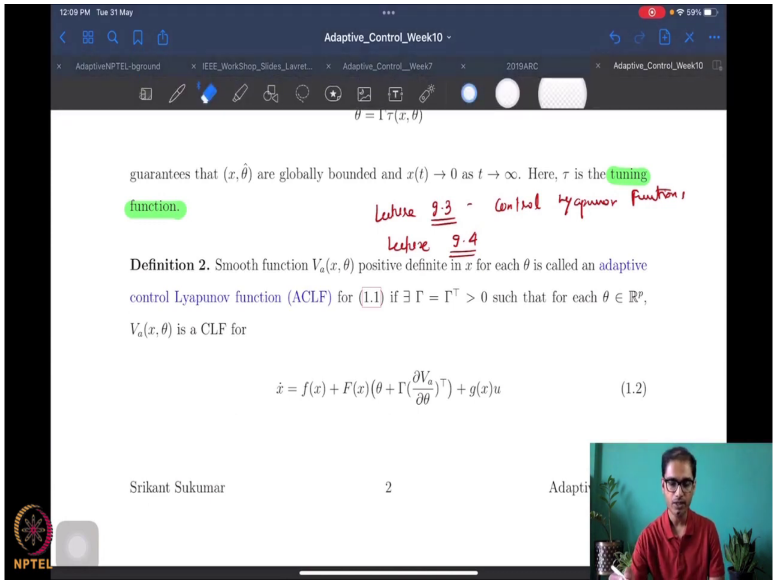
Nonlinear Adaptive Control
Professor Srikant Sukumar
Systems and Control
Indian Institute of Technology, Bombay
Week 9
Lecture No: 53
Adaptive Backstepping via Control Lyapunov Function (CLF)

(Refer Slide Time: 00:17)

Hello, everyone. Welcome to yet another session of our NPTEL on Nonlinear and Adaptive Control. I am Srikant Sukumar from Systems and Control, IIT Bombay. We are now pretty much close to ending the ninth week of lectures for this course, and I hope that all of you with me have learned several analysis and design techniques for designing algorithms that will fly autonomous systems such as what we see in our background.

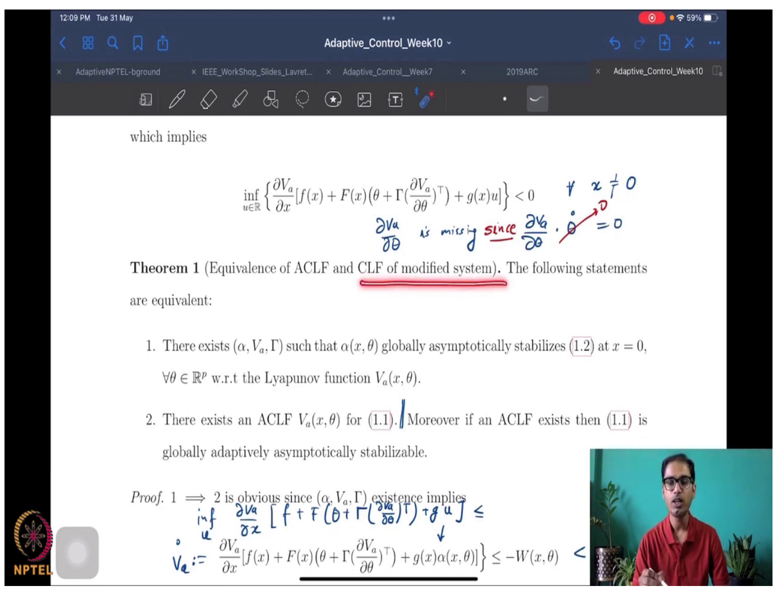
What will be rather interesting and useful for me is to get feedback from all of you on the kind of applications that you envision using it or even start to use it. And I would really like to see some your test bed results, at least on how, or the kind of improvement that you could achieve using adaptive design methods.

(Refer Slide Time: 01:32)



So, what we were doing last time was essentially in this week 10 lecture notes and we had started looking at the notion of adaptive CLF, adaptive control Lyapunov functions.

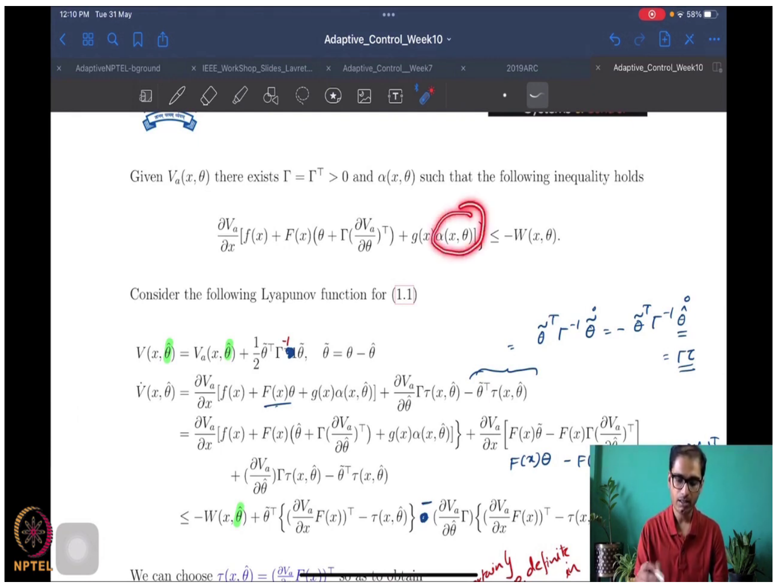
(Refer Slide Time: 01:45)

