Nonlinear Adaptive Control
Professor Srikant Sukumar
Systems and Control
Indian Institute of Technology, Bombay
Week 10
Lecture No: 57
Parameter Projection in Adaptive Control - Part 1

Hello everyone. Welcome to yet another session of our NPTEL on Non-linear and Adaptive Control. I am Srikant Sukumar from Systems and Control, IIT Bombay. So, we are already into the 10th week of our lectures on Non-linear and Adaptive Control and we already learned several algorithms that will help drive systems such as the SpaceX satellite that we see in the background autonomously.

In the previous week, sorry, in the previous week, of course, we learned tuning functions method and we spent a few weeks on adaptive integrator back stepping based methods and I am quite certain that a lot of practical systems that all of you would have in mind do fall under the category of systems that for which adaptive algorithms can be designed using these methods. So, I would strongly encourage all of you to give it a shot and try to formulate your problem and design adaptive controllers for systems using these methods that we have studied.

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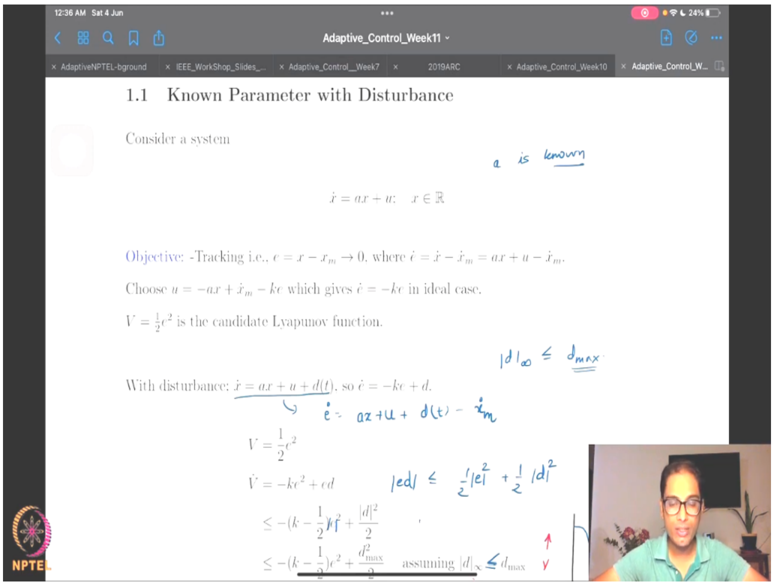

In this week we started talking about the robustness issue in adaptive control. We got a pretty fair idea of what happens in the absence of disturbance and the fact that any Lyapunov-based control design, any strictly Lyapunov function based control design, to be specific, gives us this very nice disturbance robustness property.

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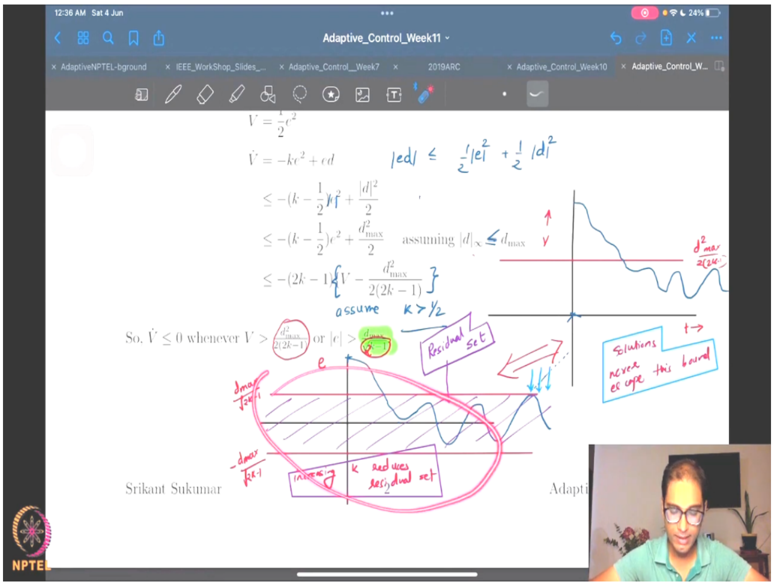

So, essentially, what you will get is a convergence to a residual set which looks something like this picture here. And we can also reduce the size of this residual set by increasing the control gain.

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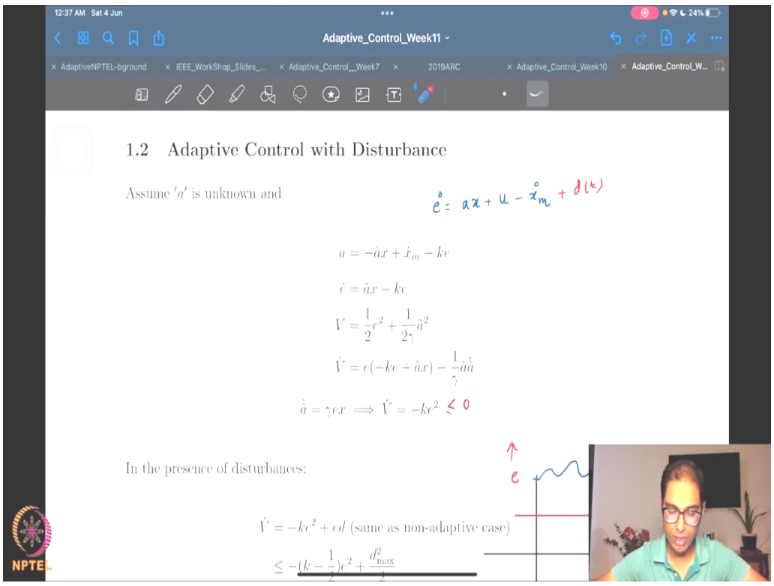

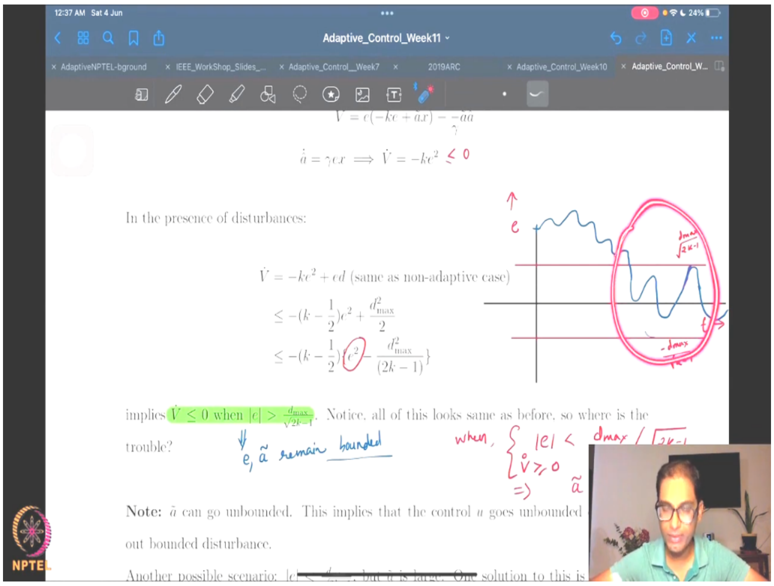
Adaptive_Control_Week11
In the presence of disturbances:
$\dot V = -ke^2 + ed$ (same as non-adaptive case)
$\le -(k-\frac{1}{2})e^2 + \frac{d^2_{\max}}{2}$
$\le -(k-\frac{1}{2})\{e^2 - \frac{d^2_{\max}}{(2k-1)}\}$
implies $\dot V \le 0$ when $|e| > \frac{d_{\max}}{\sqrt{2k-1}}$. Notice, all of this looks same as before, so where is the trouble?
$e, \tilde a$ remain bounded
when, $|e| < d_{max}/\sqrt{2k-1}$, $\dot V \geq 0$ ⟹ $\tilde a$
Note: $\tilde a$ can go unbounded. This implies that the control u goes unbounded
out bounded disturbance.
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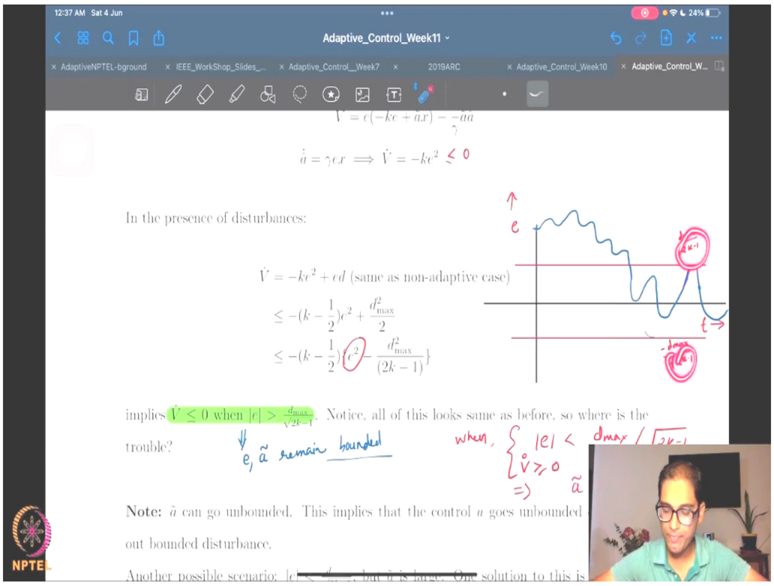
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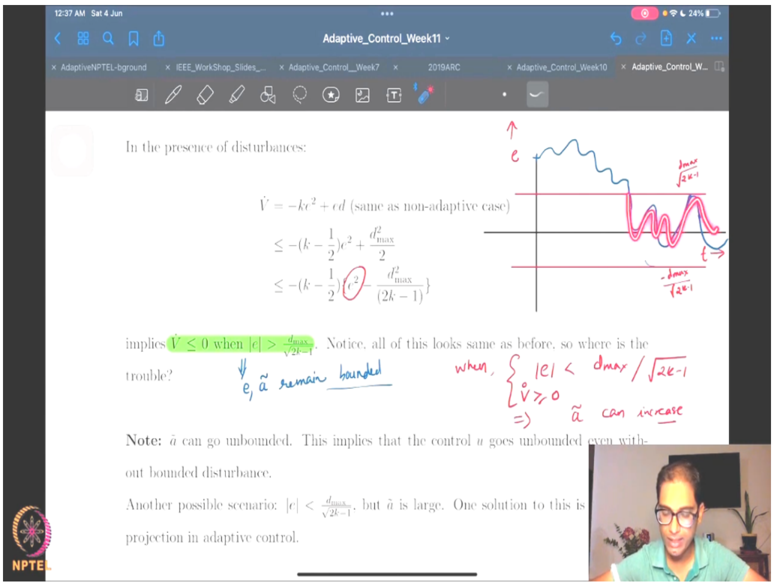
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$e, \tilde a$ remain bounded
when, $|e| < d_{max}/\sqrt{2k-1}$, $\dot V \geq 0$ ⟹ $\tilde a$ can increase
Note: $\tilde a$ can go unbounded. This implies that the control u goes unbounded even with-
out bounded disturbance.
Another possible scenario: $|e| < \frac{d_{\max}}{\sqrt{2k-1}}$, but $\tilde a$ is large. One solution to this is projection in adaptive control.
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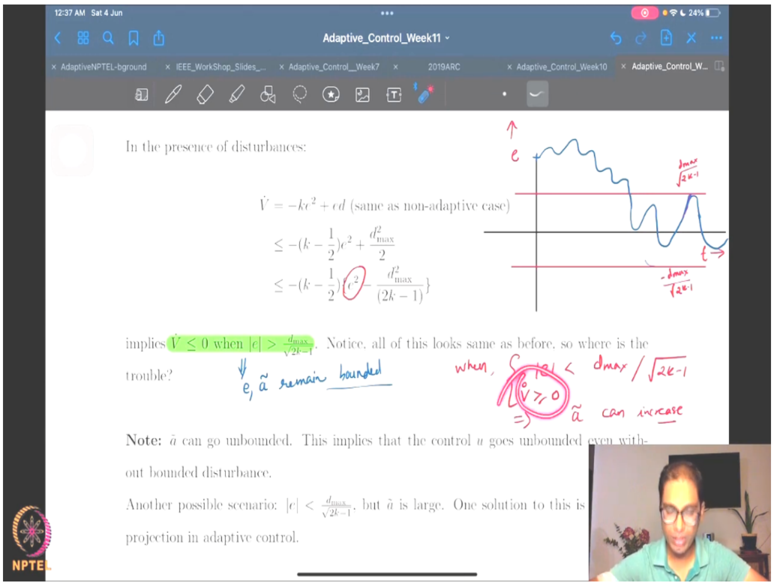

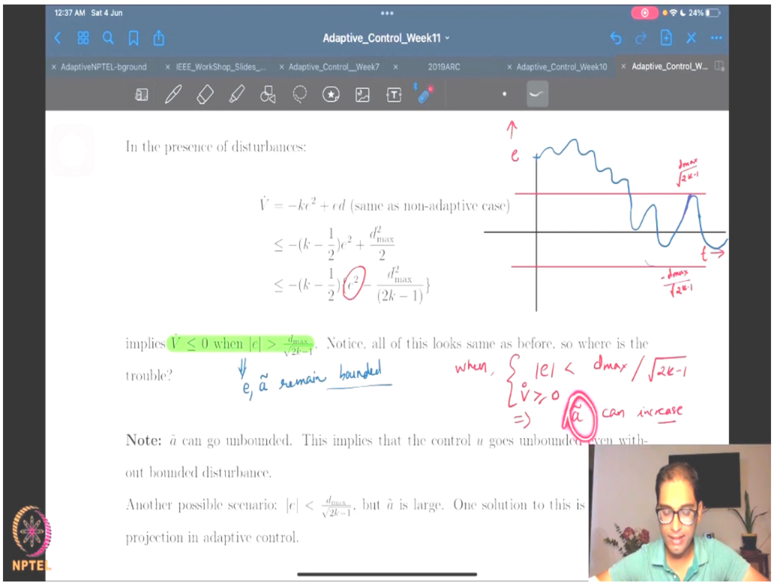

However we saw that the same does not hold true when an adaptive controller is introduced. What happens in that case is that you can show that your errors do get into a residual set which is still nice. It does look like by changing the control gain you can, in fact, reduce the size of the residual set. So, both of these are of course very nice properties that we retain.

However, the problem that happens is that this boundedness of e does not actually guarantee the boundedness of a tilde. In fact, it can so happen that once e has entered this nice residual set, your v dot becomes non-negative and therefore V can increase and if e cannot increase beyond a certain point and if v has to continue to increase, the only possibility is that a tilde keeps increasing. And if a tilde increases it means that the parameter errors and the parameter estimates themselves can go to infinity.

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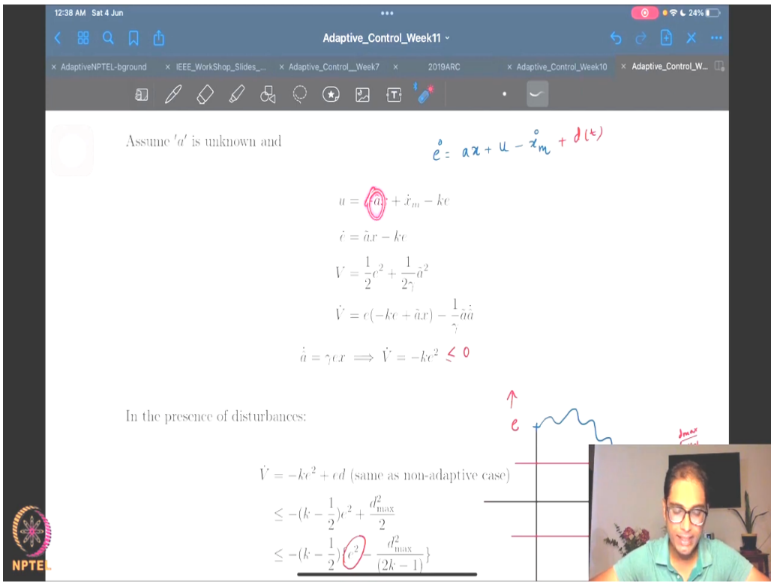

And since, this parameter estimate enters the control expression; your control will also become unbounded. And this is one of the most undesirable properties of adaptive control. This is one of those reasons which really restricted the growth of adaptive control in the 80s and 90s of course. And so what we want to know is how to get rid of this explosion of parameter value issue.

Now, one of the things that sort of an obvious sort of answer or obvious solution to this problem is that if you know some bounds on your parameters value, parameter values, you want to make sure that you search only within these bounds therefore your estimates just evolve in between these bounds. And the process of doing this kind of an adaptation is called parameter projection in adaptive control. So, again there are several ways of doing parameter projection in adaptive control and we will focus on one particular way right now.

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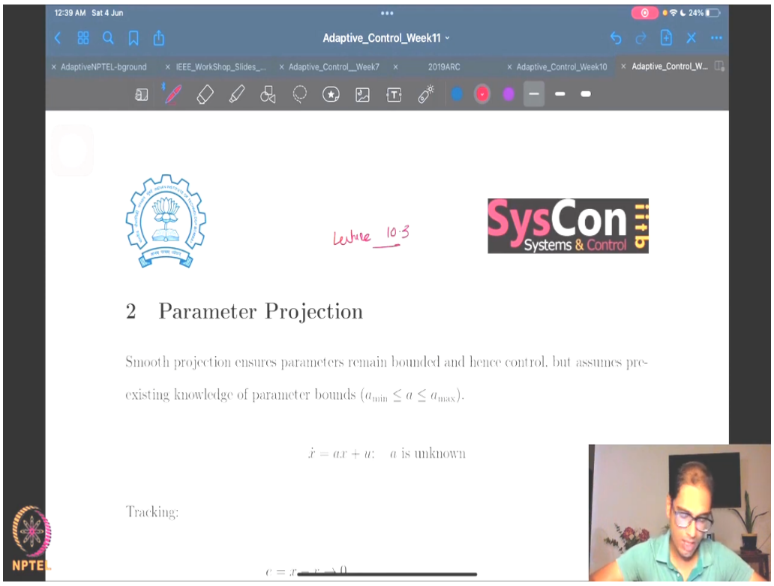

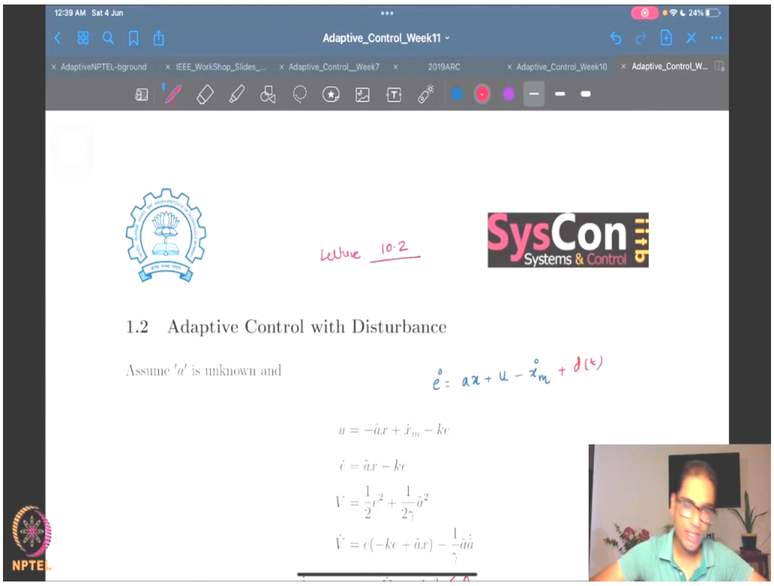

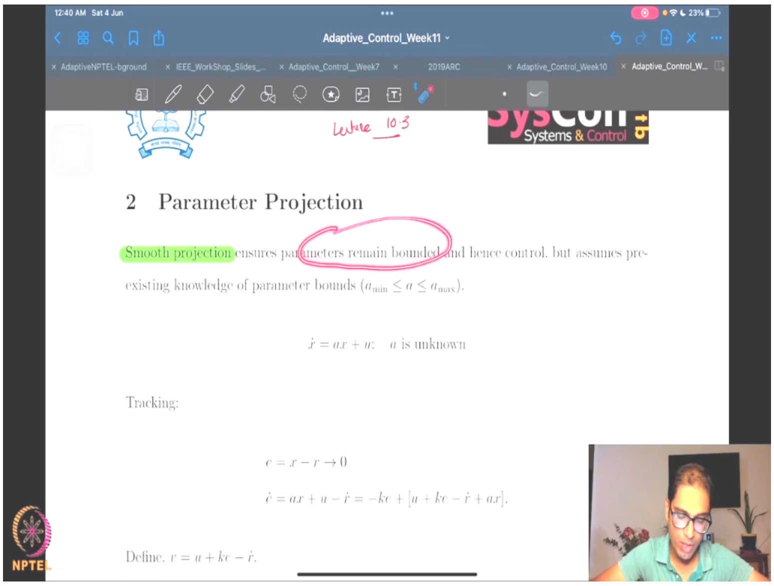
Adaptive_Control_Week11
Lecture 10.3
Systems & Control
2 Parameter Projection
Smooth projection ensures parameters remain bounded and hence control, but assumes pre-existing knowledge of parameter bounds ($a_{\min} \le a \le a_{\max}$).
$\dot{x} = ax + u$; a is unknown
Tracking:
$e = x - r \to 0$
$\dot{e} = ax + u - \dot{r} = -ke + [u + ke - \dot{r} + ax]$.
Define, $v = u + ke - \dot{r}$.
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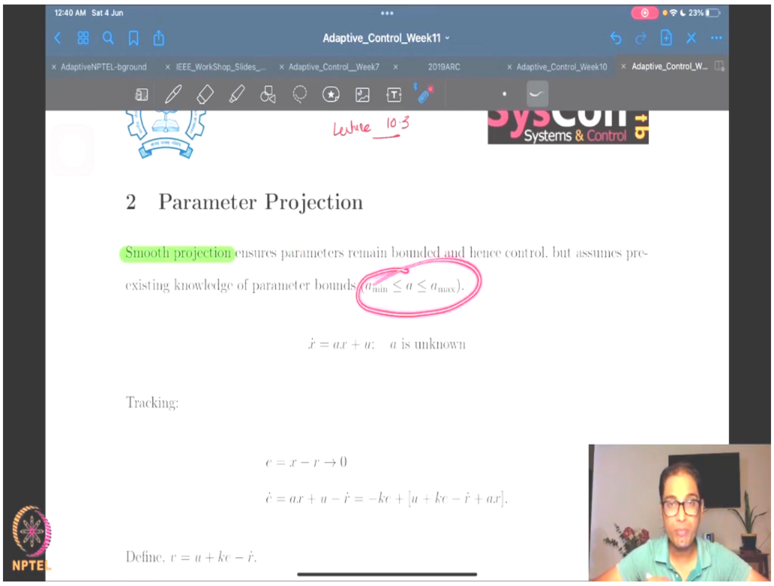
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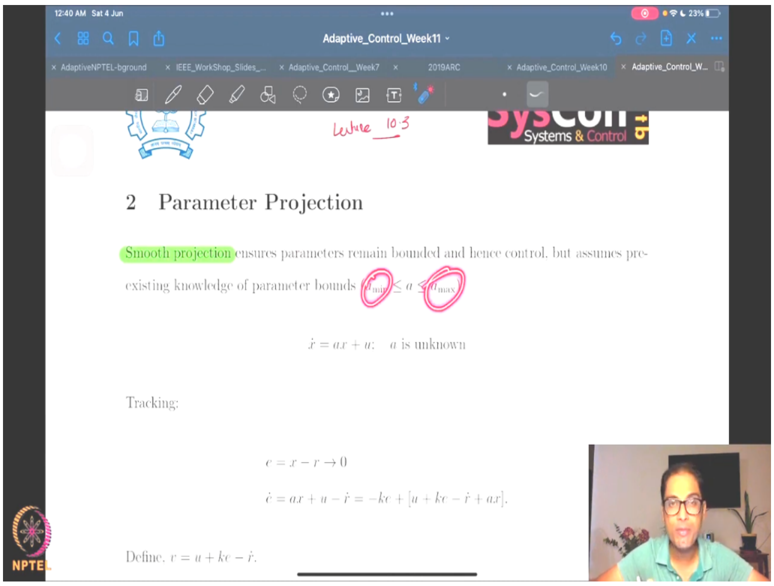
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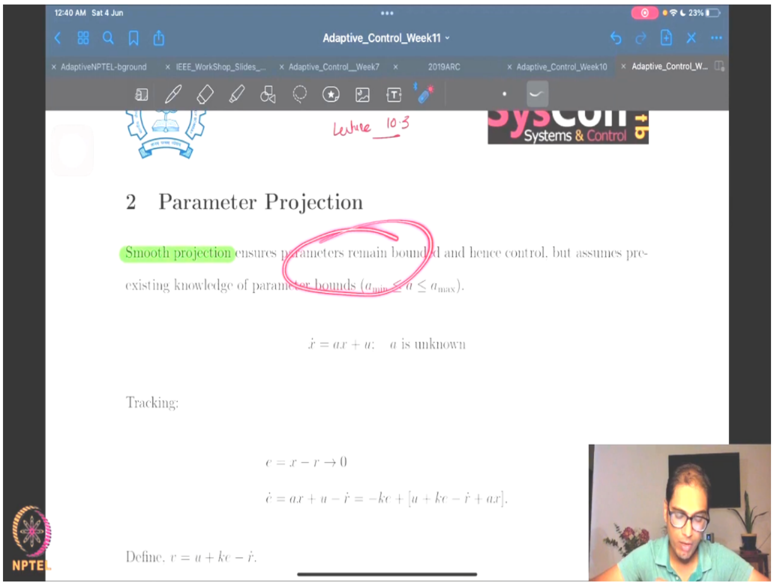

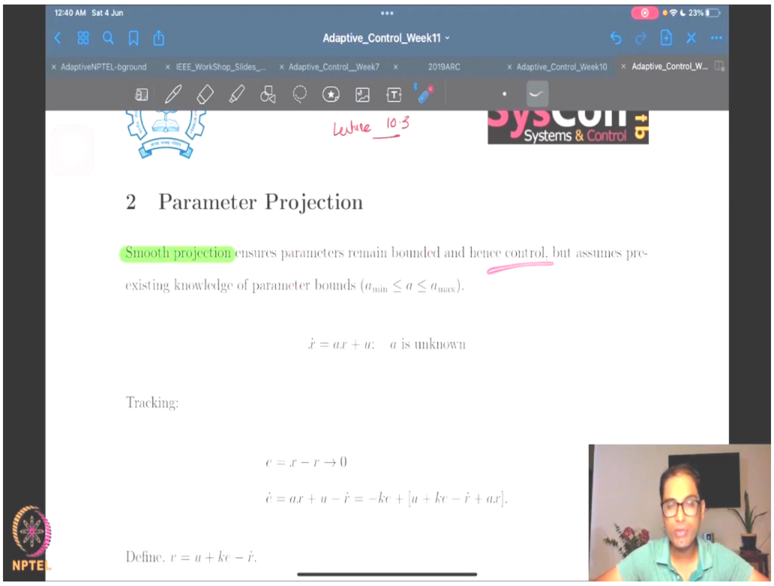

And that is where we start today. So, let me mark this as lecture 10.3. In fact, I want to see that I marked last time. I will mark here but I forgot to mark last time. So, I am going to mark this as lecture 10.2 and today we are at lecture 10.3. So, what is parameter projection is what we are looking at is called smooth projection. There is also a non-smooth version which we are not looking at right now, but yes, a non-smooth version also exists.

