Nonlinear Adaptive Control
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Systems and Control
Indian Institute of Technology, Bombay
Week 10
Lecture No: 56
Robustness in Adaptive Control (Part 2)

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Hello! Welcome to yet another session of our NPTEL on Nonlinear and Adaptive Control. I am Srikant Sukumar from Systems and Control, IIT Bombay. So we have started our week number 10 in a very interesting fashion talking about robustness in adaptive control. Robustness is a sort of rather critical issue for any nonlinear control, especially for systems such as what you see in a background, just like a spacecraft orbiting around the Earth.

Any small error or instability in your control can lead to complete deorbiting of the satellite and you might lose it forever. So robustness is a very, very key to real performance, in fact, in reality almost nobody is, no applied engineer would be concerned about asymptotic convergence and things like that. They really look for bounded performance around an equilibrium.

And that is essentially what most of the systems, such as what you see in this background also do, because it is impossible in a real system with un-modelled dynamics, with disturbances and such, an actuator, inexact actuators and so on and so forth, in order to achieve exact asymptotic convergence. So such an idealized scenario is not possible.

Therefore, robustness is critical. And this is the discussion that we are having on the nature of robustness that you have in non-linear control.

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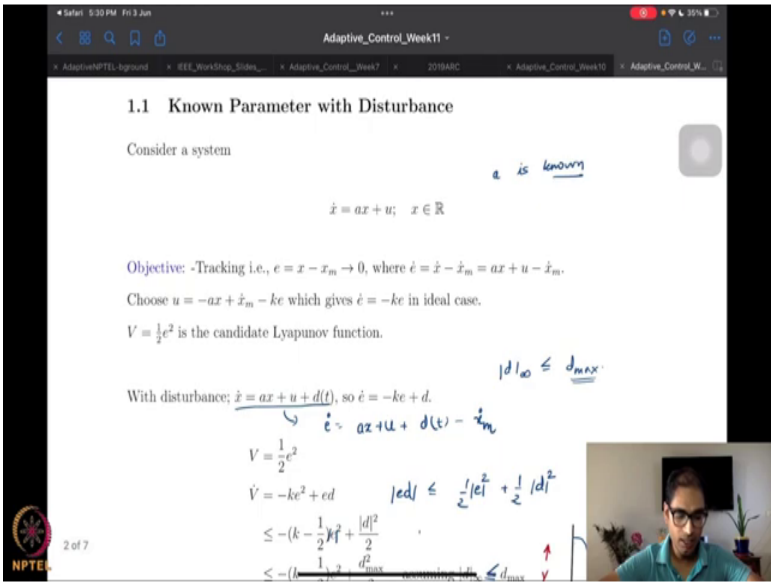