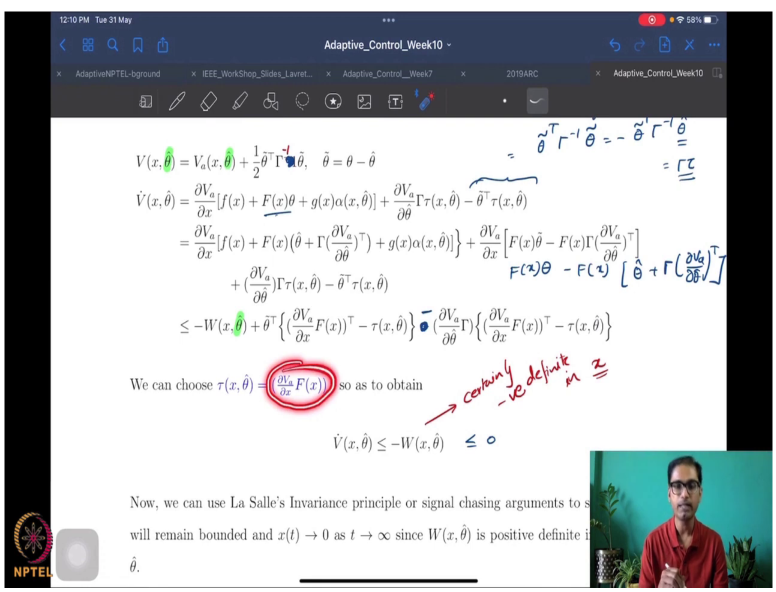


And we talked about the equivalence of the adaptive control Lyapunov function and the control Lyapunov function for the modified system. And finally, we also showed that if an adaptive CLF exists for a system of the form 1.1, then the system is globally adaptively asymptotically stabilizable, which means that I can design an adaptive controller for system 1.1, which ensures that x and theta tilde states remain bounded, and further that the x states converge to 0 as t goes to infinity.

So, that is rather nice. So, the power of ACLF is now, I hope, well understood. So, if one is able to construct an ACLF, then I mean, you already have sort of achieved a big goal because just by using this ACLF, you can now design a controller. And it is also very clear and evident in some sense.

(Refer Slide Time: 02:50)

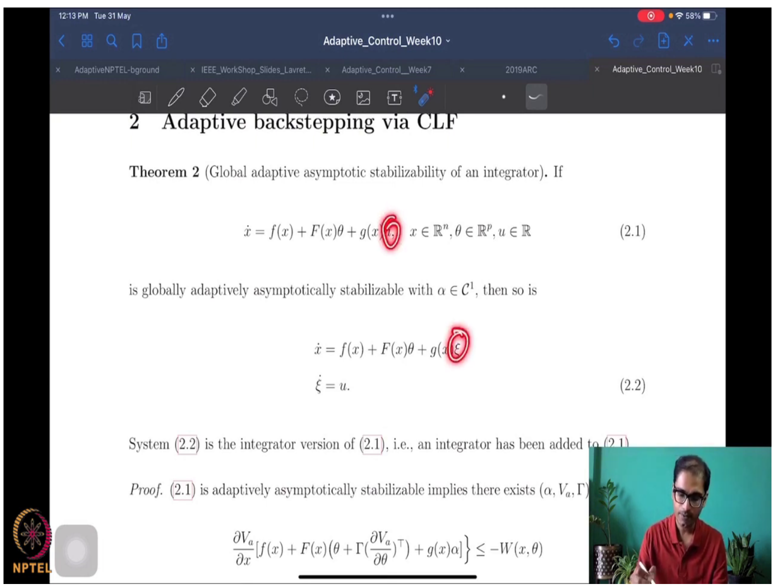


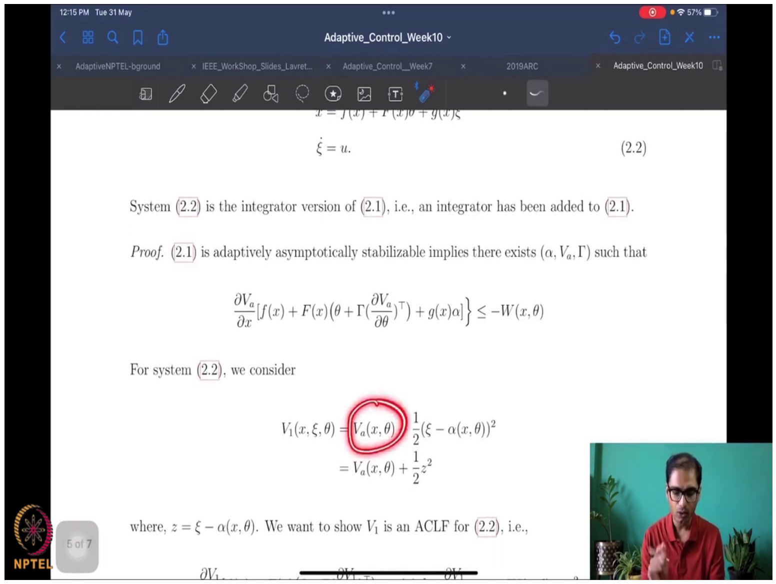


The alpha, the control that the feedback law, alpha in this case is obtained just by, you can simply use like a Artstein-Sontag formula to get an alpha, a stabilizing controller. And the tuning function tau is obtained using this expression. In fact, there is a clear expression for what would be the tuning function.

So, so this is the cool thing. I mean, the expression for the tuning function is clearly known. The expression for alpha can be obtained by an Artstein-Sontag type universal formula. I mean, of course, you can do all of this intuitively, might be even better, easier analysis wise. But even if you do not, you can almost automate it. You can have a computer compute these symbolically and simply implement it. So, having a adaptive control Lyapunov function for a system or for a class of systems is a rather powerful tool.

(Refer Slide Time: 04:00)





$$\dot{x} = f(x) + F(x)\theta + g(x)\xi$$
$$\dot{\xi} = u. \qquad (2.2)$$

System (2.2) is the integrator version of (2.1), i.e., an integrator has been added to (2.1).

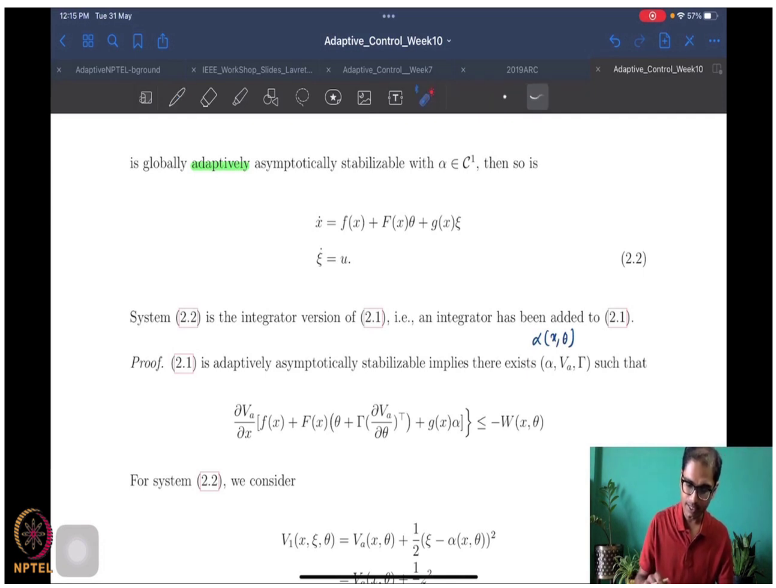
Proof. (2.1) is adaptively asymptotically stabilizable implies there exists (α, V_a, Γ) such that

$$\frac{\partial V_a}{\partial x}[f(x) + F(x)\big(\theta + \Gamma(\frac{\partial V_a}{\partial \theta})^\top\big) + g(x)\alpha]\Big\} \leq -W(x,\theta)$$

