Nonlinear Adaptive Control
Professor Srikant Sukumar
Systems and Control
Indian Institute of Technology, Bombay
Week 7
Lecture No: 40
Backstepping in Adaptive Control: Parameters Unmatched with Control
Part 2

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Hello, welcome to yet another session of our NPTEL on Nonlinear and Adaptive Control. I am Srikant Sukumar from Systems and Control IIT Bombay. So we are well into the seventh week of our course. And we have this very nice and interesting background image of this satellite from Space X orbiting the Earth. And we are well underway into learning how to desired algorithms that will drive systems such as these.

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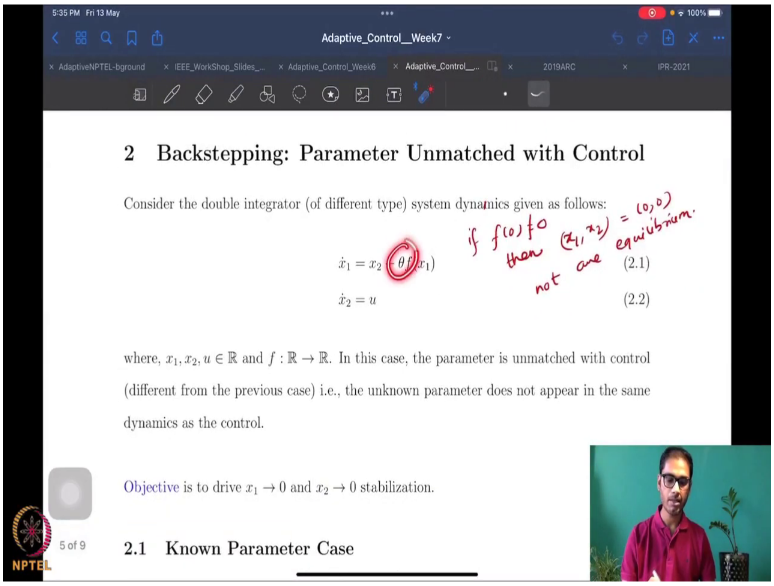

So, what we were doing last time was it we had started discussing the backstepping approach for the unmatched parameter case, in this case, what we have is that the unknown parameter appears in the state where there is no control. And we actually started with a known parameter case. So, in order to simplify the treatment, we just had stabilization as the objective that is, we just want both states to go to 0. But as we had stated last time, this does not really change anything, if you want the states to in fact, track a reference trajectory. So, we started off with the known parameter cases always.

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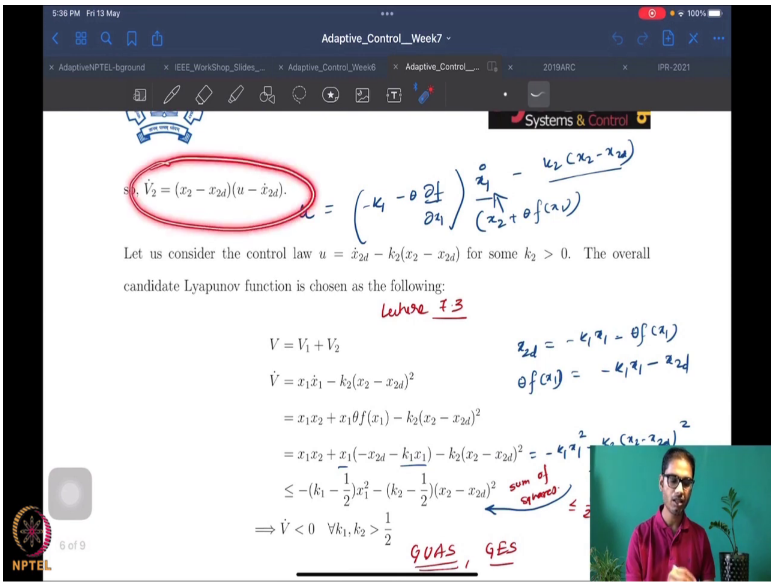

