**Nonlinear Adaptive Control**

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**Systems and Control**
**Indian Institute of Technology, Bombay**

**Week 10**
**Lecture No: 59**

# Parameter Projection in Adaptive Control (Part 3)

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Hello, welcome to yet another session of our NPTEL on Nonlinear and Adaptive Control, I am Srikant Sukumar from Systems and Control, IIT Bombay. We are well into the 10th week of our lectures on Nonlinear adaptive control and we are now in the process of learning about not just design of adaptive laws for uncertain parameters, but also robust adaptive laws, which are not impacted because of disturbances that appear in the system due to unmodeled dynamics or external reasons that may impact a dynamical system such as what you are see in the background.

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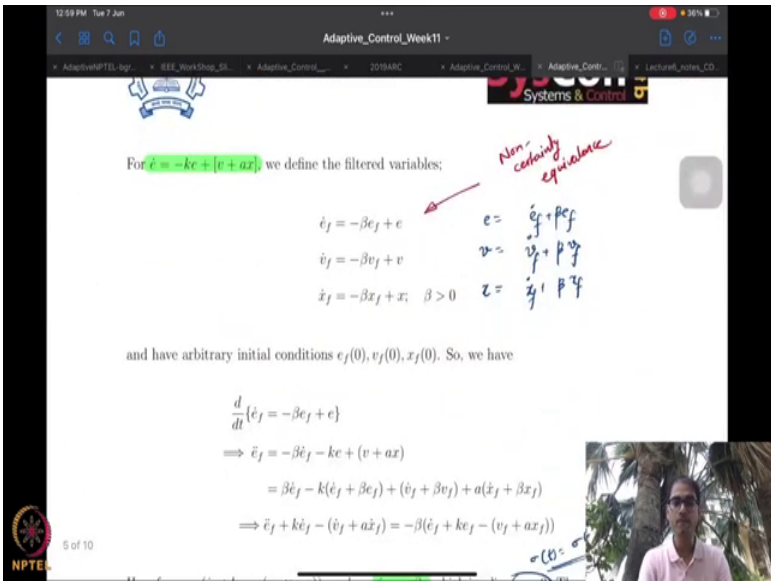


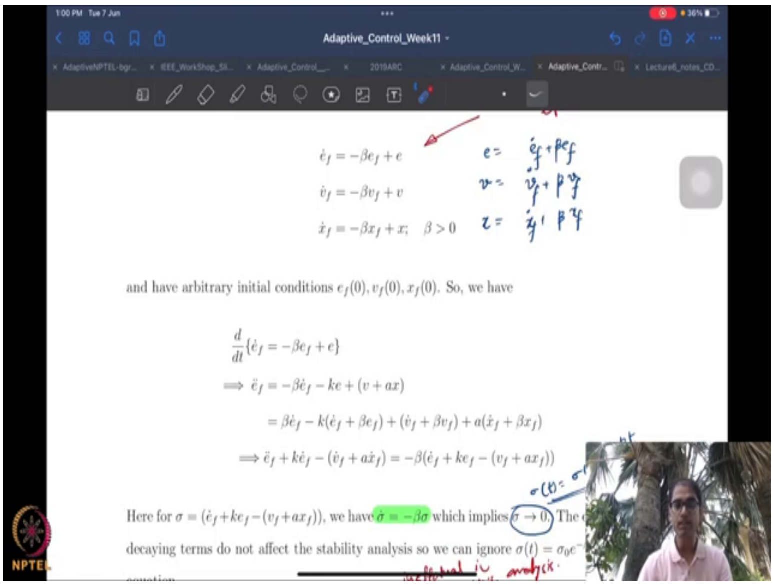

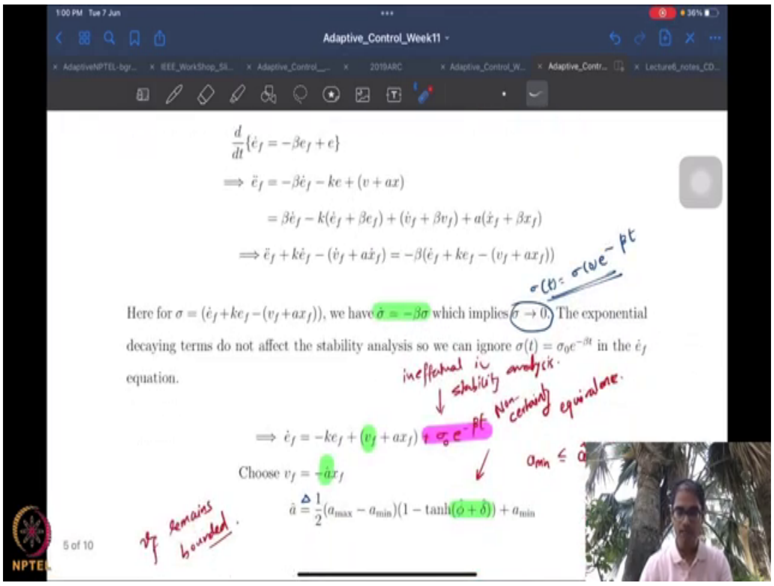

So, what we were doing in this previous session is, we had sort of started to look at the stability and adaptation design, or update law design for this projection based adaptive controller. And the idea was that the framework was based on filtering the closed loop signals.

So, we actually filtered the known closed loop signals first. And they were, of course, the important thing to remember was that these were all identical filters there is identical gains. And we then, of course, computed the dynamics in this filtered variable. This was another very critical step.

And then we figured out that, this dynamic has an additional exponential decay term, which we ignore, in standard stability analysis, this is pretty common, you can also keep it and continue your analysis, but then those terms are anyway going to go to 0, so, does not matter. We will, essentially, we end up with a lot of exponential decay terms, which you do not care about. So, we do not carry it any further anyway. So, we sort of ignore this one.

So, we have a dynamic in the filtered variables, which looks very much like the dynamics, the original system dynamics, and this is what it is rather key. And we choose a V f in terms of this what we call a hat, which is in fact the projected version of. So, we actually have a phi hat plus delta hat here. And so this results in a hat always lying between a min and a max. And these are the bounds that are already given to us, but these are bounds on the parameter data given to us, and this a hat is what appears in the control law V f and so, therefore, the controller remains bounded.

So, and then we also of course had a short discussion and I mentioned that if the control V f is the filtered control is bounded, then V is also bounded because V f dot is bounded and beta V f is bounded. So, this is again something rather important, this is what is the robustness aspect and even in the presence of disturbance we are not wild and nothing changes here.