For system (2.2), we consider

$$V_1(x,\xi,\theta) = V_a(x,\theta) + \frac{1}{2}(\xi - \alpha(x,\theta))^2$$
$$= V_a(x,\theta) + \frac{1}{2}z^2$$

where, $z = \xi - \alpha(x,\theta)$. We want to show V_1 is an ACLF for (2.2), i.e.,



is globally adaptively asymptotically stabilizable with $\alpha \in \mathcal{C}^1$, then so is

$$\dot{x} = f(x) + F(x)\theta + g(x)\xi$$
$$\dot{\xi} = u. \qquad (2.2)$$

System (2.2) is the integrator version of (2.1), i.e., an integrator has been added to (2.1).

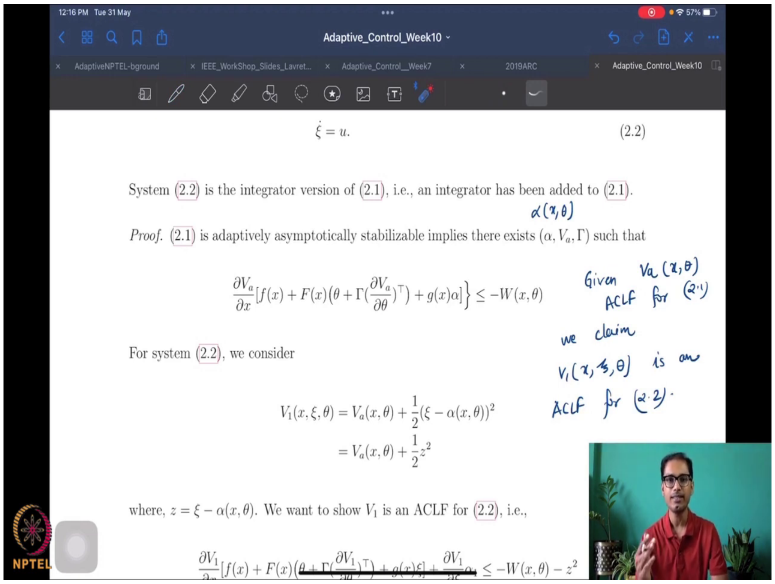
$\alpha(x,\theta)$

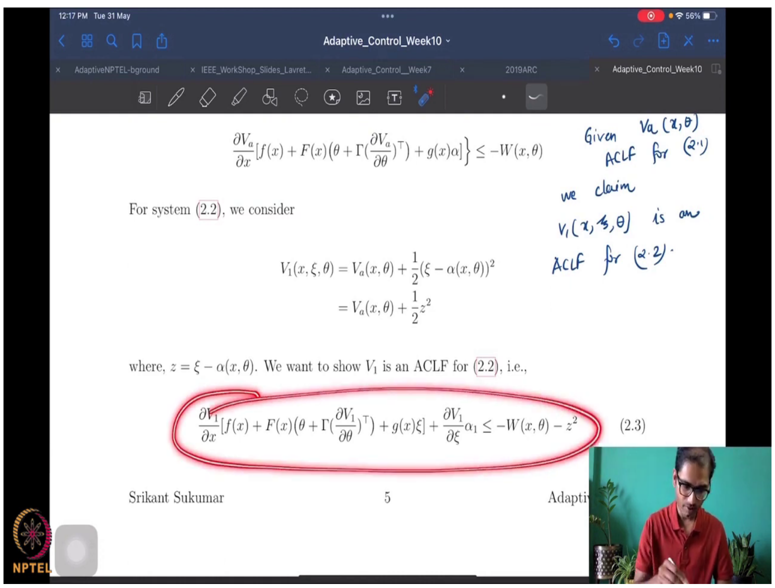
Proof. (2.1) is adaptively asymptotically stabilizable implies there exists (α, V_a, Γ) such that

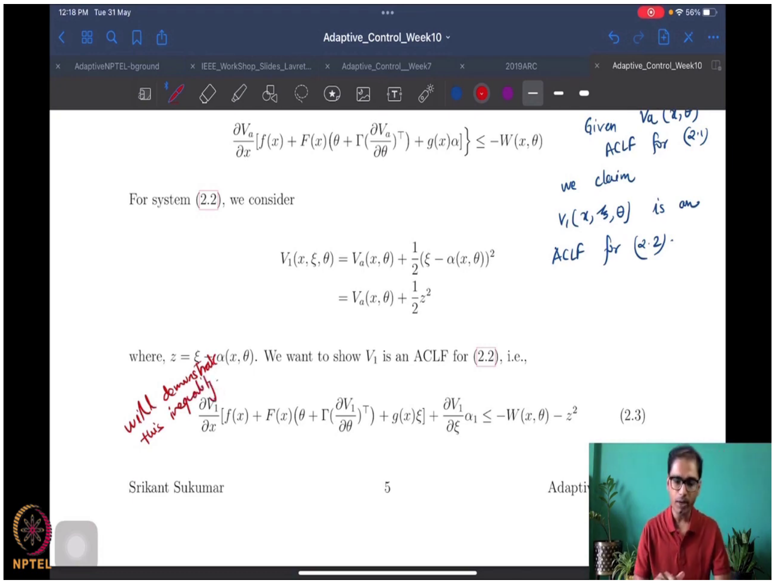
$$\frac{\partial V_a}{\partial x}[f(x) + F(x)\big(\theta + \Gamma(\frac{\partial V_a}{\partial \theta})^\top\big) + g(x)\alpha]\Big\} \leq -W(x,\theta)$$