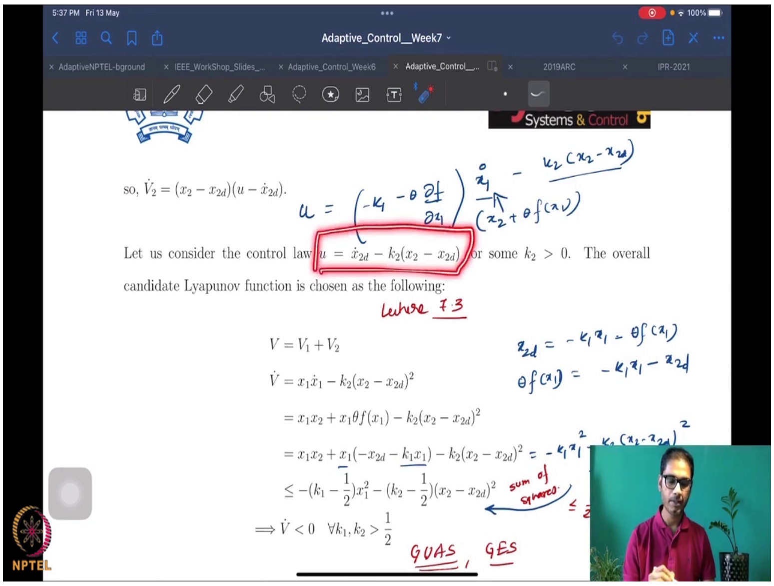

Adaptive_Control__Week7
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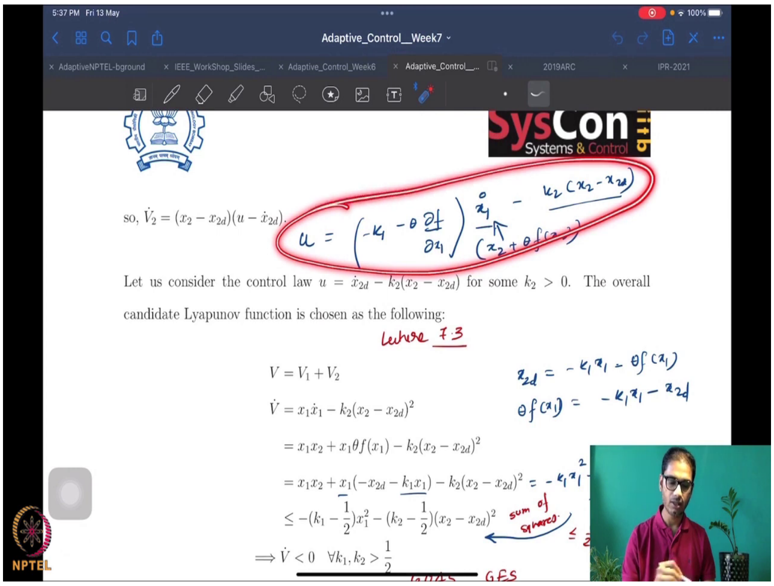
so, $\dot{V}_2 = (x_2 - x_{2d})(u - \dot{x}_{2d})$.
Let us consider the control law $u = \dot{x}_{2d} - k_2(x_2 - x_{2d})$ for some $k_2 > 0$. The overall
candidate Lyapunov function is chosen as the following:
$V = V_1 + V_2$
$\dot{V} = x_1\dot{x}_1 - k_2(x_2 - x_{2d})^2$
$= x_1x_2 + x_1\theta f(x_1) - k_2(x_2 - x_{2d})^2$
$= x_1x_2 + x_1(-x_{2d} - k_1x_1) - k_2(x_2 - x_{2d})^2$
$\leq -(k_1 - \frac{1}{2})x_1^2 - (k_2 - \frac{1}{2})(x_2 - x_{2d})^2$
$\implies \dot{V} < 0 \quad \forall k_1, k_2 > \frac{1}{2}$
GUAS, GES
NPTEL



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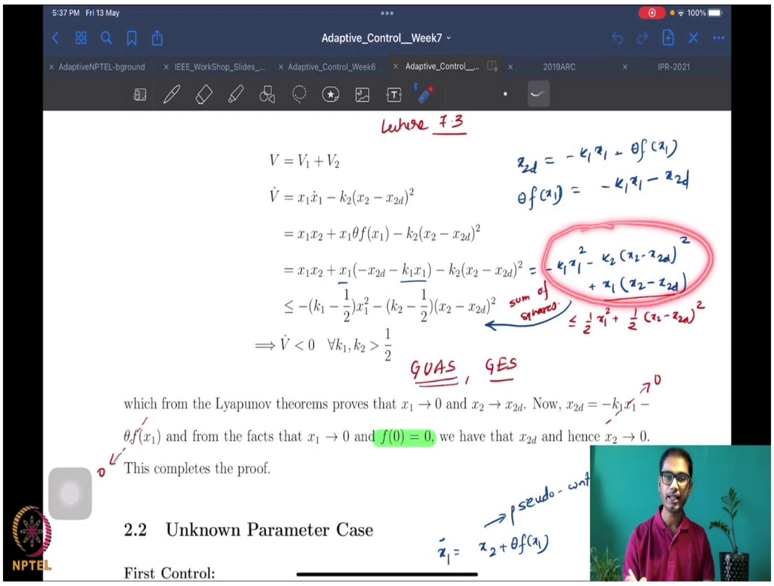

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GUAS, GES
which from the Lyapunov theorems proves that $x_1 \to 0$ and $x_2 \to x_{2d}$. Now, $x_{2d} = -k_1x_1 - \theta f(x_1)$ and from the facts that $x_1 \to 0$ and $f(0) = 0$, we have that x_{2d} and hence $x_2 \to 0$.
This completes the proof.
2.2 Unknown Parameter Case
First Control:
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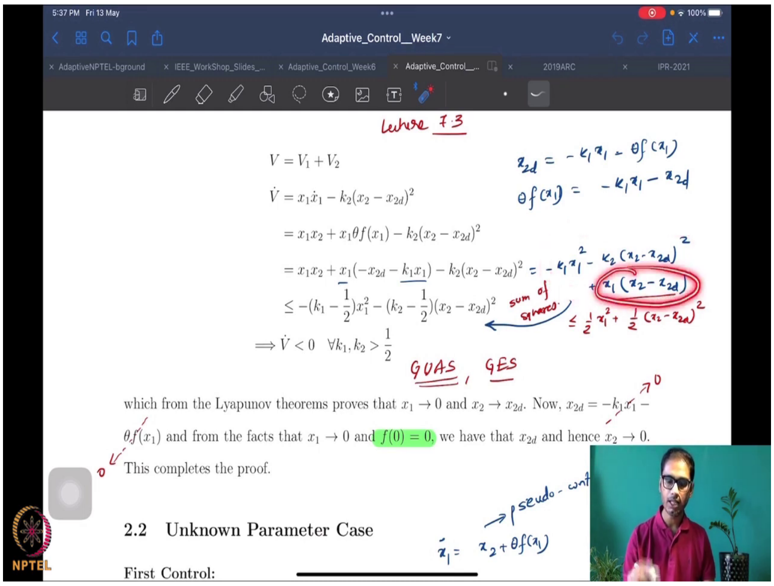