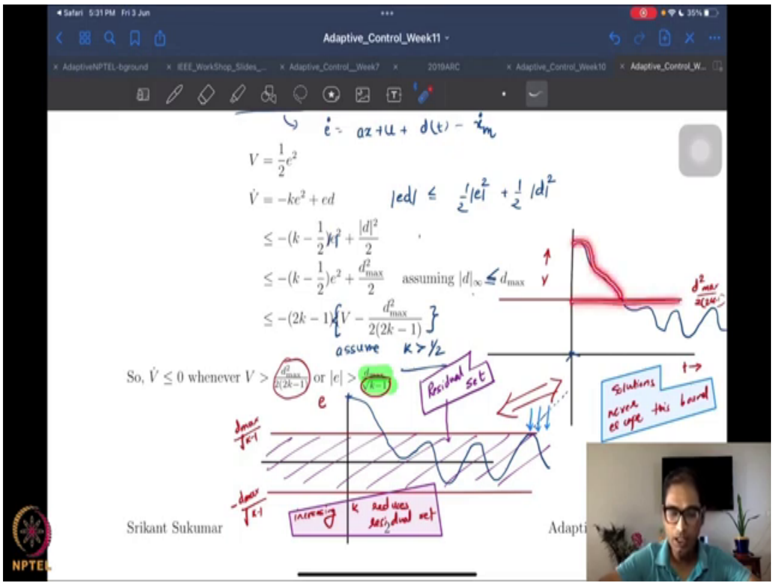

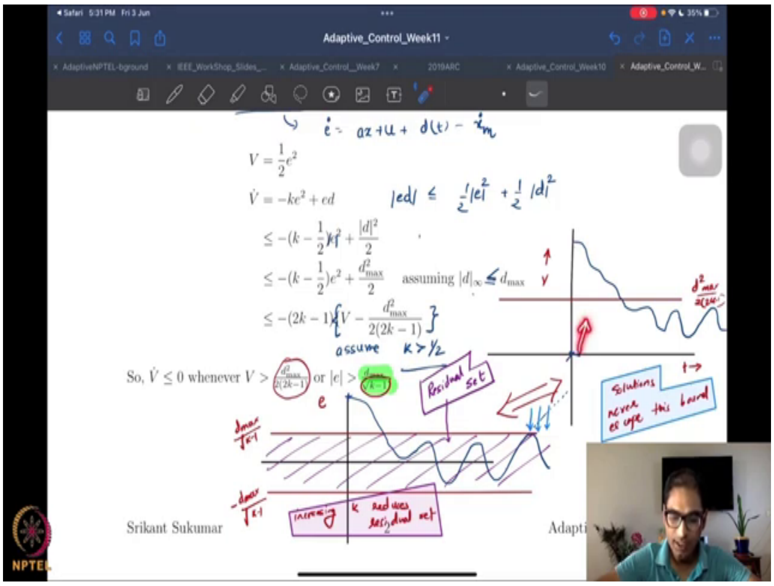

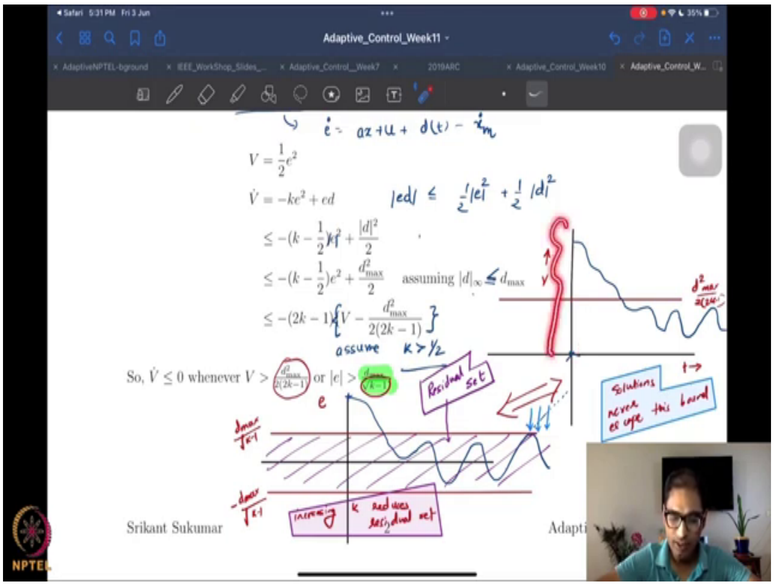

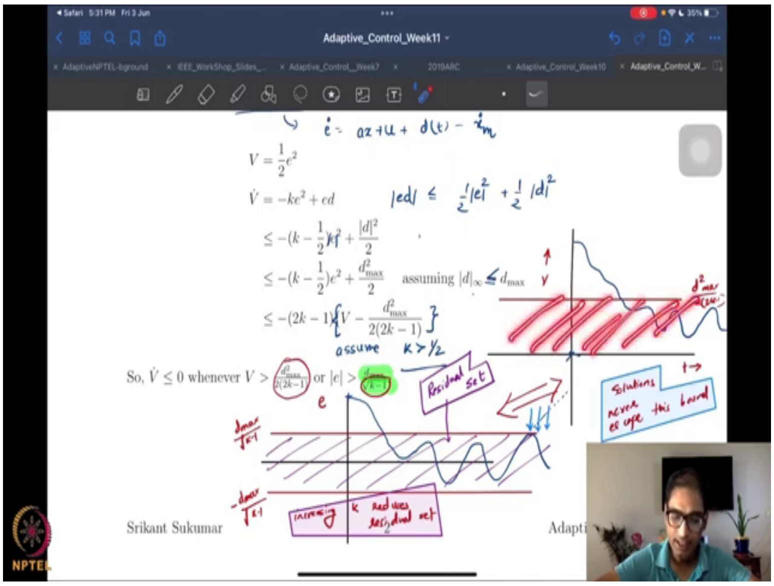

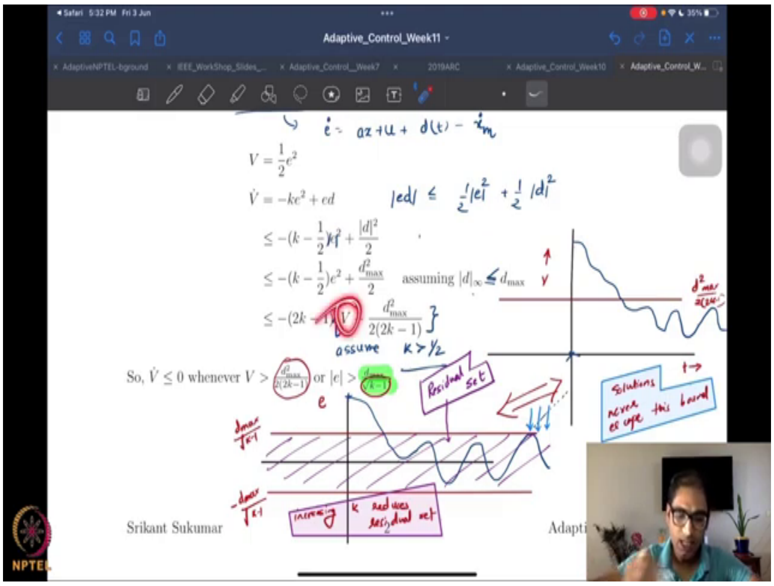
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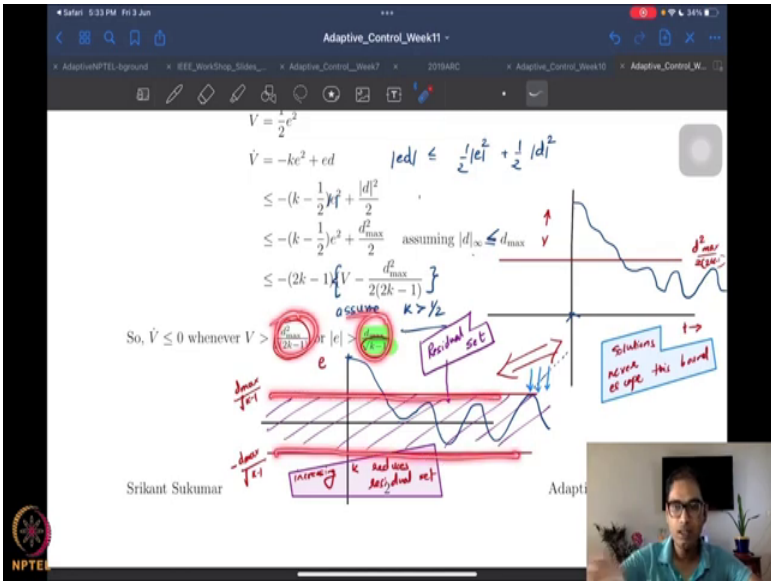
So, $\dot{V} \leq 0$ whenever $V > \frac{d_{max}^2}{2(2k-1)}$ or $|e| > \frac{d_{max}}{\sqrt{k-1}}$
Srikant Sukumar
Residual set
increasing k reduces residual set
Solutions never escape this band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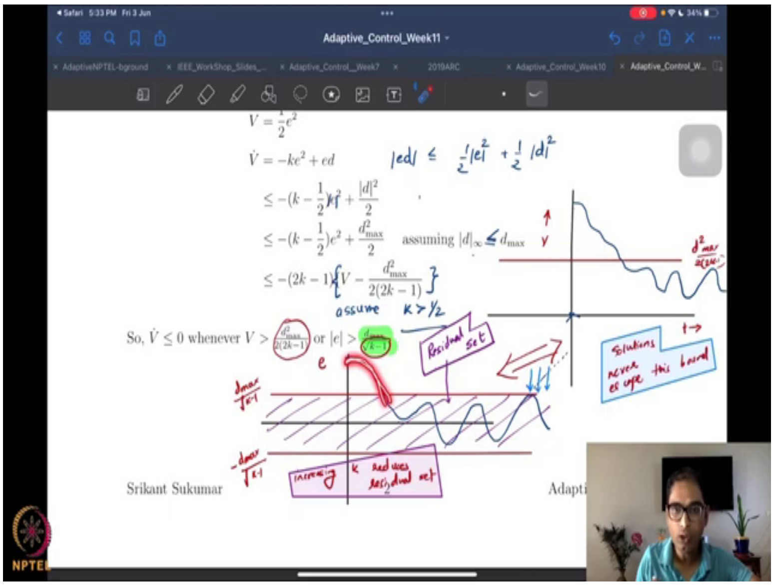
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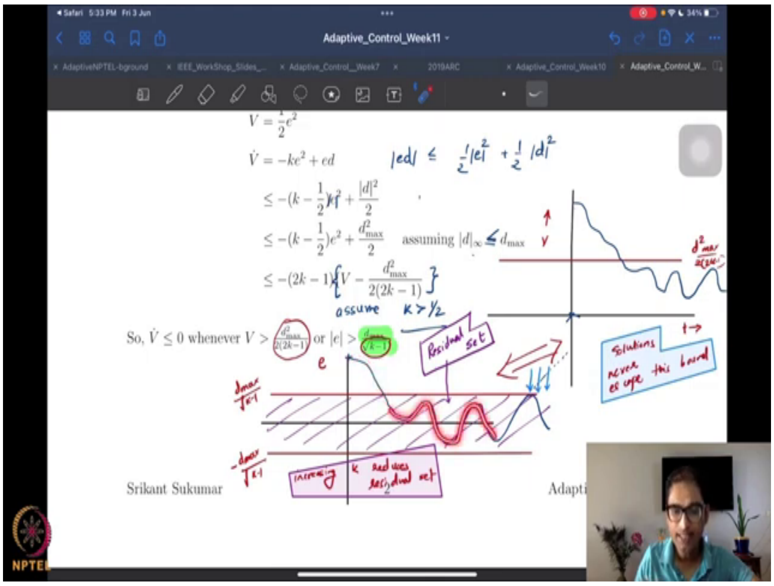
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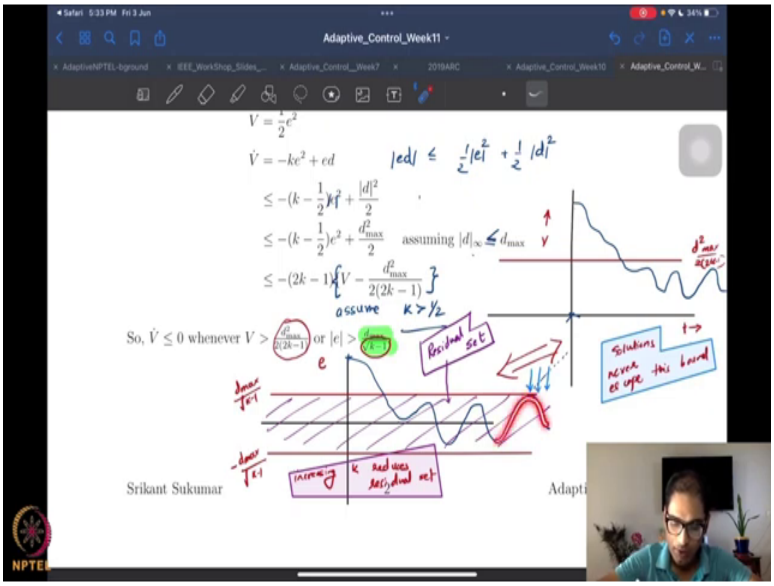
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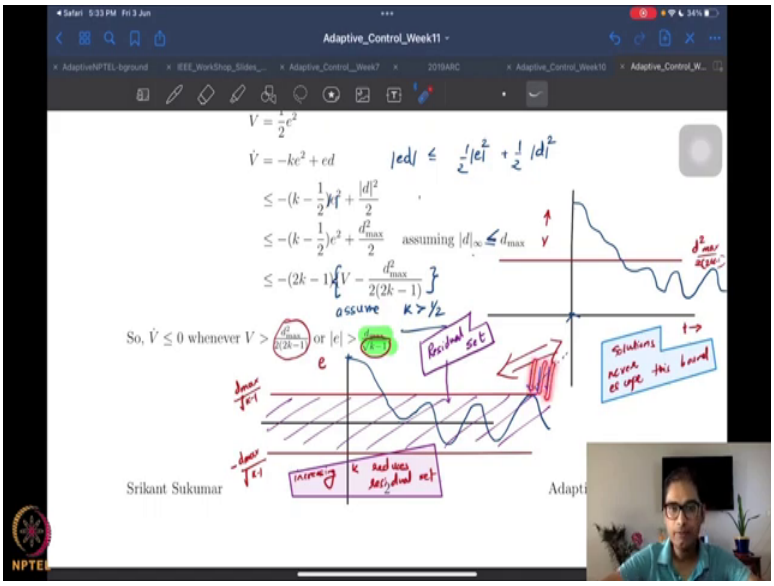
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So what we have seen until now is that if you have a very simple system in this case, but in general if you have a strict Lyapunov function for a system, that is a system with V for which V dot turns out to be negative definite in the absence of any kind of a disturbance, then it can, we can actually show that in the presence of disturbance also we have rather nice performance. And what is this nice performance?

How do we specifically quantify this nice performance? The first thing we can actually see is that your V function, whenever it is outside a particular set, remember that V is always greater than 0, so the range of V is just this, so anything below this is irrelevant. So whenever it is inside this set, sorry, whenever it is outside this set, the function V is decreasing and as soon as it gets into this set anything can happen, does not matter.

So that is the idea here. So again in this case it can increase or decrease, sorry, in the scalar case it has to decrease here and so on, very monotonically, but this is not generally the case for vector case, for vector situations or vector states, and so on and so forth, therefore, I have shown you more general plot.

So once it is inside, of course, it can oscillate and things like that. Here again, I mean, you have to sort of understand that, I mean, anyway the idea is a little bit of oscillation is anyway possible, because once you are inside you can sort of overshoot here and then you can increase, go to the boundary, overshoot, and things like that, all of that is possible. So there is an oscillation possible inside this set, but outside this set it is monotonically decreasing. And that is really the idea here.

And the same sort of feature of the Lyapunov function is mimicked in the performance of the error also, the only difference being that V is related to the error with the square, so e squared by 2, forget, I mean, the scaling divided by 2 does not change too much of the property of the plot but because V is e squared, therefore e can go positive and negative, therefore the sign is not captured in this plot and that is what gets captured here.

So the bound is just a square root of the V bound but it is on both sides of the 0, it is on both sides of the 0. And if the e starts outside it again monotonically decreases, because V decreases e decreases, identical, and it gets into this set, and then it is allowed to oscillate and it can oscillate. Now the important thing to remember is that the solutions cannot escape this bound, because if you get to the boundary and just instantaneously or immediately outside this boundary you are supposed to move inside.

Therefore, you cannot escape this boundary. This can again, like I mentioned last time this can be analytically proven. So it will always remain inside this thing. So this is called a residual set. The other important thing to remember is that the size of this residual set is governed by the control gain. So it is inversely proportional to the control gain in general. Not just in this problem, but in general it is inversely proportional to the control gain.

And therefore by increasing the control gain you can reduce the size of the residual set. Although it is well known in control design also, it is not advisable to arbitrarily increase the control gain, but up to a reasonable limit because increasing the control gain can also increase the frequency of the control.

So maybe the control may no longer be applicable and all that jazz, but within constraints of what your system can do you can still push up the control gain to get smaller and smaller and smaller residual sets, so that you can get the performance of your real system, for example, the satellite, to within the necessary performance bound that you want.