For system (2.2), we consider

$$V_1(x,\xi,\theta) = V_a(x,\theta) + \frac{1}{2}(\xi - \alpha(x,\theta))^2$$


Adaptive_Control_Week10
$\dot{\xi} = u.$ (2.2)
System (2.2) is the integrator version of (2.1), i.e., an integrator has been added to (2.1).
Proof. (2.1) is adaptively asymptotically stabilizable implies there exists (α, V_a, Γ) such that
$\frac{\partial V_a}{\partial x}[f(x) + F(x)(\theta + \Gamma(\frac{\partial V_a}{\partial \theta})^\top) + g(x)\alpha]\} \leq -W(x,\theta)$
For system (2.2), we consider
$V_1(x,\xi,\theta) = V_a(x,\theta) + \frac{1}{2}(\xi - \alpha(x,\theta))^2 = V_a(x,\theta) + \frac{1}{2}z^2$
where, $z = \xi - \alpha(x,\theta)$. We want to show V_1 is an ACLF for (2.2), i.e.,
$\alpha(x,\theta)$
Given $V_a(x,\theta)$ ACLF for (2.1)
we claim $V_1(x,\xi,\theta)$ is an ACLF for (2.2).
NPTEL


Adaptive_Control_Week10
$\frac{\partial V_a}{\partial x}[f(x) + F(x)(\theta + \Gamma(\frac{\partial V_a}{\partial \theta})^\top) + g(x)\alpha]\} \leq -W(x,\theta)$
For system (2.2), we consider
$V_1(x,\xi,\theta) = V_a(x,\theta) + \frac{1}{2}(\xi - \alpha(x,\theta))^2 = V_a(x,\theta) + \frac{1}{2}z^2$
where, $z = \xi - \alpha(x,\theta)$. We want to show V_1 is an ACLF for (2.2), i.e.,
$\frac{\partial V_1}{\partial x}[f(x) + F(x)(\theta + \Gamma(\frac{\partial V_1}{\partial \theta})^\top) + g(x)\xi] + \frac{\partial V_1}{\partial \xi}\alpha_1 \leq -W(x,\theta) - z^2$ (2.3)
Given $V_a(x,\theta)$ ACLF for (2.1)
we claim $V_1(x,\xi,\theta)$ is an ACLF for (2.2).
Srikant Sukumar
5
NPTEL



Now what, we want to do is to move on to the backstepping version of things because what does backstepping mean? It means that I add some integrator layers and I can still apply the same method. So, the adaptive integrator backstepping was essentially meant that I can design a controller for the first stage but then if there is an integrator layer then I can just move this controller to the second stage using a smart construction of a Lyapunov function.

Here also, we do something similar. Extended matching was slightly different, of course, because we did not actually declare a Lyapunov function for the first stage separately. We directly looked at a constructing, we directly constructed a Lyapunov function for the second stage. So, that is why that is a little bit different, and this is a little bit different.

However, the point being that you can, you can do Backstepping. And that is what we want to do. We want to do adaptive backstepping via CLF starting this session. So, I am going to mark this as lecture 9.5. This is where we are. And so, so what is this? So, you are saying that you, if you have a system again which is looking like the system 1.1 where x is the state in Rn, theta is the parameter in Rp and the control is still a scalar control, the system is said to be globally adaptively asymptotically stabilizable.

I am sorry, if this system is globally adaptively asymptotically stabilizable, then the claim is that the system with the integrator is also globally adaptively asymptotically stabilizable. So, so anyway, so, so that is what we will, of course, prove or try to prove. So, so that is the idea. So, the system 2.2 as you can see is essentially an integrated version.

So, the control is replaced by the state xi and then there is a xi dot equal to u. So, it is essentially the standard backstepping form that you are used to seeing. There is nothing new here in this structure. So, this is what we want to prove that if I am given that this is globally adaptively asymptotically stabilizable, then I want to claim that this is also globally adaptively asymptotically stabilizable.

So, let us look at the proof. So, if you know that system 2.1 is in fact globally adaptively asymptotically stabilizable, then that there exists a feedback a V a and a gamma, positive definite, such that this sort of a relation holds. This is essentially the ACLF relationship, essentially the ACLF type relationship.

Now, what do we do? We use our backstepping idea. We know that alpha cannot directly be applied because xi is not actually a control but xi is in fact a new state. So, we try to do the next best thing. We try to make xi to chase alpha. So, we make xi to chase alpha. And how do we do that? We add a quadratic term in the Lyapunov candidate.

So, so we consider this V 1 or a candidate function V 1, which is essentially the same V a. Notice again, that V, although I talked about adaptive, I am not really putting theta hat anywhere. I did not really put theta hat anywhere. This alpha, though I have not specified it, alpha is just x theta. So, what we are going to try to prove is that given V a x theta ACLF for 2.1, we claim that V 1 x xi theta is an ACLF for 2.2. That is, it.

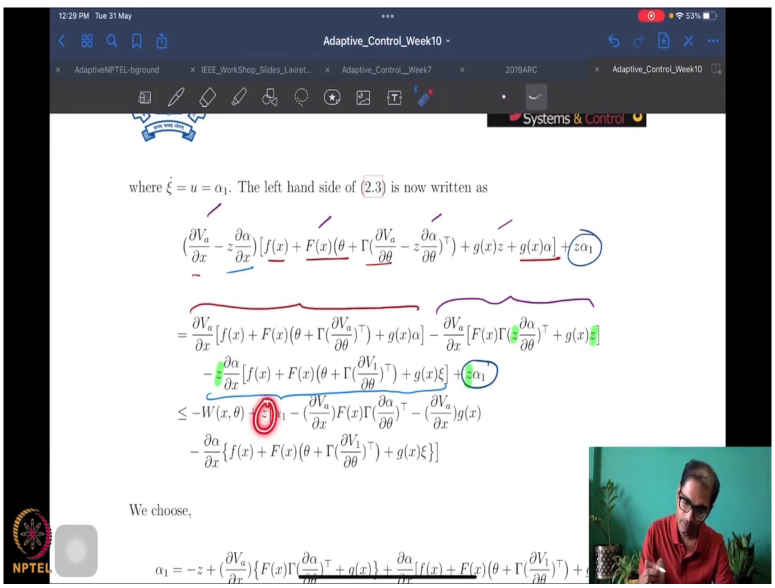
We are not going to worry about, because what do? We know we know that once I have an ACLF, I can design controllers and adaptive laws and all that jazz. Yes, I do not worry about unknown theta right now. I just worry about finding an ACLF. Now, because the system is globally adaptively asymptotically stabilizable, existence of a V a is guaranteed, guaranteed, that is what it means.