So, we are looking at smooth projection. This ensures that the parameters remain bounded within pre-specified parameter bound. So, these bounds a min and a max have to be provided by the end user. So, it is expected that the end user does have some knowledge of these unknown parameters. And once we ensure using some smooth projection algorithm that your parameter estimates remain bounded then the control itself also remains bounded.

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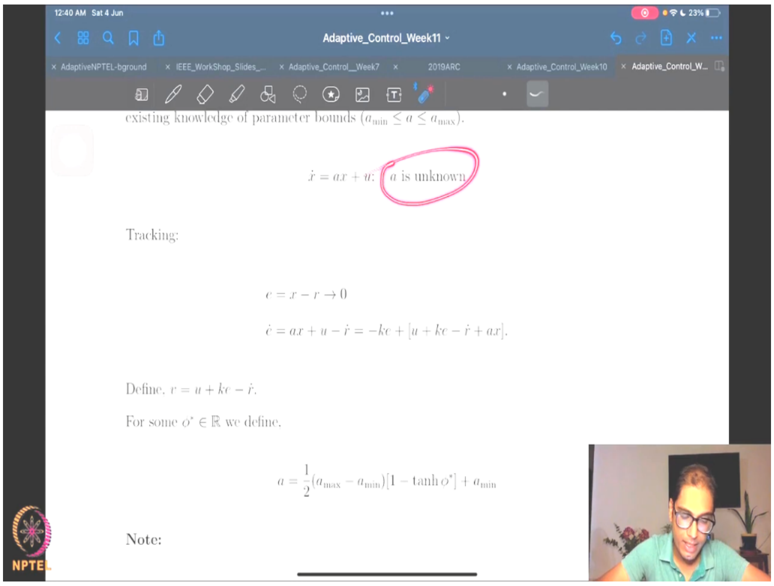

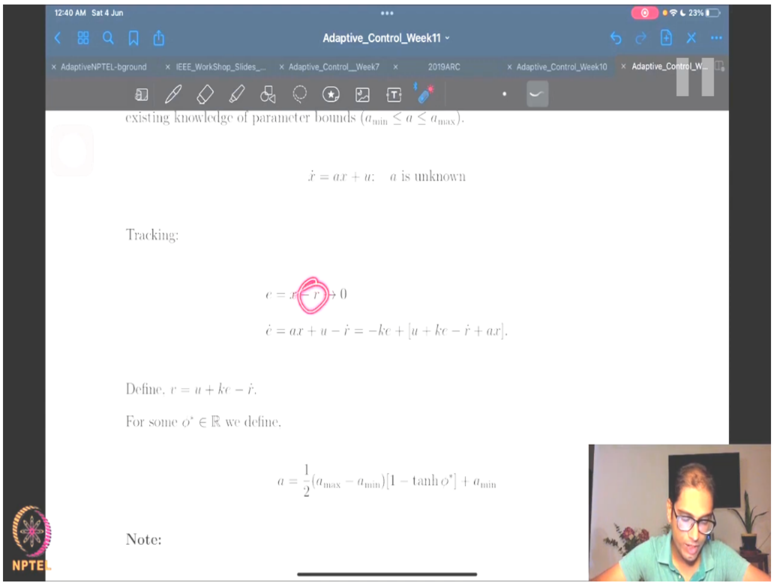

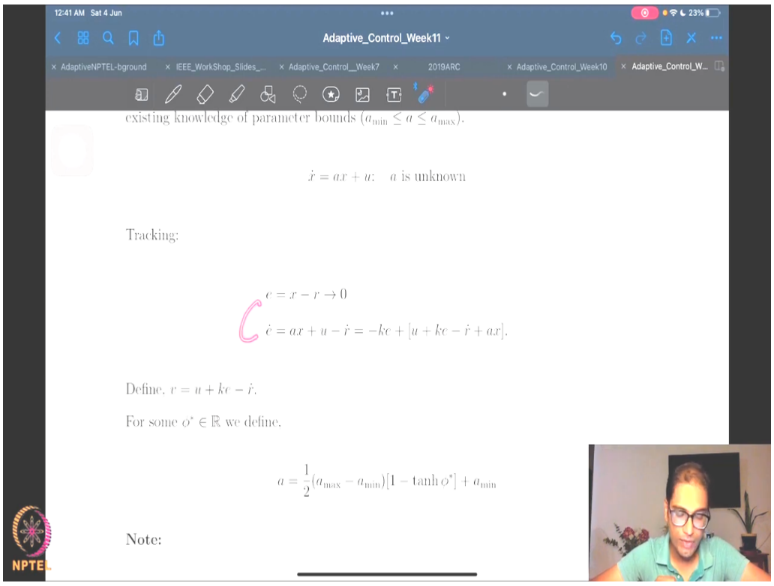
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For some $\phi^* \in \mathbb{R}$ we define,
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Note:
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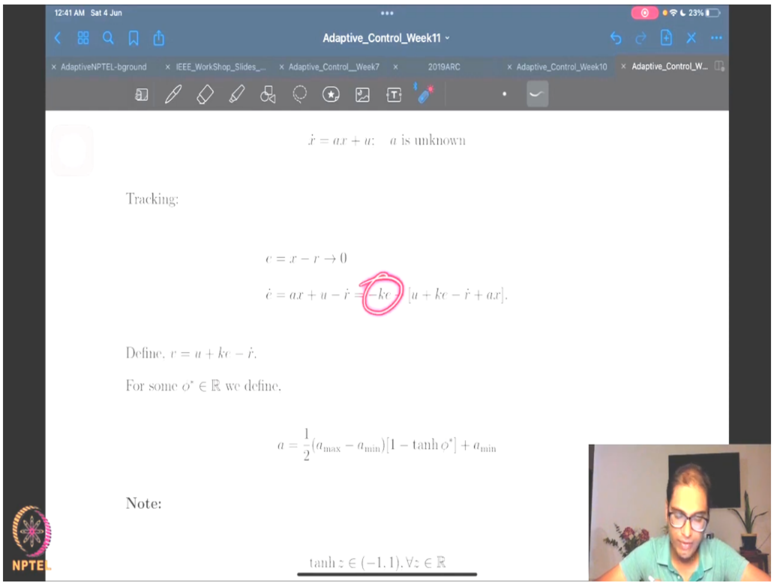
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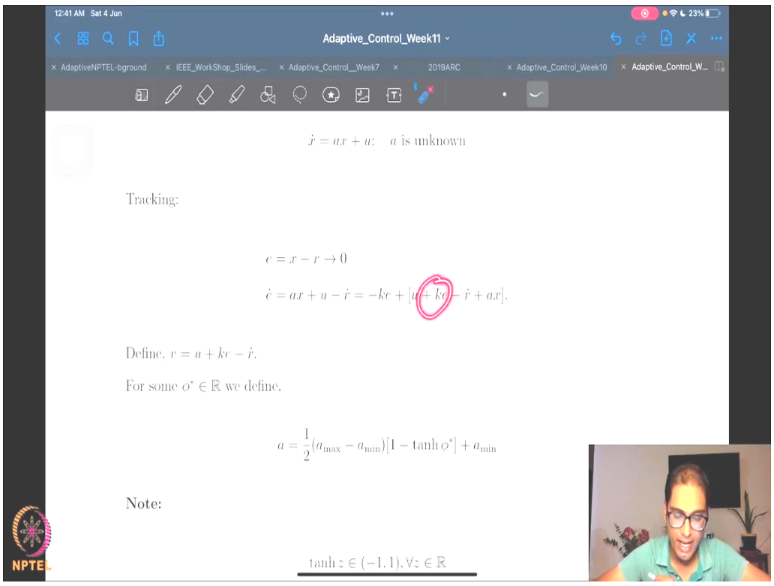
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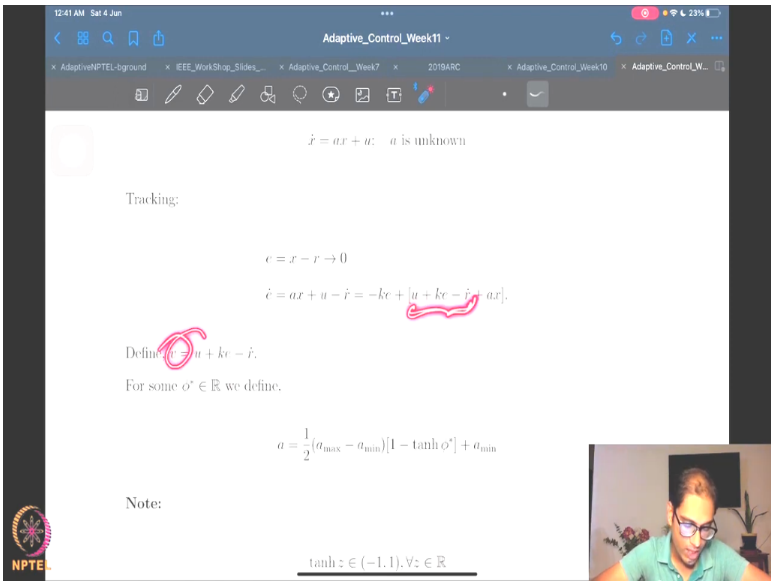

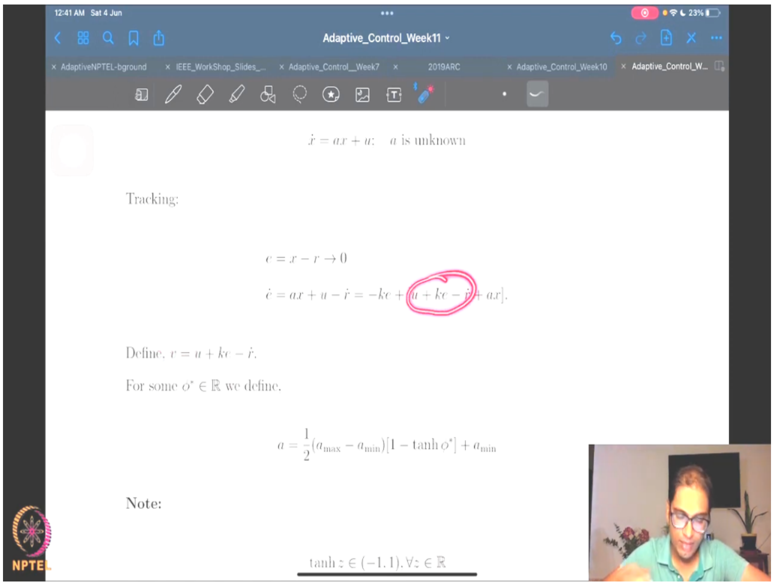

So, as usual, what we do is we look at this dynamical system, x dot is a x plus u where a is unknown and what we want to do is always is to track a reference trajectory which is nice smooth boundary infinitely differentiable and all that nice stuff. So, then we compute the dynamics of e dot which is just a x plus u minus r dot.

We sort of do a little bit of reshuffling of terms and we introduce the nice negative k e term here and add the k e term here. And this entire quantity which is a known quantity because I will specify the control is now denoted as v. So, this entire thing is denoted as a new term v or new control v which remain.

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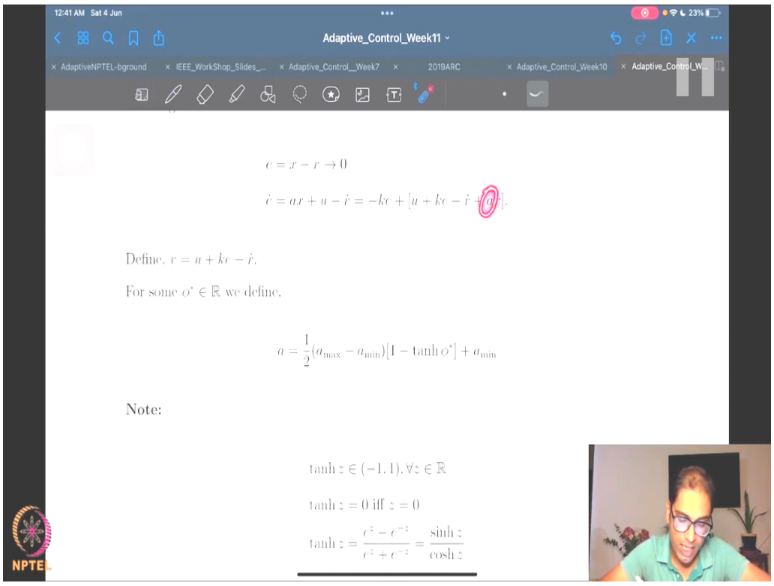
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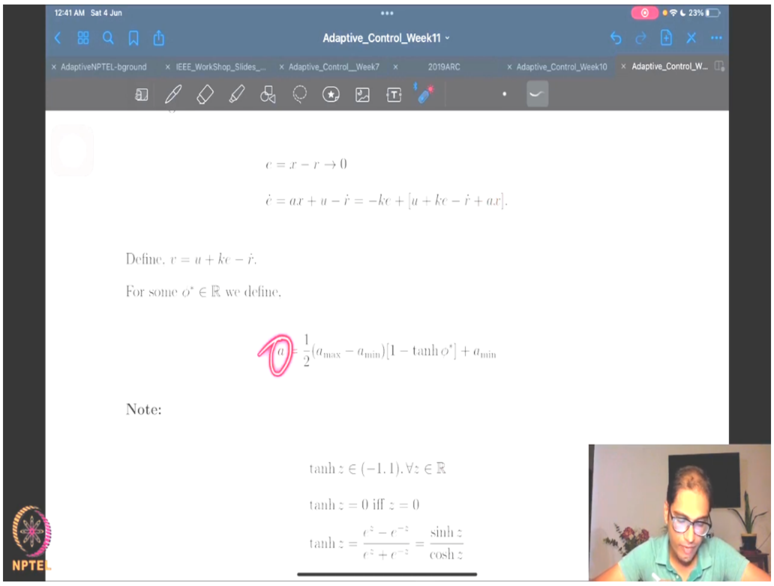
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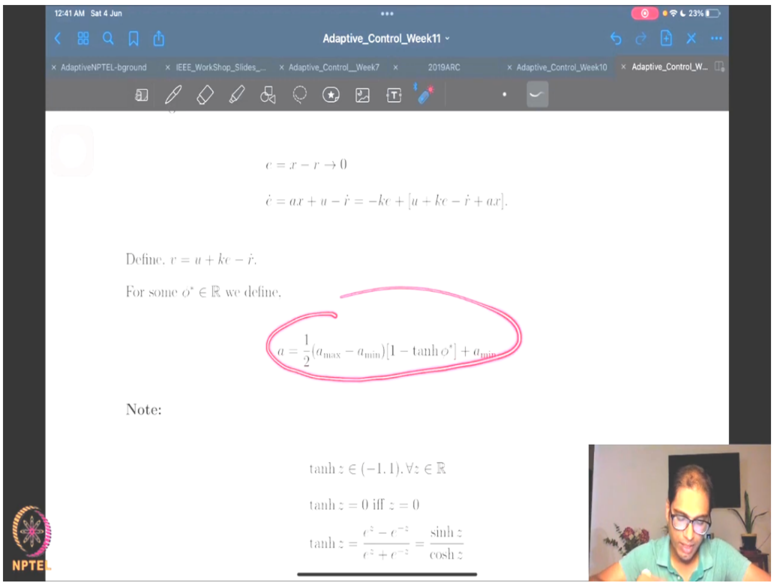
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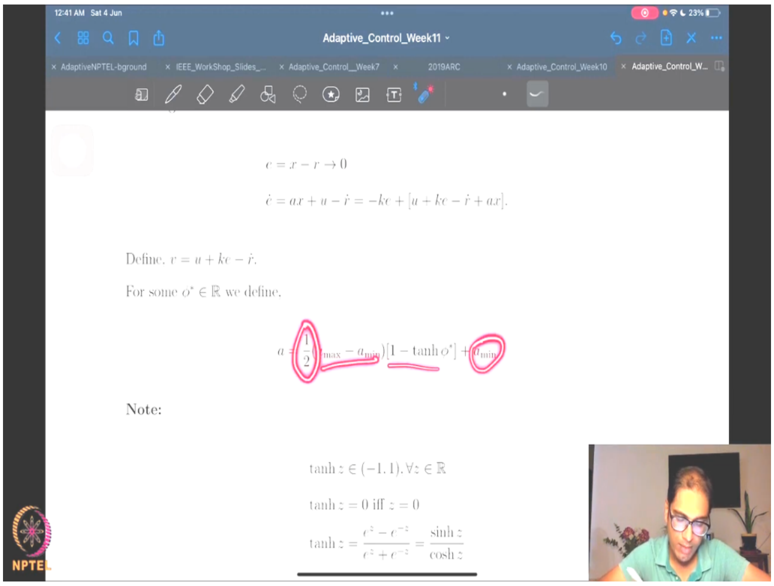

Now, how we implement projection is using tan hyperbolic functions. So, how do we do that? The unknown quantity a we claim can always be written as this sort of an expression. That is it is a min plus 1 half a max minus a min 1 minus tan hyperbolic phi star.

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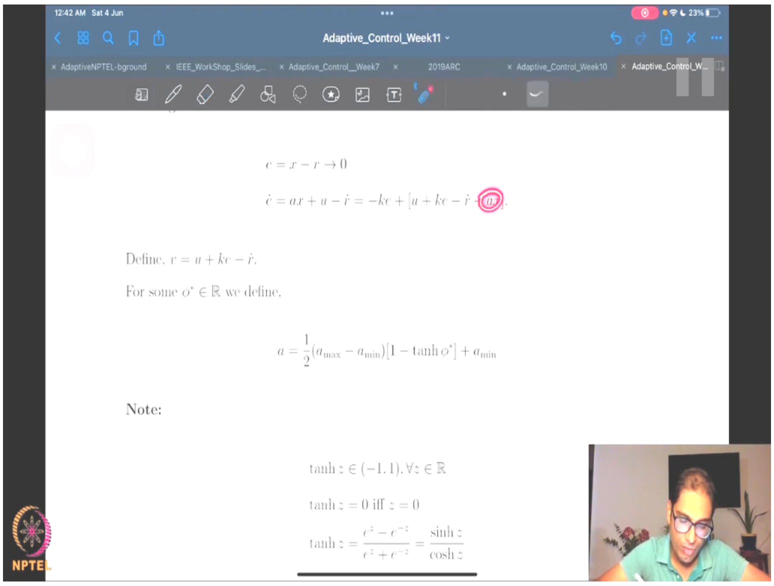

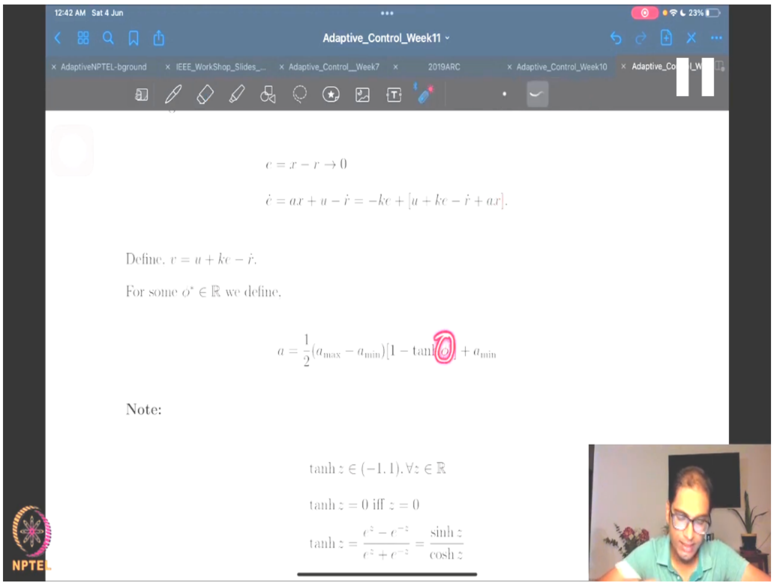
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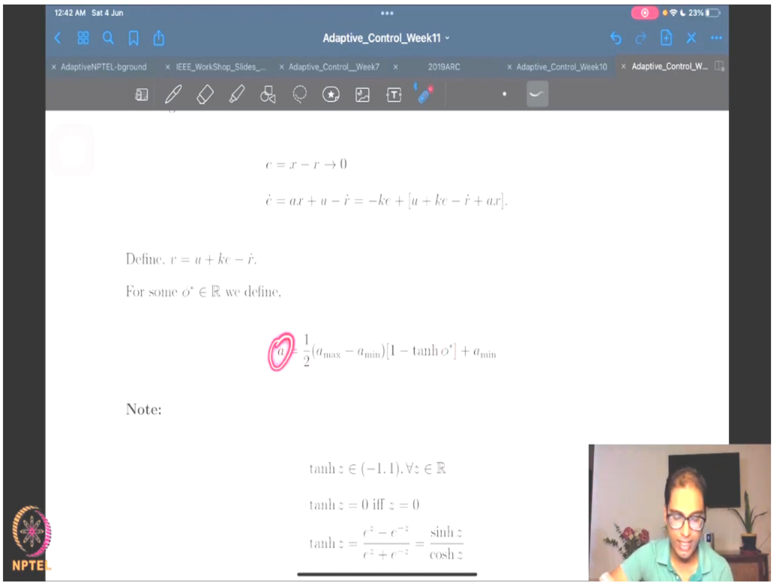
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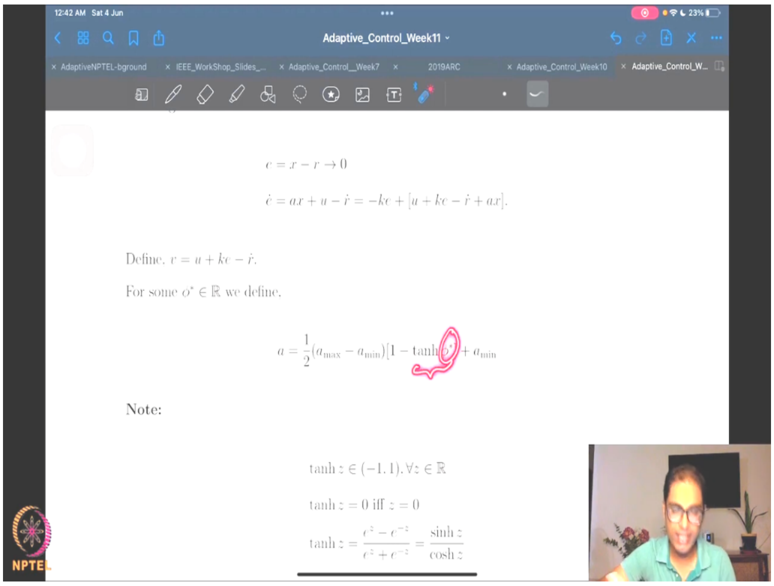
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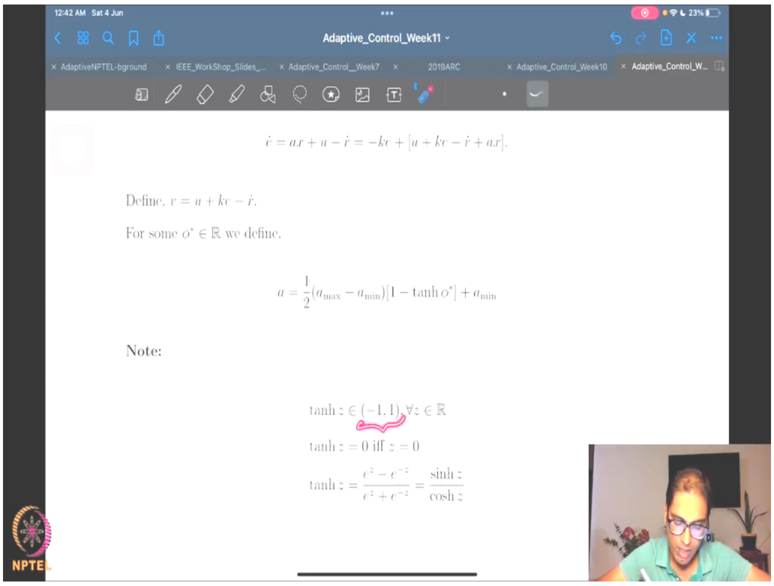