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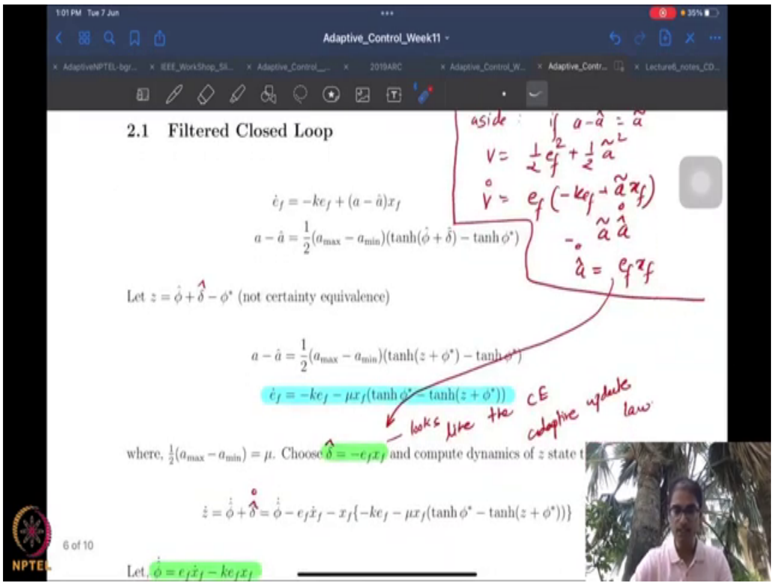


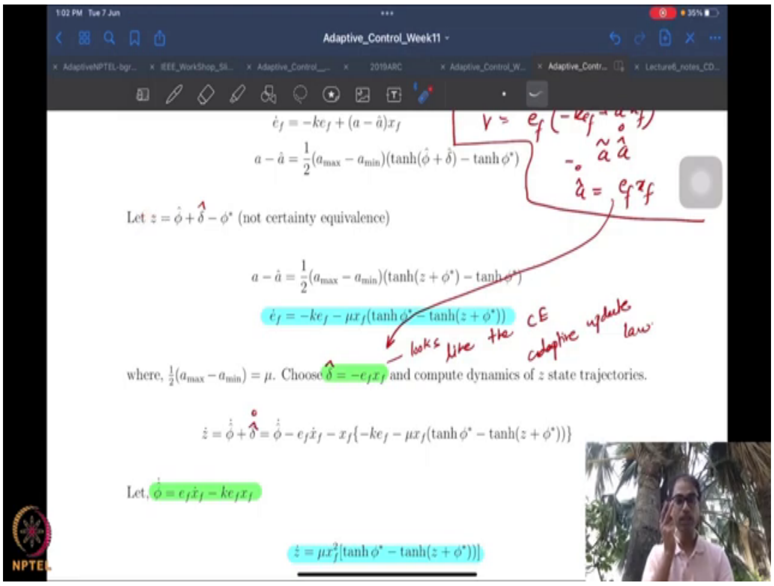

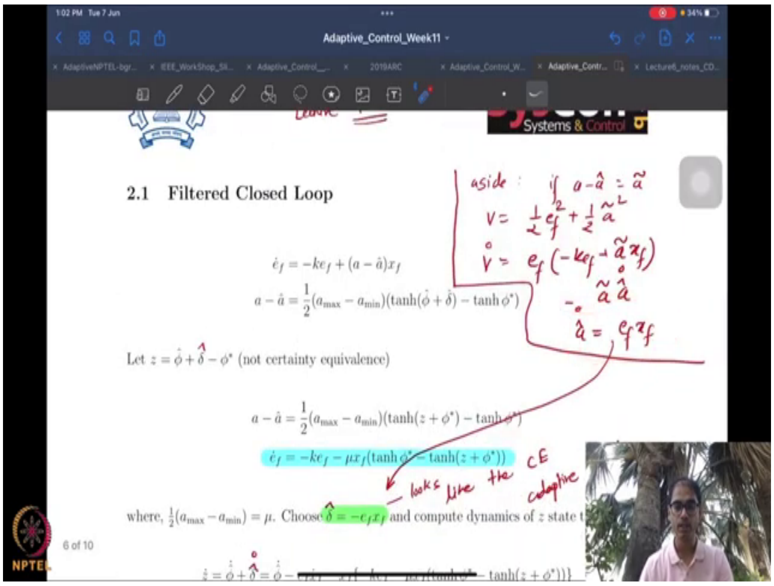

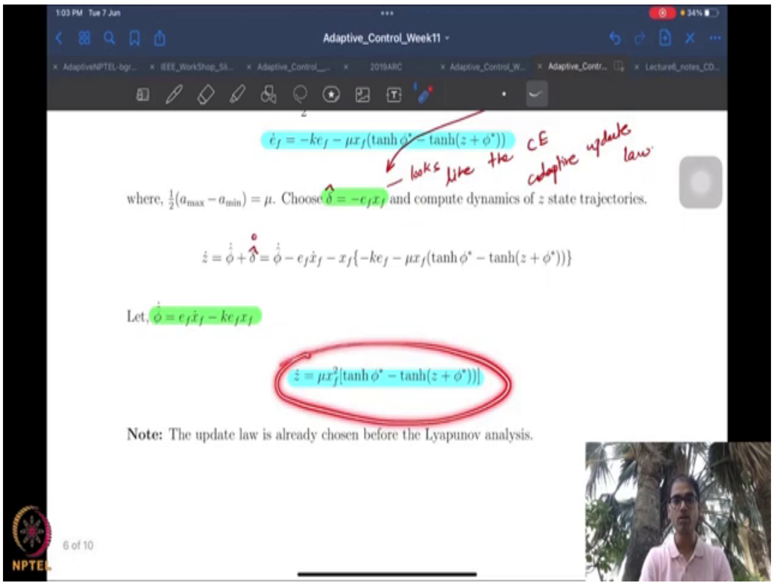

So, then if we look at this, we started looking at the filter closed loop of course, we had the e f dynamics written in terms of the new parameters z, again a new sort of expression instead of a tilde because we are using a non-certainty equivalence paradigm.

So, we had an e f dot dynamics and we had a z dot dynamic corresponding to the parameter error if you may, and the important step or the interesting thing to remember was that, the there were 2 terms phi hat and delta hat that had to be sort of defined, and delta hat was more was just a directly an expression not a dynamical system, and it was motivated by the certainty equivalence adapted law.

And after that phi hat is chosen just by computing a z dot and cancelling terms that can be cancelled in the z dot using a phi hat dot. So, there was no Lyapunov analysis in order to compute a z dot so, this is again another interesting, perspective in non-certainty equivalence where you are update law for phi hat is not computed using a Lyapunov function, so once we have cancelled whatever we could, we get a nice z dot law.

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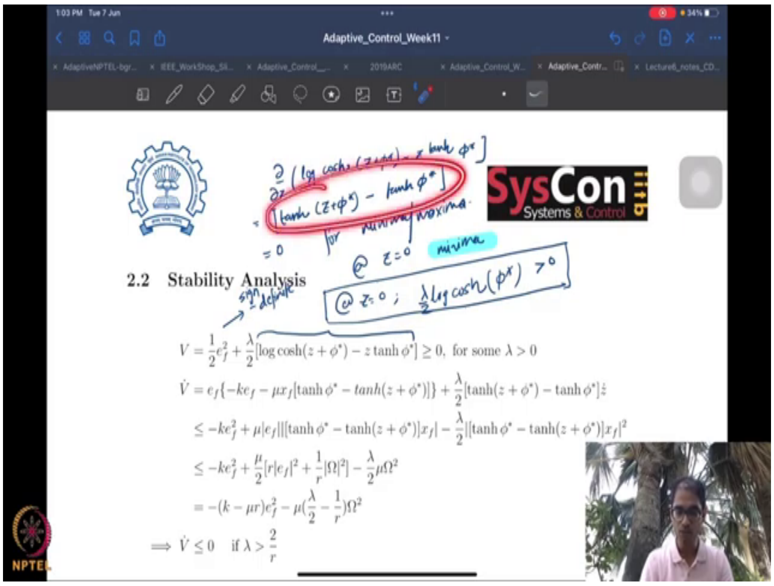