So this is one of the big, big powers of the Lyapunov method that as soon as you made a control design using a strict Lyapunov function disturbance robustness came for free. You do not have to do this analysis at all, and it is for free because you have had a strict Lyapunov. Now remember the problem that we have. In adaptive control we almost never have a strict Lyapunov function.

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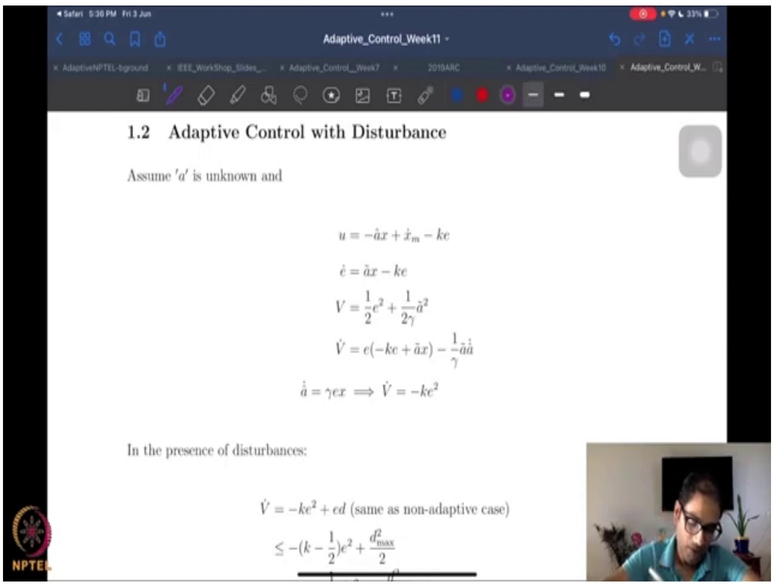