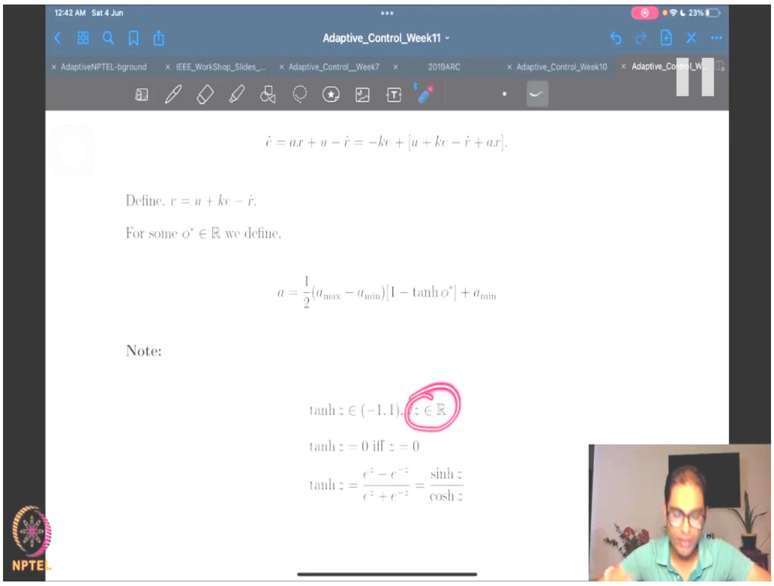

Now, what we want to do or what we are desired to, what we are going to do is to move from looking at a as the parameter to looking at phi star as the parameter. So, if you see this expression on the right hand side, the only unknown is the phi star and the left hand side is of course the unknown a.

So, what we have done is we have moved the unknown a to an unknown phi star which is inside a tan hyperbolic function. Now, let us look at why this is making sense. The first thing is that the tan hyperbolic function varies from minus 1 to 1 for all real numbers. What does it mean?

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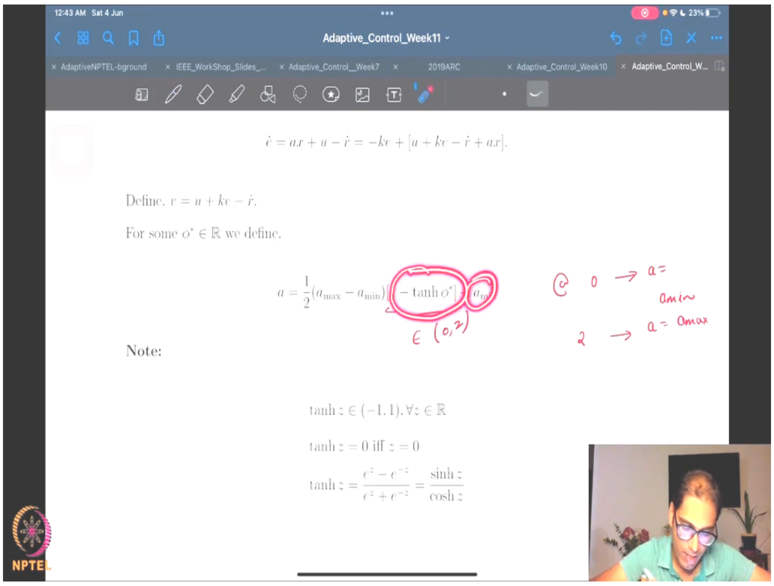
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@ 0 → a = amin
2 → a = amax
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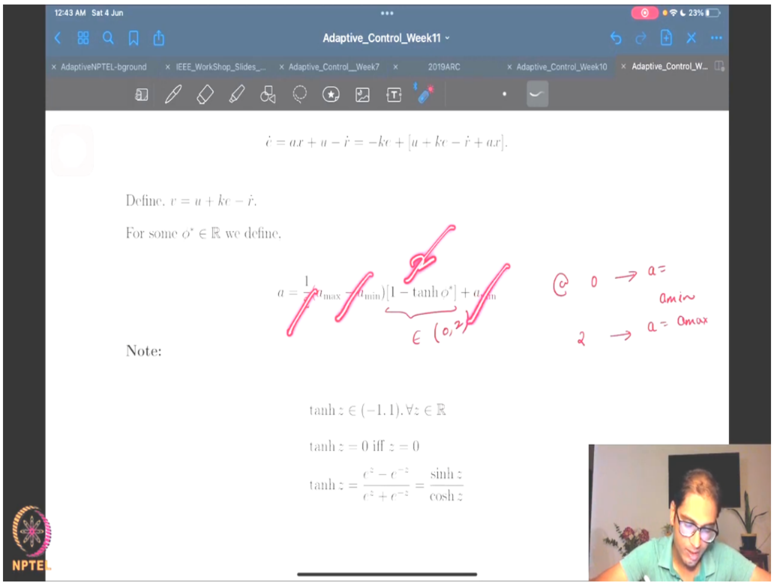
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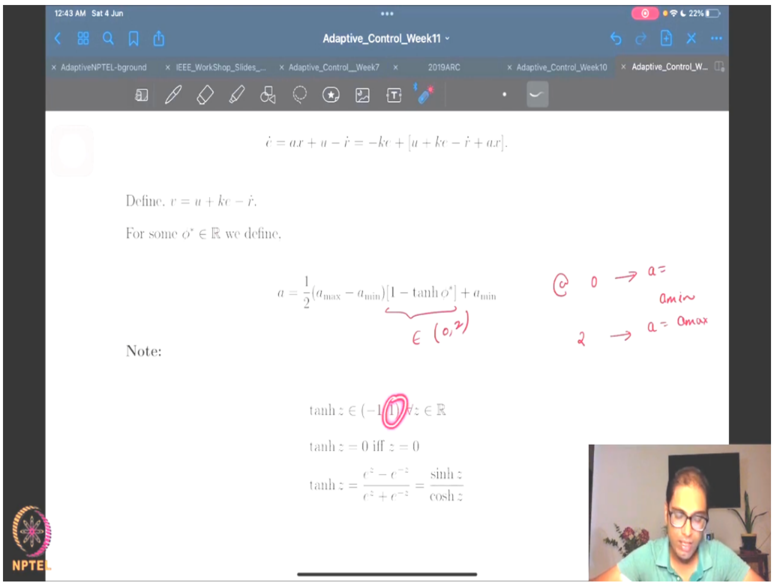
NPTEL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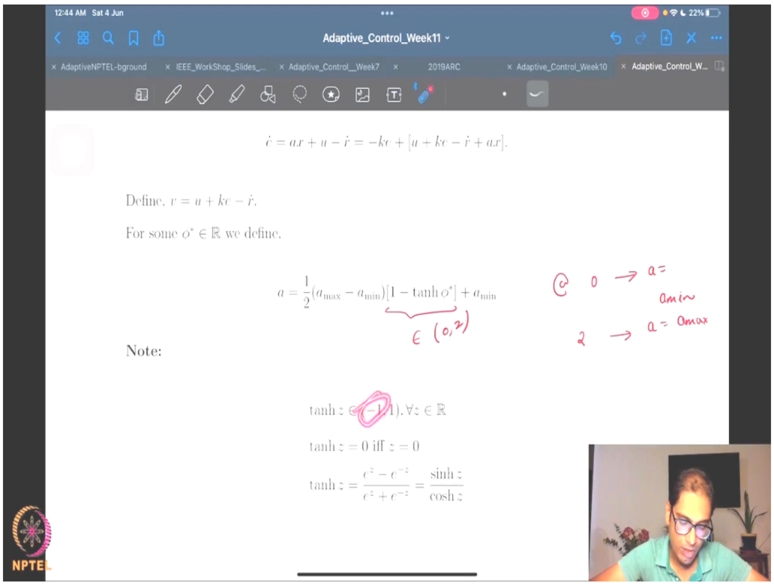

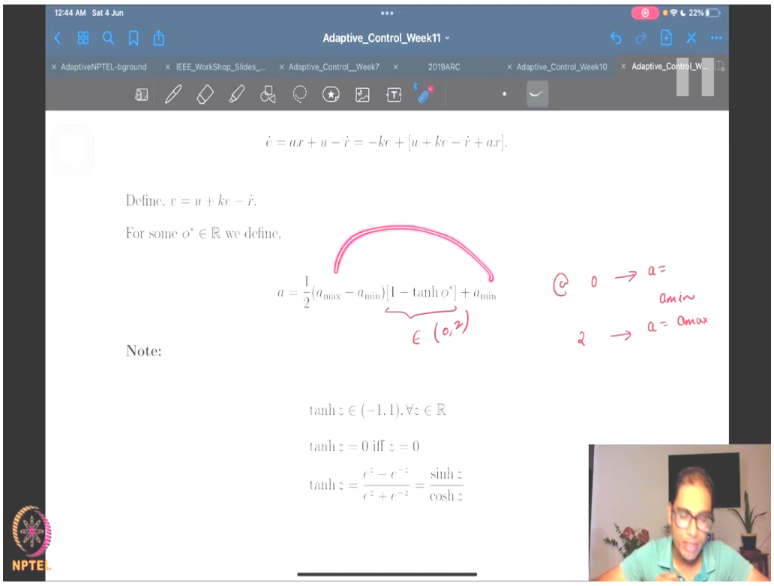

It means that this quantity lies between 0 to 2, 0 to 2. So, what happens at 0? At 0 you have the right hand side a as equal to a min and when you are at 2 you get a equals a max. This is rather easy to verify. You put 0 here you just have a min. If you put 2 here, this gets cancelled with this and this gets cancelled with this and you are left with a max. So, essentially, what does it mean?

It means that at, at one edge of the spectra and one h phi at one end when phi star tan phi star tan hyperbolic phi star takes value, one you have a min as the value of a and when tan hyperbolic phi star takes the value minus 1, you have a as a max. Therefore by scanning phi star also the phi star is of course belonging to all of real numbers. So, just by scanning in all of real numbers I go only between a min to a max. So, this is what is projection.

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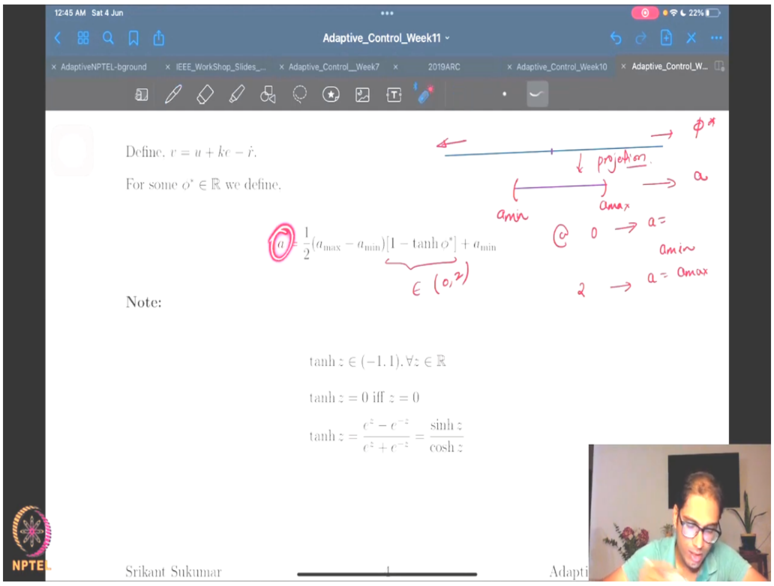

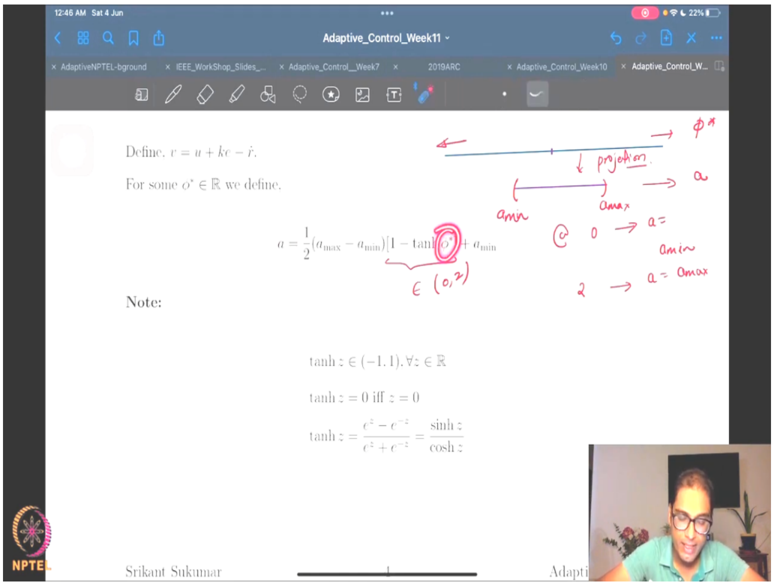

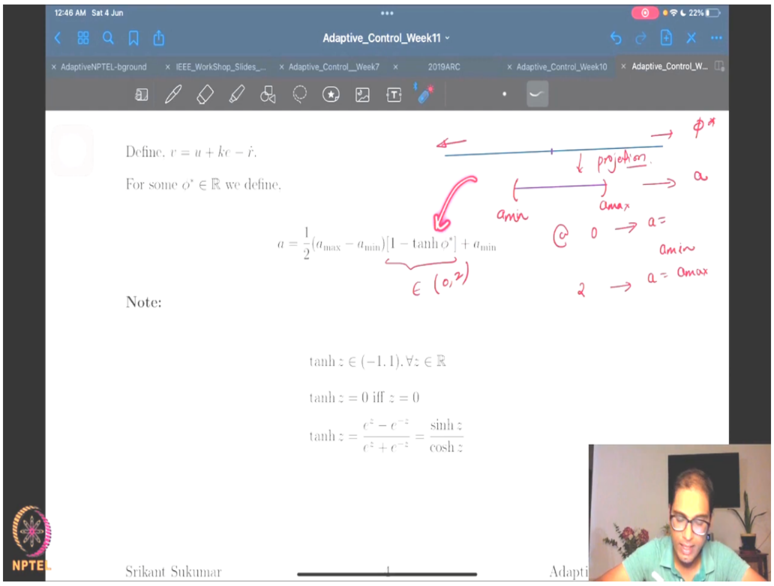

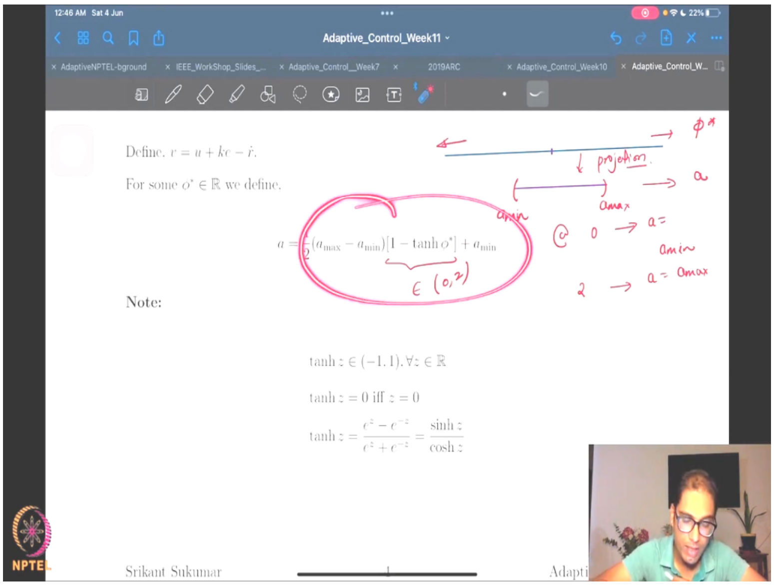

So, because, if you notice if I make a picture something like this, so the phi star is sort of, so, the phi star is ranging from all of real numbers. So, this as well as phi star and this is ranging from a min a max. This is where is a. So, even though I vary phi start from minus infinity to infinity, I am going to remain only within a min and a max. So, this is the projection. This is the projection. I hope you understand. So, this is the smooth projection using this tan hyperbolic function. Phi star ranges from minus infinity to infinity that is my search domain.

But once I do the projection, it is actually finding two valued of parameters only between a min and a max. And notice that this a is what gets implemented a hat is what gets implemented. So, you never have to worry about what the value of phi star is because once it is plugged inside the tan hyperbolic, it ranges between minus 1 to 1. So, you get a bounded quantity between a min and a max here to the right hand side.

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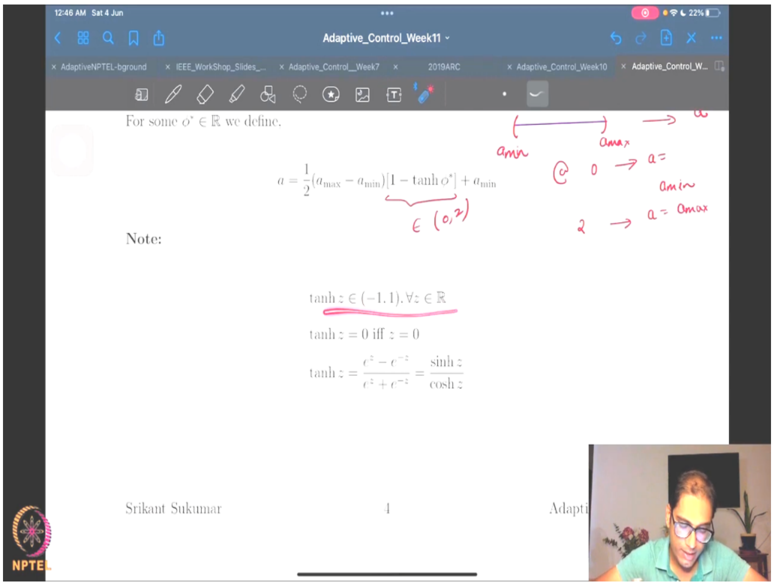

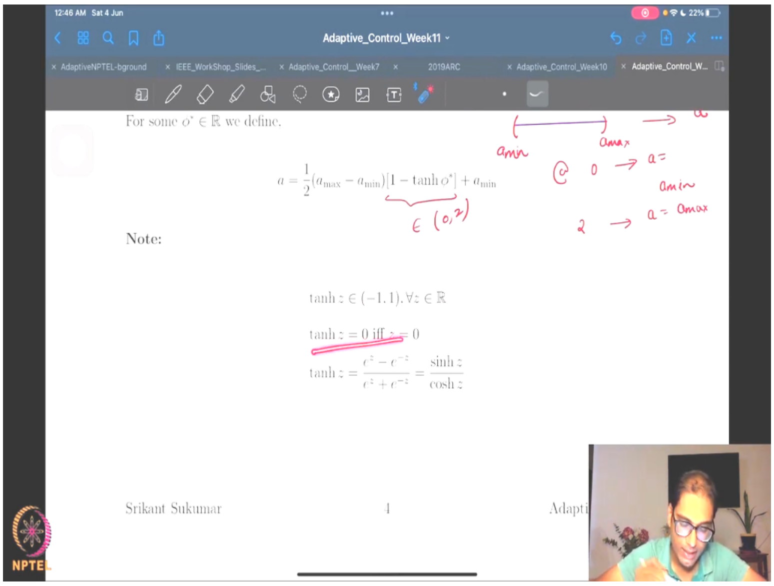

12:46 AM Sat 4 Jun

Adaptive_Control_Week11

AdaptiveNPTEL-bground | IEEE_WorkShop_Slides_... | Adaptive_Control__Week7 | 2019ARC | Adaptive_Control_Week10 | Adaptive_Control_W...