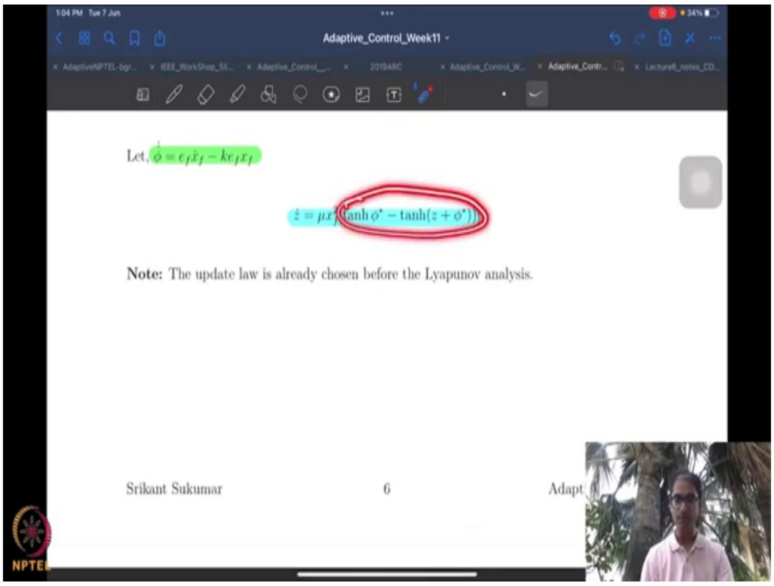

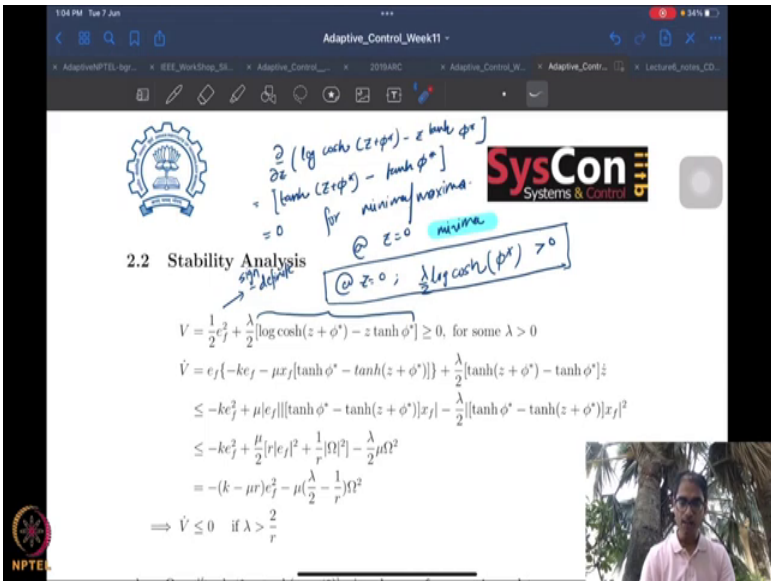


And then we sort of wanted to start the stability analysis. And in order to do so we of course needed a candidate Lyapunov function, which has a rather interesting looking expression right here, not your usual Lyapunov function. The important thing to see is that this is non negative and how we sort of claim that was by trying to find a minimum for this function. And since the e f and z terms are decoupled, we could deal with them separately, it is evident that this is anyway a non-negative term, in order to prove that this is a non-negative term, we took a partial with respect to z, that is we tried to find the minimum and then equated to 0.

When we took a partial of with respect to z, the first thing we realise was that the partial looks very much like the z dot expression, it contains a large piece of the z dot expression. And this is deliberately done, of course, because this will help us in the analysis subsequently.

And so essentially, what we do is we took the partial with respect to z in order to find a minimum or maximum, we equate this to 0. And so, it is evident that the minima is at z equal to 0. Well, it is evident that the optima z equal to 0. And we claim that it is a minima, we leave it to the audience to figure out why it is a minima and not a maximum. And if we substitute z equal to 0, in this expression, we found that the second term goes to 0. So, what we are left with is a non-negative term.

So, that is sort of nice. So, this is essentially what we are trying to say that this V is, sort of has a nice lower bound which is critical for us. So, again, let me so exact, so this is again, something I want to verify and stress on. So, at z equal to 0, we have a minima, and that minima turns out to be lambda with 2 log cos hyperbolic of phi star. And the expression for the cos hyperbolic, is something like this.

The expression for cos hyperbolic is something like this. And you can see that essentially, the cos hyperbolic is it is like we write the expression out, let us see. So, this is I believe this will lambda by 2 log of e to the power phi star plus e to the power minus phi star divided by 2. This is the expression.

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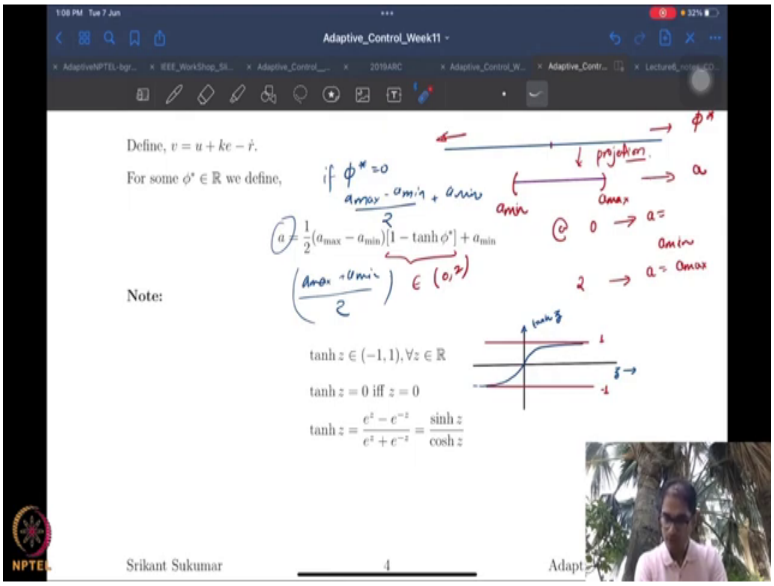


So, I am wondering if this can turn out to be negative also. I am wondering if this can turn out to be negative too it may be possible. It is not impossible for this to be negative too. But the important thing to remember so I do not want to say that this is necessarily, I do not want to say this part that just necessarily positive semi definite. I believe this is certainly lower bounded, I believe this is certainly lower bounded.

So, this is what I wanted to verify that, Tan hyperbolic z plus phi star is equal to Tan hyperbolic phi star here for the minima. And let us see if this becomes greater than or equal to 0 or not.

So, this is what I wanted to verify a little bit more carefully today. So, Tan hyperbolic z plus phi star is equal to Tan hyperbolic phi star. And because the Tan hyperbolic function looks something like this, it looks something like this. The only time the two will match is I believe when z is exactly equal to 0, I believe then z is exactly equal to 0 that is the only time they will match, only time they will match.

However, when z is 0, this term is of course, gone. And then I have this term. I have this term, which is going to give me lambda over 2 log cos phi star, log cos of phi star. And that is what will be, will give me a bound.