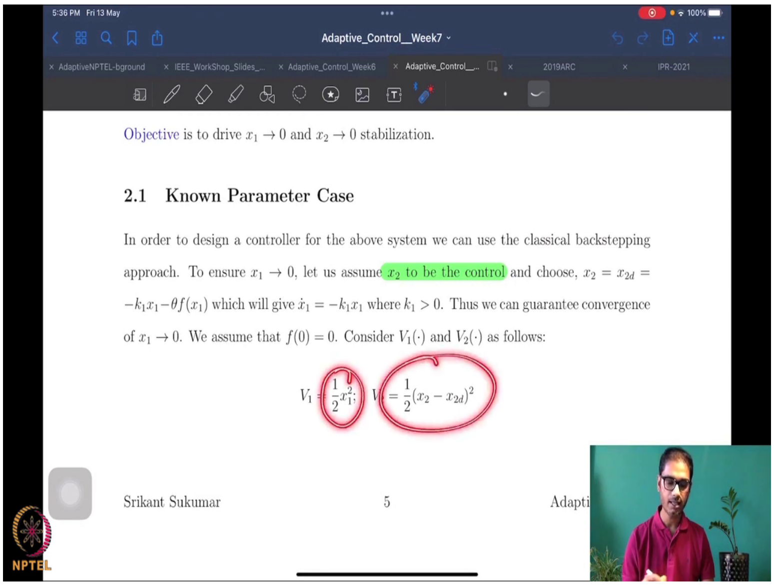

And what we did first was that in the known parameter case, so I am sorry, let me go back here. In the known parameter case, we of course constructed these 2 Lyapunov of candidate functions for the first state and the second state, where x2 desired is of course, the desired state corresponding to the first one.

So, basically, after we did this, we took a derivative and we are able to show that V2 dot looks something like this and for the known case, of course, we can prescribe this kind of a law, we can prescribe this kind of a law. And in fact, we in fact, also computed the derivative and that is we computed the derivative of x2 desired and the control expression looks something like this.

And we can see that because of x1 dot, there is a theta in the control expression also. But since theta is known, we are treating the known case. So, this is not a problem we implement this control and we can do our Lyapunov analysis with V equal to V1 plus V2 and then we are left with once we apply the control and we get to somehow this stage, we are actually left with the 2 nice negative quadratic terms and one mixed, and then we use the standard sum of squares method, which I hope by now, we have all understood very well.

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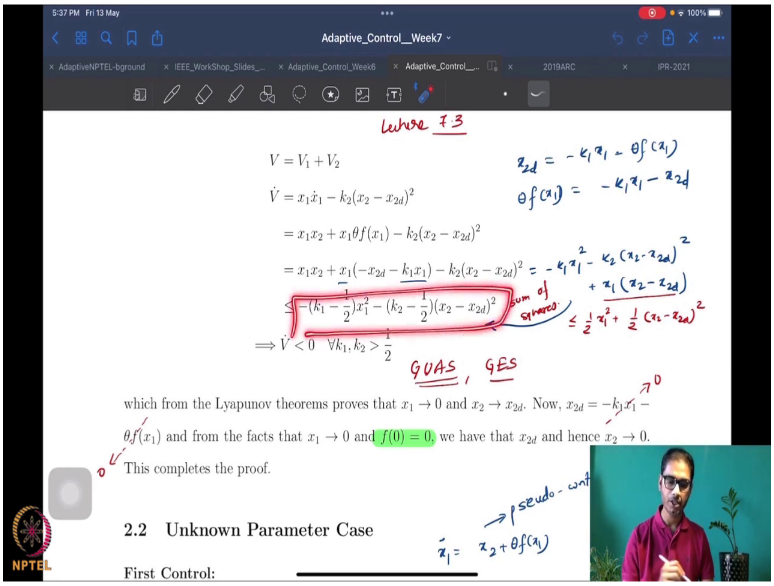

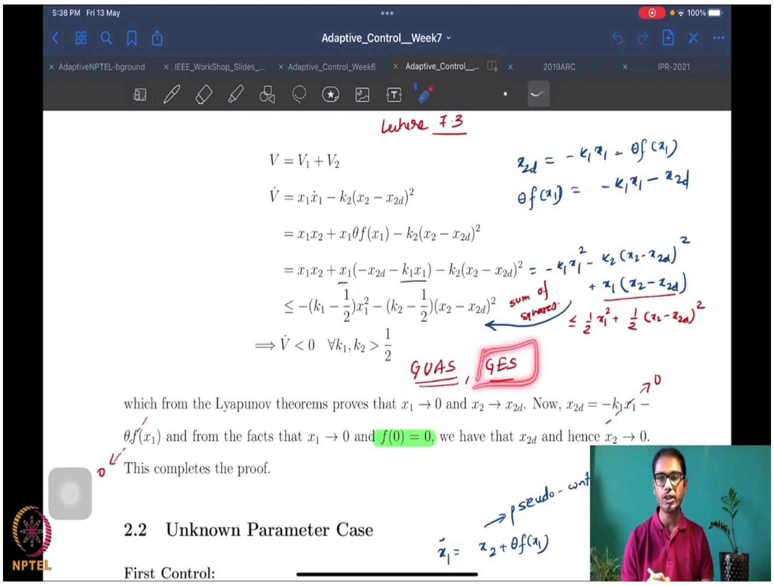

To get you this expression here. So, from inequality here, we get to an inequality here, and we can then claim that V dot is negative definite if k1 and k2 are greater than half. So, with this sort of analysis, of course, we can use the standard Lyapunov theorem to claim that all the states are stable at the origin and of course, reaching the origin asymptotically. So, that is global uniform asymptotic stability and in fact, in this kind of specific situation, global exponential stability.