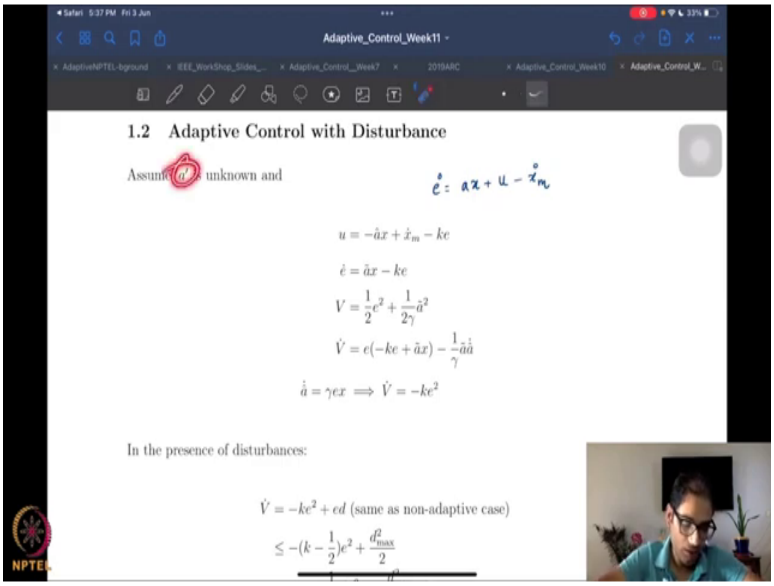
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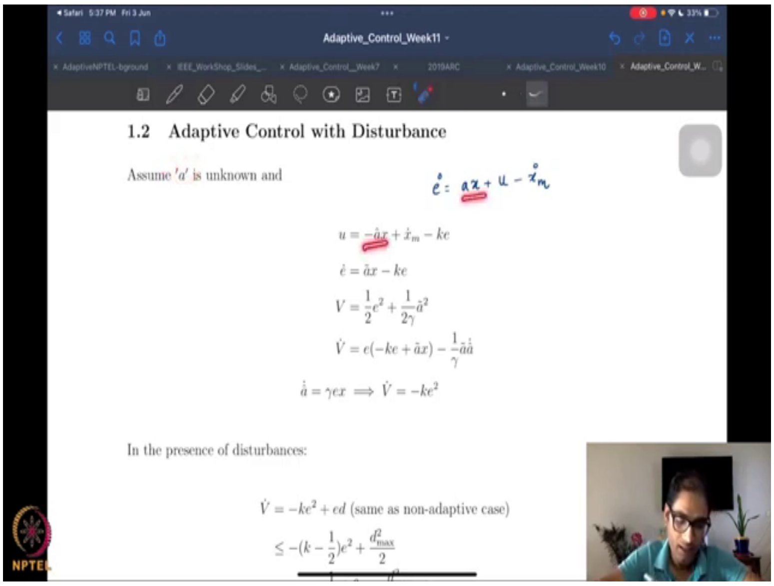
NPTEL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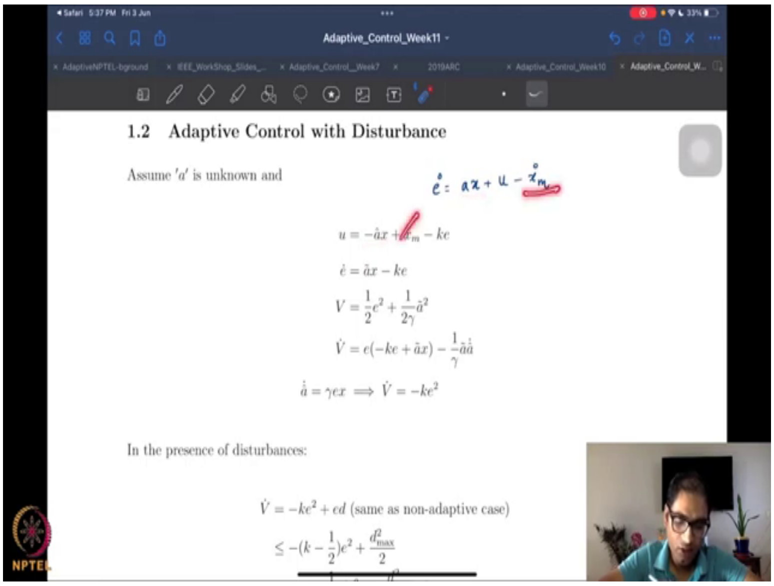
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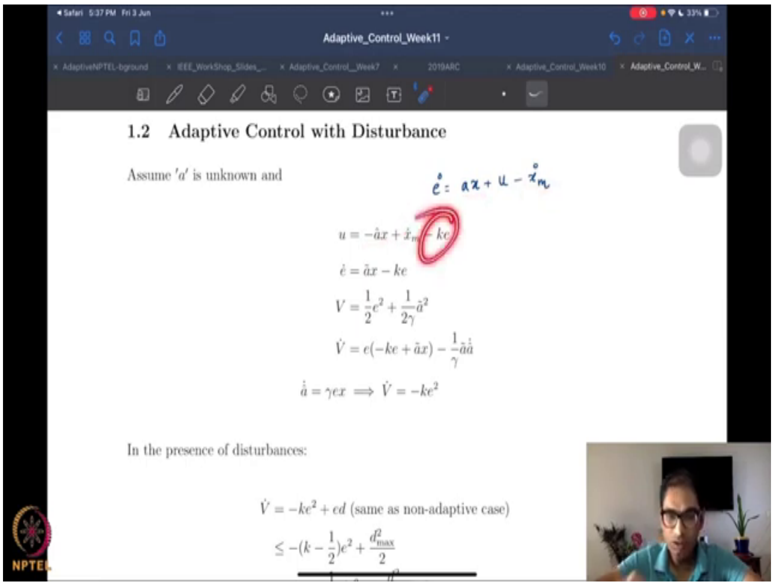
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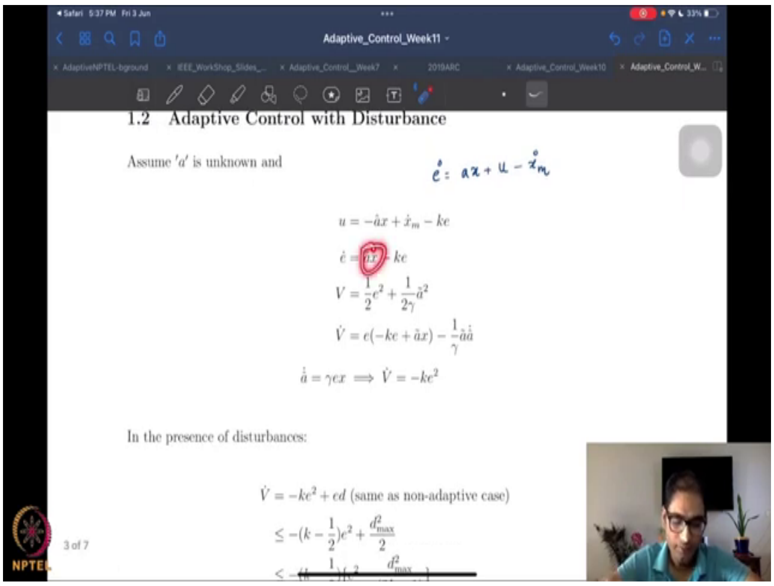
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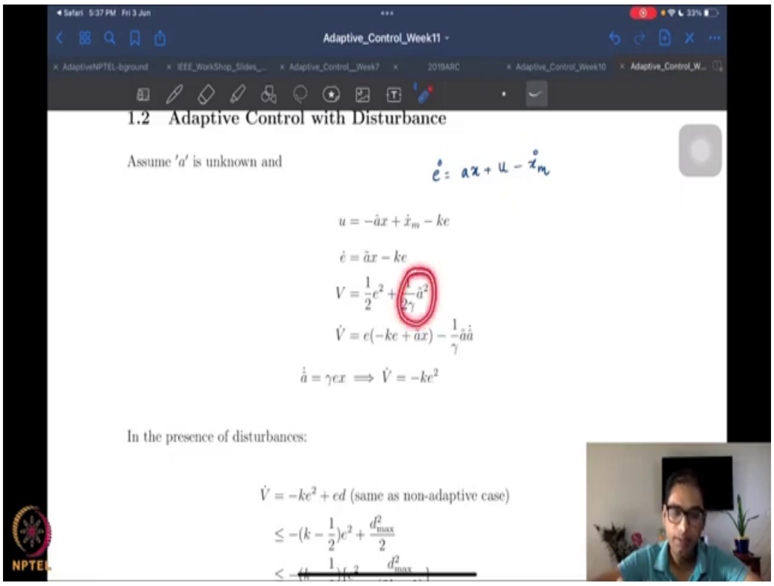
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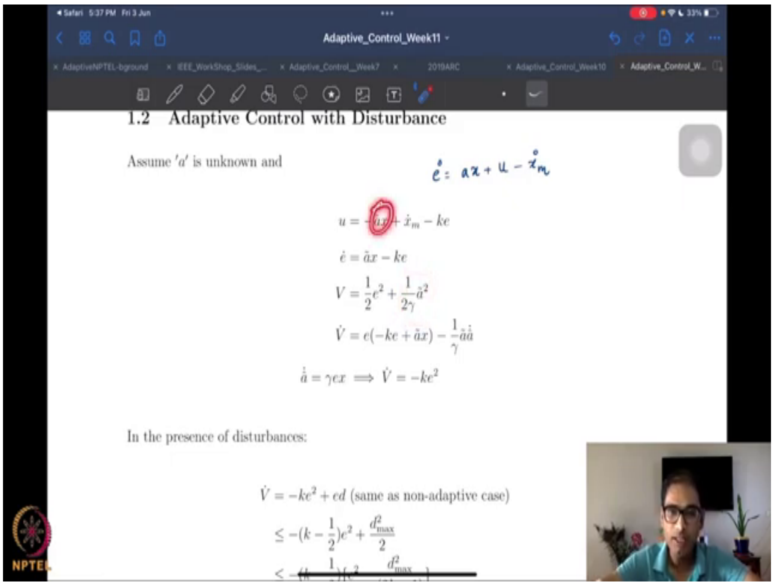
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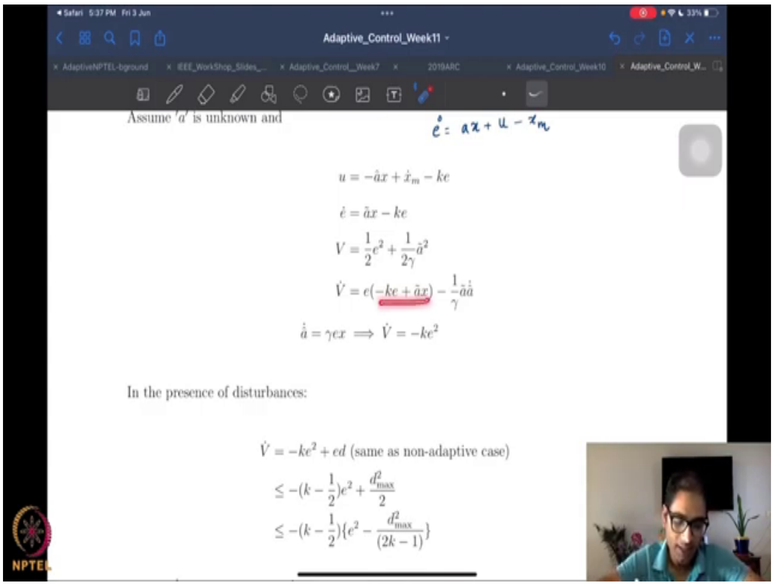
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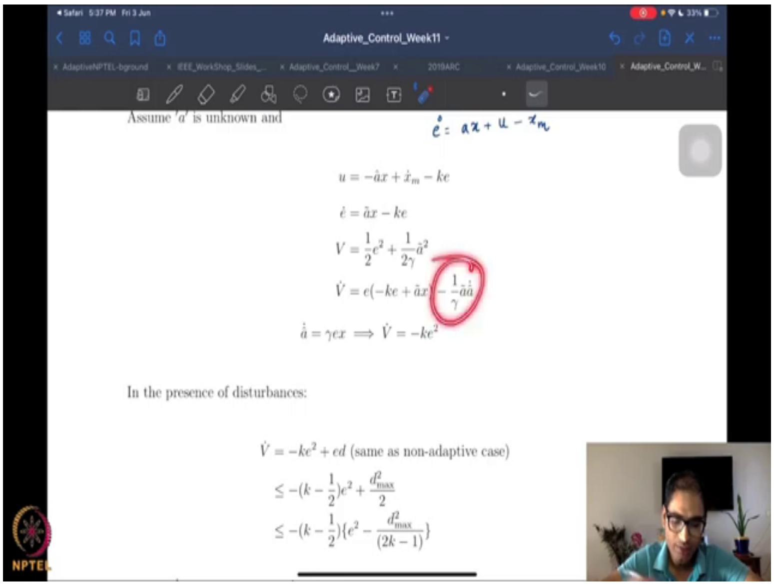
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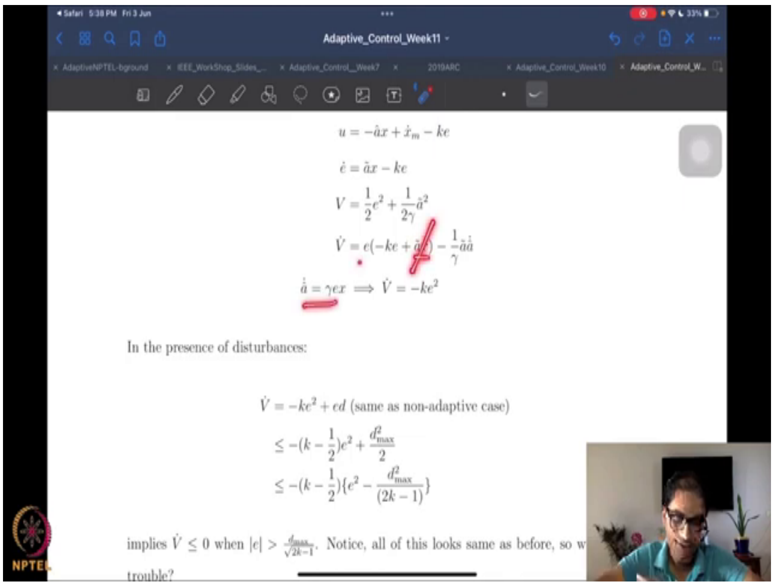
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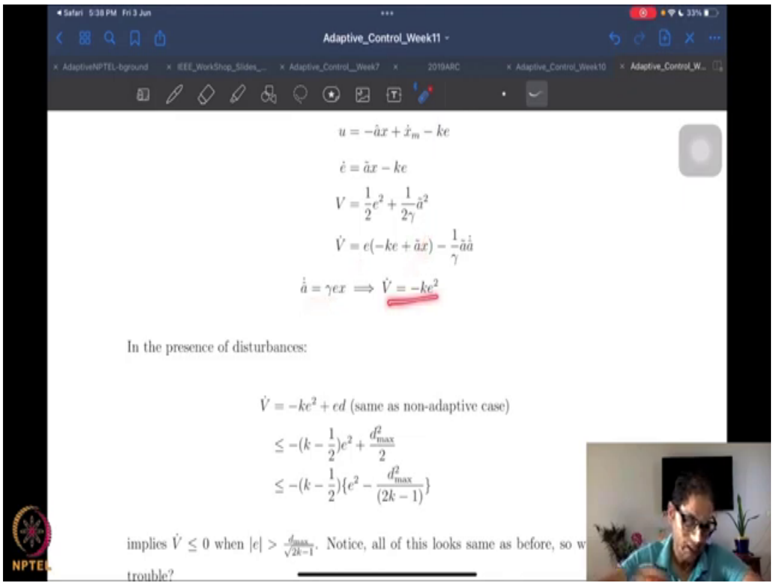
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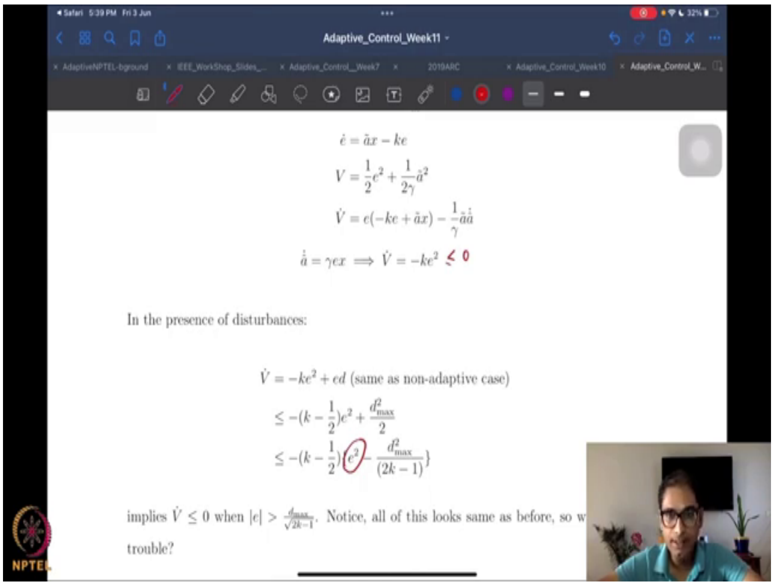
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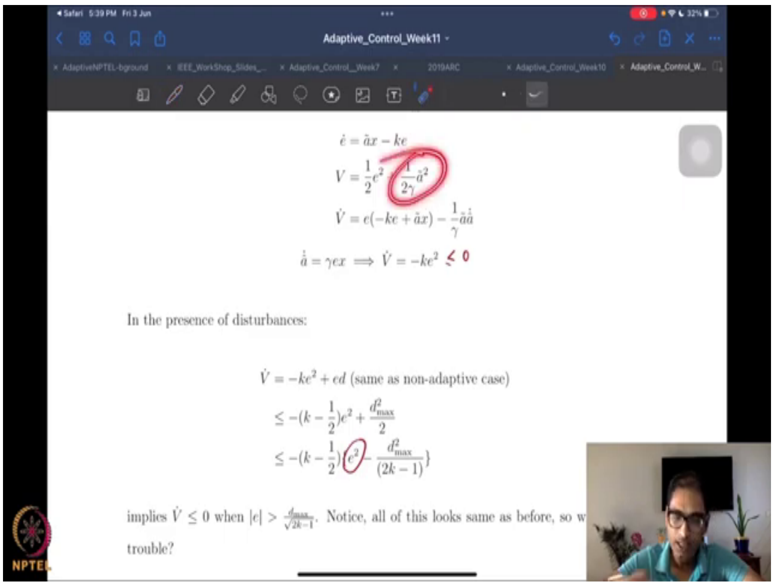
Adaptive_Control_Week11
$\dot{e} = \tilde{a}x - ke$
$V = \frac{1}{2}e^2 + \frac{1}{2\gamma}\tilde{a}^2$
$\dot{V} = e(-ke + \tilde{a}x) - \frac{1}{\gamma}\tilde{a}\dot{\hat{a}}$
$\dot{\hat{a}} = \gamma e x \implies \dot{V} = -ke^2 \leq 0$
In the presence of disturbances:
$\dot{V} = -ke^2 + ed$ (same as non-adaptive case)
$\leq -(k - \frac{1}{2})e^2 + \frac{d_{max}^2}{2}$
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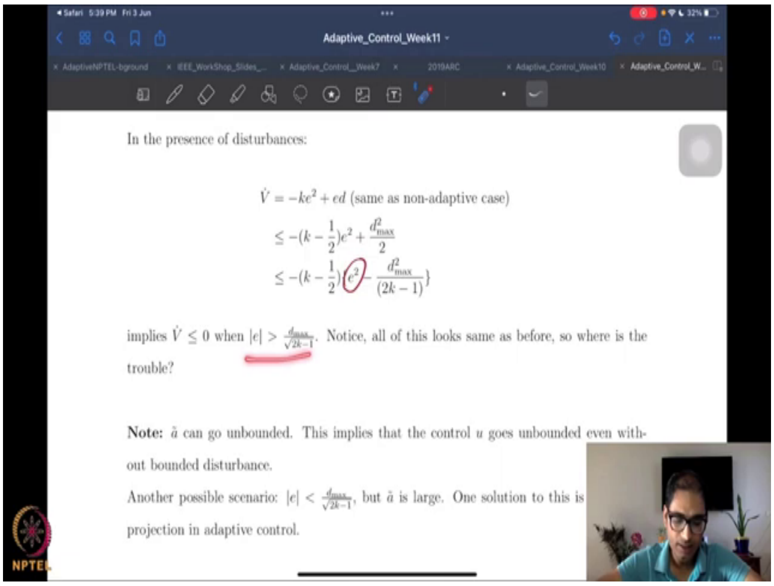