We already said that, that if you have an ACLF, I mean if the system is globally adapted asymptotically stabilizable, means the existence of such a V a function. So, this is guaranteed. So, what we are claiming is that from this V a, I can construct this V 1 which is just a standard backstepping construction, because what do I do?

I take the V a and then I add to it the backstepping error term, a quadratic in the backstepping error term. This is just xi minus alpha, that is defined as a z. And this is just half z squared amplitude. And we essentially claim that V 1 is in fact an ACLF for this new system. So, how do we do that?

We just try to compute the ACLF inequality. So, what is this ACLF inequality? We want, we essentially have this. Let us see. We essentially will show this. So, that is what, let us be careful. We will demonstrate this inequality. And how do I demonstrate this inequality? I am just going to take V 1 and, and take its derivative along this. V 1 is a function of x, xi and theta. So, all I am going to do is I am going to take the derivative of V 1 along these equations 2.2, because this is what, not along 2.2 but the modified version of 2.2, let us be careful here.

(Refer Slide Time: 11:06)



$$= \frac{\partial V_a}{\partial x}[f(x)+F(x)(\theta+\Gamma(\frac{\partial V_a}{\partial \theta})^\top)+g(x)\alpha] - \frac{\partial V_a}{\partial x}[F(x)\Gamma(z\frac{\partial \alpha}{\partial \theta})^\top + g(x)z]$$

$$- z\frac{\partial \alpha}{\partial x}[f(x)+F(x)(\theta+\Gamma(\frac{\partial V_1}{\partial \theta})^\top)+g(x)\xi] + z\alpha_1$$

$$\leq -W(x,\theta) + z[\alpha_1 - (\frac{\partial V_a}{\partial x})F(x)\Gamma(\frac{\partial \alpha}{\partial \theta})^\top - (\frac{\partial V_a}{\partial x})g(x)$$

$$- \frac{\partial \alpha}{\partial x}\{f(x)+F(x)(\theta+\Gamma(\frac{\partial V_1}{\partial \theta})^\top)+g(x)\xi\}]$$

We choose,

$$\alpha_1 = -z + (\frac{\partial V_a}{\partial x})\{F(x)\Gamma(\frac{\partial \alpha}{\partial \theta})^\top + g(x)\} + \frac{\partial \alpha}{\partial x}[f(x)+F(x)(\theta+\Gamma(\frac{\partial V_1}{\partial \theta})^\top)+g(x)\xi]$$

and finally obtain

$$\dot{V}_1 \leq -W(x,\theta) - z^2.$$

$$\leq -W(x,\theta) + z[\alpha_1 - (\frac{\partial V_a}{\partial x})F(x)\Gamma(\frac{\partial \alpha}{\partial \theta})^\top - (\frac{\partial V_a}{\partial x})g(x)$$

$$- \frac{\partial \alpha}{\partial x}\{f(x)+F(x)(\theta+\Gamma(\frac{\partial V_1}{\partial \theta})^\top)+g(x)\xi\}]$$

We choose,

$$\alpha_1 = -z + (\frac{\partial V_a}{\partial x})\{F(x)\Gamma(\frac{\partial \alpha}{\partial \theta})^\top + g(x)\} + \frac{\partial \alpha}{\partial x}[f(x)+F(x)(\theta+\Gamma(\frac{\partial V_1}{\partial \theta})^\top)+g(x)\xi]$$

and finally obtain

$$\dot{V}_1 \leq -W(x,\theta) - z^2.$$

So, V_1 is an ACLF for (2.2) since it is a CLF for the modified system (1.2). It a[...] that (2.2) is globally adaptively asymptotically stabilizable.

Proof. (2.1) is adaptively asymptotically stabilizable implies there exists (α, V_a, Γ) such that

$$\frac{\partial V_a}{\partial x}[f(x)+F(x)\big(\theta+\Gamma(\frac{\partial V_a}{\partial \theta})^\top\big)+g(x)\alpha]\Big\} \leq -W(x,\theta)$$

Given $V_a(x,\theta)$ ACLF for (2.1)

For system (2.2), we consider

we claim $V_1(x,\xi,\theta)$ is an ACLF for (2.2).

$$V_1(x,\xi,\theta) = V_a(x,\theta) + \frac{1}{2}(\xi - \alpha(x,\theta))^2$$
$$= V_a(x,\theta) + \frac{1}{2}z^2$$

where, $z = \xi - \alpha(x,\theta)$. We want to show V_1 is an ACLF for (2.2), i.e.,

will demonstrate this inequality

$$\frac{\partial V_1}{\partial x}[f(x)+F(x)\big(\theta+\Gamma(\frac{\partial V_1}{\partial \theta})^\top\big)+g(x)\xi] + \frac{\partial V_1}{\partial \xi}\alpha_1 \leq -W(x,\theta) - z^2$$

Srikant Sukumar 5 Adapti

is globally adaptively asymptotically stabilizable with $\alpha \in \mathcal{C}^1$, then so is

$$\dot{x} = f(x) + F(x)\theta + g(x)\xi$$
$$\dot{\xi} = u. \qquad (2.2)$$

$$\frac{d}{dt}\begin{pmatrix} x \\ \xi \end{pmatrix} = \begin{pmatrix} f(x)+g(x)\xi \\ 0 \end{pmatrix} + \begin{pmatrix} F(x) \\ 0 \end{pmatrix}\theta + \begin{pmatrix} 0 \\ 1 \end{pmatrix}u$$

f', F', g'

System (2.2) is the integrator version of (2.1), i.e., an integrator has been added to (2.1).