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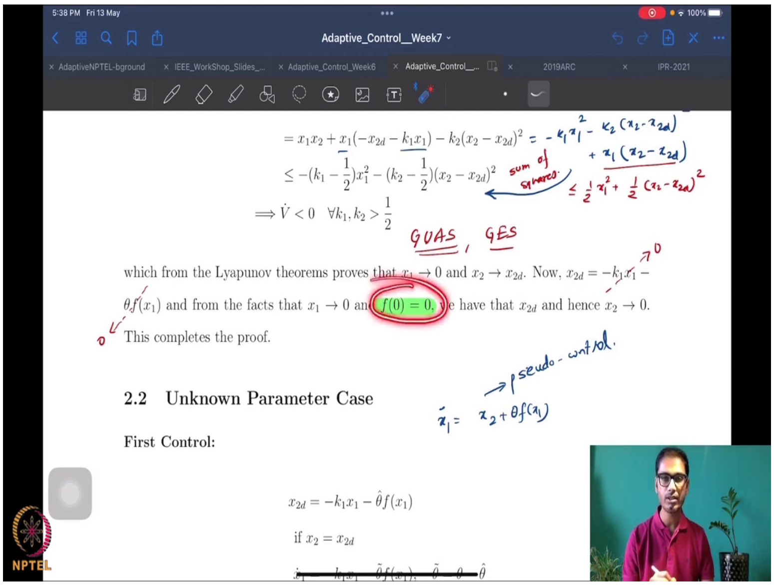

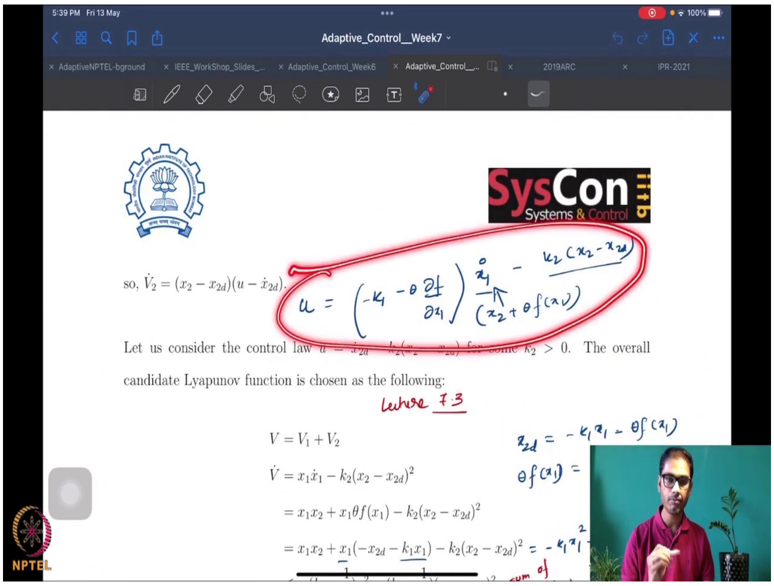

So, we of course, with this you can prove that the modified state that is the states of the backstepping, which is x1 goes to 0 and x2 goes to x2 desired and with the expression of x2 desired and the assumption that f 0 is 0, which we already said that was a reasonable assumption, you get that x1 also, sorry, x2 also goes to 0 because x2 desired also goes to 0.

Now, when we went to the unknown case, this sort of expression of the control created a problem. Because there is a theta there is a theta and so on and so forth. So, this is not the entire approach is not viable anymore. So we have to start with a different approach.

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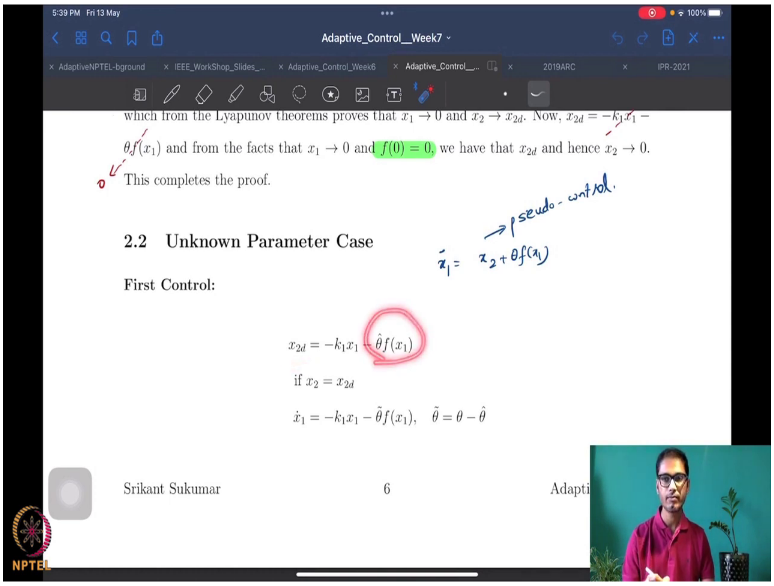