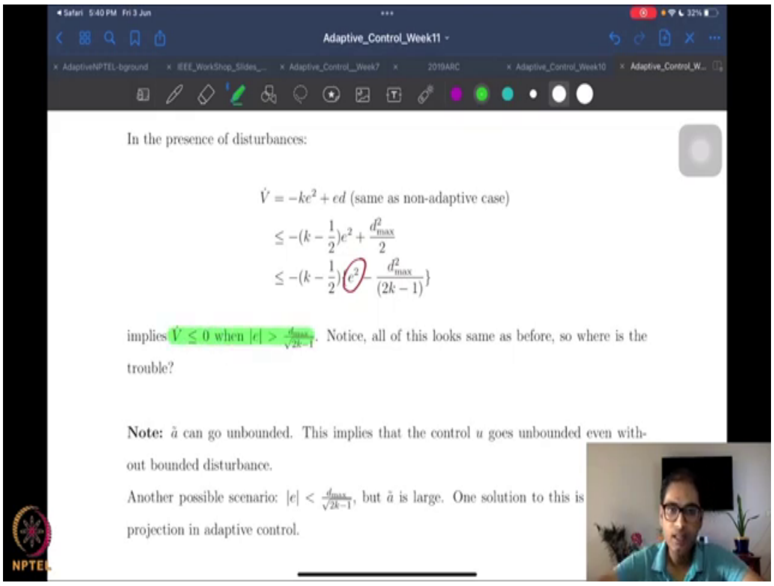

You may start with a strict Lyapunov function for the known case, but as soon as you come to the adaptive control problem the strictness of the Lyapunov function is lost, because in V dot, in fact, the V dot that you get turns out to be exactly same as the V dot for the known case. Although in V you had to have an additional term in the unknown parameter. So this is, I mean, all of you remember this.

So V can never be strict in adaptive control problem. Well, it can be strict if you have persistence of excitation and all that nice stuff, then you can create some strict Lyapunov function for an adaptive control problem, but more often that is not the case, that is why adaptive control theorist do not care about parameter convergence because persistence may not be guaranteed, therefore, you do not have strict Lyapunov functions, at least not in the setup that we have usually seen.

So, let us sort of work this out and look at what happens. Let us look at what happens. So for the same system, I mean, I will write the e dot equations again, so the e dot was a x plus u minus x m dot. And now because the a is unknown, I will use a hat x here of course xm dot is still known and introduce the good control term.

Great, what happens? Now I have an additional term in the e dot. What is my Lyapunov function? We add a term to the Lyapunov function corresponding to the parameter error and of course we have to find an update law for a hat, very simple certainty equivalence method, scalar system, all of you have done this a thousand times. I hope, by now you have.

And so I populate all the terms e times e dot minus k plus a tilde x and a tilde dot which is a tilde minus a hat dot, so I have this guy and with this, if I take a hat dot as gamma ex, this term cancels and I am left with V dot is minus k e squared. Now remember this is negative semi-definite turned out to be exactly the same as the non-adaptive case, big surprise. We already knew this was going to happen. This has always been the case, no exceptions.

Now what happens in the presence of uncertainties? In the presence of uncertainties your e-dynamics will have plus d of t and that will propagate all three. So simply stated, your V dot will have minus k e squared plus e times d. Again you will get the same V dot as the non-adaptive case. Even with the disturbance nothing changes because I did not change the control, I did not change the adaptive law, I changed nothing.

The only thing that changes is the Lyapunov analysis, because I cannot change anything since I have no knowledge of the disturbance, so there is no question of changing anything. So great, I have this V dot and now I continue my analysis. So I do the same thing minus k minus half e squared plus d max squared by 2 and I get this expression.

Now notice the key difference here, I write this as e squared and not as V, because this is not V anymore, in the non-adaptive case this was in fact my candidate Lyapunov function, but in this case it is not. It is just a piece of it, because the candidate function also has this guy. So this implies V dot is less than equal to 0 whenever your e is greater than this.

So this, now if you look at this particular conclusion, this still seems to be the same. But where is the problem, where is the big problem, I mean, problem that crashed an airplane, so big problem. So let us see what is that?

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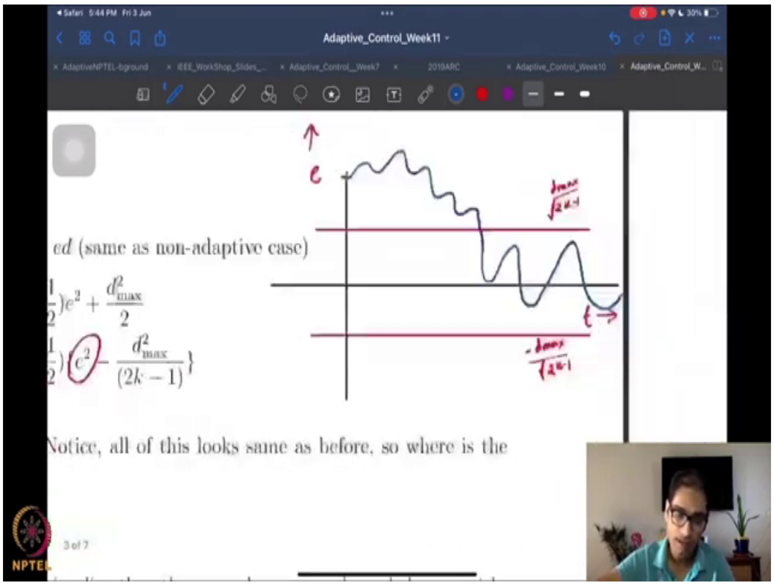
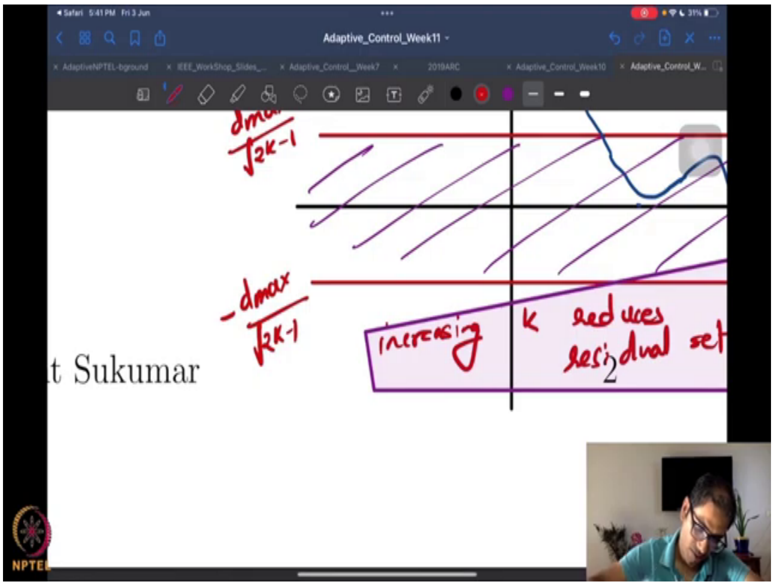
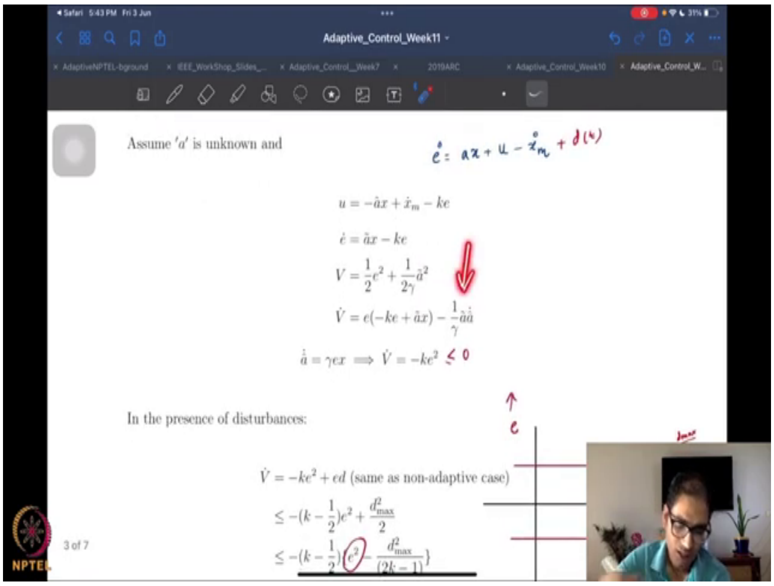
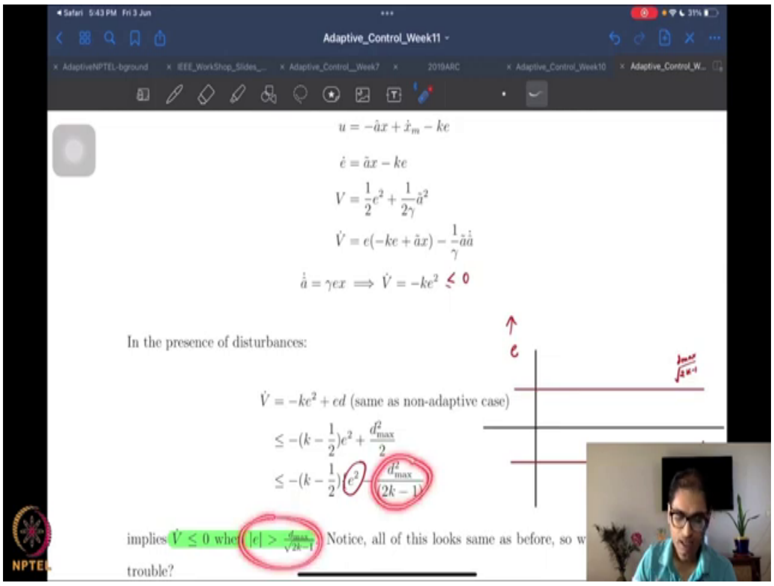