$\alpha(x,\theta)$

Proof. (2.1) is adaptively asymptotically stabilizable implies there exists (α, V_a, Γ) such that

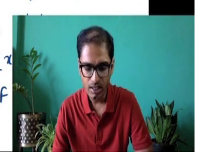
$$\frac{\partial V_a}{\partial x}[f(x)+F(x)\big(\theta+\Gamma(\frac{\partial V_a}{\partial \theta})^\top\big)+g(x)\alpha]\Big\} \leq -W(x,\theta)$$

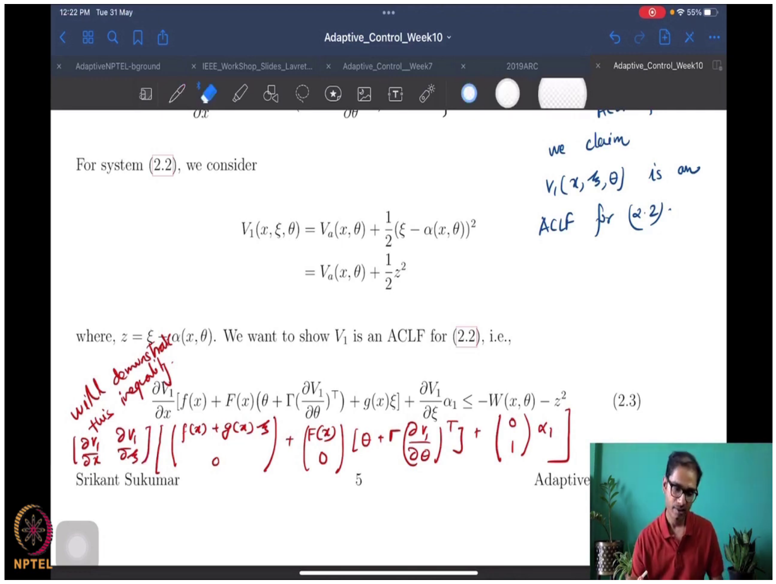
Given $V_a(x,\theta)$ ACLF for (2.1)

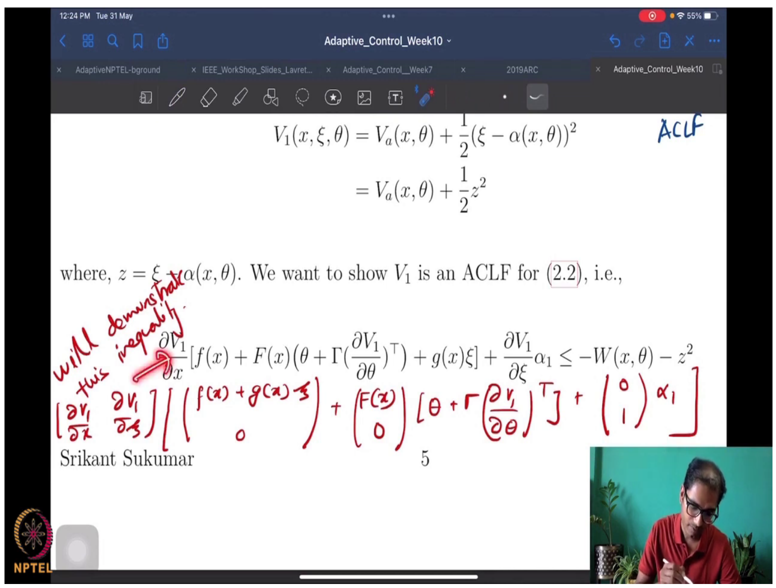
For system (2.2), we consider

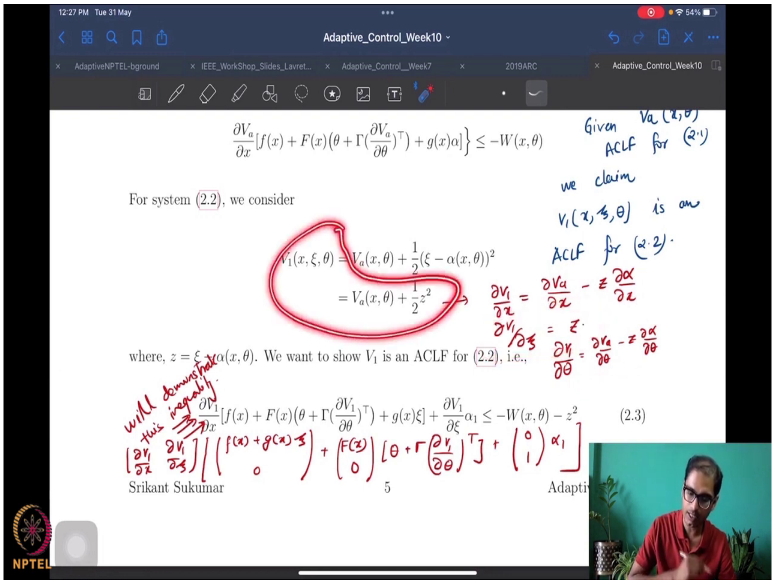
we $V_1($ ACLF

$$V_1(x,\xi,\theta) = V_a(x,\theta) + \frac{1}{2}(\xi - \alpha(x,\theta))^2$$




Adaptive_Control_Week10
For system (2.2), we consider
$V_1(x,\xi,\theta) = V_a(x,\theta) + \frac{1}{2}(\xi - \alpha(x,\theta))^2$
$= V_a(x,\theta) + \frac{1}{2}z^2$
where, $z = \xi - \alpha(x,\theta)$. We want to show V_1 is an ACLF for (2.2), i.e.,
$\frac{\partial V_1}{\partial x}[f(x) + F(x)(\theta + \Gamma(\frac{\partial V_1}{\partial \theta})^\top) + g(x)\xi] + \frac{\partial V_1}{\partial \xi}\alpha_1 \leq -W(x,\theta) - z^2$ (2.3)
we claim $V_1(x,\xi,\theta)$ is an ACLF for (2.2).
will demonstrate this inequality.
Srikant Sukumar
5
Adaptive
NPTEL


Adaptive_Control_Week10
$V_1(x,\xi,\theta) = V_a(x,\theta) + \frac{1}{2}(\xi - \alpha(x,\theta))^2$
$= V_a(x,\theta) + \frac{1}{2}z^2$
ACLF
where, $z = \xi - \alpha(x,\theta)$. We want to show V_1 is an ACLF for (2.2), i.e.,
$\frac{\partial V_1}{\partial x}[f(x) + F(x)(\theta + \Gamma(\frac{\partial V_1}{\partial \theta})^\top) + g(x)\xi] + \frac{\partial V_1}{\partial \xi}\alpha_1 \leq -W(x,\theta) - z^2$
will demonstrate this inequality.
Srikant Sukumar
5
NPTEL



So, we have assumed that xi dot is u is some alpha 1. So, now, oh I am sorry, so what is the left-hand side? The left-hand side is essentially the V dot for the modified system. So, that is del V 1 del x times f x plus cap f x theta plus this guy plus g x xi. And del V 1 del xi times xi dot which is alpha 1. So, that is it. Now, let me verify if this is correct. Correct. So, this is del V 1 del x times the modified x dot, if you may, the modified x dot, if you may. And del V 1 del xi times the current. So, this is what should be the case, I believe. This is what should be the case, I believe.