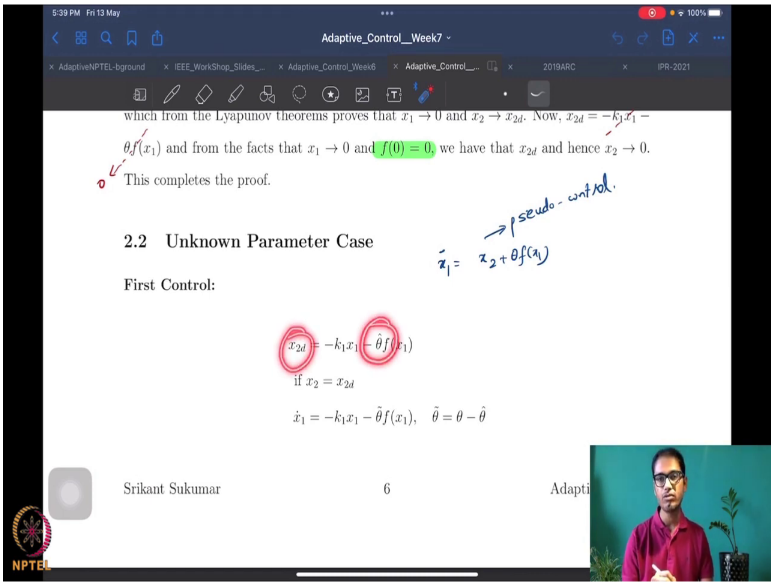

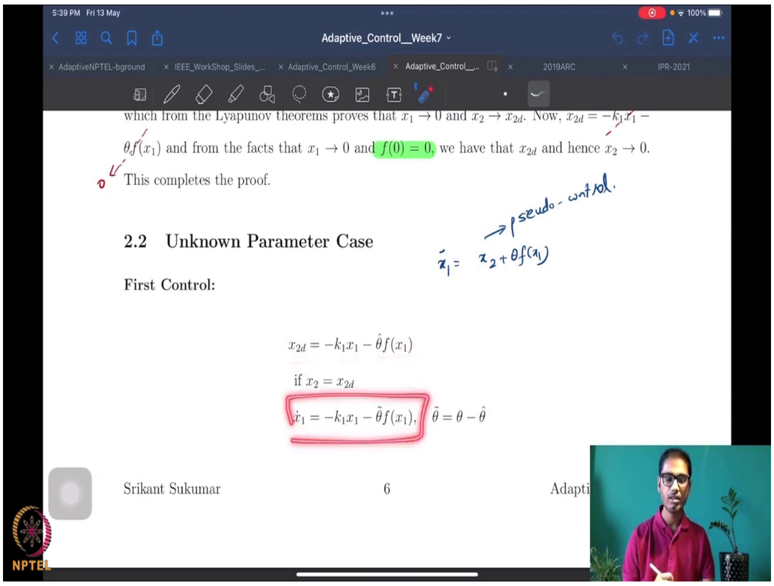

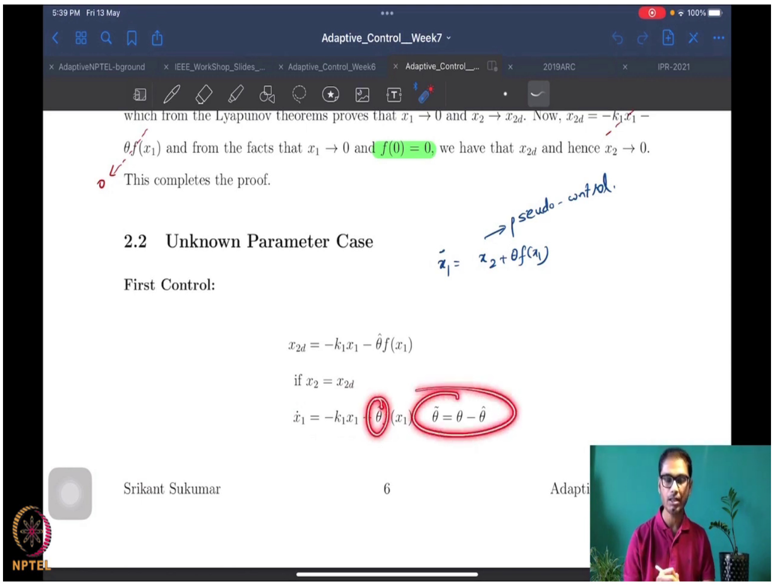

And what was that? We, in fact, specify the desired x2 with a theta hat. This is still certainty equivalence, but starting with the pseudo control itself, so x2 was the pseudo control. And at the pseudo control level itself. I have to use a estimate because that is where the unknown parameter in fact appears. And, so if x2 were in fact x2 desired, then you have this sort of a x1 dot equation, with theta tilde being the parameter error.