Adaptive_Control_Week11
Assume 'a' is unknown and
$\dot{e} = ax + u - \dot{x}_m + d(t)$
$u = -\hat{a}x + \dot{x}_m - ke$
$\dot{e} = \tilde{a}x - ke$
$V = \frac{1}{2}e^2 + \frac{1}{2\gamma}\tilde{a}^2$
$\dot{V} = e(-ke + \tilde{a}x) - \frac{1}{\gamma}\tilde{a}\dot{\hat{a}}$
$\dot{\hat{a}} = \gamma ex \implies \dot{V} = -ke^2 \leq 0$
In the presence of disturbances:
$\dot{V} = -ke^2 + ed$ (same as non-adaptive case)
$\leq -(k - \frac{1}{2})e^2 + \frac{d_{max}^2}{2}$
$\leq -(k - \frac{1}{2})\{e^2 - \frac{d_{max}^2}{(2k-1)}\}$
3 of 7
NPTEL


Adaptive_Control_Week11
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$V = \frac{1}{2}e^2 + \frac{1}{2\gamma}\tilde{a}^2$
$\dot{V} = e(-ke + \tilde{a}x) - \frac{1}{\gamma}\tilde{a}\dot{\hat{a}}$
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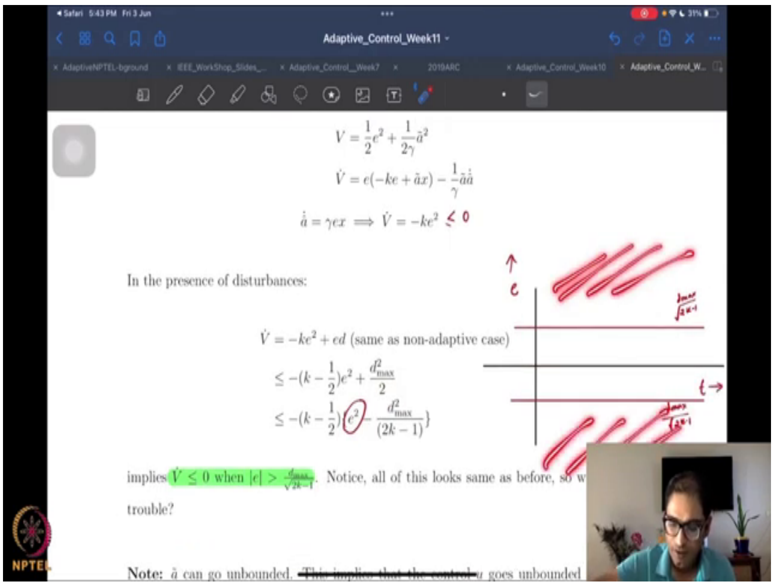
NPTEL


Adaptive_Control_Week11
$V = \frac{1}{2}e^2 + \frac{1}{2\gamma}\tilde{a}^2$
$\dot{V} = e(-ke + \tilde{a}x) - \frac{1}{\gamma}\tilde{a}\dot{\hat{a}}$
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trouble?
Note: $\hat{a}$ can go unbounded. This implies that the control u goes unbounded
NPTEL

Now I do not make the, I will not be making the, actually I do not need this to do, I can make this here. I am no longer going to be making the V plot, I only make the e plot and because V plot is no longer corresponding to the e, the V, the candidate function V and e plots are no longer corresponding, so it does not make sense for me to make the plot for the V. So I am just going to plot my e here and time here.

Now I will make the two boundaries, these boundaries are basically d max. Did I get this wrong last time? This should, I am sorry, this boundary last time should have been 2 k minus 1, so this should also have been 2 k minus 1, 2 k minus 1. So there was a 2 factor missing here. So here also this is d max, I am going to make this bigger, so I can write a little bit more easily. So I have the same boundary if you notice.

And minus d max over 2 k minus 1 at the same boundary. Now what happens? If I look at the e plot things seem okay, because when an e is larger than this quantity, then e still then, when e is larger than this quantity then V dot is still negative. Notice if e is larger than this quantity V dot is still negative. Therefore, V has to reduce. So if V reduces, eventually e also has to reduce, because notice that V is a quadratic sum of e square and a tilde.

So if V goes down, down, down, because V will not stop going down until e is larger than this guy, until e is here or here we cannot stop decreasing. It may decrease in a nonlinear way or in this case not monotonic necessarily, it does not matter if it decreases in a monotonic way or not, that is not necessarily going to be the case because in this case e beyond this requires V to be decreasing.

But V decreasing could also mean that this guy is decreasing and this is not, for some time at least, but in the long run this also has to decrease, because the negative V remains continues to be negative, when e is larger than this quantity. So this also has to decrease. So logically speaking e may start here, it could increase, but eventually it has to decrease, so it has to go inside the set. So great, I mean, nice. And inside the set, of course, any oscillations are allowed you remain within this set and all that. Why? Well, do you remain within this set? That is also a question.

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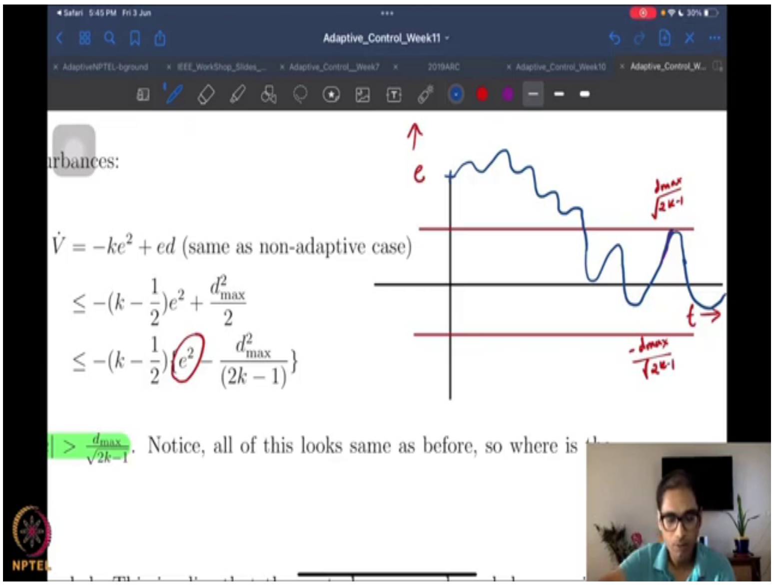
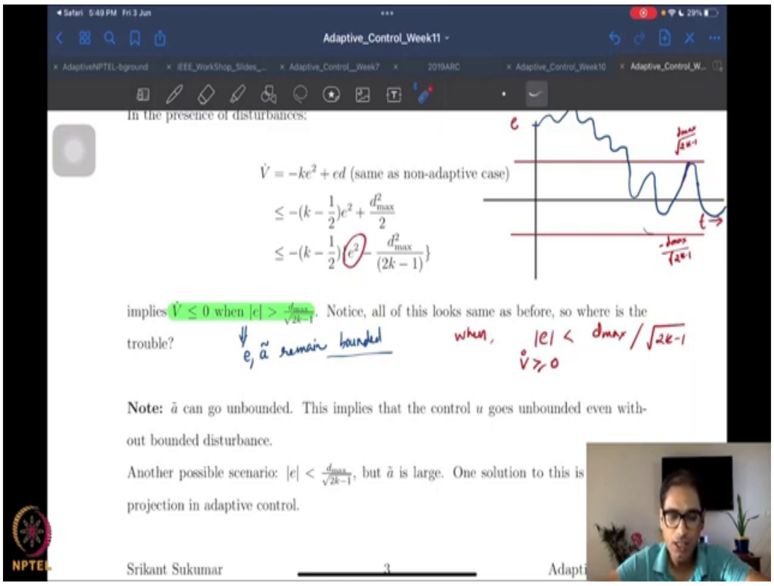

Adaptive_Control_Week11
rbances:
$\dot{V} = -ke^2 + ed$ (same as non-adaptive case)
$\leq -(k - \frac{1}{2})e^2 + \frac{d^2_{max}}{2}$
$\leq -(k - \frac{1}{2})\{e^2 - \frac{d^2_{max}}{(2k-1)}\}$
$|> \frac{d_{max}}{\sqrt{2k-1}}$. Notice, all of this looks same as before, so where is the
NPTEL