For some $\phi^* \in \mathbb{R}$ we define,

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a_{min} a_{max} $\rightarrow a$

@ $0 \rightarrow a = a_{min}$

$2 \rightarrow a = a_{max}$

Note:

$$\tanh z \in (-1,1), \forall z \in \mathbb{R}$$

$$\tanh z = 0 \text{ iff } z = 0$$

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Srikant Sukumar 4 Adapti

NPTEL

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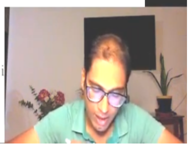

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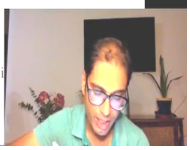

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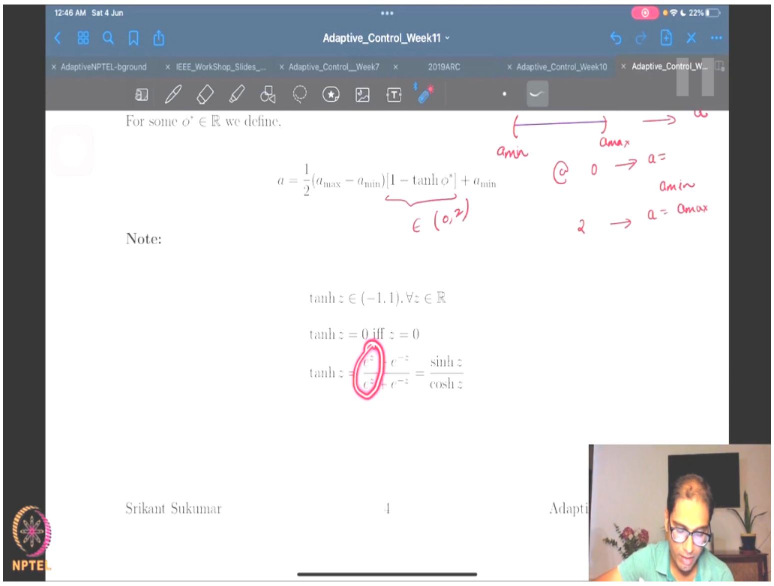
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Srikant Sukumar
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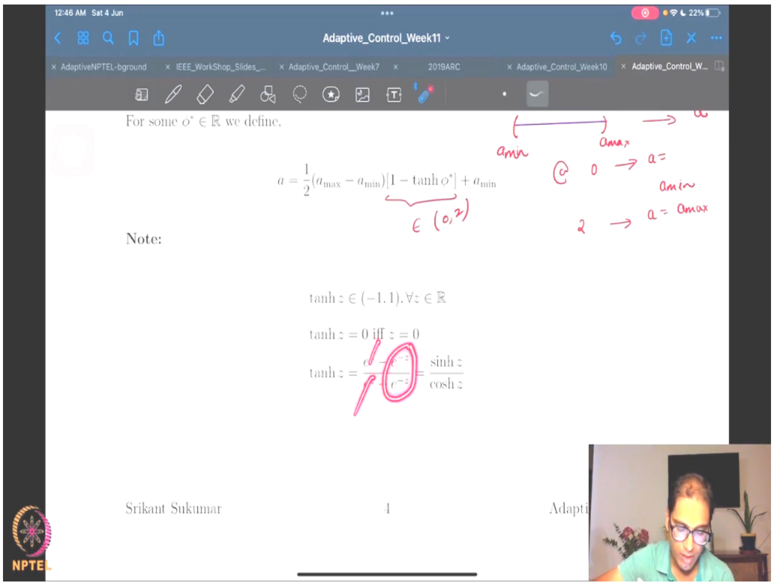
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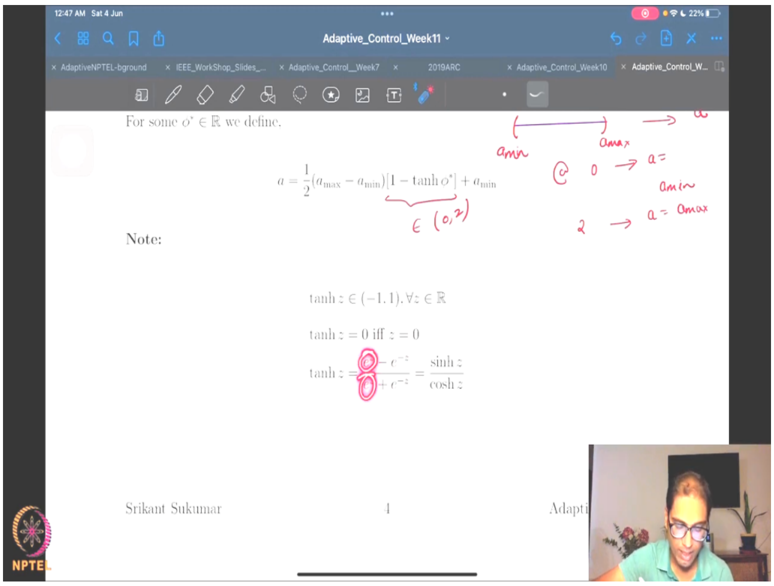
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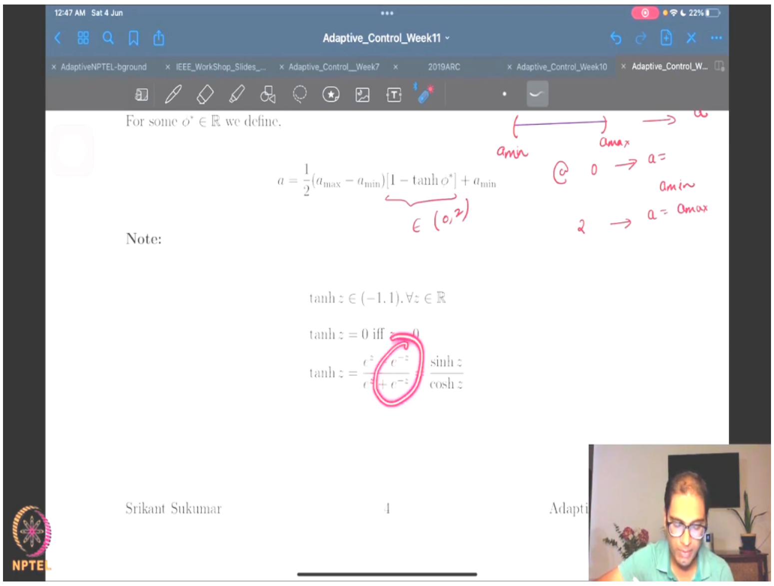
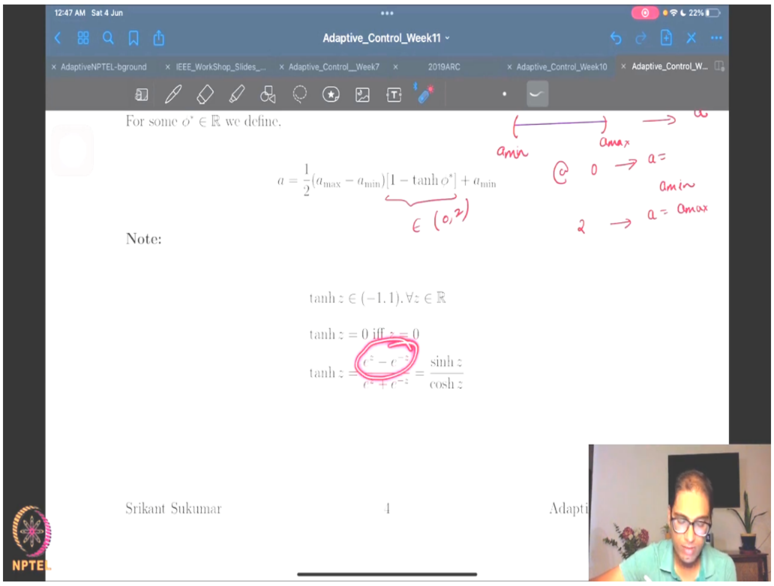
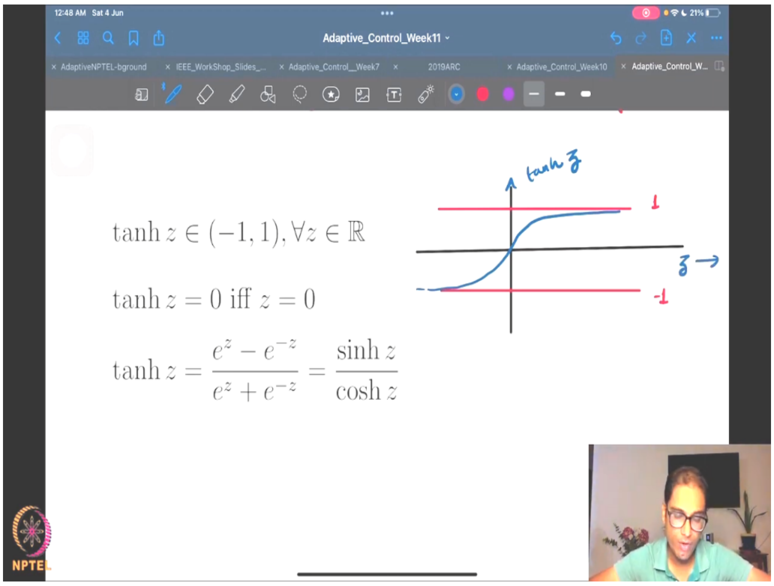




So, tan hyperbolic z lies between minus 1 to 1. It is 0, exactly 0, if and only if z is 0. And tan hyperbolic z is actually this expression. If you see when do you get tan hyperbolic equal to 1 is when let us see, you will get tan hyperbolic equal to 1 when you have z going to infinity.

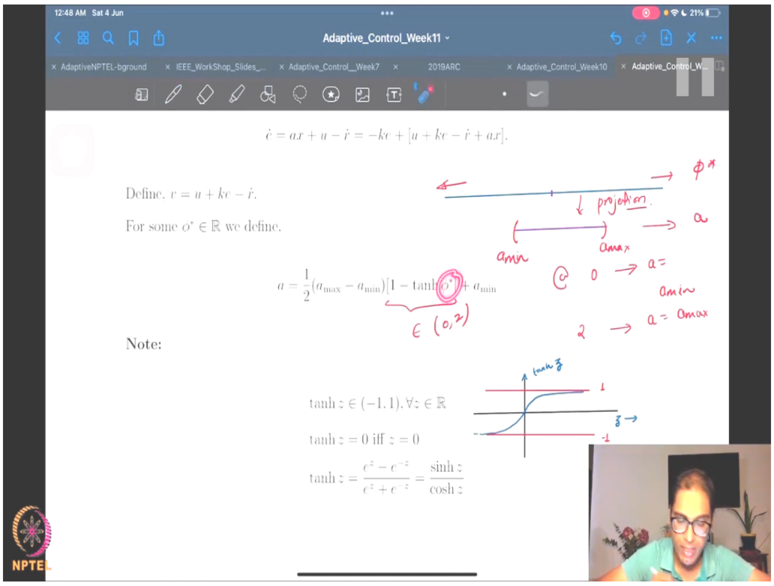
When you have z going to infinity and you will get it as minus 1 when you have z going to minus infinity, then because in that when z goes to infinity these two are 0. So, these two become 1. And when z goes to minus infinity, these two are minus 1. Sorry, these two are 0, so, this becomes minus 1. Now, what happens at z equal to 0? The numerator is 0. So, therefore you get a 0.

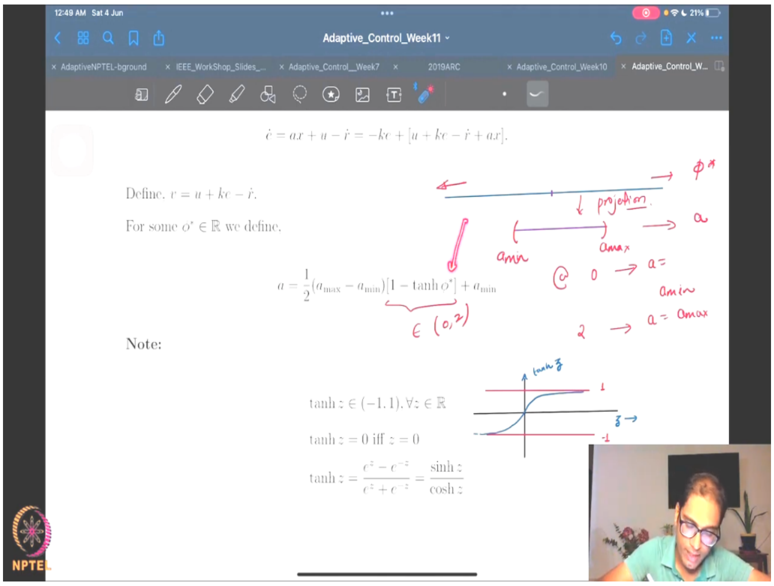
(Refer Slide Time: 12:31)

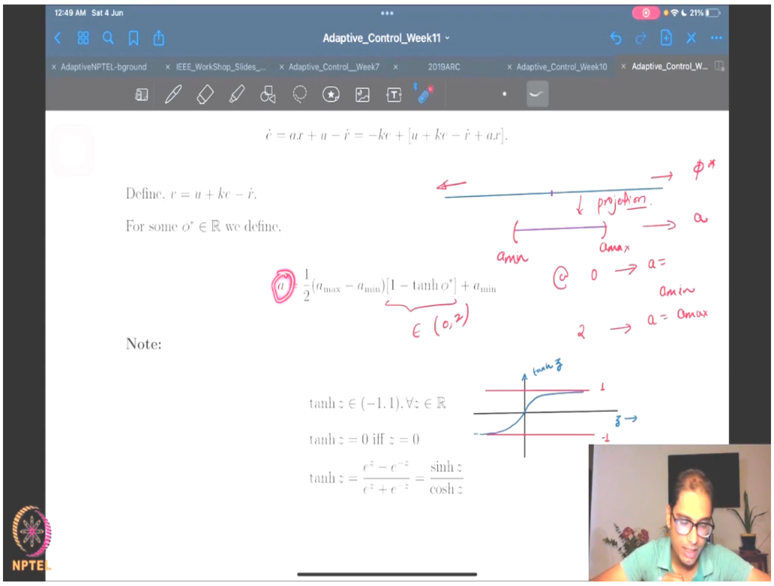


So, the picture of the, if the tan hyperbolic function if you may, looks something like this. So, the tan hyperbolic function. Sorry. So, this is basically 1 and minus 1 and the tan hyperbolic function looks something like this. So, the tan hyperbolic function looks something like this. I hope that makes sense.

(Refer Slide Time: 13:55)







So, this is essentially the projection. So, this is the key part here. So, remember, the phi star being the parameter that we are trying we will try to identify. Actually, v helps us to do projection because we adapt for phi star. So, we create a phi hat but then we implement a hat in the controller not a not phi hat. So, the control always contains the a hat and that is all we care about.

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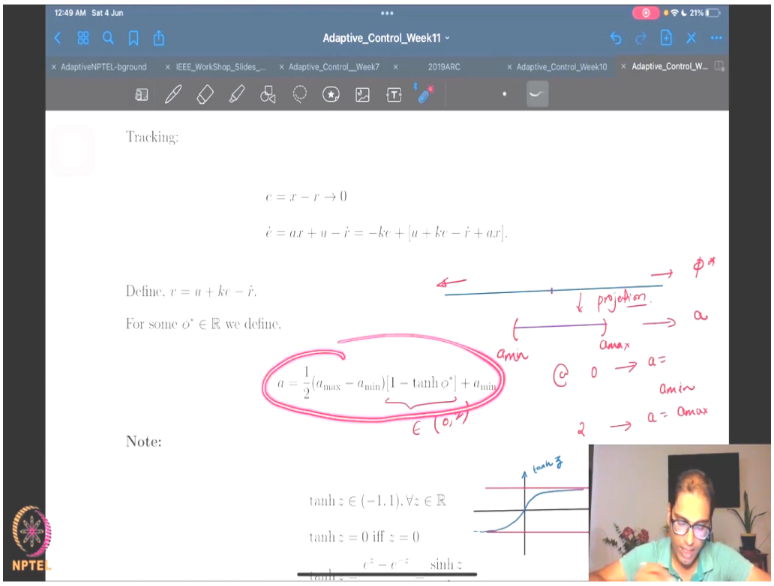

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projection
a
$a_{\min}$
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Note:
$\tanh z$
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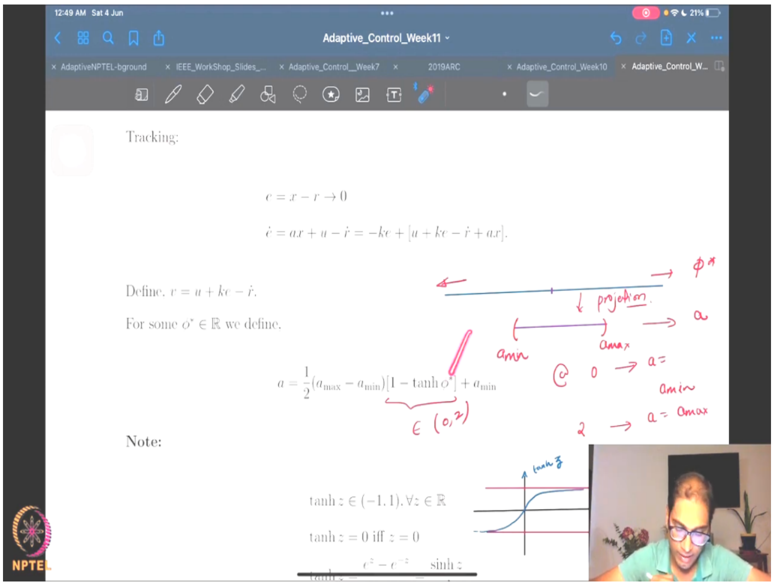
$\tanh z = 0$ iff $z = 0$
NPTEL



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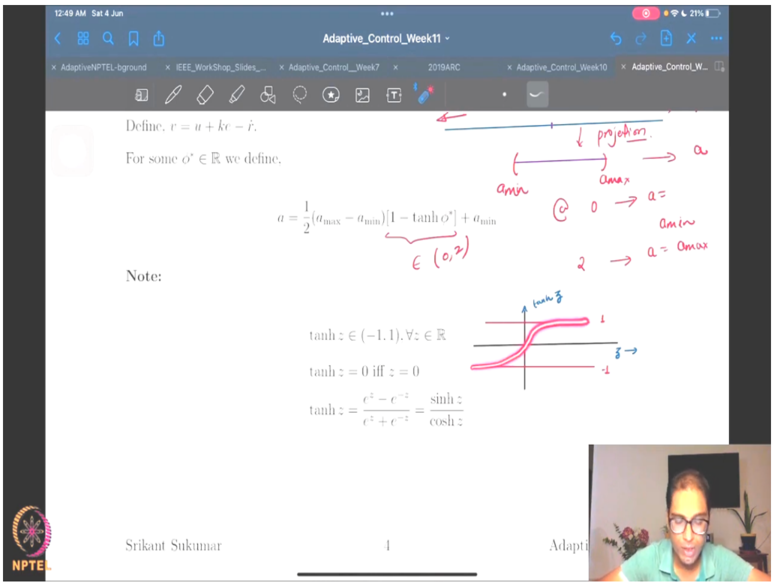

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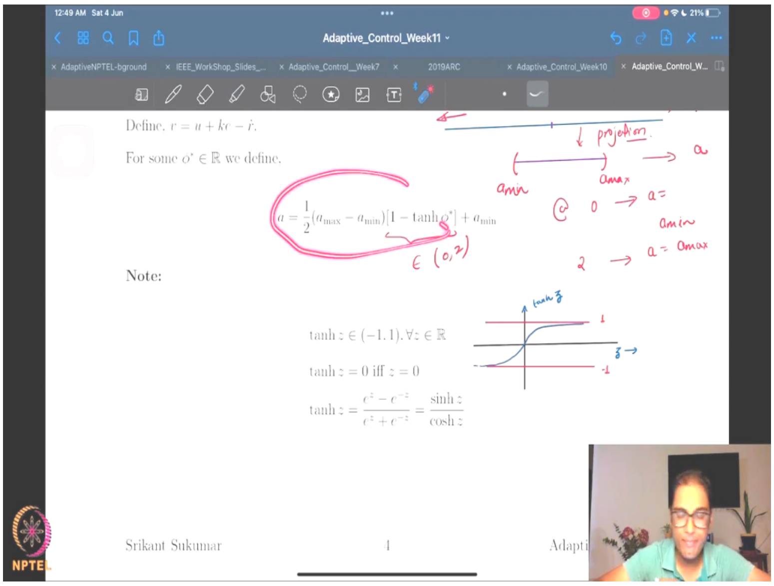

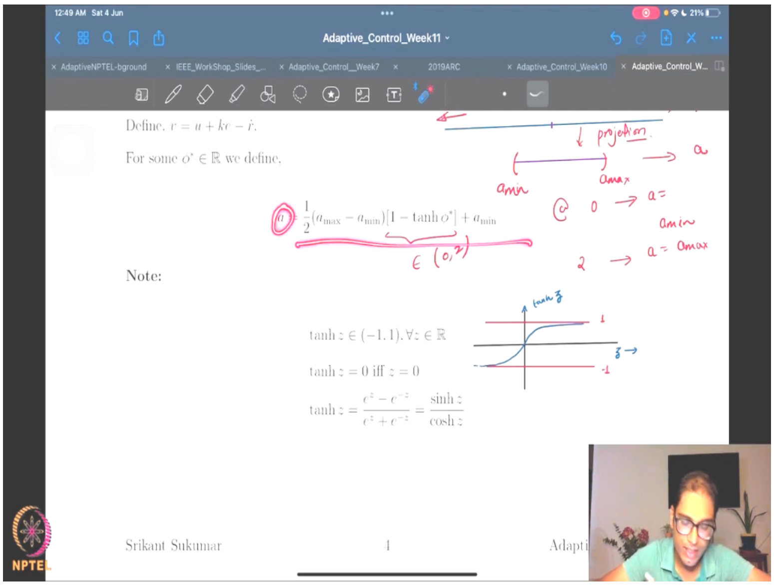

So, let us look at the adaptive problem. So, this is the definition of a. It should not be difficult to see that if a does lie between a min and a max there exists such a phi star because this tan hyperbolically takes every value from minus 1 to 1. So, obviously, you cannot miss a. So, there does exist a phi star such that a satisfies this expression.

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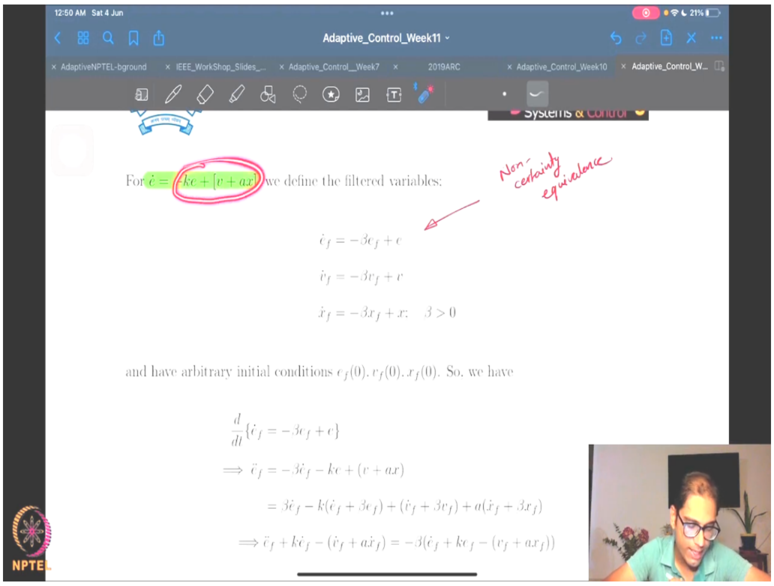
12:50 AM Sat 4 Jun
Adaptive_Control_Week11
AdaptiveNPTEL-bground
IEEE_WorkShop_Slides_...
Adaptive_Control__Week7
2019ARC
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Adaptive_Control_W...
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$\dot{x}_f = -\beta x_f + x; \quad \beta > 0$
and have arbitrary initial conditions $e_f(0), v_f(0), x_f(0)$. So, we have
$\frac{d}{dt}\{\dot{e}_f = -\beta e_f + e\}$
$\implies \ddot{e}_f = -\beta \dot{e}_f - ke + (v + ax)$
$= \beta \dot{e}_f - k(\dot{e}_f + \beta e_f) + (\dot{v}_f + \beta v_f) + a(\dot{x}_f + \beta x_f)$
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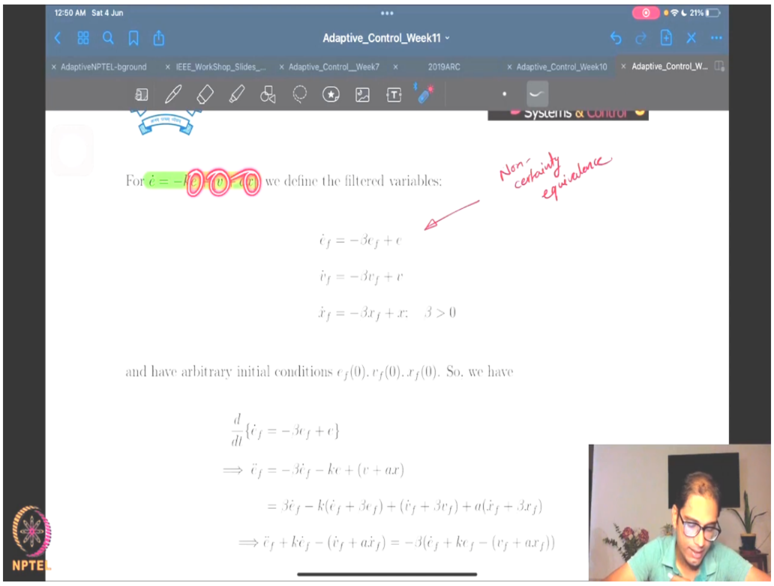
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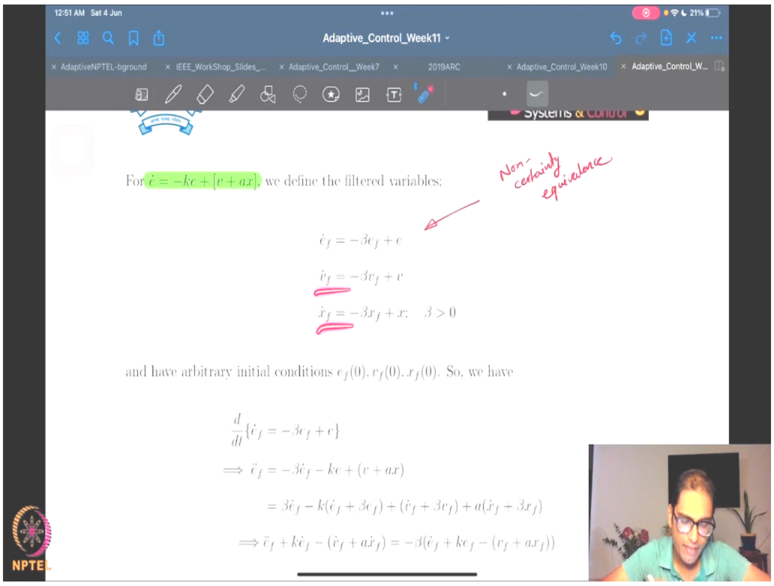
12:51 AM Sat 4 Jun
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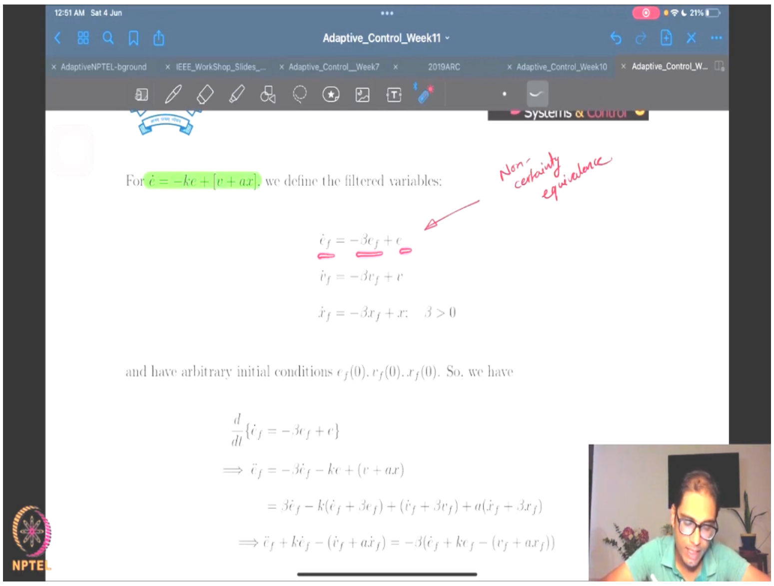