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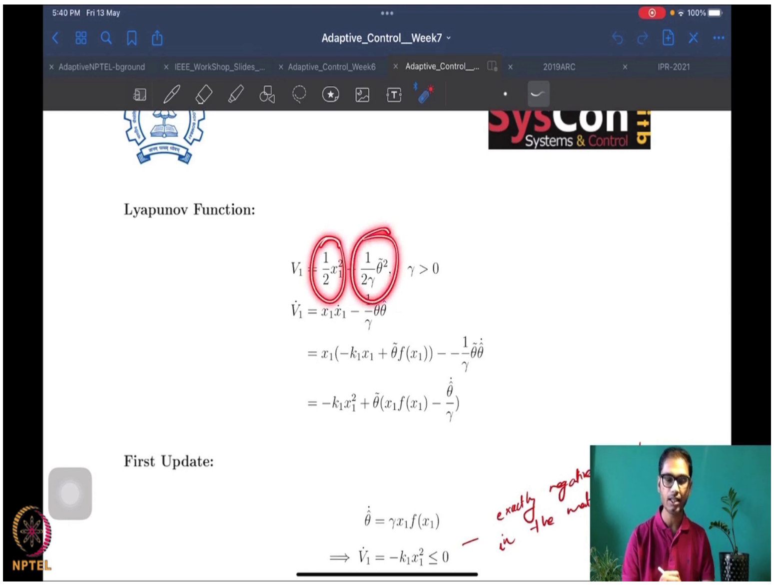
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Lyapunov Function:
$V_1 = \frac{1}{2}x_1^2 + \frac{1}{2\gamma}\tilde{\theta}^2, \quad \gamma > 0$
$\dot{V}_1 = x_1\dot{x}_1 - \frac{1}{\gamma}\tilde{\theta}\dot{\tilde{\theta}}$
$= x_1(-k_1x_1 + \tilde{\theta}f(x_1)) - \frac{1}{\gamma}\tilde{\theta}\dot{\tilde{\theta}}$
$= -k_1x_1^2 + \tilde{\theta}(x_1f(x_1) - \frac{\dot{\tilde{\theta}}}{\gamma})$
First Update:
$\dot{\tilde{\theta}} = \gamma x_1 f(x_1)$
$\implies \dot{V}_1 = -k_1x_1^2 \leq 0$
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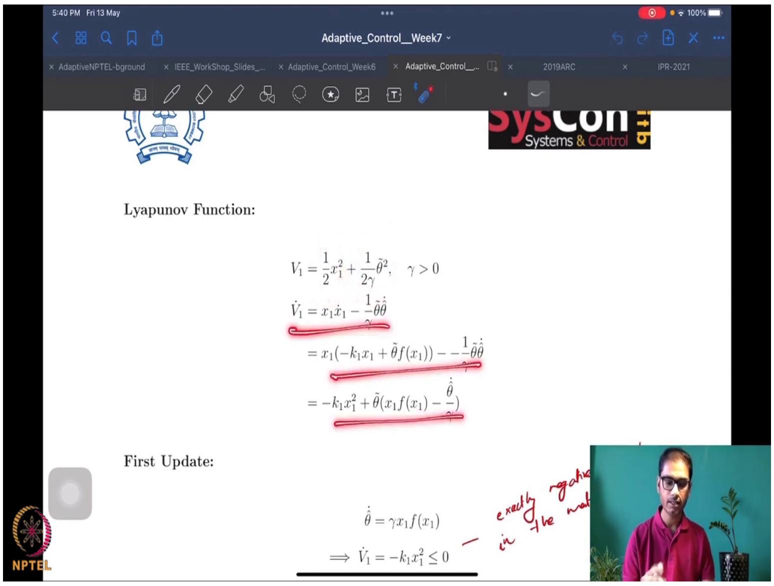
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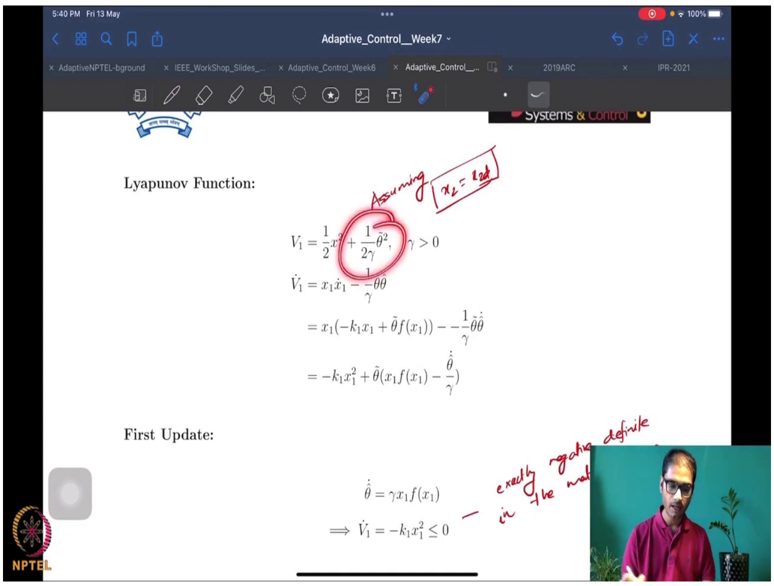
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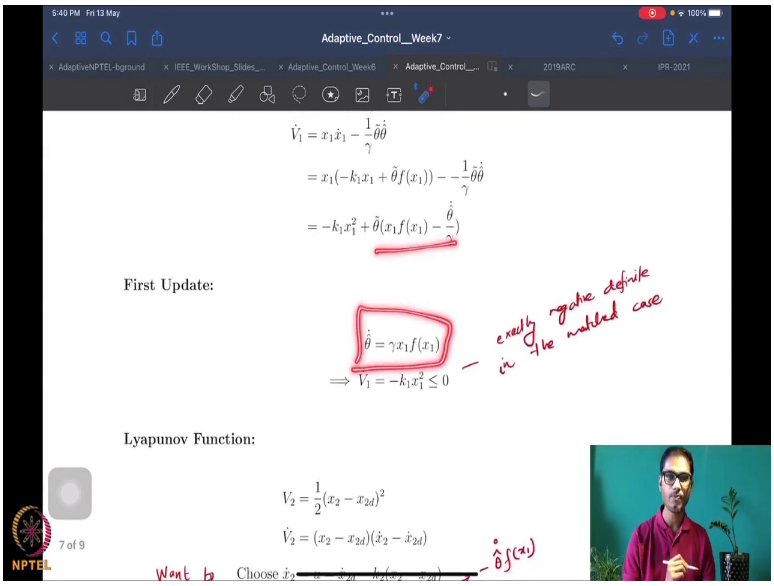

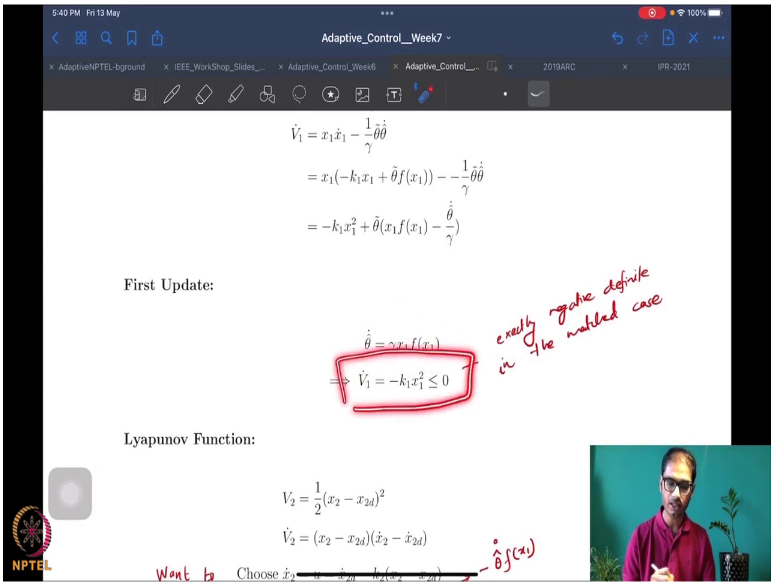

So now, we, of course, propose this kind of Lyapunov function for the first state. So the first state itself contains the theta tilde squared. And we do an analysis. This is again, assuming that x2 is exactly equal to x2 desired. So we already, this should be clear.