Adaptive_Control_Week11
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implies $\dot{V} \leq 0$ when $|e| > \frac{d_{max}}{\sqrt{2k-1}}$. Notice, all of this looks same as before, so where is the trouble?
Note: $\tilde{a}$ can go unbounded. This implies that the control u goes unbounded even without bounded disturbance.
Another possible scenario: $|e| < \frac{d_{max}}{\sqrt{2k-1}}$, but $\tilde{a}$ is large. One solution to this is projection in adaptive control.
Srikant Sukumar
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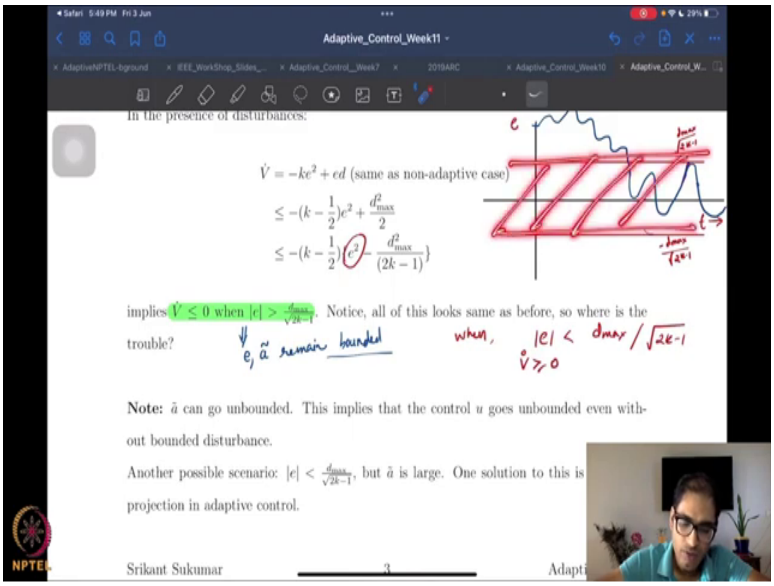
NPTEL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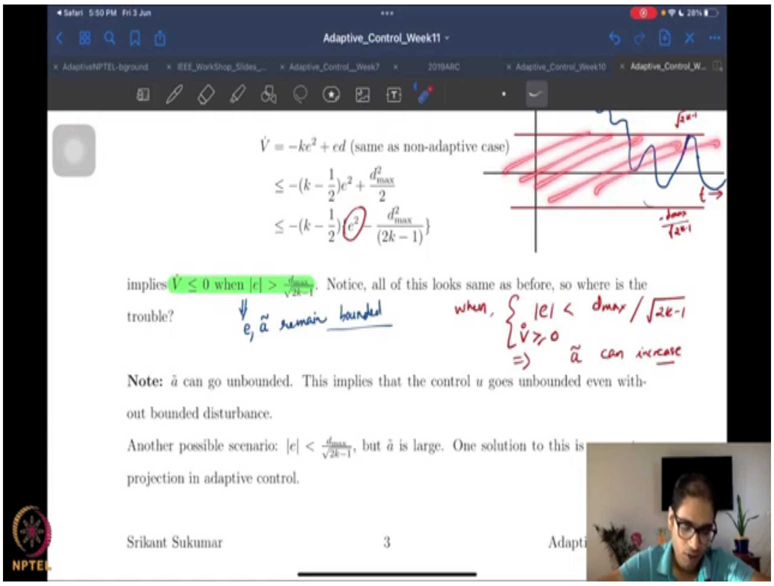
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$\Downarrow$ $e, \tilde a$ remain bounded
when, $|e| < d_{max}/\sqrt{2k-1}$, $\dot V \ge 0$ $\Rightarrow$ $\tilde a$ can increase
Note: $\tilde a$ can go unbounded. This implies that the control u goes unbounded even without bounded disturbance.
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Srikant Sukumar
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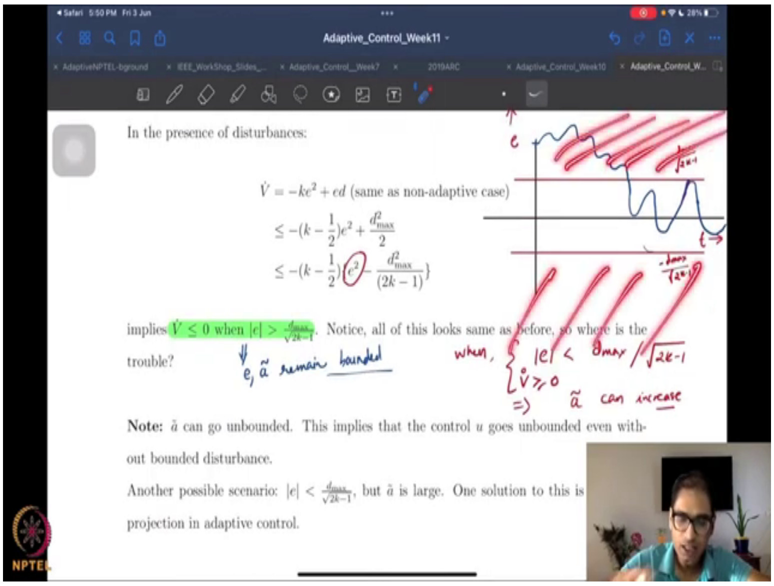
Adaptive_Control_Week11
In the presence of disturbances:
e
t
$\dot V = -ke^2 + ed$ (same as non-adaptive case)
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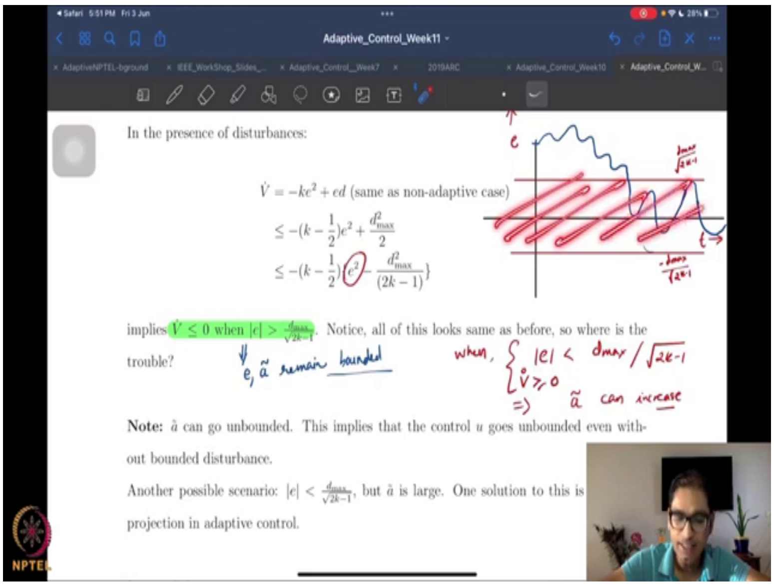
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In the presence of disturbances:
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$-\frac{d_{max}}{\sqrt{2k-1}}$
t
$\dot V = -ke^2 + ed$ (same as non-adaptive case)
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NPTEL

So let us try to answer that question also, but you do get into this set, this residual said that we the said, that we usually call the residual set. So you do get into this set. So let us see what happens if instead of doing this I do hit the boundary. Now at the boundary again as soon as I try to cross what happens, V becomes negative, V becomes, sorry, V dot becomes negative, so V has to reduce.

So very close to the boundary, as soon as you are very near the boundary V becomes almost 0, as you reach the boundary V is becoming almost 0 here. So as you reach the boundary here V is becoming almost 0, V does not change anymore, so if V does not change e does not change, and then V becomes negative as soon as you try to cross it.

So therefore, this has to come back in. So this is again I am talking about it in words, but I promise you this can also be proven. This can also be proven. So this thing works no problem. I mean, this is good to know. I mean, I kept saying that there are so many problems and now I find that e goes into this residual set. In fact, looks like the size of the residual set can also be controlled similarly using the gain k, then you guys might ask me really what were you talking about.

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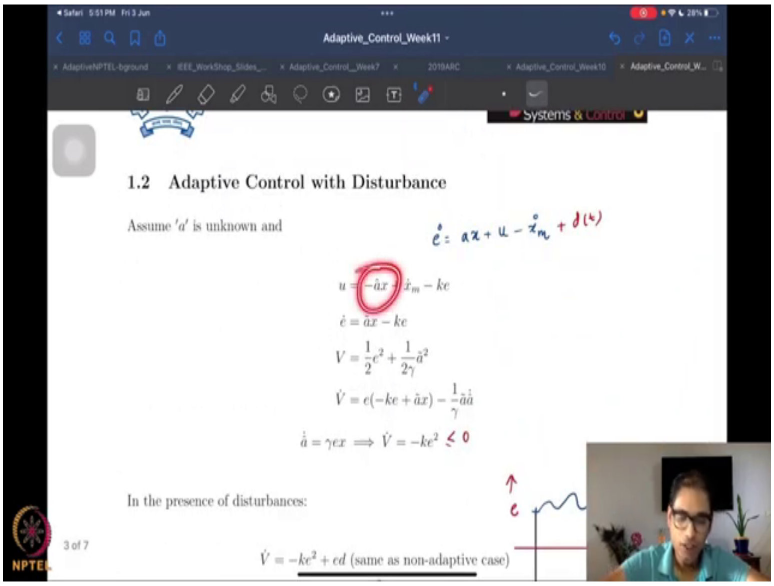

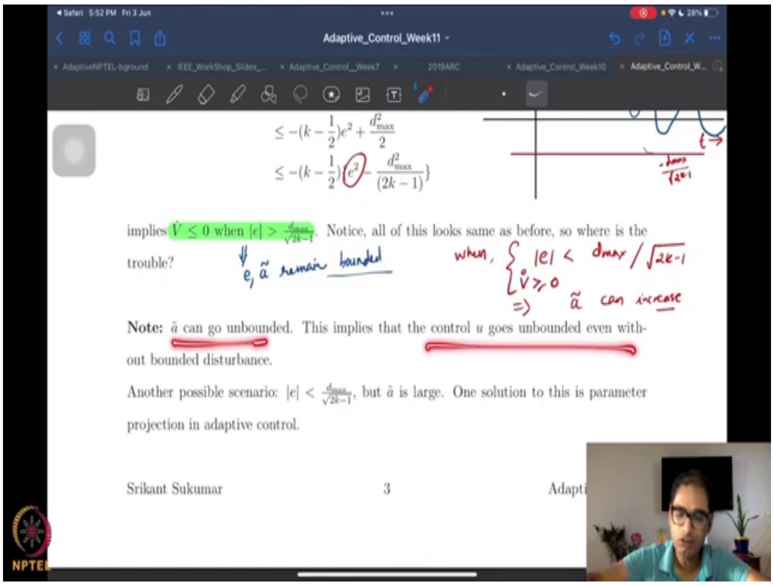

So let us come to the trouble part, let us come to the trouble part. Now once, as long as you are outside this set V is decreasing, V is less than, V dot is decreasing, negative, so V is decreasing or non-increasing. So therefore, everything here remains bounded and because if you remember, so this implies e a tilde remain bounded, when you are outside this set e a tilde remain bounded.

