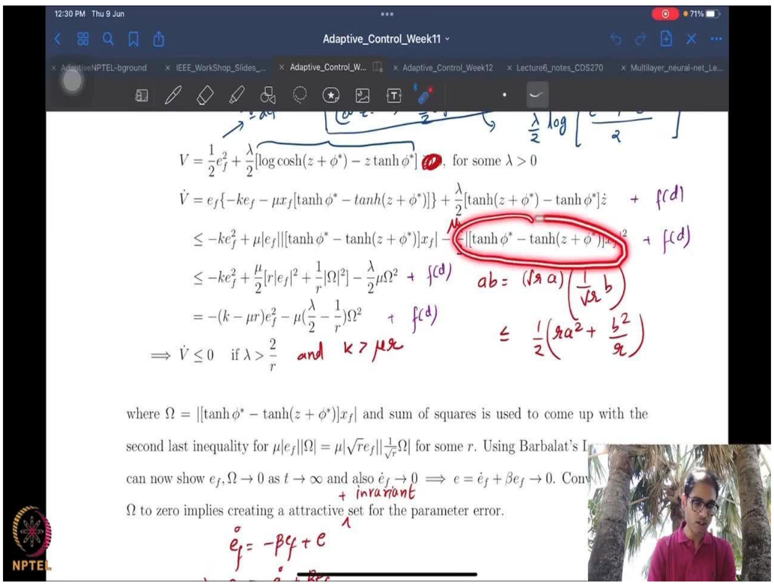
$\dot{e}_f = -\beta e_f + e$
$\Rightarrow e = \dot{e}_f + \beta e_f$
$v = \dot{v}_f + \beta v_f$
✓ boundedness
NPTEL


Adaptive_Control_Week11
$V = \frac{1}{2}e_f^2 + \frac{\lambda}{2}[\log\cosh(z+\phi^*) - z\tanh\phi^*]$, for some $\lambda > 0$
$\dot{V} = e_f\{-ke_f - \mu x_f[\tanh\phi^* - \tanh(z+\phi^*)]\} + \frac{\lambda}{2}[\tanh(z+\phi^*) - \tanh\phi^*]\dot{z} + f(d)$
$\leq -ke_f^2 + \mu|e_f||[\tanh\phi^* - \tanh(z+\phi^*)]x_f| - \frac{\mu\lambda}{2}|[\tanh\phi^* - \tanh(z+\phi^*)]x_f|^2 + f(d)$
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can now show $e_f, \Omega \to 0$ as $t \to \infty$ and also $\dot{e}_f \to 0 \implies e = \dot{e}_f + \beta e_f \to 0$.
Ω to zero implies creating a attractive + invariant set for the parameter error.
$\dot{e}_f = -\beta e_f + e$
NPTEL

And also we did in the projection based adaptive control, you remember that because of this filtered variables, you got some kind of attractive set here, you see you got an attractive invariant set here, which is omega equal to 0, we got an omega equal to 0, which was an attractive invariant set. So this is sort of what Slotine also proved and that is what you get when you have a filtered like a filter system, you can expect to get this kind of convergence to some attractive set kind of behaviour which improves the performance of the adaptive control, however, this does not allow you to get rid of persistence.

And even here in this system, if you notice, all you are guaranteed to get is that omega goes to 0. And what was omega? Omega was some complicated function of the unknown. So this quantity going to 0 does not necessarily mean the parameters are convergent. It is just that this quantity is going to 0 and you need some kind of persistence condition on probably x f or something to get parameter convergence. So that is the important thing to remember that persistence condition is not cannot be done away with just because you filter it. And this is the sort of lesson that was well understood.