The modification is basically theta plus gamma del V 1 del theta transpose. It is now del V 1 del theta transpose. Is this quite correct, is what I am wondering. So, if I write this system out carefully I want to write it a little bit more carefully, x xi d d t is, and the drift term is f x plus g x xi and 0 plus the parameter term is F x 0 times theta plus the control term is 0 1 times u.

So, for this system if I want, so this is my sort of this F prime, and this is F prime, and this is sort of my, sorry, this is not quite right. Sort of my g prime. So, what I will have is everything remains the same but this gets modified. So, technically, this should be, so this should be, if I was to write this carefully, I have to be a little bit careful. I am sorry.

And this may not, so this is, this should be del V 1 del x times f x plus g x xi 0 plus F x, F x 0 times theta plus gamma del V 1 del theta transpose plus 0 1 alpha 1. And this should be multiplied not by just del V by del x but by del V 1 del x and del V 1 del xi. So, is this correct is what I am thinking.

Yes, this is what it should be because if you look at, I am just trying to make sure this is precise. I think it will come out to be the same expression, but I want to make sure this is the correct expression. So, what do I do, how do I get the modified system? I just take the original system and just in the term connected to the parameter, I make this change. And so then everything is multiplied by the del V a del x.

Now, in this case there are two states that V 1 depends on, x and xi. Therefore, there has to be, I apologize, therefore, there has to be del V 1 del x and del V 1 del xi. Correct. And the states, again the term in the theta is just modified everything else remains exactly the same, everything else remains exactly the same. So, I really hope you can all see this.

And this essentially, I believe boils down to exactly this guy, because this will give me del V 1 del x f x plus g x xi plus del V 1. So, yeah, yeah, essentially this, from here I, will get this. So, essentially, the same thing. Just that, this is how it is been obtained. So, this term has been obtained using this kind of calculation. I hope this much is clear. I wanted to make this more precise here.

So, so, basically, we want to show that this is an ACLF. So, now we evaluate the left-hand side more carefully. So, how do we do this? So, again, so this is a correct expression, this is correct. I was just not sure if, if all the terms are correct so I just wanted to write the original expression.

So, if you look at the left-hand side, what I can do is I can write partial of V 1 with respect to x and partial of V 1 with respect to xi in terms of the components using this expression right here. And that is what we do. So, del, so if you look at this expression here, from here you can write del V 1 del x is basically del V a del x. And is there anything else? Yeah, minus, sorry, plus z times, or actually minus z times del alpha del x.

And similarly, del V 1 del xi is equal to z times del, no del V 1 del xi is just z, is just z. I think that is correct. So, this I can substitute for here. So, this is del V 1 del x. And this is del V 1 del xi. This is del V 1 del x. So, that is what we do. And this is also del V 1 del x right here. Sorry, this is del V 1 del theta, I believe. So, we also have del V 1 del theta in the expression here.

So, del V 1 del theta is equal to what? Del V a del theta minus z del alpha del theta. Very similar. So, this expression and this are very similar. And del V 1 del xi is just z. So, all of this is just computed from this expression. And so, then we substitute all of that here. Once we make this substitution, we again know that some relation on del V on V a, and so we write that out.

So, we know that del V a del x plus f x, f x theta plus gamma del V a del theta transpose so I, I will tell you which terms we take. We take this guy along with this, this and this term, and this g x times xi is written as g x times z plus g x times alpha. So, we take this term. So, all of that goes here. And then we are left with these terms, the del V a del x along with this guy, this guy and this guy, that goes here.

Then I am left with the entire del z a del x terms, that is here. And this term is just taken as it is. This term is taken as it is. And now, what do I know? By our ACLF result I know that this quantity is of course minus W x theta. And then I have, again taking z common with this, I have alpha 1 minus all of this mess. So, I, so you can see that, the good thing you can see is that all the other terms contain z in it.

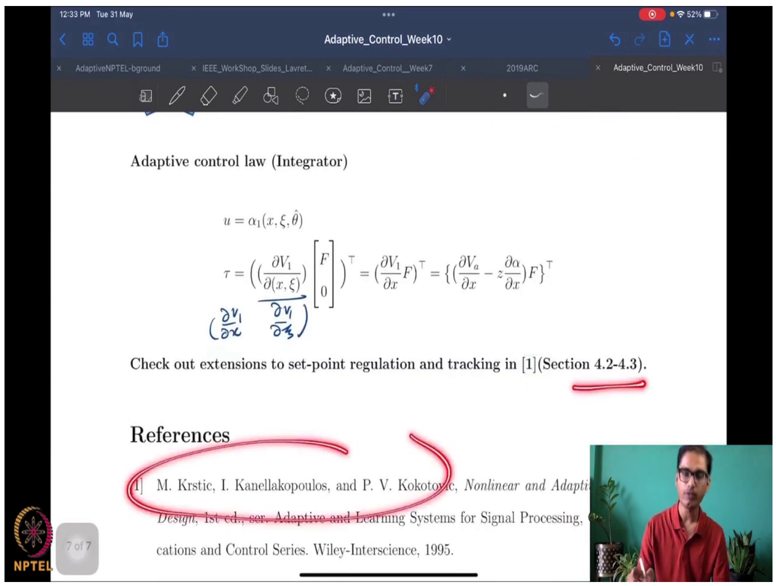
Apart from this guy in red, apart from this guy, everything else, there is a z here in the end, there is a z here in the end, there is a z here in the beginning and there is a z here. So, I can take the z common in all the other terms. That is what I do. And I get this big expression. So, now if I choose my alpha 1 to cancel all of this, because, again, remember as of now we are not talking about theta being unknown or anything like that. Theta is completely known.