So, this is what will be the expression, so, let us not worry about, it is not exactly 0, I believe, not necessarily 0. So, this is not necessarily 0 but this is essentially the lower bound that you will get. And this is essentially a constant lower bound which you will get, I do not think we have to worry too much about what the exact values because for my Barbalat's Lemma type analysis, all I need is a lower bound.

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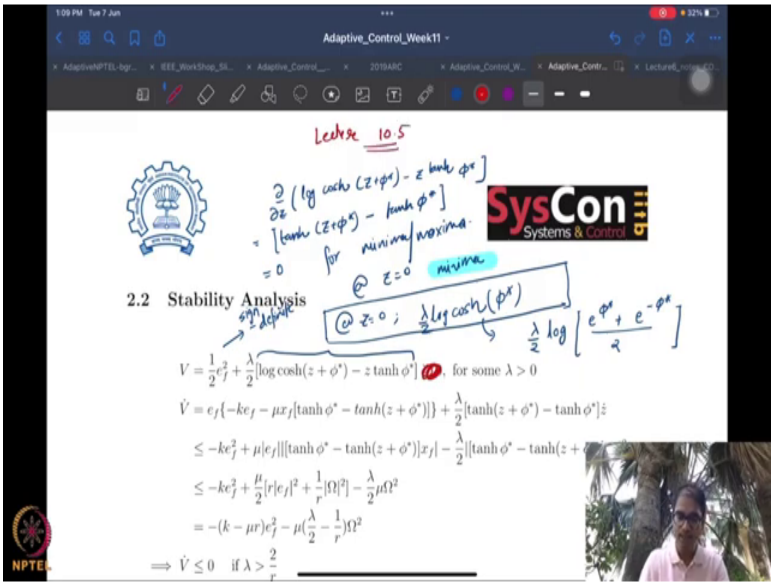

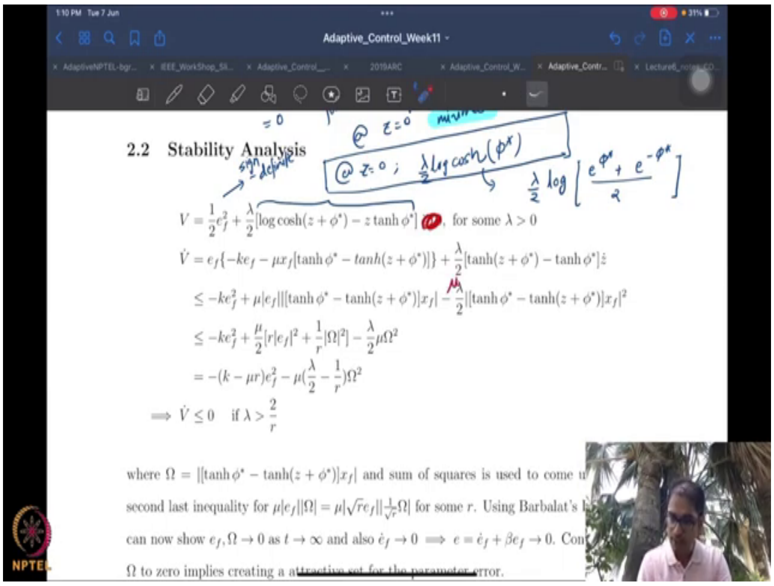

## 2.2 Stability Analysis

$$V = \frac{1}{2}e_f^2 + \frac{\lambda}{2}[\log\cosh(z+\phi^*) - z\tanh\phi^*], \text{ for some } \lambda > 0$$

$$\dot V = e_f\{-ke_f - \mu x_f[\tanh\phi^* - tanh(z+\phi^*)]\} + \frac{\lambda}{2}[\tanh(z+\phi^*) - \tanh\phi^*]\dot z$$

$$\le -ke_f^2 + \mu|e_f||[\tanh\phi^* - \tanh(z+\phi^*)]x_f| - \frac{\lambda}{2}|[\tanh\phi^* - \tanh(z+\phi^*)]x_f|^2$$

$$\le -ke_f^2 + \frac{\mu}{2}[r|e_f|^2 + \frac{1}{r}|\Omega|^2] - \frac{\lambda}{2}\mu\Omega^2$$

$$= -(k-\mu r)e_f^2 - \mu(\frac{\lambda}{2} - \frac{1}{r})\Omega^2$$

$$\implies \dot V \le 0 \quad \text{if } \lambda > \frac{2}{r}$$

where $\Omega = |[\tanh\phi^* - \tanh(z+\phi^*)]x_f|$ and sum of squares is used to come u
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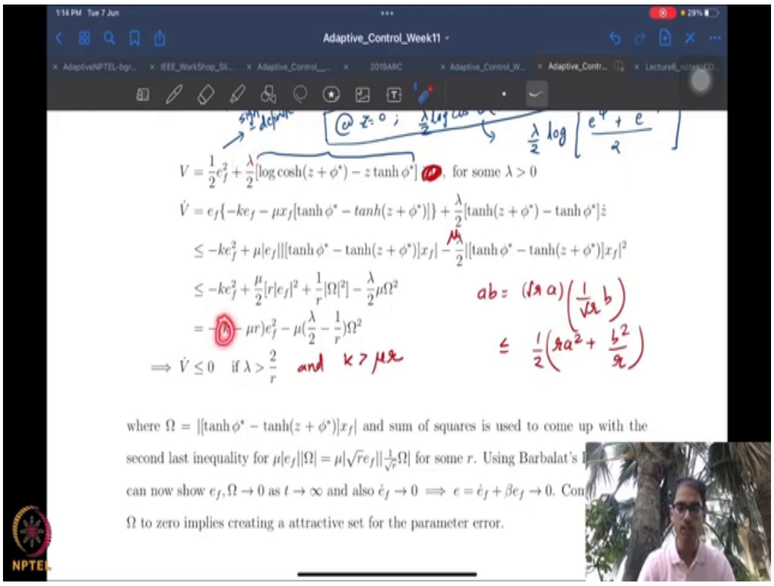

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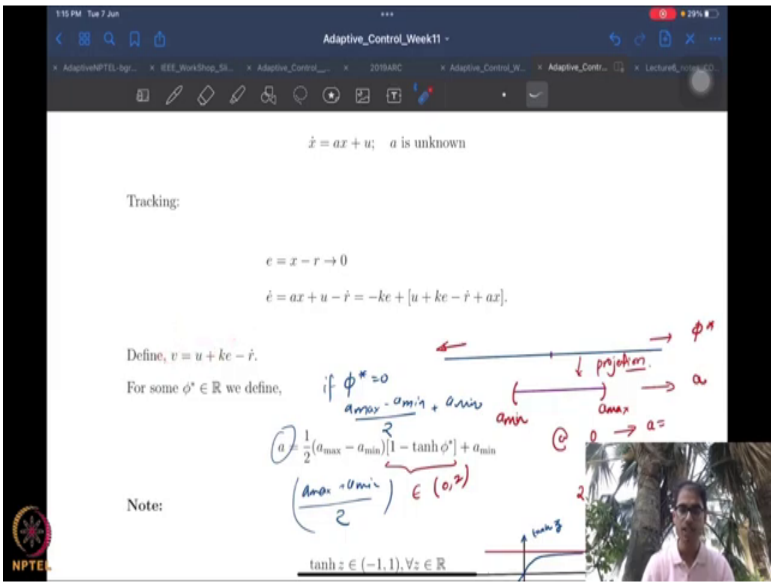

$$\dot x = ax + u; \quad a \text{ is unknown}$$

Tracking:

$$e = x - r \to 0$$

$$\dot e = ax + u - \dot r = -ke + [u + ke - \dot r + ax].$$

Define, $v = u + ke - \dot r$.