Assuming x2 is identical to x2 desired. So this is, of course, a big assumption. This is just a pseudo control not the real control. But of course, everything goes through because we choose a theta tilde, theta hat dot, which sort of cancels this term. So that is what we do. And then we get a V1 dot, which is negative semi definite. So on the other end of the known case, this was actually negative definite, correct. So here, we already have an unknown parameter. Therefore, the derivative is V1 dot is negative semi definite. Although the expression is again the same. All assuming that the pseudo control is in fact, the real control.

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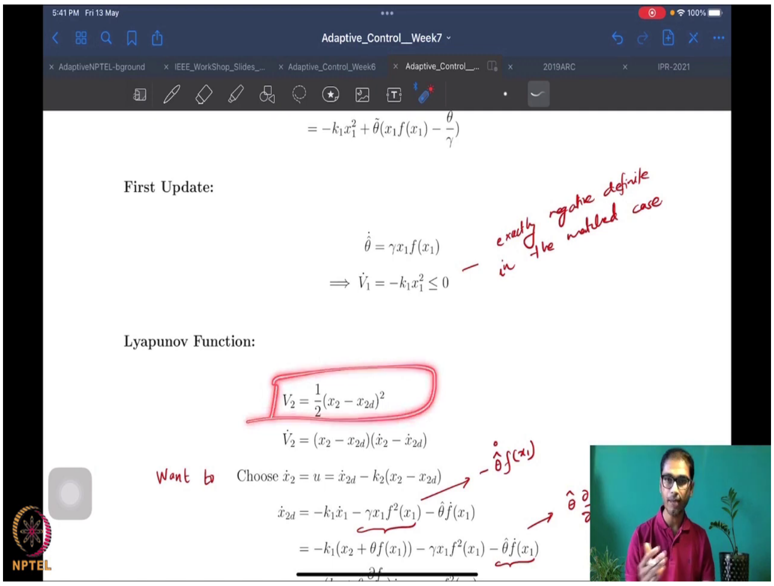