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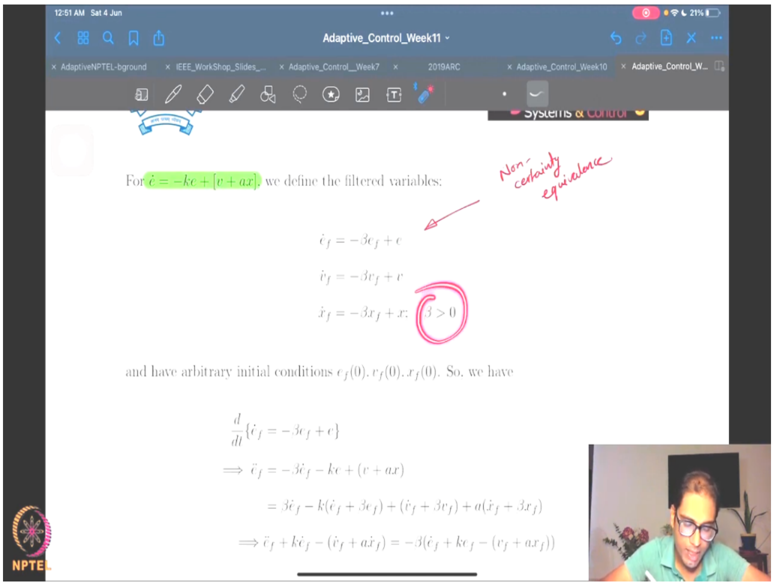

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$\implies \ddot{e}_f = -\beta\dot{e}_f - ke + (v + ax)$
$= \beta\dot{e}_f - k(\dot{e}_f + \beta e_f) + (\dot{v}_f + \beta v_f) + a(\dot{x}_f + \beta x_f)$
$\implies \ddot{e}_f + k\dot{e}_f - (\dot{v}_f + a\dot{x}_f) = -\beta(\dot{e}_f + ke_f - (v_f + ax_f))$
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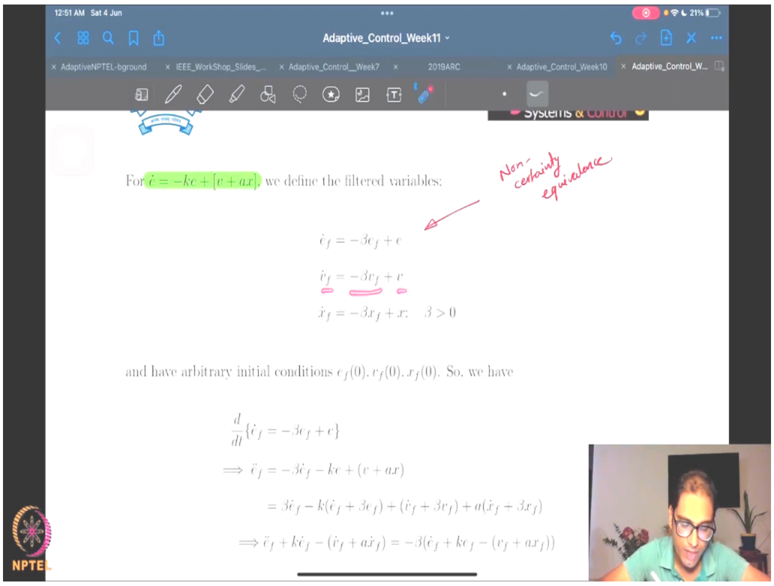

Adaptive_Control_Week11
Systems & Control
For $\dot{e} = -ke + [v + ax]$, we define the filtered variables:
Non-certainty equivalence
$\dot{e}_f = -\beta e_f + e$
$\dot{v}_f = -\beta v_f + v$
$\dot{x}_f = -\beta x_f + x; \quad \beta > 0$
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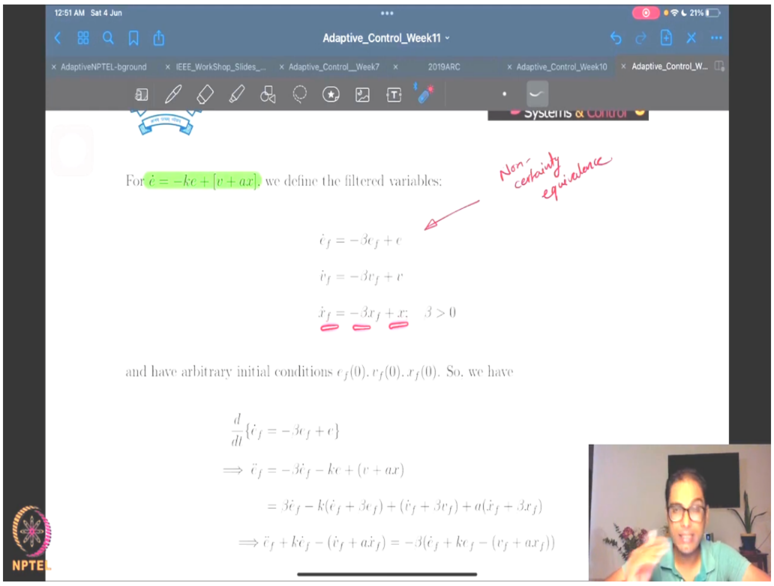

So, before we move on, so, this becomes a dynamic because we redefined the control. So, before we move on, we define some filtered variables. There is a very particular unique value to these filtered variables. This is follows and I mean falls under the purview of what is called non-certainty equivalence. You will see why very soon. You will see why very soon. You will see why very soon.

But before that we define these filtered variables. It is not very uncommon. Slotine also used these filter variables a long time ago but here the purpose is different. So, what do we do? We look at the dynamics. How do we create filtered variables is the important question. What do we create filtered variables for is the important question.

We create filtered variables of all the terms, we all the known terms on the right hand side. So, the known terms are e, v and x. So, a is the unknown. So, we create filtered variables for e, v and x because all three are known quantities because it does not make any sense to filter unknown quantities since these filtered quantities will get used in actual control implementation and you cannot if you do not know that.

So, you cannot filter a quantity which is unknown. So, therefore I create a filter for e, v and x and the filters are very straightforward. It is a standard low pass type filter. Basically it is like e f dot is minus beta e f plus e. For the same beta you create three filters, v f dot is minus beta v f plus v, x f dot is minus beta x f plus x.

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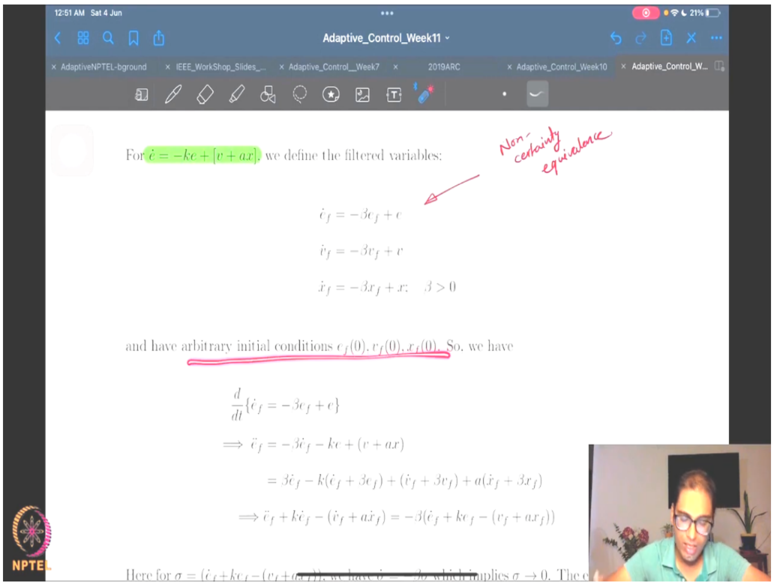

Adaptive_Control_Week11
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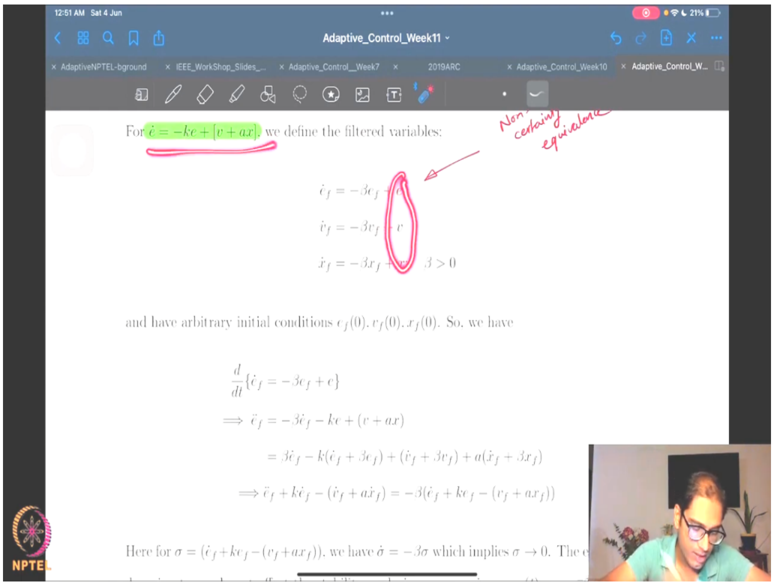
NPTEL



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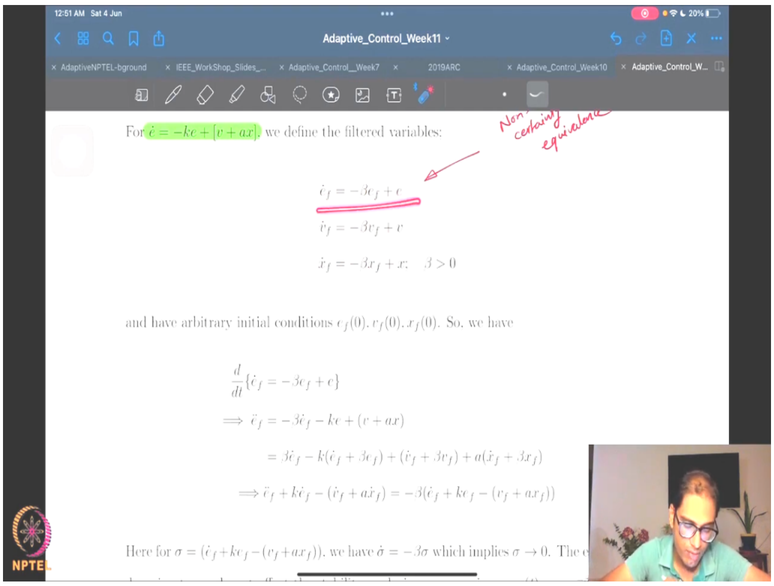

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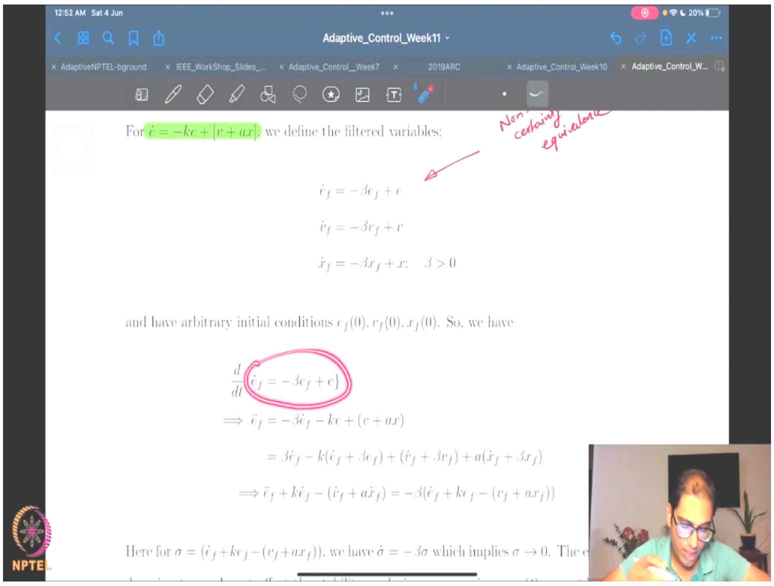
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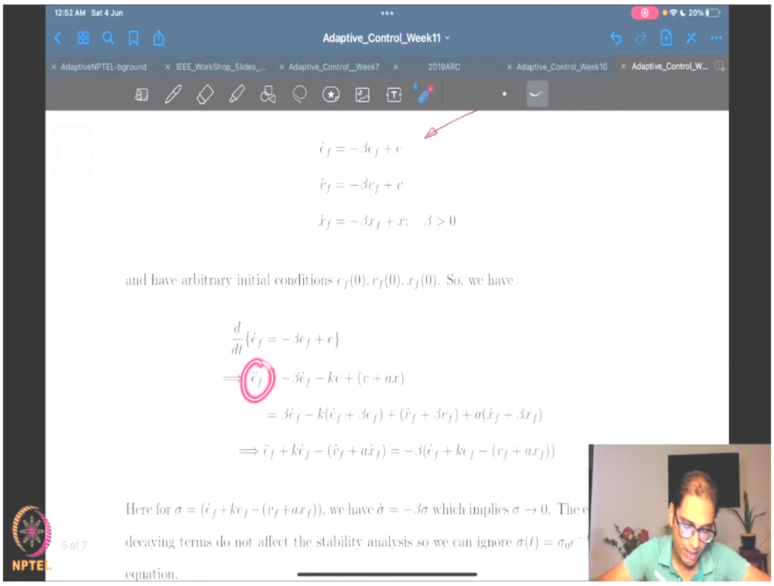
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equation.
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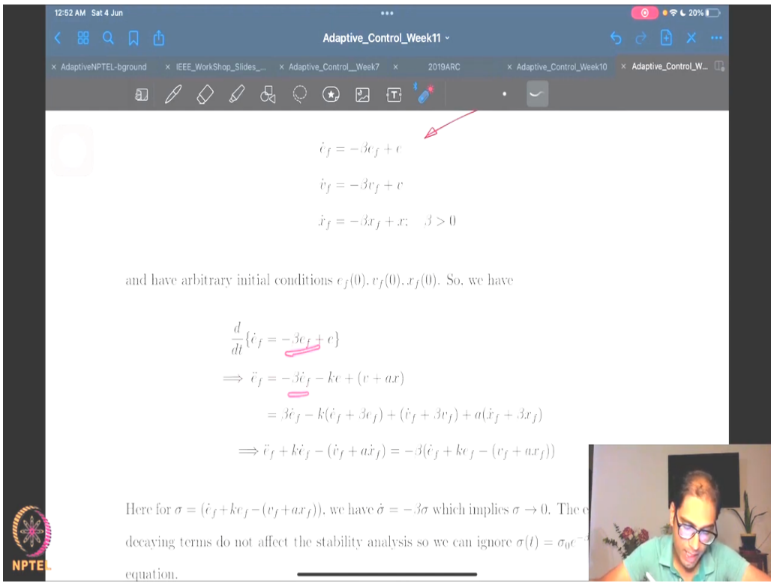
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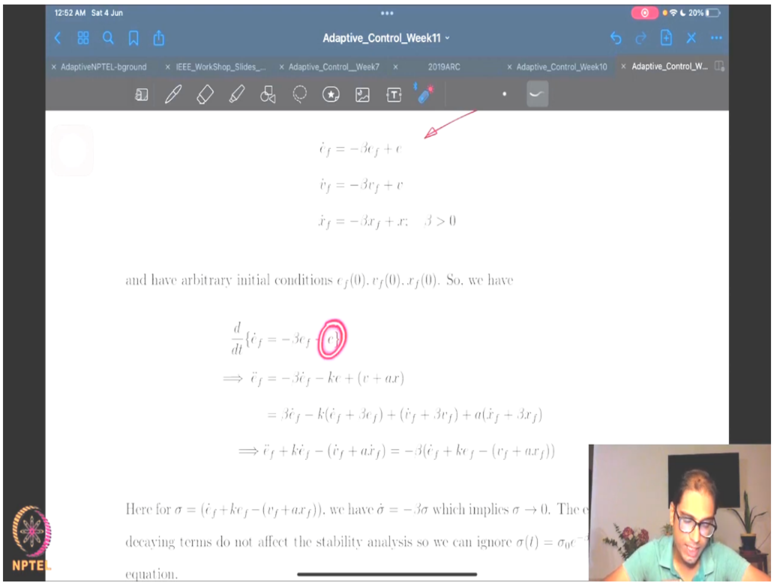

And the initial conditions are chosen to be arbitrary. We do not worry about; we do not have to worry about what the initial conditions are. Now, what do we do? I want to sort of establish a relationship between the original variables and their dynamics which is this and the filtered variables and their dynamics.

In order to do this what I do is I take this first equation and I take a derivative on both sides. So, that is it I take the first equation and I have taken derivative on both sides. So, on the left hand side I get a second derivative e f double dot and on the right hand side I get minus beta e f dot plus e dot and I get an e dot.

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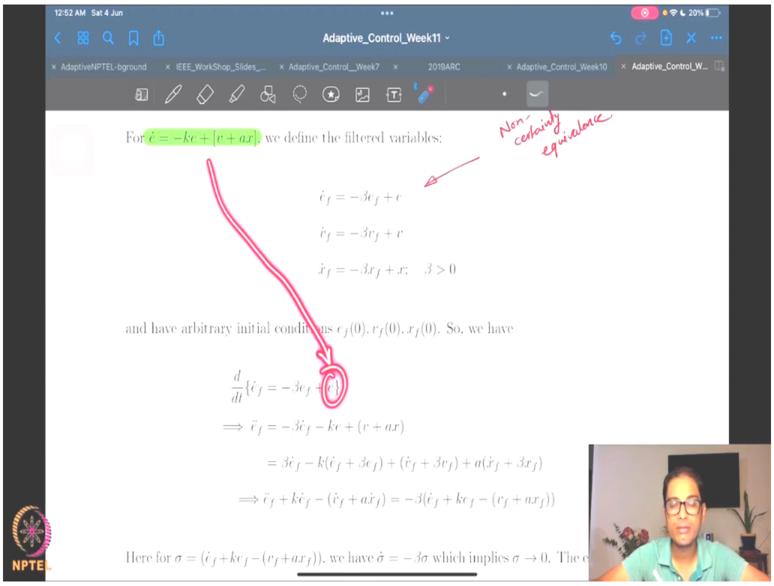

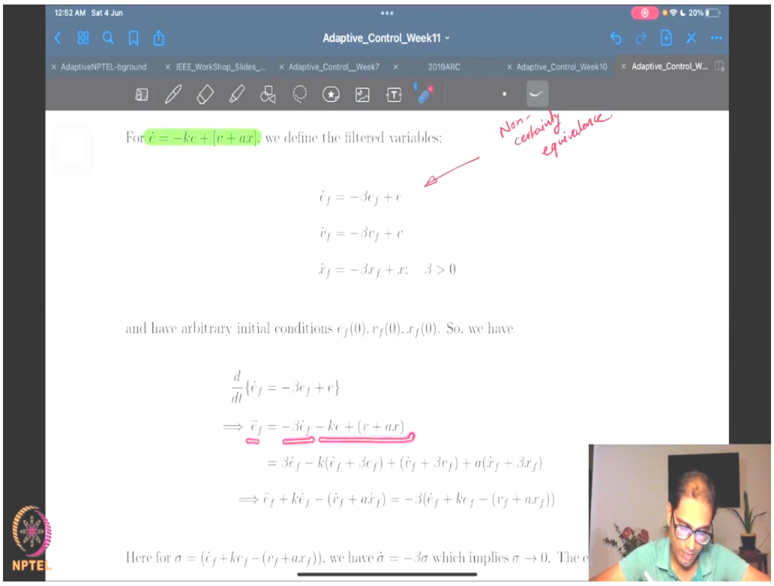
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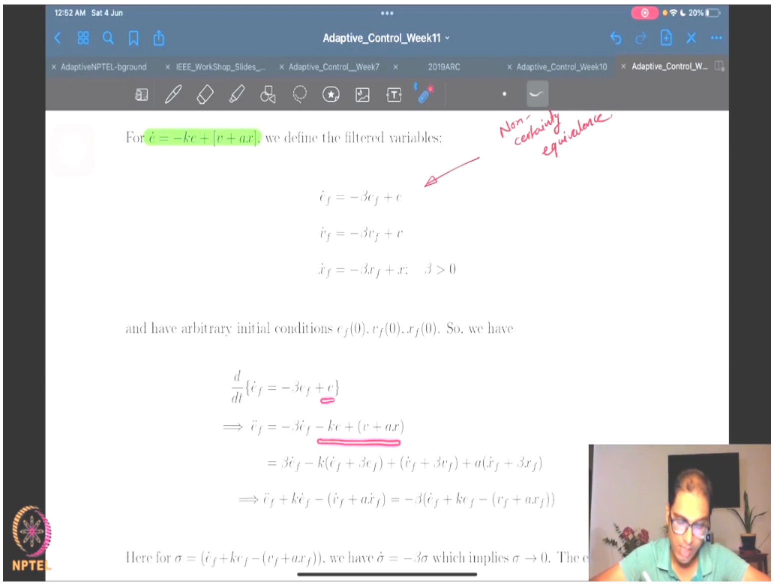
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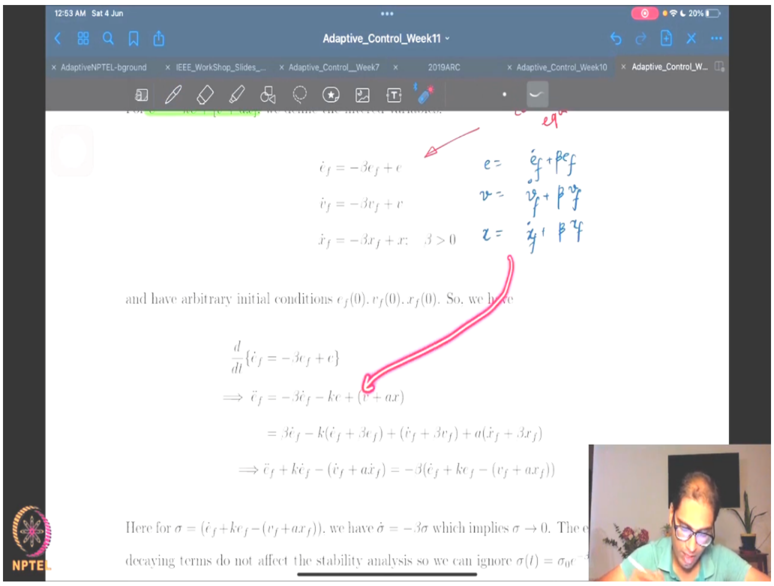
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NPTEL

Why did I take the derivative of this term only? Because it brings in the e dot. And once I have the e dot, I can substitute from here. And that is what is happening. This is essentially that, e f double dot is minus beta f dot plus e dot. So, this is the entire thing is plus e dot.

So, once I have these what I want to do is I want to write everything in terms of the filtered quantities. So, I want to write e in terms of filtered quantities, v in terms of filtered quantities and x in terms of the filtered quantities. So, what do I know? I know that e is e f dot plus beta e f, v is v f dot plus beta v f and x is x f dot plus beta x f. And this is what we substitute here.