For some $\phi^* \in \mathbb{R}$ we define,

$$a = \frac{1}{2}(a_{\max} - a_{\min})[1 - \tanh\phi^*] + a_{\min}$$

**Note:**

$$\tanh z \in (-1, 1), \forall z \in \mathbb{R}$$

So, this is where we, this is where we were. So, I will start here on lecture 10.5. So, now all I do is take careful derivatives. So, I take any e f ,e f dot, which is this guy. And then I take the derivative of this guy, which is lambda by 2, partial with respect to z, which is this times of z dot. And now it will substitute z dot, z dot has exactly this term in the negative, with a mu x f square, so that is what you get, you get a z dot here.

So, this is already a mu here. So, this z dot also contains a mu here. So, the mu x f square and so what you will have been a mu lambda by 2 Tan hyperbolic phi star minus Tan hyperbolic z plus phi star times x f squared and this is also squared here.

And there is a square here. I believe this square is on this whole term this is fine. So, now you see I have nice negative terms, negative definite term minus k e f square, minus k e f square. And I also have a square term here in x f, times Tan hyperbolic phi star minus tan, hyperbolic z plus phi star.

So, I am going to call this term as omega. In fact, this whole term as omega, that is what we do here. And so, you have minus k e f square minus lambda by 2 mu omega square. And then this term can be written using our sum of squares as mu by 2 times e f squared with a r, some gain r. What I am using here is that ab is less than equal to well, I am actually doing this ab is equal to like the square root r a times a times 1 over square root r b, this is less than equal to half r a squared plus b squared by r that is what is being used here.

If you notice from here, so mu remains as it is, and from these 2 terms, I get r e f squared plus 1 by r omega square. So, then once we do this, you get this, k minus mu times r e f square and mu times, lambda by 2 minus 1 by r omega square, so you get this term and therefore you get V dot negative semi definite. If lambda is greater than 2 over r and of course k greater than mu times r.

Now, one might ask, why did we, one of the important questions to ask is why did we need the lambda. Why did we choose put the lambda here, remember one thing, even though we get this nice term, this is not a negative definite, V dot remember. Because V was a function of e f and z and it is not very clear that this term is negative definite inside because there is an x f also. So, negative definite with respect to z is not very clear here. So, this is at best negative semi definite only.

Now, because it is negative semi definite and I would like this to sort of dominate some other term. So, I have to make sure that this term is smaller than this term and so I take an arbitrary lambda, I do not know the value of lambda, I took some lambda positive. Anyway, this lambda does not enter anywhere in the controller, it is not like we need to know the value of lambda is just for the purpose of analysis.

So, this lambda now has a 1 by r, so I can choose r large, by choose r to be big, this is a small term, and I have dominated this with some whatever lambda. And correspondingly, I have this r here so, k can be large.

Then if I choose a large enough, k I am done. So, that is so if r is small sorry, if r is large, then k is large and lambda needs to be small vice-versa. I mean, if you do not want to really care about choosing large k, you do not know how large the k should be, you can choose any k, arbitrary k, then r has to be really small.

So, if r is really small, then any k will work. And you choose any k, correspondingly there is a small enough r for which this will be positive. And once this is positive, this becomes very large. So, then lambda has to be very large, but we do not care. This lambda does not appear in the control law so lambda being large, does not change anything for us, if just for the purpose of analysis.

So, this is a rather neat trick. Then in analysis, when you do not know how big this gain is, and you do not want to push the control gain k large because k actually appears in the control law is so if k is not here in v f, k is not in v f, but if you remember, k is there in the control law. Because we have to implement u and v contains k.

So, k being arbitrary large is not a good idea. So, if you choose a, any small positive k, there is always one can always say that there exists an r, such that k minus mu r is positive. And if the r is small, then all you need to do is have a large lambda in the analysis. But again, it is just for the analysis, so, it does not change anything for us. So, we are more than happy.

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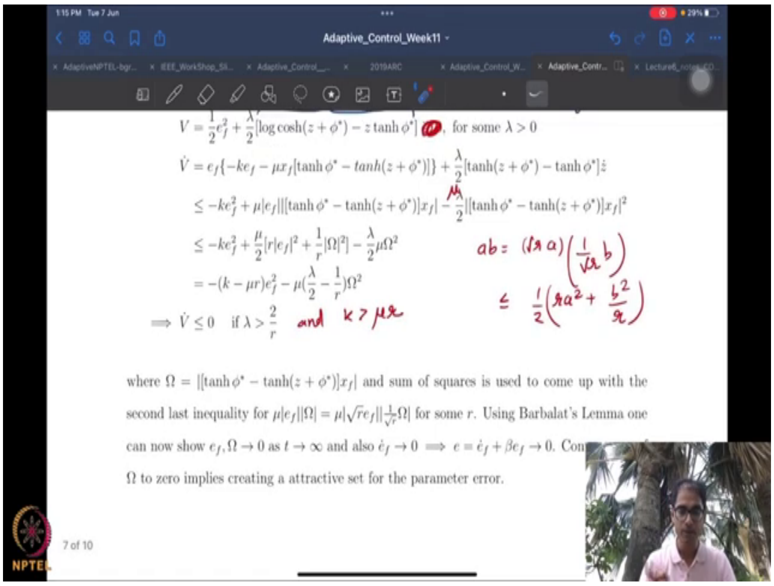


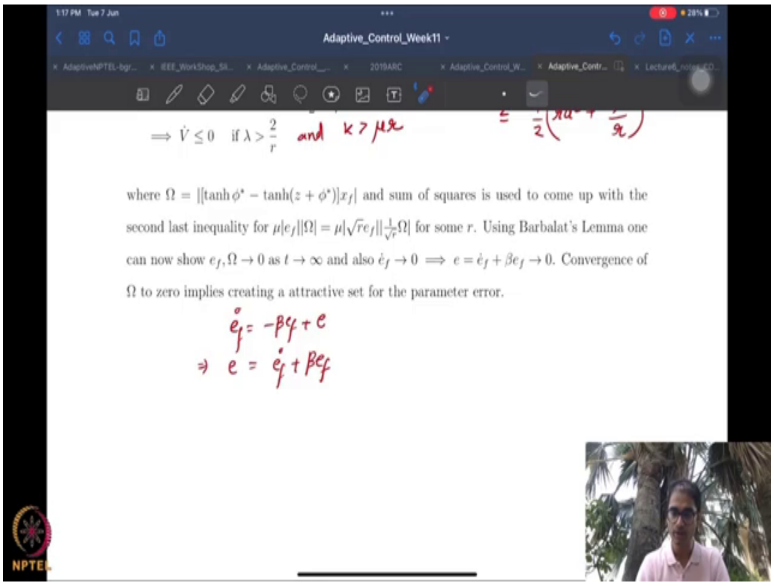

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where, $\frac{1}{2}(a_{\max} - a_{\min}) = \mu$. Choose $\dot{\delta} = -e_f x_f$ and compute dynamics of $z$ state trajectories.