All nice, because why do they remain bounded because V dot is negative semi-definite so V is non-increasing V is non-increasing then, whatever value you started with, I mean, you cannot, none of the terms can go to, e or a tilde cannot go to infinity because if you started with finite value you will remain in a finite value set. Now once e gets inside this residual set is when problems begin.

Because then also anyway inside this residual set that e will never escape this set, so it will always remain inside the set as long as you apply the control, of course. You stop applying the control, no guarantees, but if you continue to apply the control e will remain inside this set. Great and the second thing that happens is that inside this set V is not necessarily, V dot is not less than equal to 0.

Now when, so what happens, when e less than d max over square root 2k minus 1 V is greater than equal to 0, sorry, V dot is greater than equal to 0, correct, just using this guy. If V dot is greater than equal to 0, so V can increase, it can either increase or stay equal to 0, V can increase. Now the important thing to remember again, be very careful, e cannot increase beyond this guy, e has to remain in this set only.

Because I just said that residual set property, it will remain inside. But V can increase, not unintuitive. The only thing that is happening, when e is inside this, is it stops e from increasing beyond a certain point, e remains in this cage sort of, remains in this cage, but V, because V dot is non-negative and if V dot is non-negative then V can increase and if V can increase but e cannot increase beyond a certain value, what does it mean?

If I take these two into account that e cannot increase beyond a certain point, but V can increase, what does it mean? Implies a tilde can increase. And there is nothing stopping it, nothing, absolutely nothing stopping it, because e is bounded here and as long as e is bounded here, V dot is non-negative and if V dot is non-negative V can increase and which means a tilde can increase, nothing stopping it, never does it have to become negative definite.

Because why? Why did all this happen? Because, V for this, the same V for this adaptive case is not a strict Lyapunov function, the V dot has only the e term and not the theta tilde term or the k a tilde term. If the a tilde term was present here this would be the same analysis as the as the previous case and no issues, everything will be nice and bounded, but here as long as e remains outside, that is here everything is bounded.

Because V dot is negative semi-definite, therefore V is not increasing, so if you started with finite value you remain in a finite value, but as soon as e gets into this residual set the nice cage which we made for this e we want the e to get into this cage and we know that it cannot escape this cage, but we know that inside this V dot function, which contains e becomes non-negative, which means V can increase.

The only way V can increase and without having e increase itself is if a tilde increases and a tilde can increase to arbitrary values now, nothing stopping it. So this means essentially that a tilde can, in fact, go unbounded. So what do we care? We care, because it contains the, the control contains a hat and a hat is just a tilde with a constant offset, a hat is just a tilde with some constant offset, nothing different than that.

So once I have this issue, I know that if a tilde goes unbounded, I know the control goes unbounded. So the issue is although the, if you implement this control then your error will go into this nice residual set, which you can even make smaller with the increasing gain, but interestingly you may end up needing infinite control. And as you can imagine no actuator can provide infinite control.

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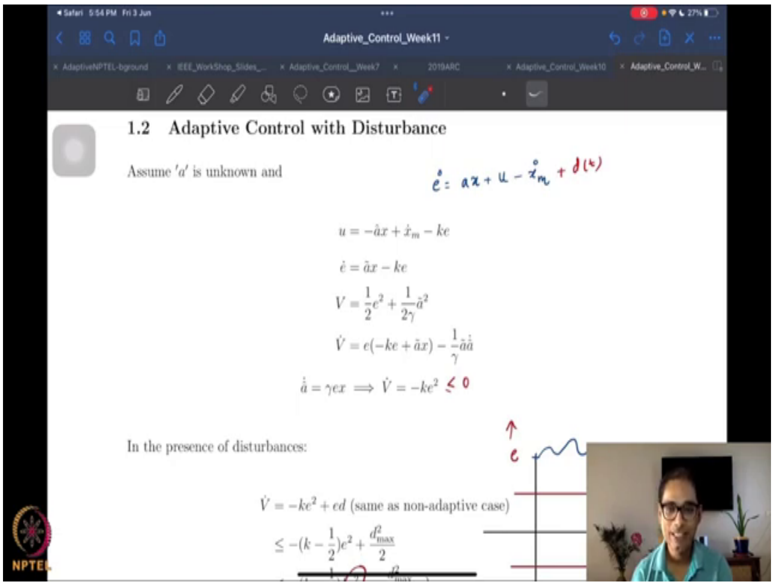

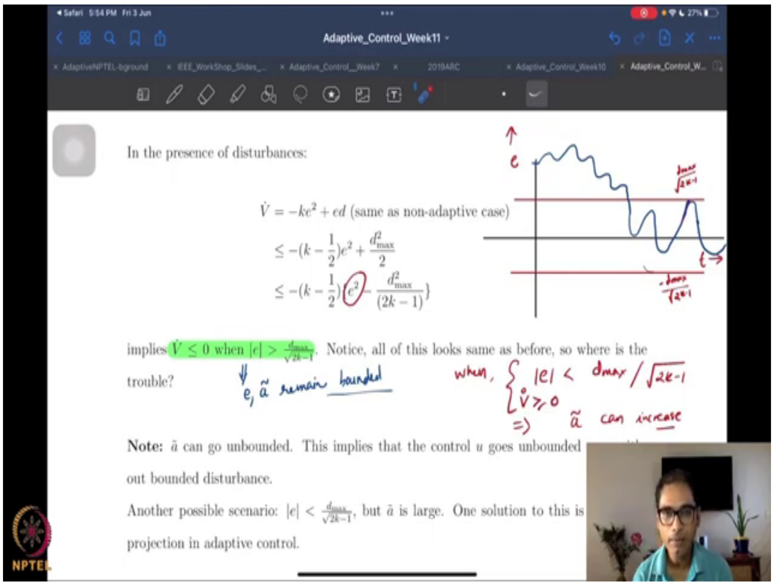

Until now in all our adaptive design all the terms are bounded, boundedness of these things was sort of a given, we did not even have to think about it, but just in this very simple scalar case with one disturbance term added here, you see that my control can go unbounded to maintain a residual set or a bounded performance for the tracking error and this is if nothing mind-blowing, this is if nothing mind-blowing.

I mean, I cannot the first time I saw this, I was, I mean, absolutely baffled as to how I could have missed it or how is this even possible, am I doing the calculations correctly or is my professor doing the calculations correctly, maybe he is completely wrong, but this is it. I mean, just a very, very simple sort of a situation, seeming situation and things become ugly, and therefore, you can see why there was a big hiatus in adaptive control.

And with such ambitious project of implementing the controller on a fighter jet and even with such a simple scalar problem my disturbance and with some basic disturbance bounded disturbance and things go awry, then you can imagine why a lot of folks would have given up adaptive control. I am sure some of us were there in trying to fly the fighter plane, we would have given up adaptive control ourselves and moved on to others cooler fields, maybe learning.

So anyway, so that is the idea, this is this is the problem. Simple problem is we did not have strict Lyapunov functions in adaptive control, even if we started with a strict Lyapunov function in the known case, in the unknown parameter case it becomes a non-strict Lyapunov. So what are the solutions? So one of the solutions is called parameter projection.

The idea here is until now we have assumed that we know nothing of the parameter, we do not know anything about the bounds of the parameter and such, now this is again not super realistic because in a real applied engineer has a very good idea of what kind of parameters this system, a system would have, so it is not unnatural, it is not unreal to assume that you do know a little bit about the parameter values.

Especially if the bounds and the parameter values then what we do in adaptive control is to make sure that the parameter estimates a hat remain within this pre-specified bound. And that directly tackles the problem, directly tackles the problem. Because what is the problem in our adaptive controller that the parameter goes to infinity or can go to infinity and it has happened.

So, and what we want to do? We want to attack this. So what do we do? We make sure that the parameter search in some sense, a hat is essentially searching for the right value of the parameter, it is an update law. But in reality what is it doing? I mean, in fact, a lot of the earlier adaptive control laws are gradient descent laws. So essentially the a hat dot is designed in such a way so that you just search for the true value of the parameter.

Now if I know a range for the true value of the parameter it makes sense that I search in this range only, not search everywhere, that is what we do you do parameter projection in adaptive control so that the a hat lies between the given bounds. Again no guarantees of convergence to true value, it does not matter how small your range is, but still it does not allow the parameter to go to infinity and therefore, guarantees that you have robustness.

So this is what we will look at in the upcoming session that is how to do parameter projection, at least one way of doing parameter projection in adaptive control, there are quite a few ways, but we will look at one way.

So what did we look at today? We looked at the issue at hand that is how do, basically what happens when you do adaptive control on a very simple scalar system? And then there is disturbance introduced into the system that we cannot account for. Then we saw what happens to the adaptive performance, we almost stared with disbelief that and found that you can have unboundedness of the parameters and therefore, unboundedness of the controller and the whole system can get ruined, if you are not careful about implementing a robust adaptive control.

So implementing an adaptive controller without some kind of robustness to disturbance can be fatal. So obviously this is what we are going to look at in the subsequent session. So I hope you will join me in this interesting session. Thank you.