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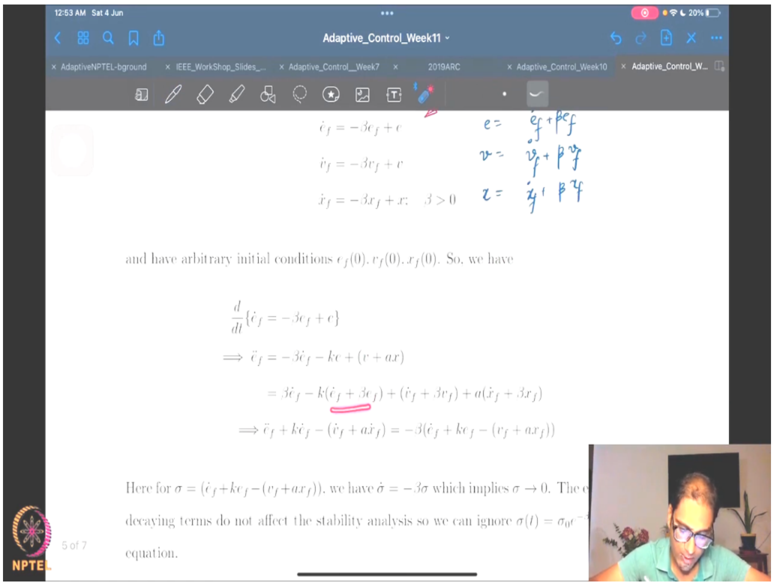

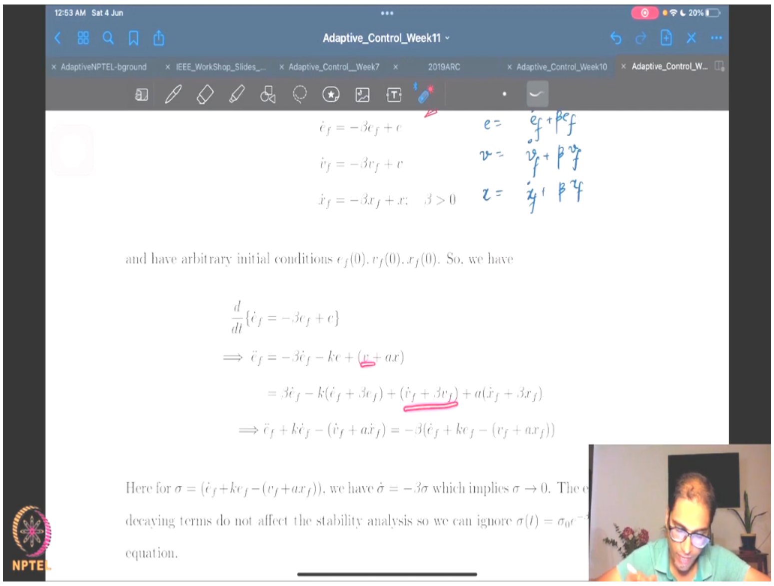

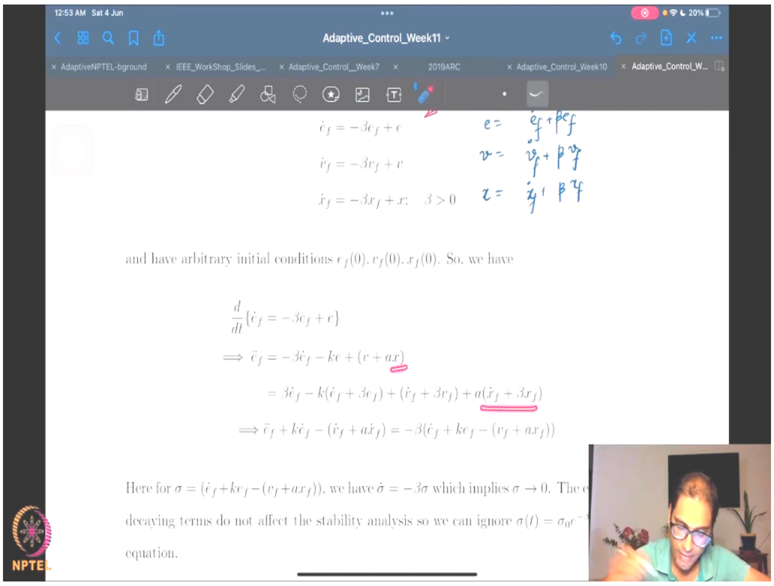

So, e is written as e f dot plus beta e f, v is written as v f dot plus beta v f, x is written as x f dot plus beta x f.

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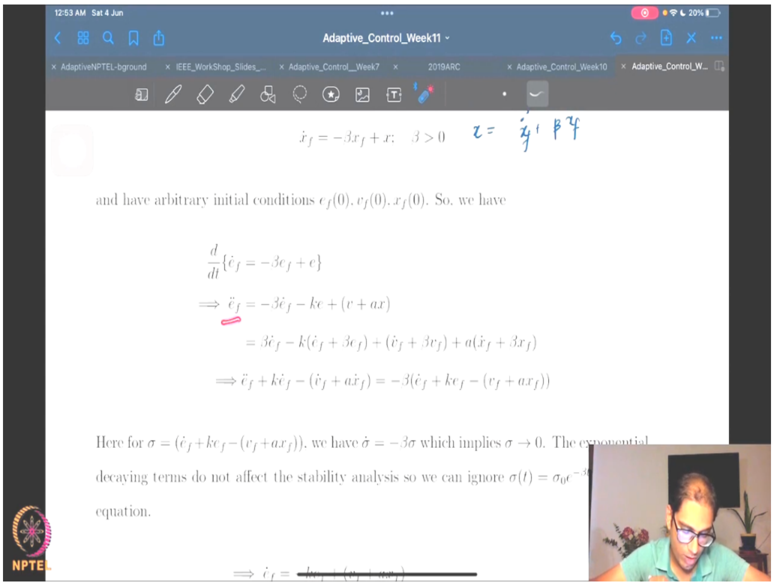

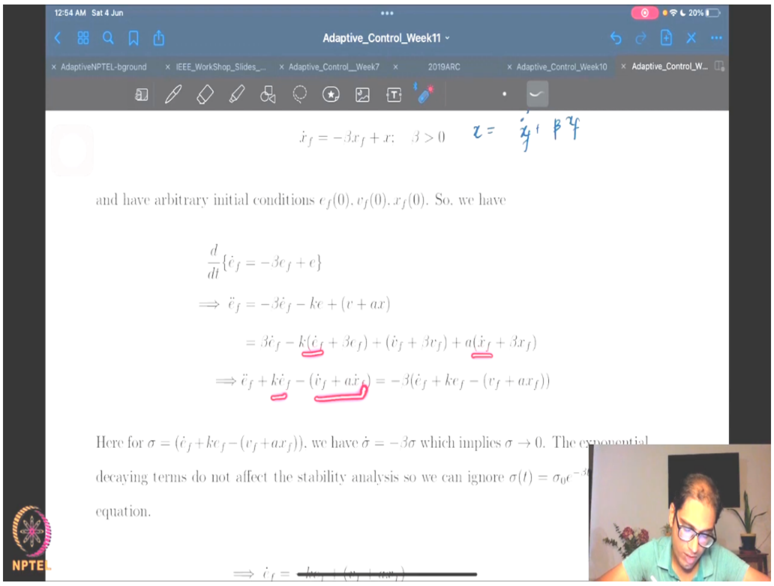

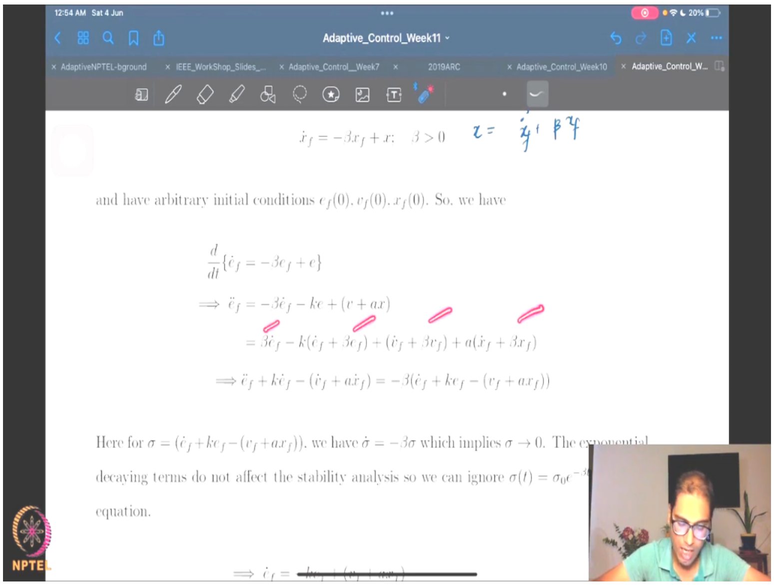

Then what I do is I take the higher derivatives on one side. So, e f double dot was already on the left hand side, e f double dot was already on the left hand side and then the v f dot plus a x f dot is taken here and then k e f dot is also taken here. So, basically or to put it simply all the beta terms, beta containing terms are kept on the right hand side and all the rest of the terms are brought to the left hand side.

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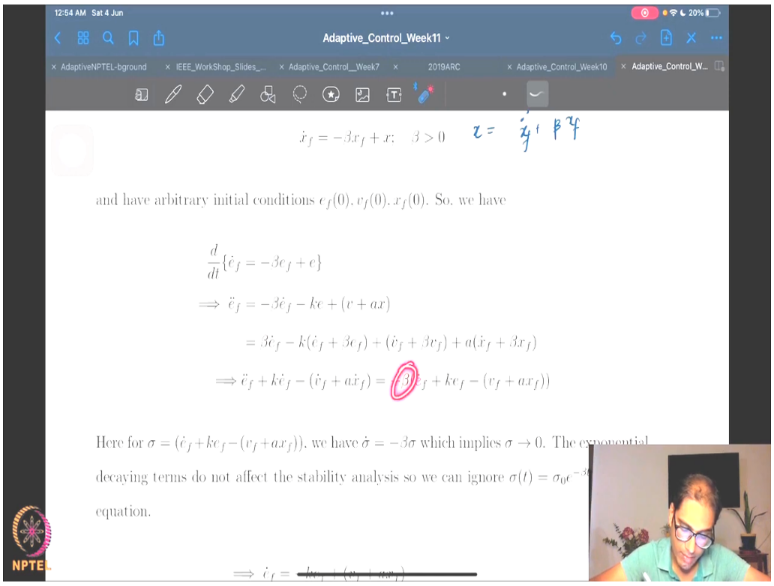
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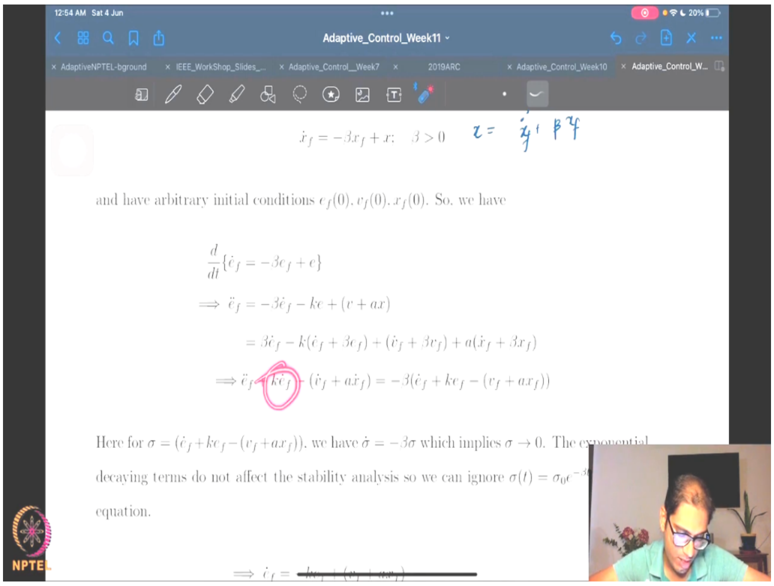
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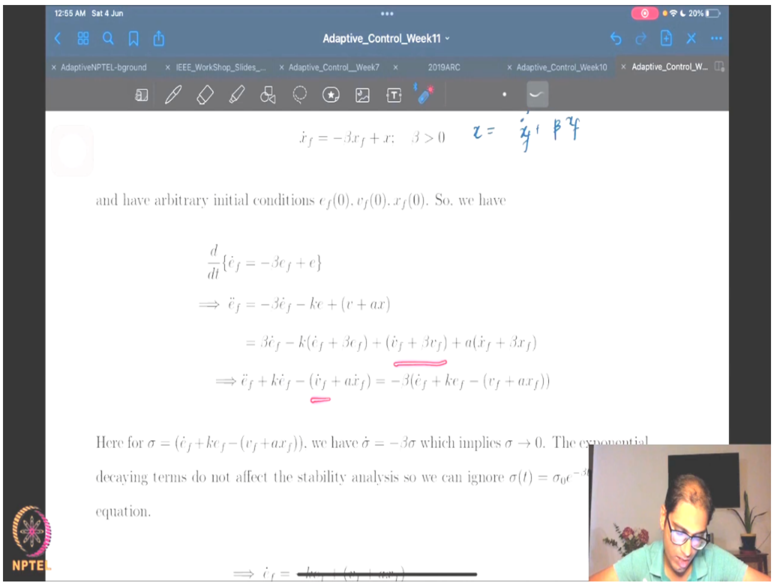
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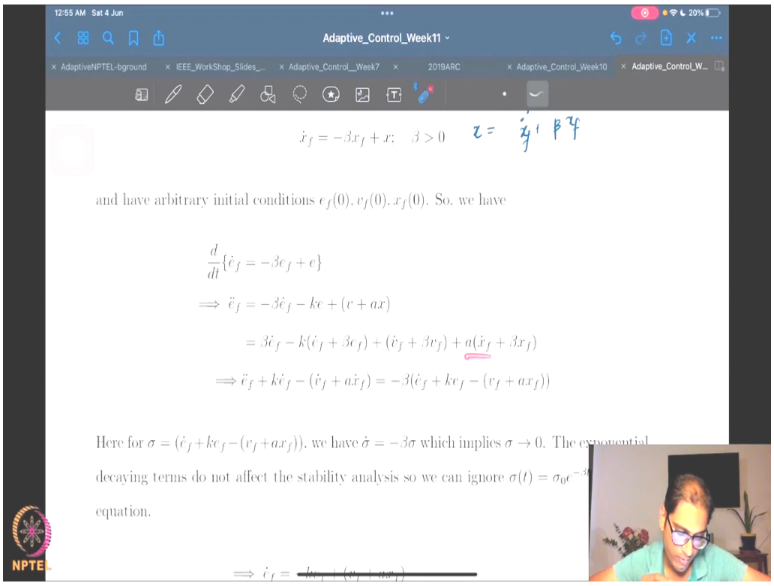
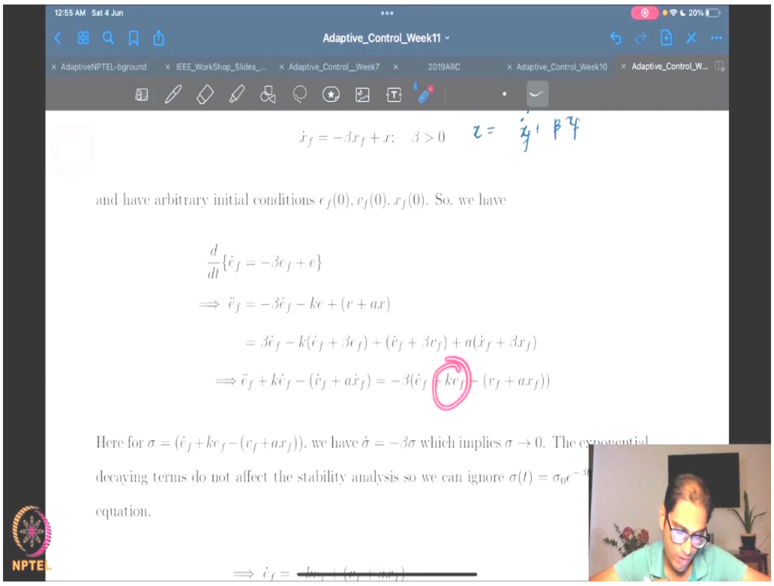
NPTEL





So, all the terms on the right hand side are scaled by beta which is e f dot from here, e f k from here v f from here and a x f from here. Just checking if the signs are correct, minus beta e f, I am just checking the signs are correct. I am starting I am not sure let us see. We have e double dot plus k e f dot plus v f dot plus a x f dot and on the right hand side I have beta k e f, sure, minus, minus beta v f correct plus a x f.

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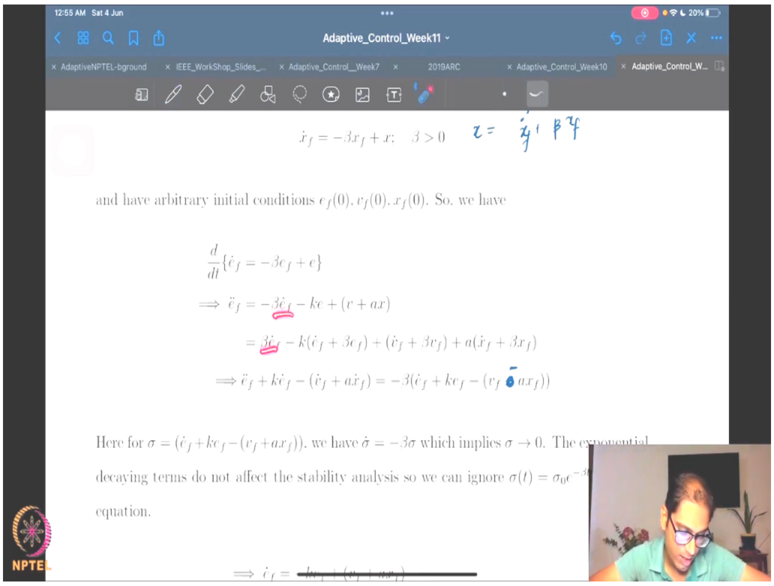
Adaptive_Control_Week11
$\dot{x}_f = -\beta x_f + x; \quad \beta > 0$
$z = \dot{y} + \beta y$
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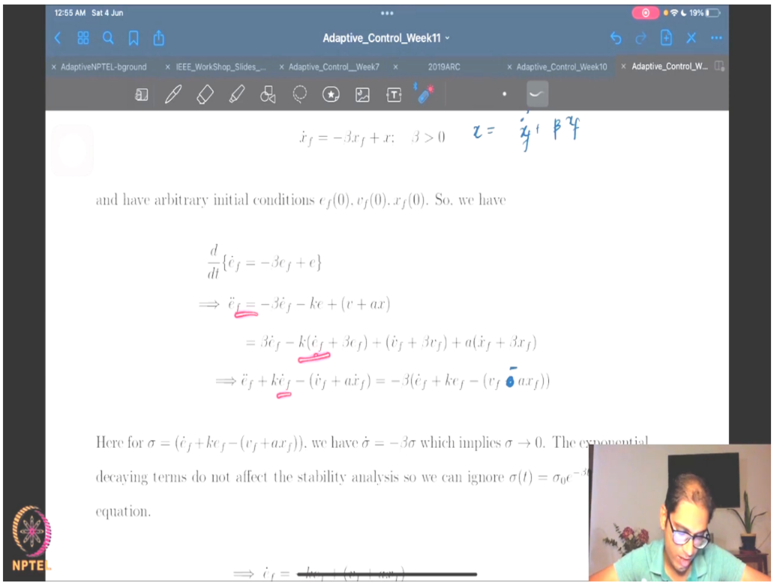
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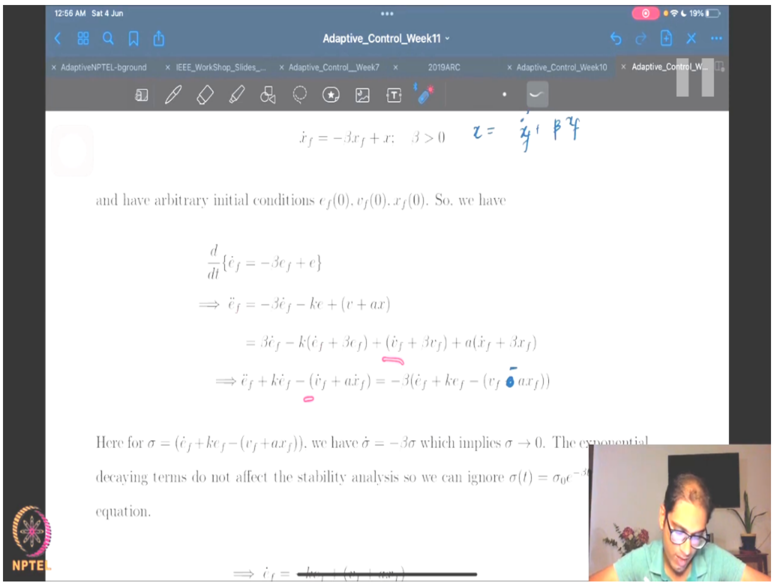
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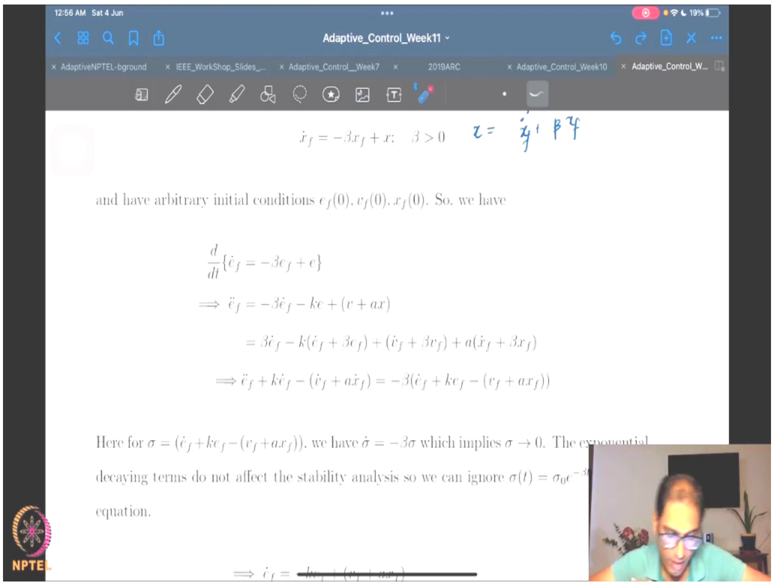

So, this is not correct, does not seem correct. And this sign does not seem correct. Again this sign does not seem correct I need to verify this. So, beta e f dot is same minus k e f dot plus beta e f plus v of dot plus beta v f plus a x f plus beta a x f all this is good. Then what goes to the left hand side e f double dot plus k e f dot minus v f dot minus a x f dot. Then what do I have is minus beta e f dot plus k e f minus v f minus a x f. So, this sorry. So, this was correct. This was correct. So, this is fine.