$$\dot{z} = \dot{\hat{\phi}} + \dot{\delta} = \dot{\hat{\phi}} - e_f\dot{x}_f - x_f\{-ke_f - \mu x_f(\tanh\phi^* - \tanh(z+\phi^*))\}$$

Let, $\dot{\hat{\phi}} = e_f\dot{x}_f - ke_f x_f$

$$\dot{z} = \mu x_f^2[\tanh\phi^* - \tanh(z+\phi^*))]$$

**Note:** The update law is already chosen before the Lyapunov analysis.

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$$\dot{e}_f = -ke_f + (a - \hat{a})x_f$$

$$a - \hat{a} = \frac{1}{2}(a_{\max} - a_{\min})(\tanh(\hat{\phi} + \delta) - \tanh\phi^*)$$

Let $z = \hat{\phi} + \delta - \phi^*$ (not certainty equivalence)

$$a - \hat{a} = \frac{1}{2}(a_{\max} - a_{\min})(\tanh(z+\phi^*) - \tanh\phi^*)$$

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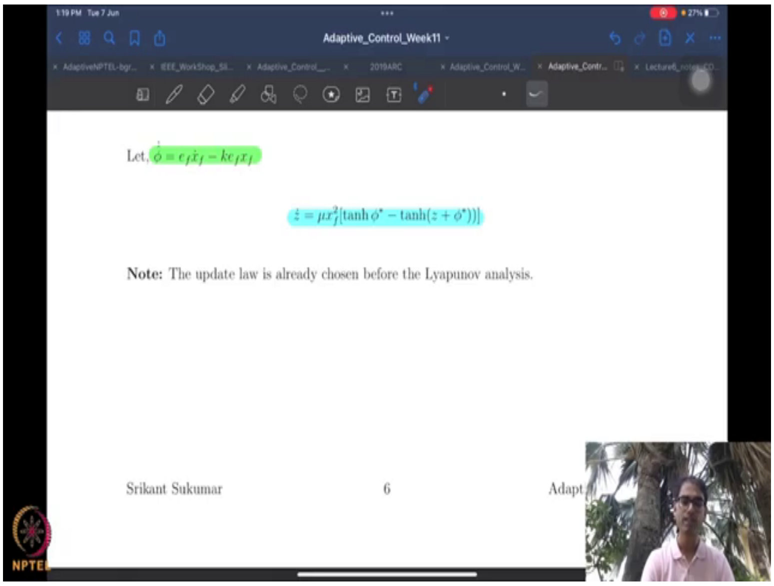


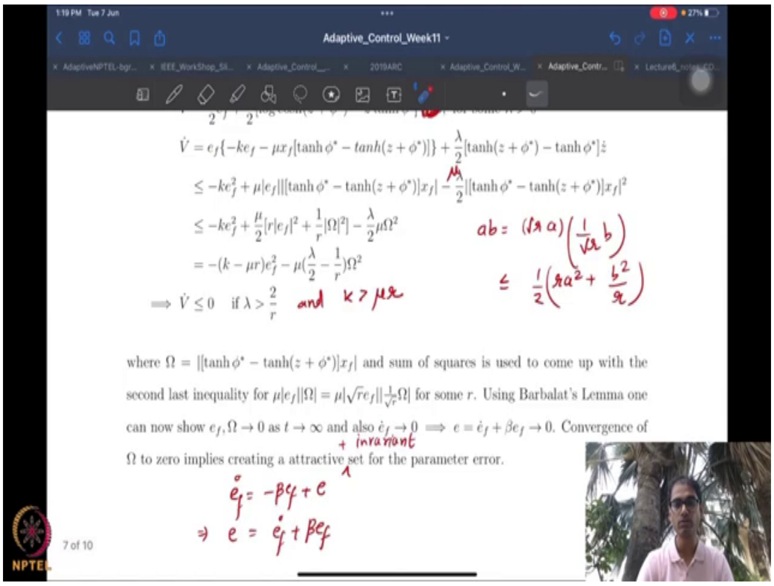


So, that is the purpose. So, what we have now is we have a V dot which is V of course, which is lower bounded and V dot which contains these 2 quadratic terms. So, it should be sort of obvious to you for all of those who have done Barbalat's Lemma signal chasing analysis several times that, you can prove that e f and Omega go to 0 as t goes to infinity.

So, this is something you can always prove from this kind of Lyapunov like analysis, we are not claiming that V is positive definite and lower bounded at 0 and all that, but we know it has a lower bound it is not increasing.

So, we can prove all the quadratic terms that appear here are going to go to 0 using Barbalat's Lemma signal chasing and that is what we do, like right here, we can also show that e f dot goes to 0. Not difficult, we have done this before to once if e f goes to 0 we can prove e f dot goes to 0. And if e f dot and e f both go to 0, what do we know? We know from our filter construction that you have e f dot equals minus beta e f plus e, which implies e is e f dot plus beta e f so both e f and a beta e f dot go to 0 mean e goes to 0. Fine that is what we have.

The other additional thing we sort of get, which we do not get in certainty equals remember, we never get a term in parameter error in certainty equivalence, but here we do have a term in parameter error. It is a very nonlinear term. So, not very obvious what this term means or what it looks like physically, but we at least have some kind of a term which goes to 0.

We know that this guy goes to 0. And this is a sort of an attractive set for the parameter error. The important thing to remember is that if you start with 0 parameter error, that is when z is 0, suppose z is 0, then this quantity is 0.

So, omega is 0. If you start with 0 parameter error, you will stay with 0 parameter error because look at this, I mean in the absence of disturbance and errors, etcetera, if you look at this term, this is 0, 0 is z equal to 0 is an equilibrium, so therefore, this right-hand side becomes 0.

Now, this is not true in certainty equivalence. In certainty equivalents, this would have been your update law. So, even if you started with a tilda 0, the right-hand side has no a tilda in it. So, it does not matter if you started with a 0 a tilda, you will still change the parameter value which is of course, seems unhealthy, but when that is the sort of solutions we will have.

In this setup, if we started with 0 parameter error in this case z being 0 then the right-hand side is 0 and you do not move from it. So, that sort of creates an attractive set in fact attractive and invariant set, it creates an attractive and invariant set.

So, if you stay in that set, you will remain if you start in that set attractive plus invariant set for the parameter errors, if you start in the set, you will remain in the set and you will be attracted towards this set also. So, this is a rather interesting property, which actually improves the adaptation behavior for the system.

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$\Omega$ to zero implies creating a attractive set for the parameter error.