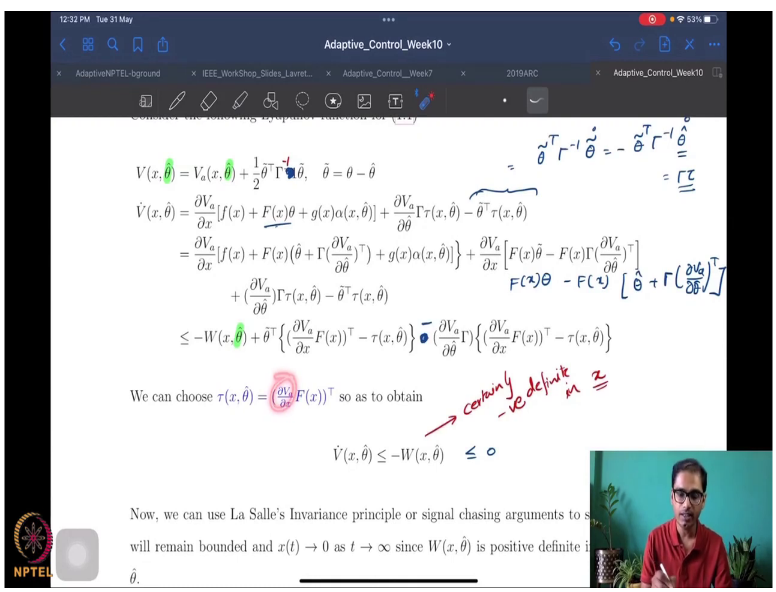
So, we can use theta. So, everything is known. So, I cancel all this mess which is what is all of this. And then I introduce a good term in the z. And once I can do that, I know that I am left with V 1 dot is minus W x theta minus z square. Which means what? This is also turns out to be negative definite for all theta in x comma z.

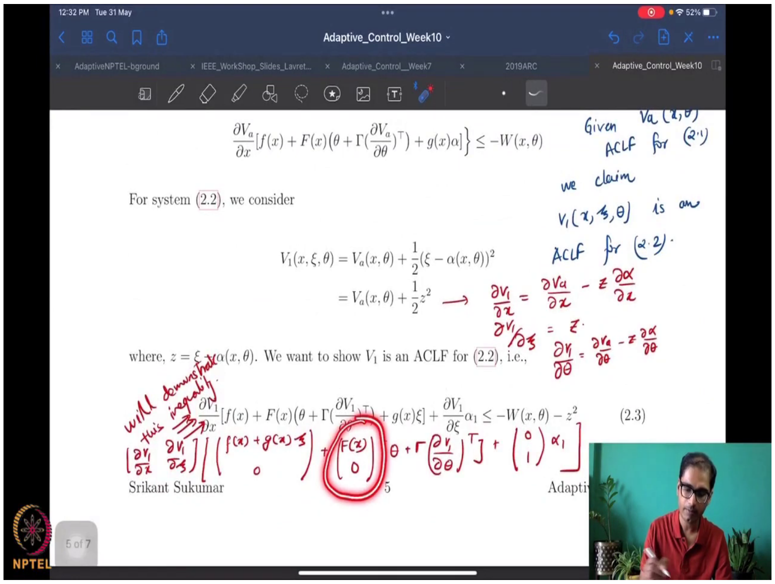
So, this is a ACLF because it is a CLF because if you got the derivative to be negative definite, it means that V 1 is a CLF for the modified system by choosing an appropriate alpha 1, of course. So, if you could choose a one alpha 1, you, you, when you take an infimum over all possible u, that is again smaller than, necessarily smaller than this, or necessarily not larger than this, at least, and therefore you have that V 1 is a CLF for this modified system. And if you, V 1 is a CLF for the modified system, it means its an ACLF for the original system.

Again, what was the original system? Original system was this guy. So, that is pretty great. We have essentially been able to prove that if the original, if the system 2.1 was globally adaptively asymptotically stabilizable, then so is this guy. Why? Because from the ACLF for this system 2.1, I could construct an ACLF for system 2.2 just by modifying the control, alpha 1 and choosing V 1 in a smart way from V a. And that is, that is great, that is essentially what you want. So, this is what you need in typical backstepping adaptive control also. So, just that we are, we now have a different sort of expression.

(Refer Slide Time: 24:11)







Now, what would be the control law. The control law is alpha 1, we just computed it, complicated. And what would be the tuning function? Now, this is interesting. The tuning function is, has a proper expression because we already have an expression for the tuning function. It is the partial of the ACLF times f. f is the term multiplying the unknown in the dynamics. In this case, term multiplying the unknown in the dynamics is this guy, and the ACLF is this guy.

So, what do you have for the tuning function? It is partial of V 1 with respect to both states, which is basically, this is the same as saying times F 0 term multiplying the unknown. And if you actually compute this, the second term goes to nothing. So, I will just have del V 1 del x F, and again I can expand del V 1 in this form. So, this is essentially my tuning function for this integrator system.

So, so if you want to look at extension to again set point regulation and tracking problems you have to look at the K. K. K. book. And this is the reference, section 4.2 and 4.3. So, this is essentially the tuning function design method. What we want to do is of course we want to look at some examples also in the subsequent sessions, that is the idea. So, let us, we will do that.

But, but just to summarize, what we did in this session is that we had started, we had already looked at ACLF ideas in the last session. In this session, we essentially wanted to look at the extension of the ACLF idea to backstepping, that is when I add an integrator layer. Then, how do you apply the ACLF idea is what we wanted to look at, and we did.

We actually proved that if you have an adaptive CLF for the original system, then if you add an integrator you can construct an adaptive CLF for the integrator system using the original ACLF itself. And we also showed how this is an ACLF by slightly modifying the feedback law. And of course, we can then construct a nice tuning function which will help us construct the parameter estimator also.

So, what we intend to do in the subsequent session is to take up the same example, if you remember we had taken up an example for an unmatched case, and we solved it using the adaptive integrator backstepping and also the extended matching method. Now, we want to solve the same problem using the tuning function method also. And so that is what we will do.

Of course, we will try to get the ACLF motivation from the earlier examples and so on and so forth. But yeah, essentially, we will, that is sort of the problem we want to work out. And because it is a vector problem, we are again hopeful that well, we can do a good job. So, that is what we will try. And we will try to solve that particular problem in the vector context, and we will try to design a tuning function based adaptive control law, and try to compare it with our other two case control adaptive control laws. So, I hope all of you will join me again in the next session, and I hope you enjoyed whatever we discussed today. So, these are one of the more advanced methods of adaptive control, and rather useful methods also. So, this is where we stop. Thank you.