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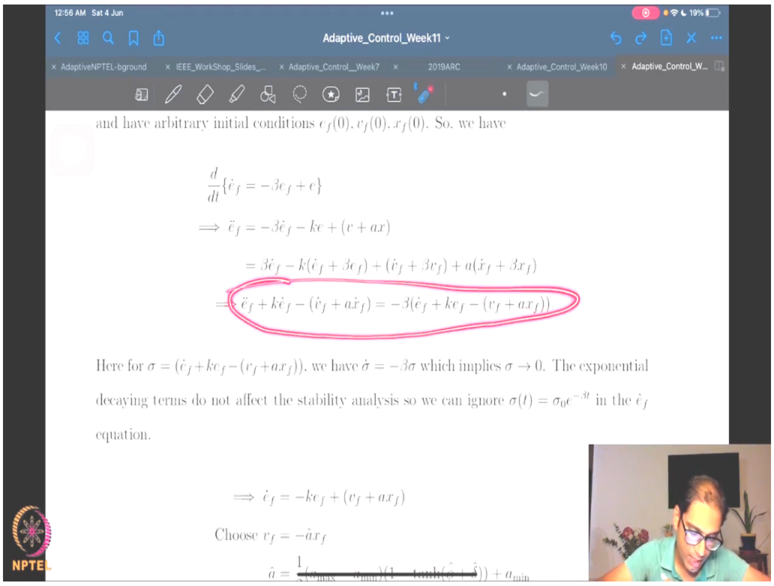

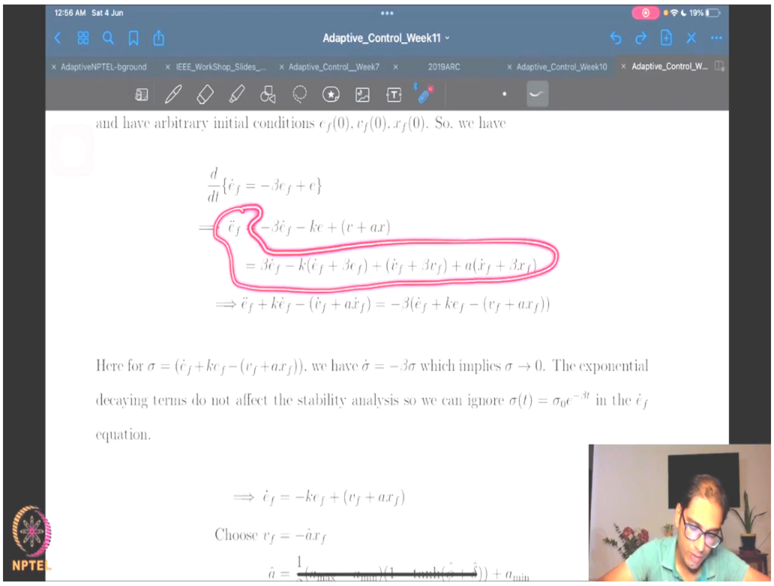
12:56 AM Sat 4 Jun
Adaptive_Control_Week11
AdaptiveNPTEL-bground
IEEE_WorkShop_Slides_...
Adaptive_Control_Week7
2019ARC
Adaptive_Control_Week10
Adaptive_Control_W...
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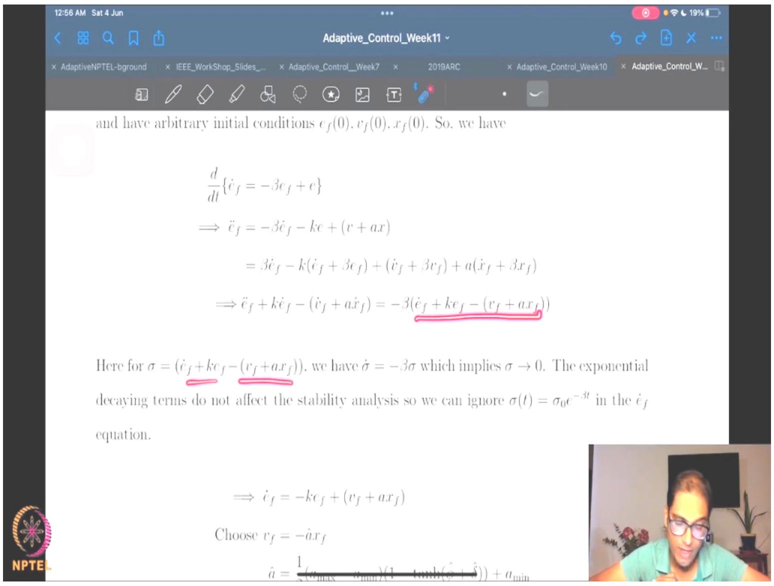
Choose $v_f = -\hat{a}x_f$
NPTEL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Adaptive_Control_Week11
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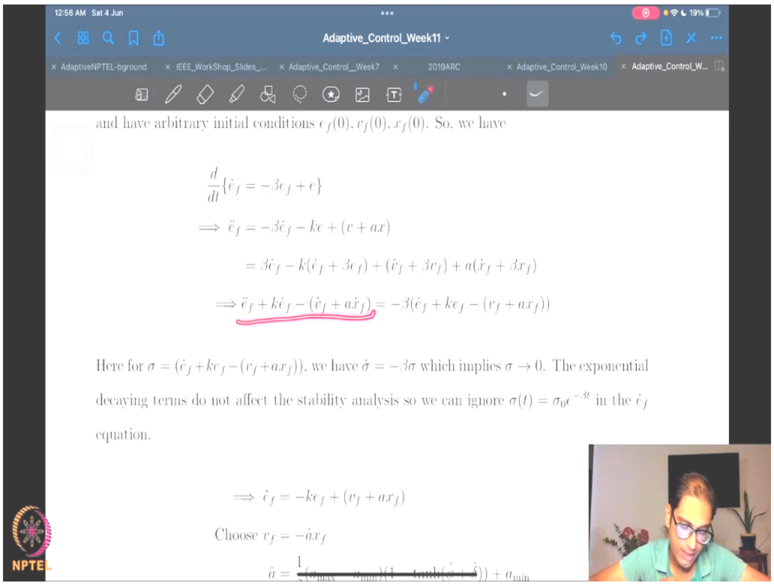
12:56 AM Sat 4 Jun
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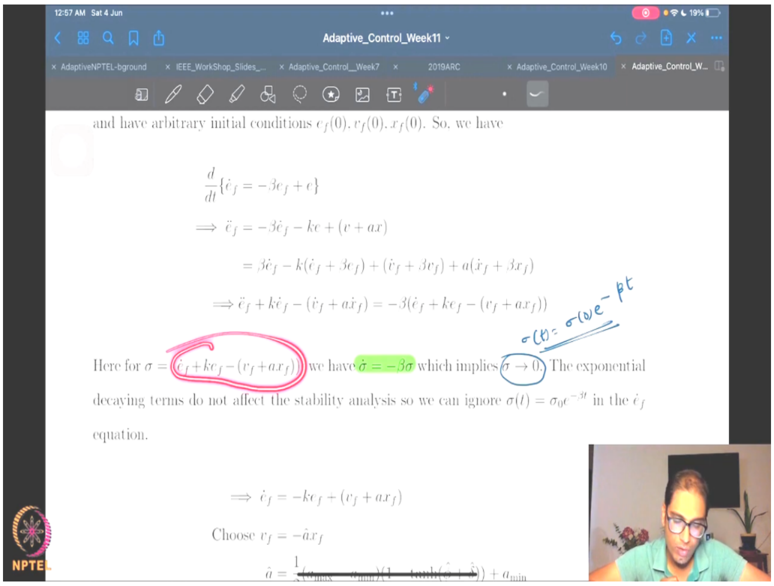

12:57 AM Sat 4 Jun
Adaptive_Control_Week11
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IEEE_WorkShop_Slides_...
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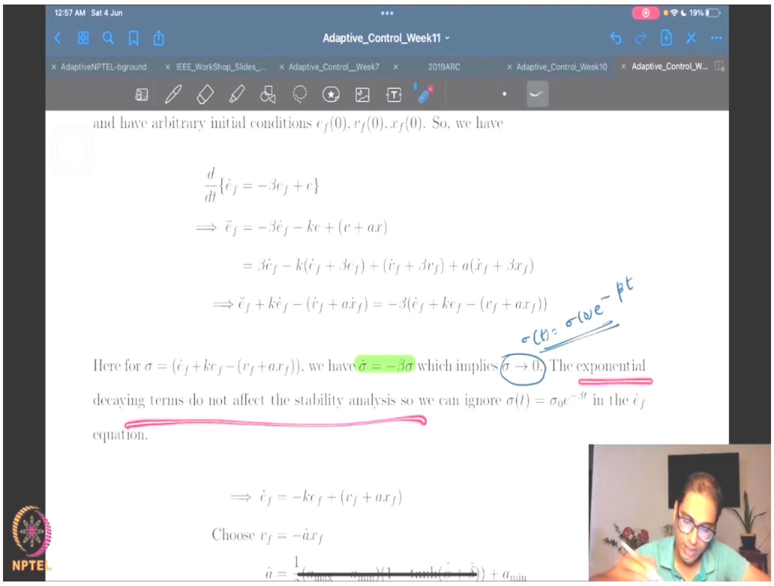
NPTEL



12:57 AM Sat 4 Jun
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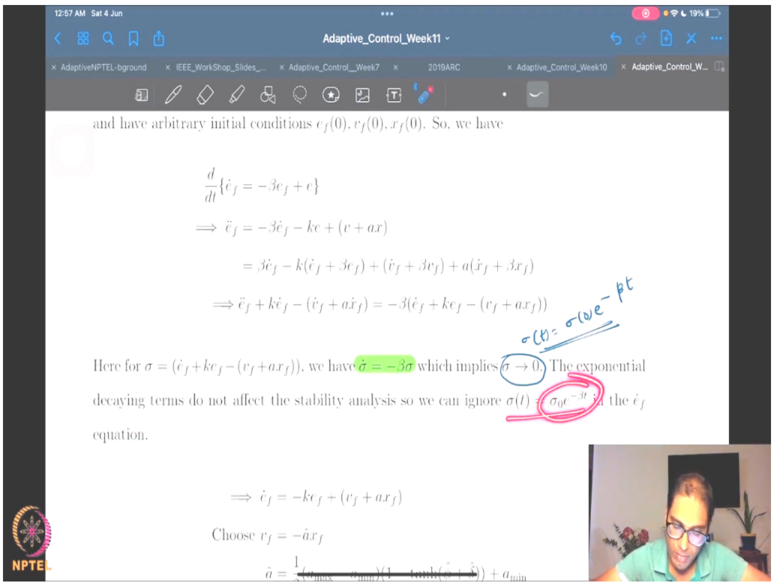
NPTEL



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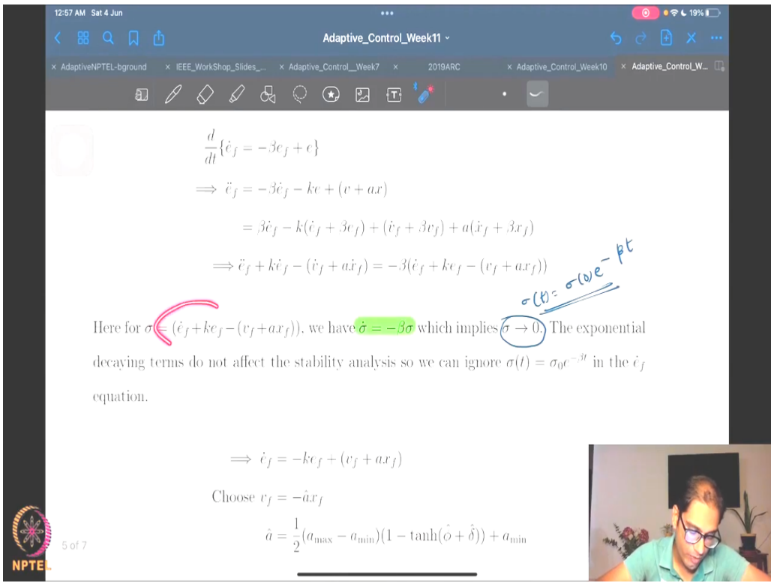

So, now, what you notice is something rather interesting. So, this expression is correct, absolutely right. All I have done is in this guy; right here I have taken the terms with the beta on the right hand side and the term without the beta scaling on the left hand side that is all. Now, if you look at this very carefully and I choose sigma to be whatever is in the bracket here that is e f dot plus k e f minus v f plus x a x f, if I do that, I notice that the left hand side is just sigma dot. So, the equation that we get is that sigma dot is minus beta sigma. And what do we know from here?

It means that sigma is exponentially decaying which implies not just sigma goes to 0; it implies that sigma t equals sigma 0 e to the power minus beta t. So, this is rather cool. Essentially, means that this quantity sigma is going to 0 as t goes to infinity. Now, it is very well known in control that the exponentially decaying terms do not affect the stability. So, we can ignore the sigma equal to sigma 0 e to the power minus beta t type of term in this equation.

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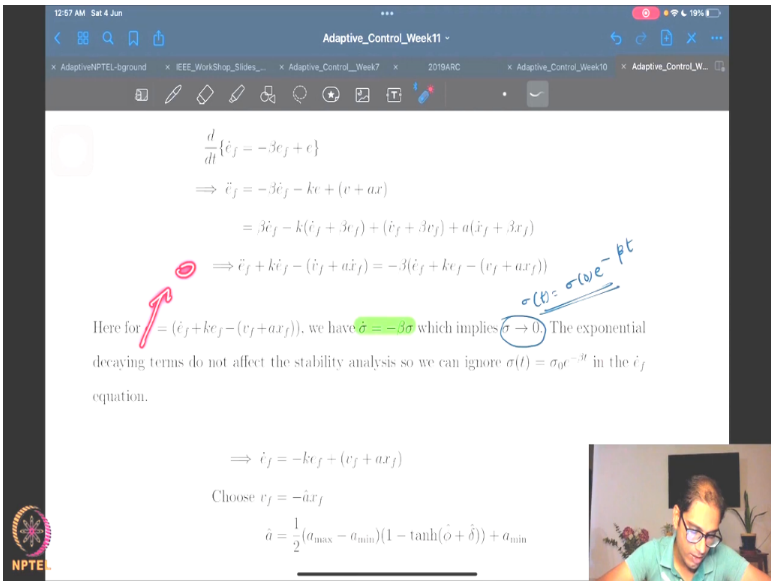
Adaptive_Control_Week11
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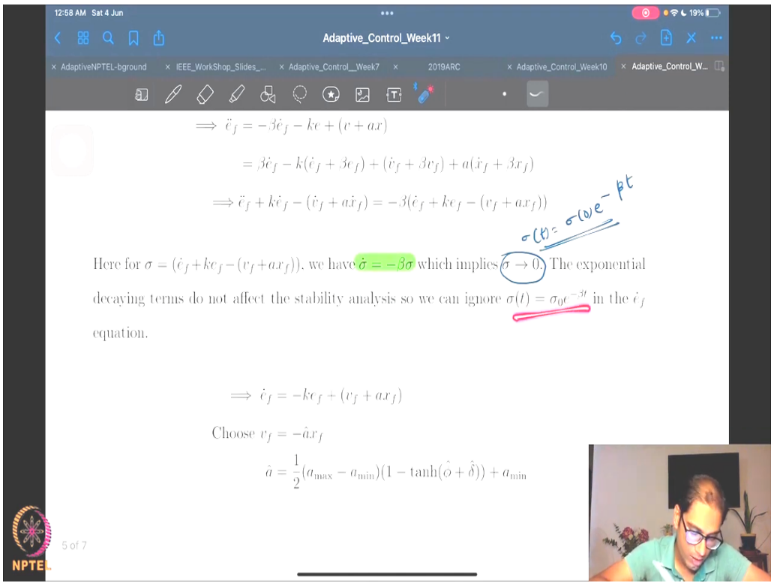
Choose $v_f = -\hat{a}x_f$
$\hat{a} = \frac{1}{2}(a_{\max} - a_{\min})(1 - \tanh(\hat{\sigma} + \hat{\delta})) + a_{\min}$
NPTEL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Choose $v_f = -\hat{a}x_f$
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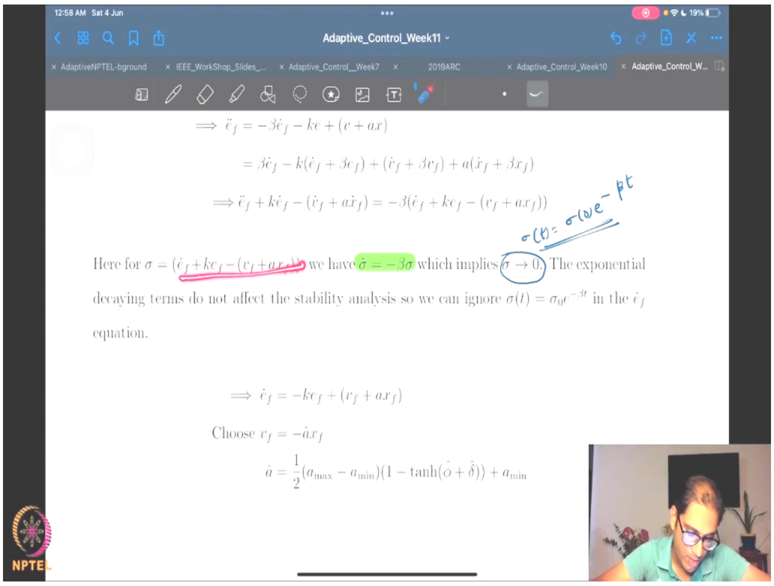
5 of 7
NPTEL


Adaptive_Control_Week11
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NPTEL

So, what we know is that we can just assign this to 0. We can simply assign this to 0 because this is simply an exponentially decaying term that can be ignored. I mean I can very be very careful and write this as sigma 0 e minus beta t. But that is not required. It does not affect our stability analysis at all and that is the key point. So, we ignore this term. So, which means we put this to 0. And if you do put this to 0, you will notice that you will get the equation from here as e f dot is minus k e f plus v f plus a x f.

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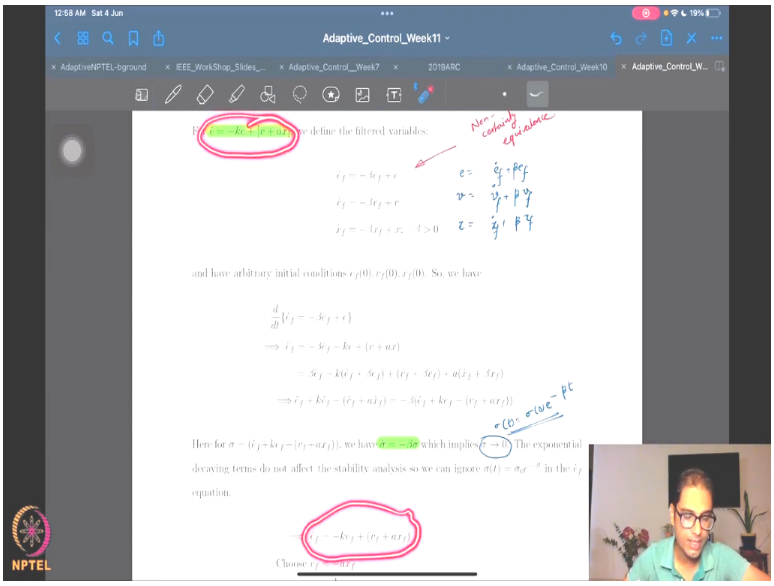

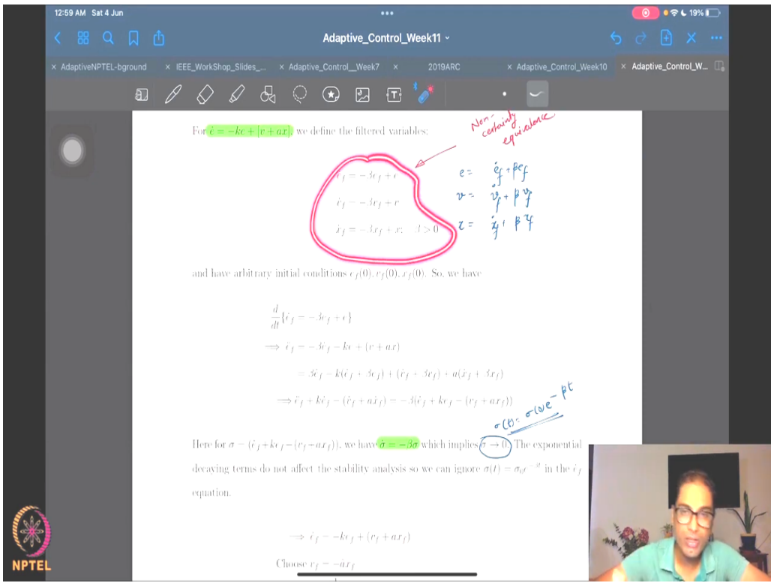

Now, notice very carefully. I am going to make this smaller so we can see both of them together. If you look at this equation, I apologize; look at this equation and this equation, simultaneously, we notice that they look exactly the same modulo the filtered variables in place of the original variables. So, this is very very critical.

So, this original dynamics and the filtered dynamics are precisely the same, except for the filtered variables appearing in place of the original variables. And this is what is the magic of this kind of a filter construction. So, this sort of a nice neat outcome happened because of the filter construction. And you want these to have the same structure. We want this to have the same structure. And so, what we do is now in this filtered state, we prescribe our control

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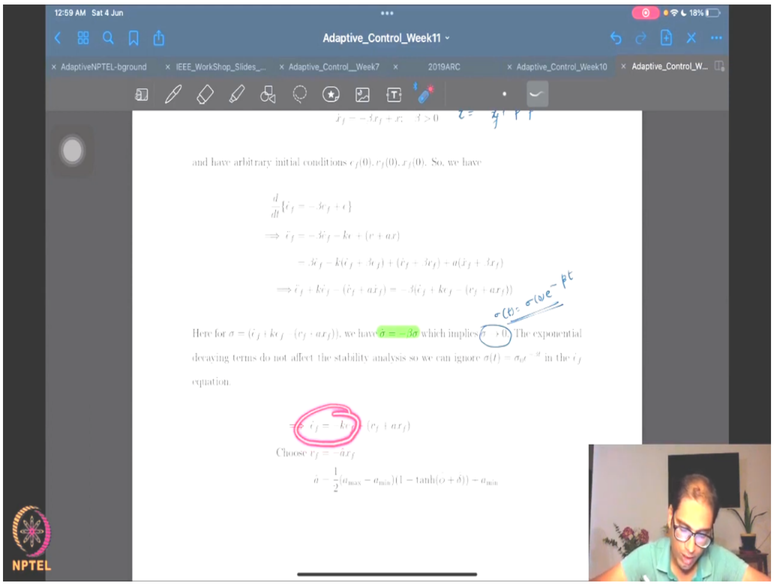