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$$a - \hat{a} = \frac{1}{2}(a_{\max} - a_{\min})(\tanh(\hat{\phi} + \hat{\delta}) - \tanh\phi^*)$$

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$$\dot{z} = \dot{\hat{\phi}} + \dot{\hat{\delta}} = \dot{\hat{\phi}} - e_f\dot{x}_f - x_f\{-ke_f - \mu x_f(\tanh\phi^* - \tanh(z + \phi^*))\}$$

Let, $\dot{\hat{\phi}} = e_f\dot{x}_f - ke_f x_f$

$$\dot{z} = \mu x_f^2[\tanh\phi^* - \tanh(z + \phi^*))]$$

## 2.2 Stability Analysis

$$V = \frac{1}{2}e_f^2 + \frac{\lambda}{2}[\log\cosh(z + \phi^*) - z\tanh\phi^*], \text{ for some } \lambda > 0$$

$$\dot{V} = e_f\{-ke_f - \mu x_f[\tanh\phi^* - \tanh(z + \phi^*)]\} + \frac{\lambda}{2}[\tanh(z + \phi^*) - \tanh\phi^*]\dot{z}$$

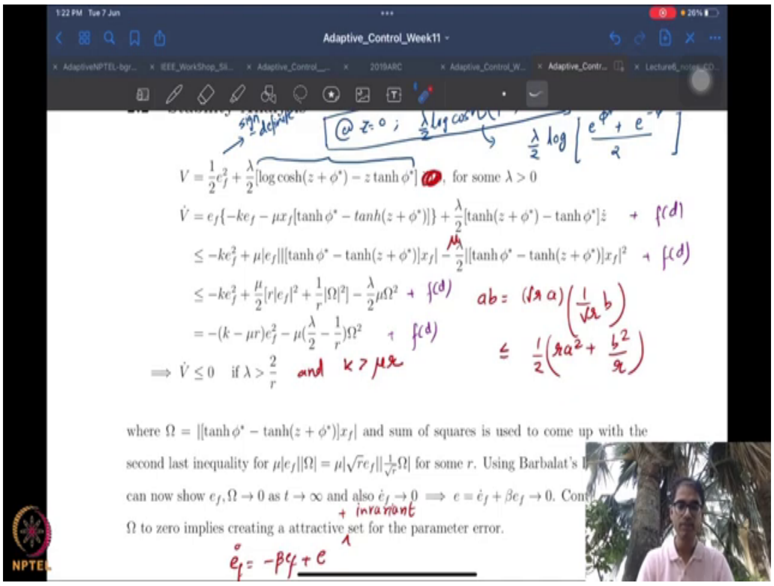
$$\leq -ke_f^2 + \mu|e_f||[\tanh\phi^* - \tanh(z + \phi^*)]x_f| - \frac{\lambda}{2}|[\tanh\phi^* - \tanh(z + \phi^*)]x_f|^2$$

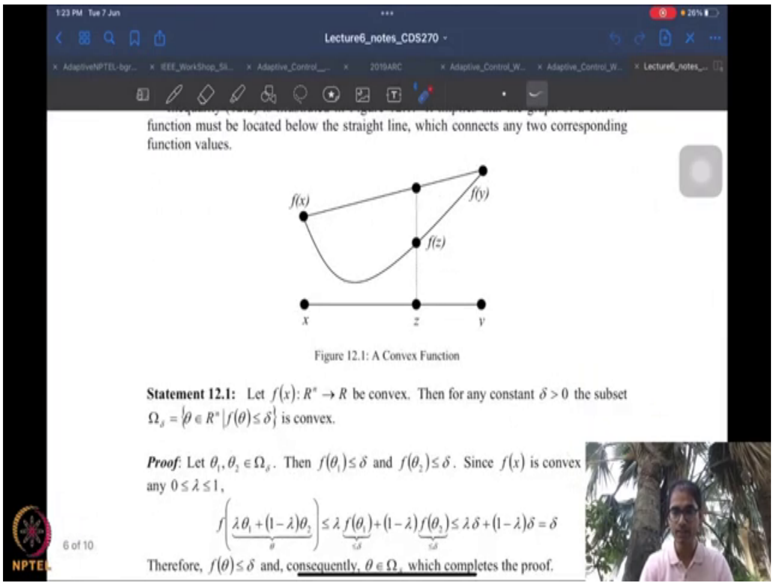
$$\leq -ke_f^2 + \frac{\mu}{2}[r|e_f|^2 + \frac{1}{r}|\Omega|^2] - \frac{\lambda}{2}\mu\Omega^2$$

$$= -(k - \mu r)e_f^2 - \mu(\frac{\lambda}{2} - \frac{1}{r})\Omega^2$$

$$\implies \dot{V} \leq 0 \quad \text{if } \lambda > \frac{2}{r}$$

where $\Omega = |[\tanh\phi^* - \tanh(z + \phi^*)]x_f|$ and sum of squares is used to come u second last inequality for $\mu|e_f||\Omega| = \mu|\sqrt{r}e_f||\frac{1}{\sqrt{r}}\Omega|$ for some $r$. Using Barbalat's l can now show $e_f, \Omega \to 0$ as $t \to \infty$ and also $\dot{e}_f \to 0 \implies e = \dot{e}_f + \beta e_f \to 0$. Con




So, I hope that was clear. And you understand that, now, of course, we did not go into any further details. So, but remember that in order to compute the actual control law, you have to compute v, which is equal to v f dot plus beta v f and we already have shown that v f is going to be bounded and v f is going to be bounded because all closed loop signals are bounded and if v f is bounded, then v f dot is also going to be bounded, I mean, that also can be proven, we are not proving it.

So, I will actually say prove boundedness I would leave it to you as an exercise to prove boundedness of this once you have the boundedness for v f dot and v f also, then you know that v is going to remain bounded and so is u and this is not going to be affected in the presence of disturbance again, we have not done any analysis with disturbance notice there is no disturbance analysis here, but what is happening here is the fact that the parameter, the design is not going to change in the presence of disturbance like only the Lyapunov analysis changes.

So, basically you will start getting some terms corresponding to the disturbance here, that is what will happen, this will happen. And so, again if I was to sort of do this, there f of d, some function of disturbance appearing here, everywhere.

So, all these steps remain the same, but then I will have something like this. But the good thing is, because I have negative definite terms in both e f and omega, nothing changes, regarding I will still get like a residual set in the e f which will of course, move to a residual set in the original states and all that mess.

And of course, my parameters remain bounded. So, my control remains bounded control will never become unbounded, it is irrelevant what happens, the control is never going to become unbounded. Neither is the parameter estimate. So, the disturbance may affect how the residual set, I mean instead of going to 0 errors. You may not go to 0 errors, but it does not really, I mean, you will get a residual set as you expect, but you do not change anything in your parameter boundedness nor in your control boundedness. So, that is the sort of important thing to remember.