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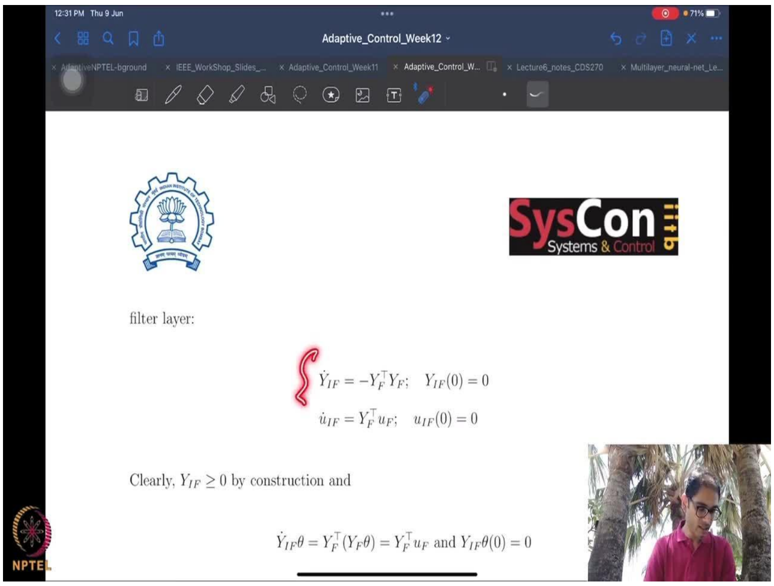

And in order to relax the persistence condition, these authors, that is Shubhendu and Shayon and Indrakar from IIT Delhi, they proposed the introduction of second layer filters. So this is the innovation, the addition of a second layer filter. So earlier it was just filtering Y, now they filter Y f, how? It has a very specific structure, Y IF now is the new variable, Y IF dot is just minus Y F transpose Y F, again with 0 initial condition and u IF dot is just Y F transpose u F with 0 initial conditions. So this is the second layer filter, which will in fact help us to get rid of the persistence of excitation requirement. Excellent.

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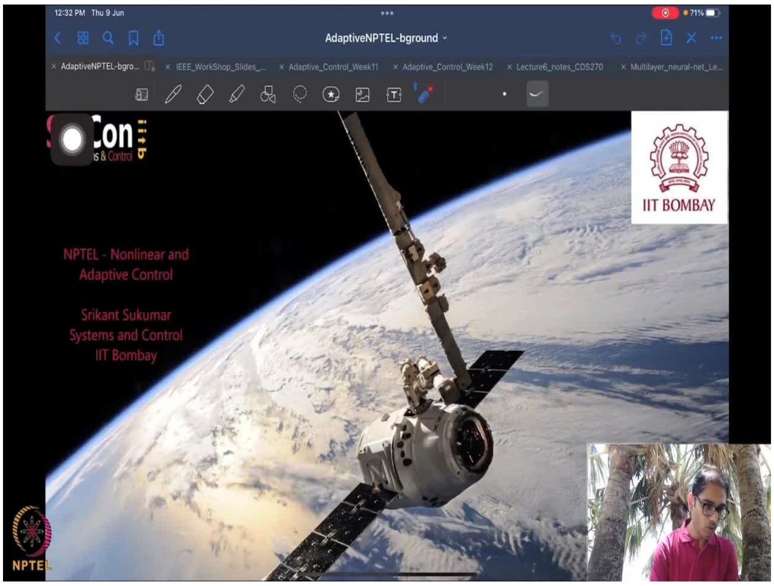

So what did we look at today? We started talking about one particular method, which helps relax persistence of excitation condition, which may be a rather steep demand on a dynamical system, because having excitation infinitely is not a very feasible requirement. Therefore, in order to relax such a requirement, there is some recent work which uses which relies on the filter ideas that we have already seen in projection based adaptive control. The only thing is, instead of a one layer filter that you have in the projection based adaptive controller, which is again motivated by Slotine's work in the 80's.

Here we have a 2 layer adaptive filter. And we will see how this will result in construction of an adaptive law without requiring a Lyapunov function. And subsequently, we will also see how it relaxes the requirement for persistent excitation. So that is really the idea and we will see how this helps improve, the system performance. So we are looking at a single integrator system, of course to keep things simple, but once we understand it, once we can follow this, we can very well follow the work on double integrators and we can also work with Euler-Lagrange systems, spacecraft dynamics, and so on, things like, again, autonomous systems, such as what you see in the background, great. So I will see you again in the next session. Thank you.