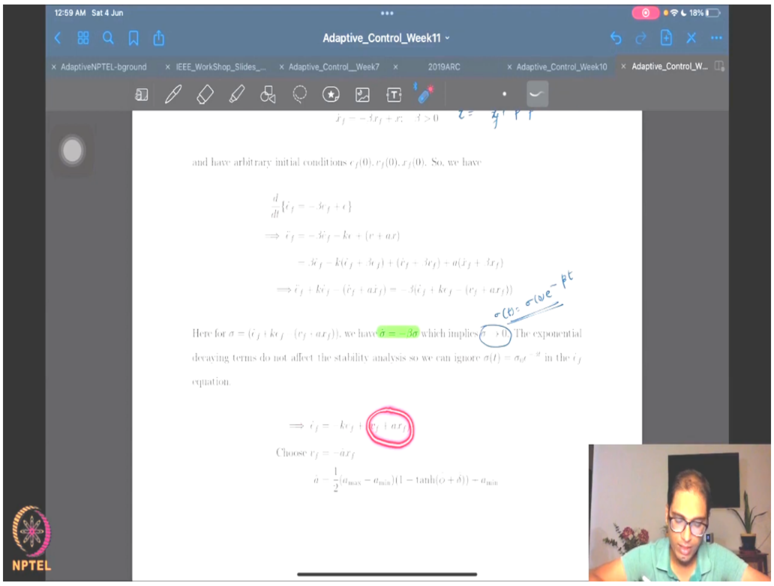

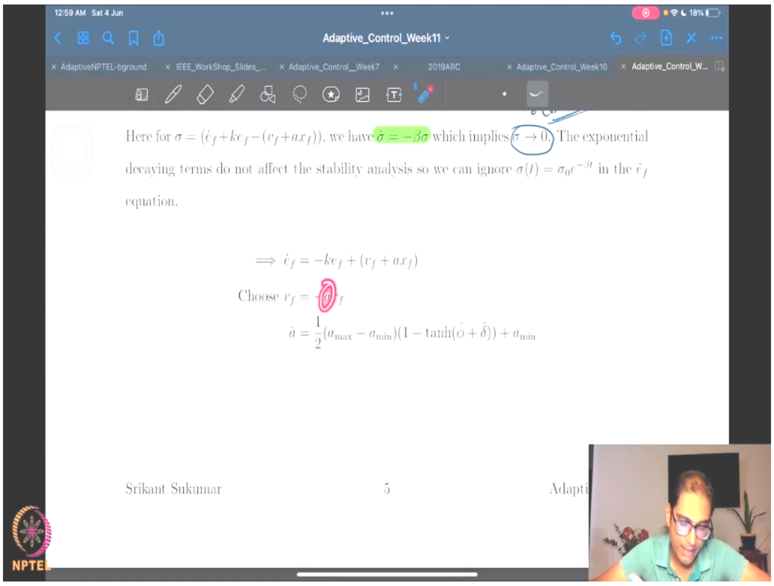
Adaptive_Control_Week11
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Srikant Sukumar
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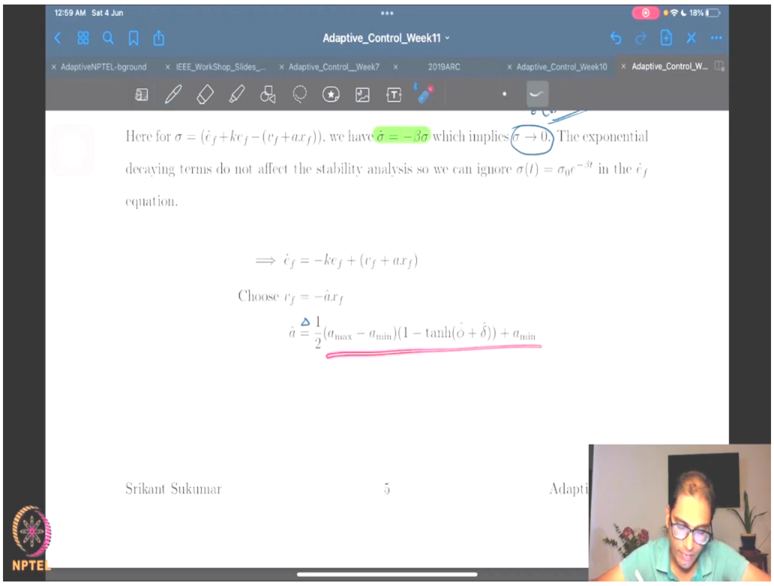
Adapti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Choose $r_f = -\hat{a} x_f$
$\hat{a} \triangleq \frac{1}{2}(a_{\max} - a_{\min})(1 - \tanh(\hat{\phi} + \hat{\delta})) + a_{\min}$
Srikant Sukumar
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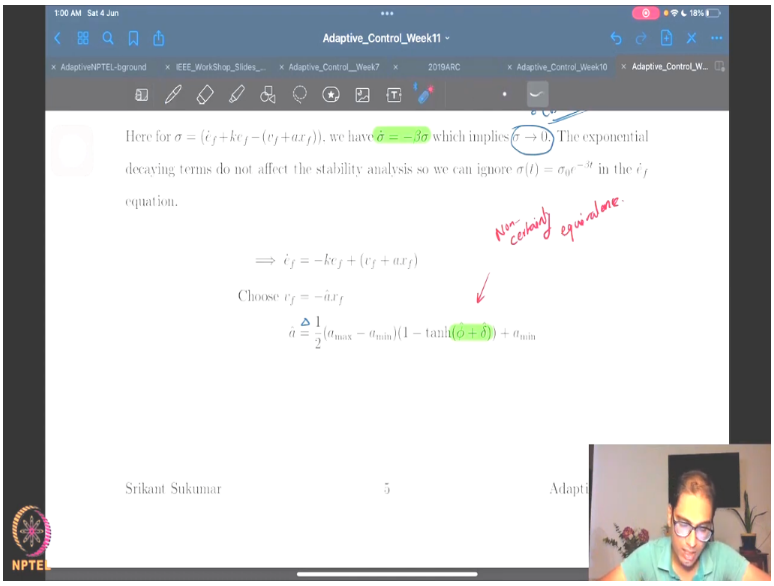
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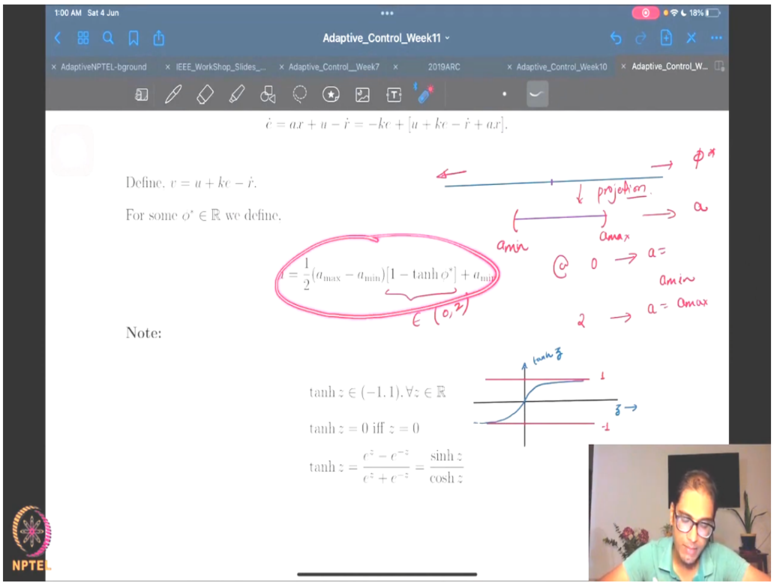

Notice that this is already a stable system. So, all I want to do is sort of get rid of this. But the problem is I do not know a. So, I cannot really implement a. So, I implement an a hat. Now, a hat is just a placeholder which means it is just a symbol to define this guy. Now, this is where your non-certainty equivalence becomes very apparent.

So, I am going to mark this as non-certainty equivalence. So, you see that the expression is very identical to what we had for the original system, what for the actual a. If you see the actual a expression is something like this a max minus a min by 2 1 minus tan hyperbolic phi star plus a, that is the expression plus a min.

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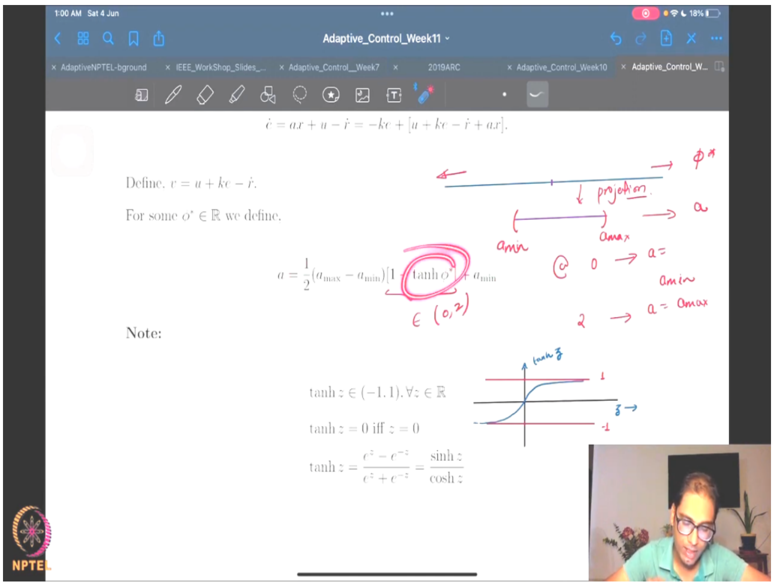

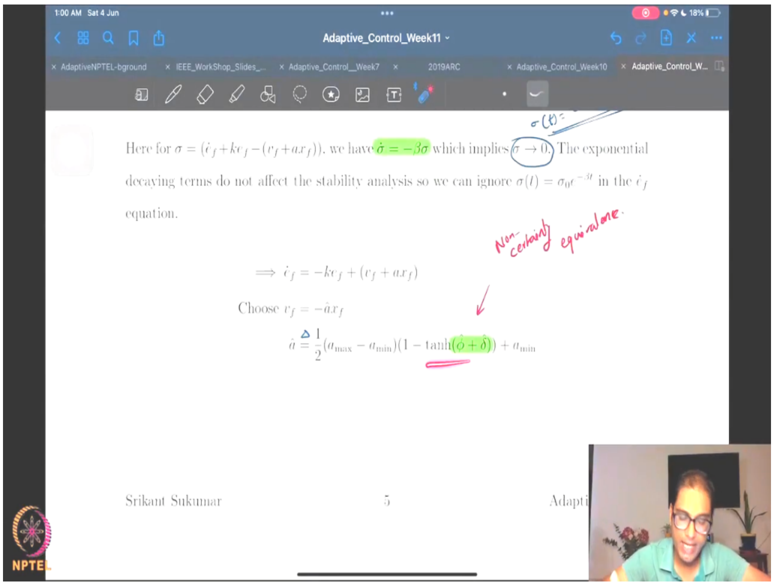

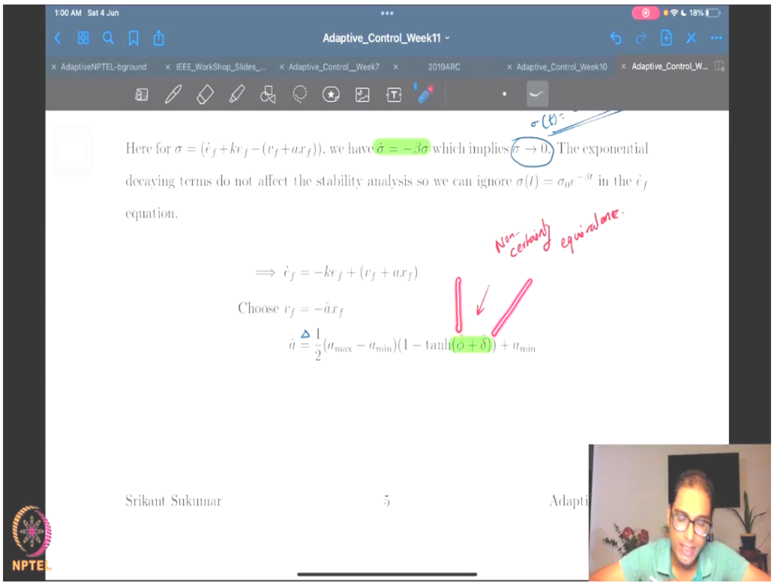

So, in place of this thing, we have tan hyperbolic phi hat plus delta hat. So, in general, you expect that phi will get, phi star will get replaced just by a single phi hat but it does not in this case. It is replaced by phi hat and also by a delta hat. So, two terms, not two estimates or anything but just two different terms. And this is what makes it non-certainty equivalence. So, it is not exactly following certainty equivalence principle.

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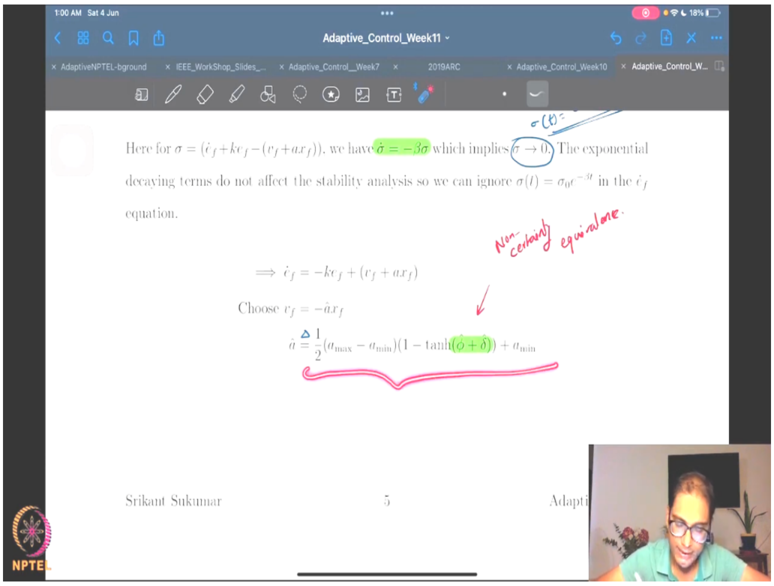
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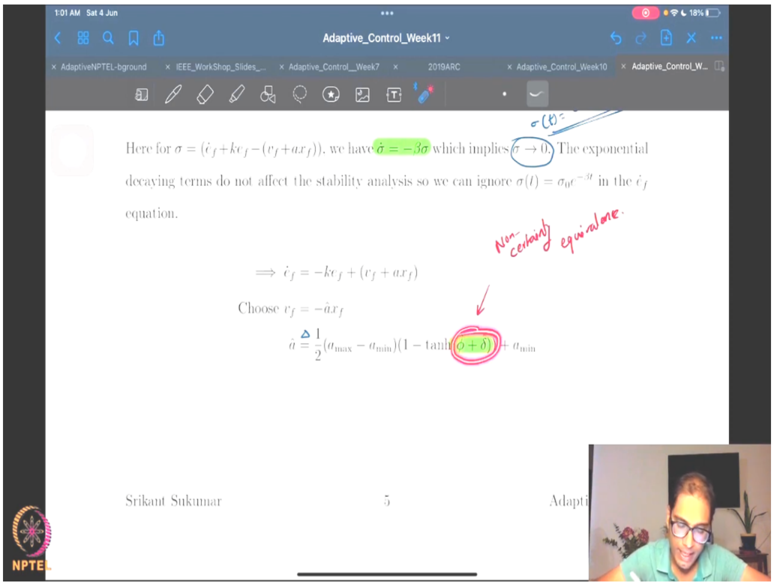
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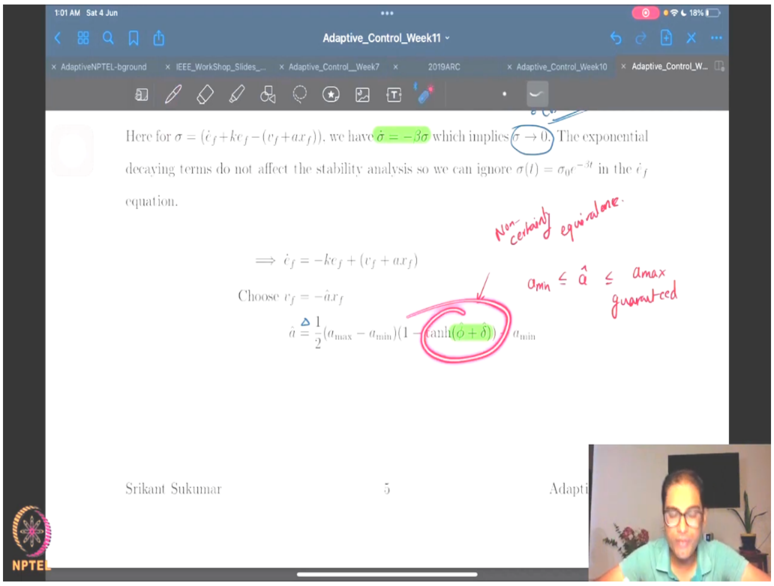
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$a_{min} \le \hat{a} \le a_{max}$ guaranteed
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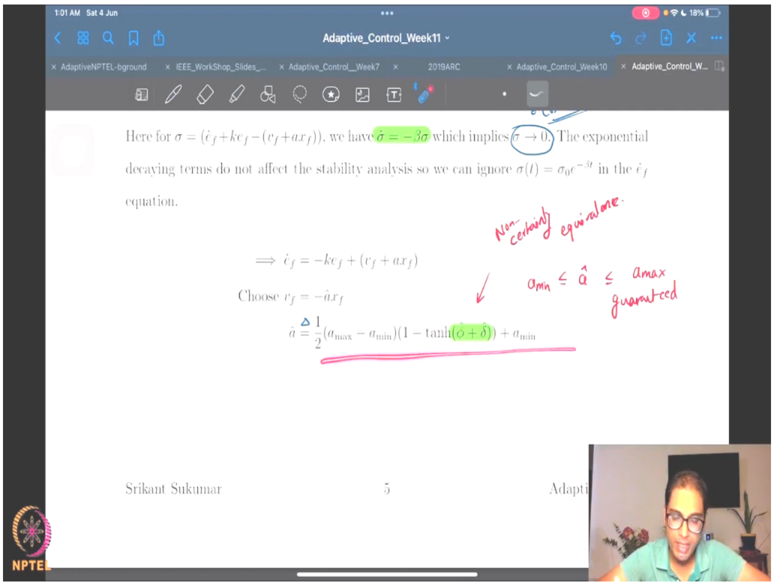
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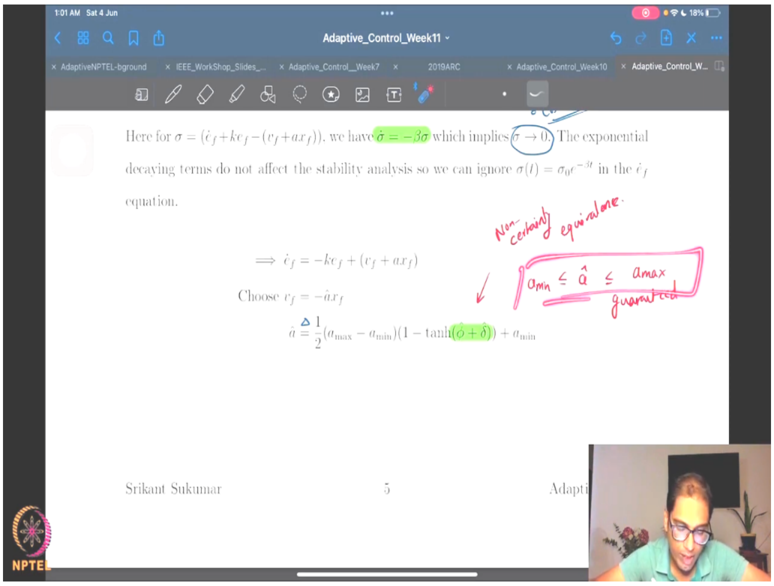
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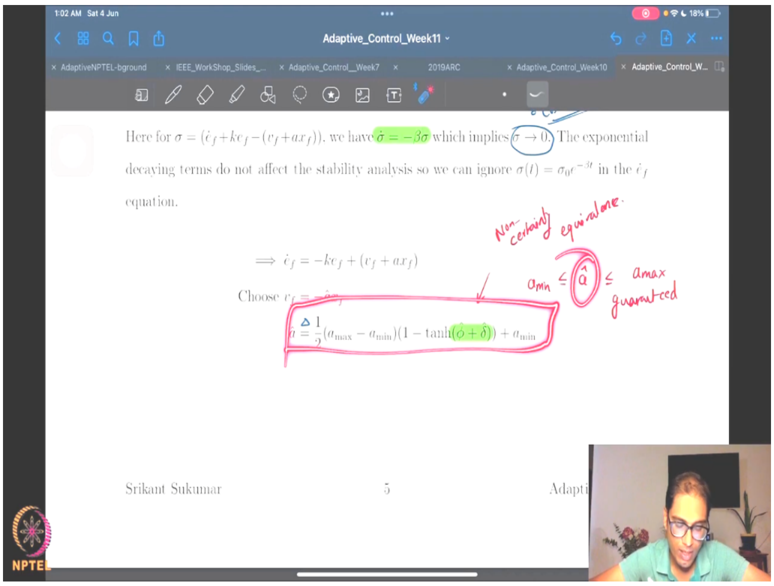
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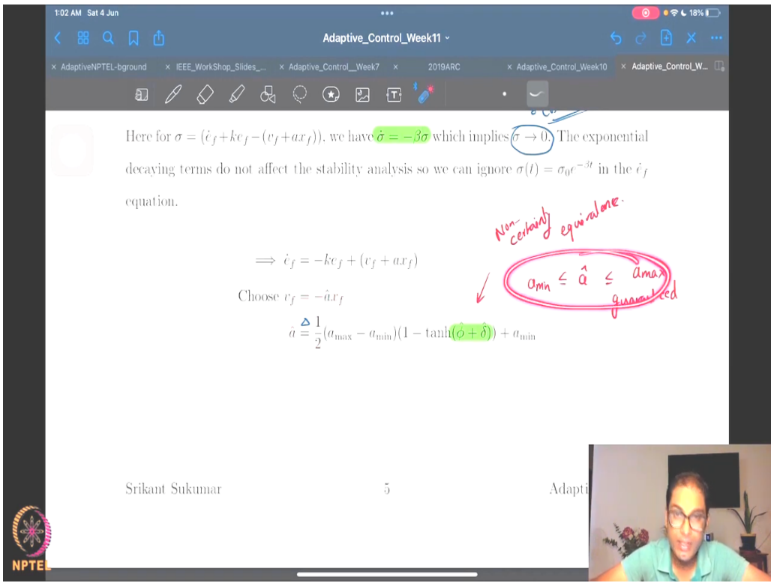

But the idea is the overall expression is exactly the same. Whatever the phi hat plus delta hat this a hat is going to lie between. So, a hat is definitely going to lie between a min and a max, this is guaranteed by virtue of the property of the tan hyperbolic that it lies between minus 1 to 1 irrespective of what the argument is. So, you are bound to lie between a min and a max and this is what we want. This is essentially projection.

Although we are trying, we are going to try to update phi hat that is whatever is inside between in all of real numbers, there is no bound on that. But what we do, can do is we create a bound or what we want to do is get a bound on a hat and we do have it. Whatever be phi hat and delta hat which you specify subsequently, your estimate a hat the given by this formula and which is what actually shows up in the control is bounded.

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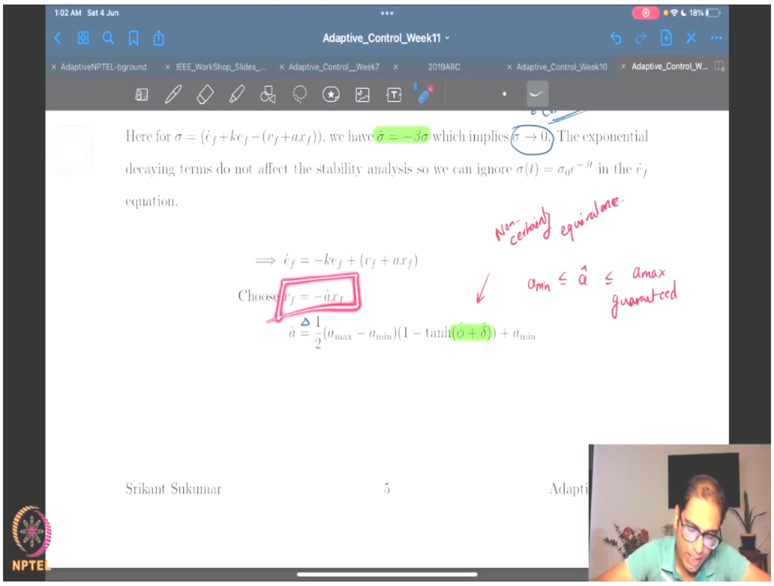

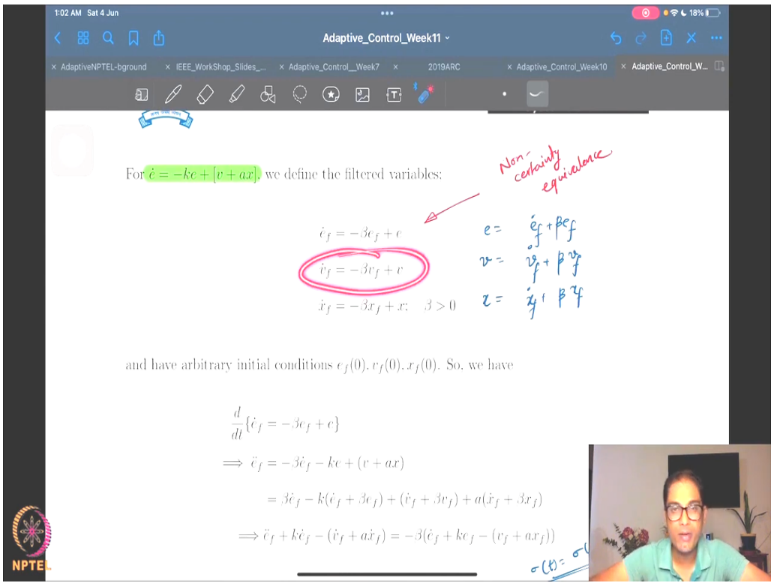

So, it does not happen, it does not matter what happens to v, v dot, negative definite, positive definite and all that. As soon as I have this kind of an expression, I am guaranteed to have v f to be bounded. And if v f is bounded, it is not difficult to see that v will also be bounded because v f is just, v is just v f dot plus beta v f. So, everything is going to be nice and bounded, just like you want it. You will not get unbounded controls and so on and so forth.

So, what did we talk about today? We sort of started talking about the parameter projection method for adaptive control. So, what it entails is first of all of course knowledge of a bound and secondly a we try to do what is called smooth projection and this smooth projection is really implemented using tan hyperbolic functions which are essentially saturation functions of a kind and lie between minus 1 to 1 for all possible value of the arguments.

Therefore we redefine our parameter in terms of tan hyperbolic of argument of estimate phi. And this phi is what we try to estimate later on. So, we actually propose an update of a phi hat. And because of this what happens is that your actual estimate a hat that enters the control law remains bounded due to this projection and therefore robustness issue is completely resolved.

Because this is what happens when you introduce a disturbance that the parameters start to fluctuate or can start to grow really large and unbounded. But in this case it is simply impossible because we are guaranteeing by construction that a hat remains bound. So, all of these, of course implemented using filtered variables and origin of non-certainty equivalence.

And so in the subsequent sessions, we will see how this non-certainty equivalence method gets implemented and how you do then, how do you do the Lyapunov analysis for the same. So, I hope you enjoyed this session and I definitely hope to see you all in the next one. Thank you.