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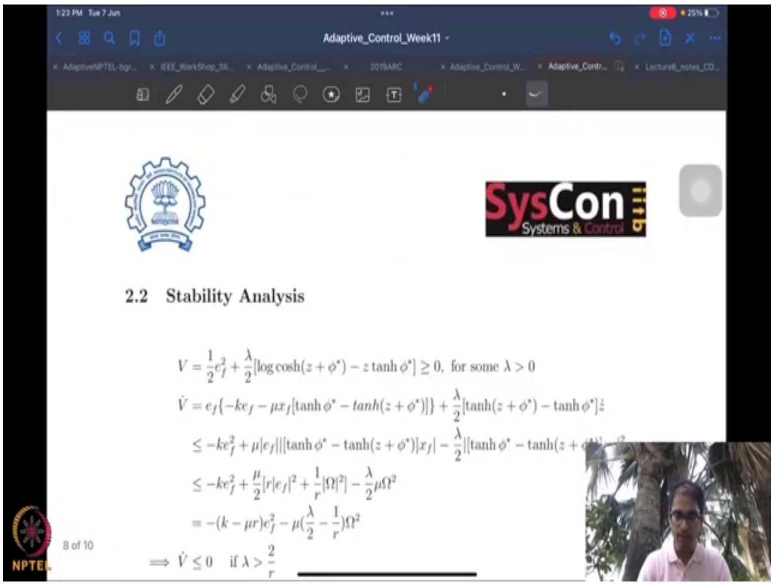

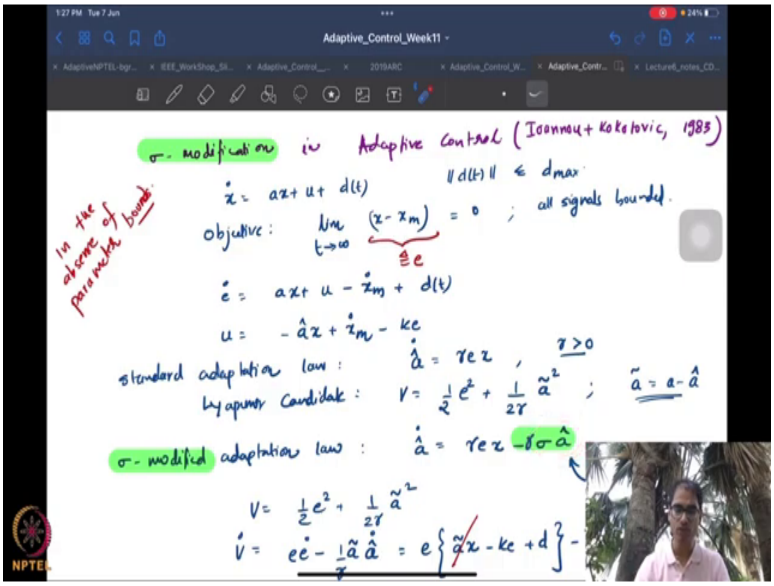


So, this is the projection method. By the way, this is only one kind of projection method that I have specified. There are other ways to do parameter projection. In fact, this is a rather unusual way, the more common way is using the notion of convex sets, which I am not presenting right here. But it is available in I mean, in results from Ioannou et cetera. So, you have references, which talk about convex sets and how to do projection on convex sets.

I am not going into that right now, because, for the sake of time, we have to restrict are material. But there is a pretty solid, pretty good way of doing parameter projection and parameter projection, as you can see is critical in adaptive control.

Now, one of the concerns that some of you might have is that projection requires knowledge of your bounds on the parameter. Without parameter bounds, there is no way you can do projection. And if you remember, in the beginning of adaptive control, you did not assume any kind of bounds on the parameters. So, what happens if you do not have parameter bounds available to you, and you still want to impart robustness to the adaptive control.

So, these were, so this discussion began, of course, when we had that the fighter jet crash, and results came about in a few years after that. And the first one of them was, first whatever first seminal result was by Ioannou and Kokotovic, which in 1983 and this is called this was called a sigma modification in adaptive control.

So, I will sort of classify this as in the absence of parameter bounds. In the absence of parameter bounds, so the question is, what happens if you do not have parameter bound, and you still want to impart some kind of robustness to your adaptive control?

So, we start as usual with the same system. And this is called the sigma modification in adaptive control by Ioannou and Kokotovic, we start with the same system x dot, a x plus u plus d. And we know that there is a bound on the disturbance, we do not know the value of the disturbance at each instant in time. But of course, we do have knowledge of a bound, very reasonable assumption.

If anything, you can just keep a very large bound, just to be conservative, but we do know about on most disturbances. For example, if I am flying a drone, I have a pretty fair idea of how much the wind velocities will be. I mean, I have a pretty good idea of the range of what the wind velocities would be.

If I am flying a satellite, and, I have actuator issues, I do not know how well the actuators perform if they precisely produce the amount of torque that I expect them to. I may not know the exact values, but I will still know how far off they can be.

So, that is essentially what is the job of a good engineer, to have a pretty good range in which your system will operate. So, what is the objective? The objective is, as usual to try to drive the error between x and some bounded trajectory x m is 0. So as usual, what do we do? We create the error dynamics. And we have this certainty equivalence adaptation law. So, we are back to the certainty equivalence adaptation law.

So, the idea is, when you do if you do a certain equivalence adaptation law, you will take a V, which is something like 1 half e squared plus 1 over 2 gamma a tilda squared, pretty standard, where a tilda is a minus a hat. And if you do the V dot and compute everything and try to find a hat dot, we will see that a hat is gamma times e times x, where gamma positive is the adaptation gain.

So, what Ioannou and Kokotovic proposed was the sigma modification, which meant that you add a damping sigma a hat term to this adaptive law so the whole problem was that a hat dot update law never contain the term in a hat or a tilda, of course, can’t contain a term a tilda because that is not implementable. So, Ioannou and Kokotovic did the next best thing they added a nice negative term in a hat. And then of course, we did some analysis to show what happens and this analysis is what we will see in the subsequent session.

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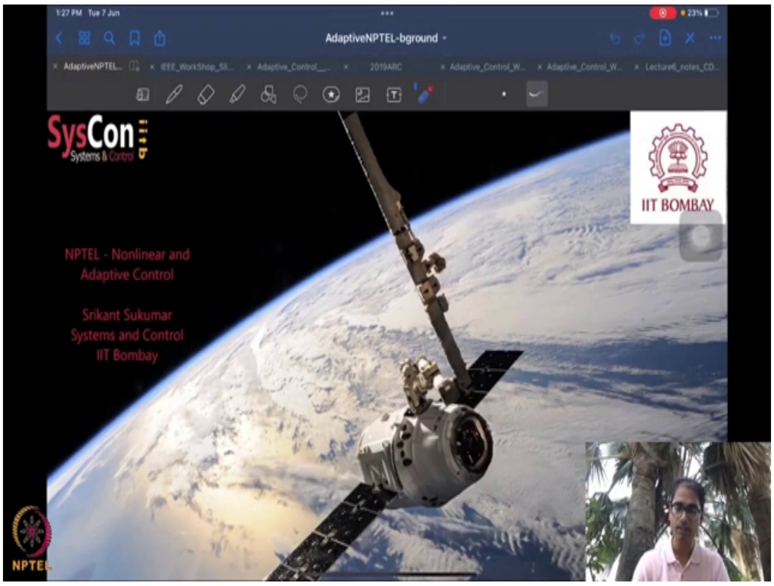


So, what we looked at today is the sort of completion of proof of a parameter projection based adaptive control law. We had rather interesting Lyapunov candidate and we took its derivative. We also had a sort of lambda coefficient on this Lyapunov candidate which we saw was very useful for analysis and played no role in the control.

So, it did not matter how large we took it. And we also realized that this non certainty equivalence type paradigm in production gave us a sort of attractive and invariant set, which has nice properties such as if you start with 0 parameter error, you remain at 0 parameter error in the absence of disturbance. So, these are the kind of properties that were absent in the certainty equivalence adaptive control.

And finally, we sort of started to look at the sigma modification in adaptive control, which would be a way of imparting robustness if you did not have knowledge of upper and lower bounds of your parameters. So, we will continue with our discussion on sigma modifications in the sub sequent session, so I invite you to attend. Thank you.