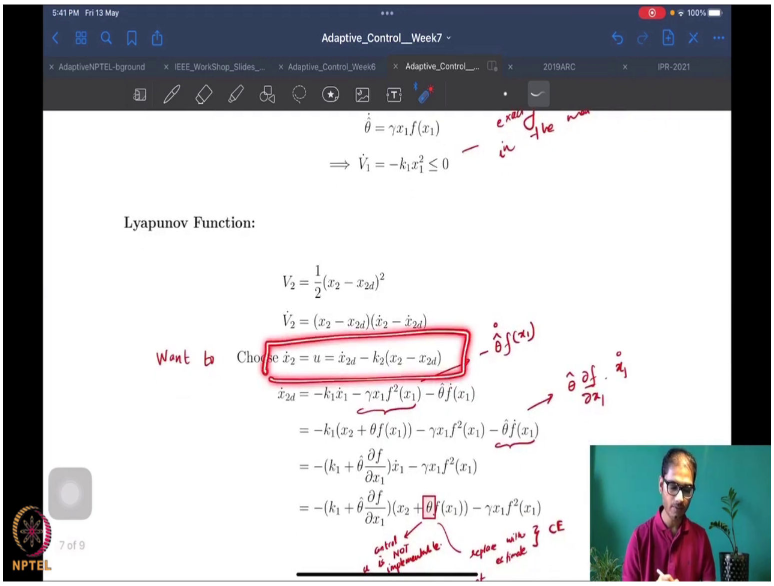

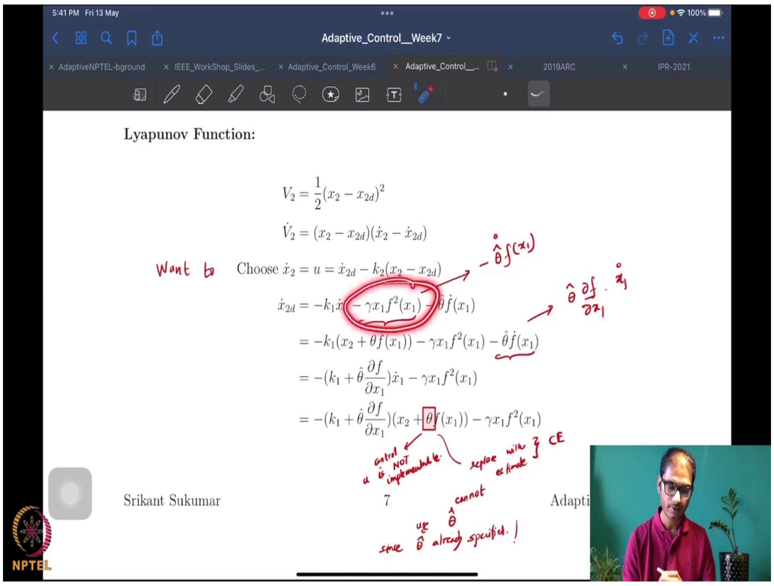
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$-\dot{\hat{\theta}} f(x_1)$
$\hat{\theta}\frac{\partial f}{\partial x_1}\cdot\dot{x}_1$
control u is NOT implementable
replace with CE
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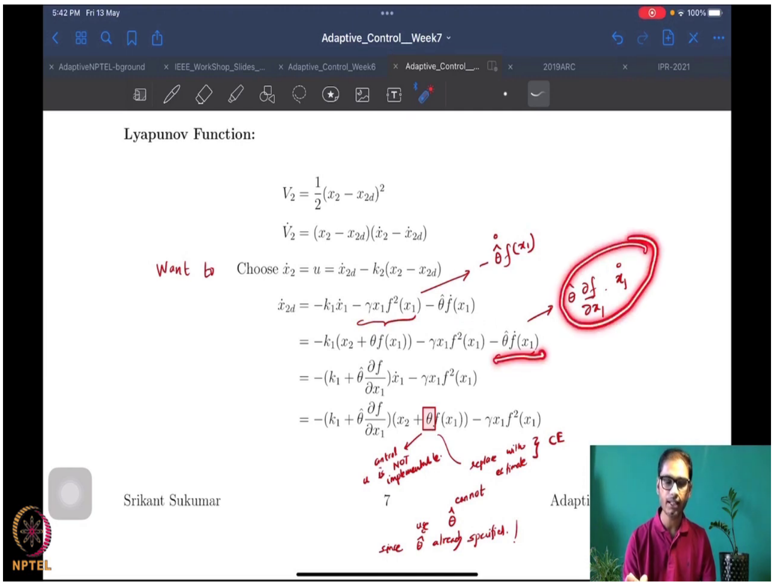
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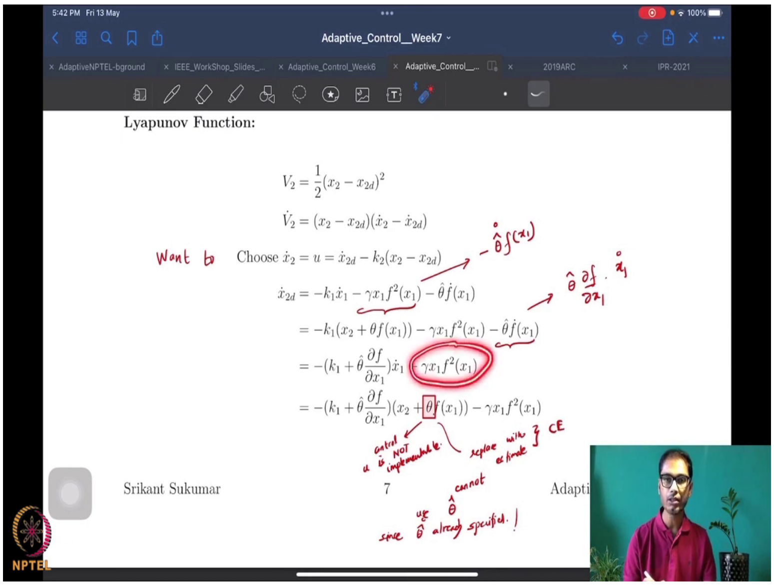
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Then we go to the second state, where V2 is the same, we have not changed V2, so V1 changes, but V2 has not changed yet, I would use the word yet. We get V2 dot to be the same in the control, we want to prescribe the same.

So, we want to choose, but then what happens we compute the expression. So now, the expression is slightly different, of course, because x2 desired contains theta hat and not theta itself.

So, let us look at x2 desired dot, so this expression is basically theta hat dot f x1. And so basically, it is minus k1 x1 minus theta hat f x1, so it is just the derivative of minus k1 x1 which is minus k1 x2 and derivative of theta hat f x1 using the product rule. So that is what we get 2 terms, and the derivative of the first with respect to the first term and then a derivative with respect to the second term.

When we take the derivative with respect to the second term, you again get an x1 dot. We already had an x1 dot here, and we get an x1 dot here too. And that is what we club this and this gets clubbed here this and this term gets clubbed to give me this term, and this is of course, this term, so these are the 2 distinct terms. Now it is easy to see that this is of course, an implementable term, no problem, but then x1 dot contains the unknown again.

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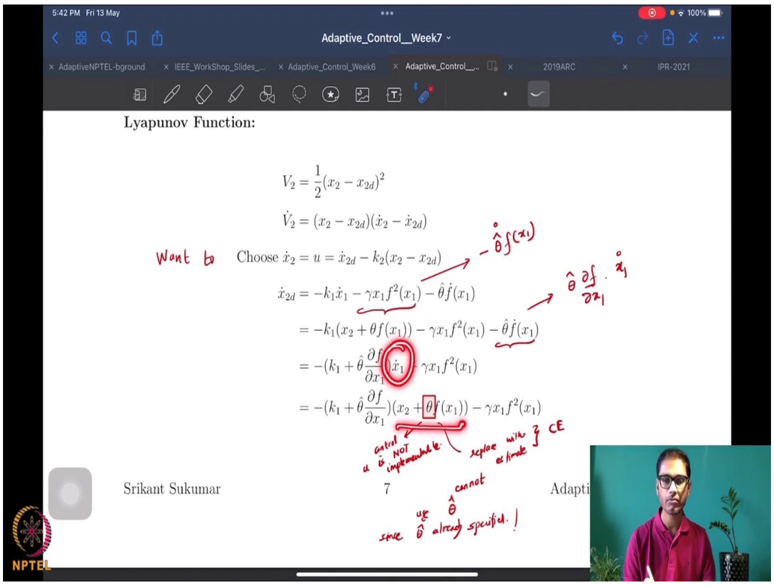

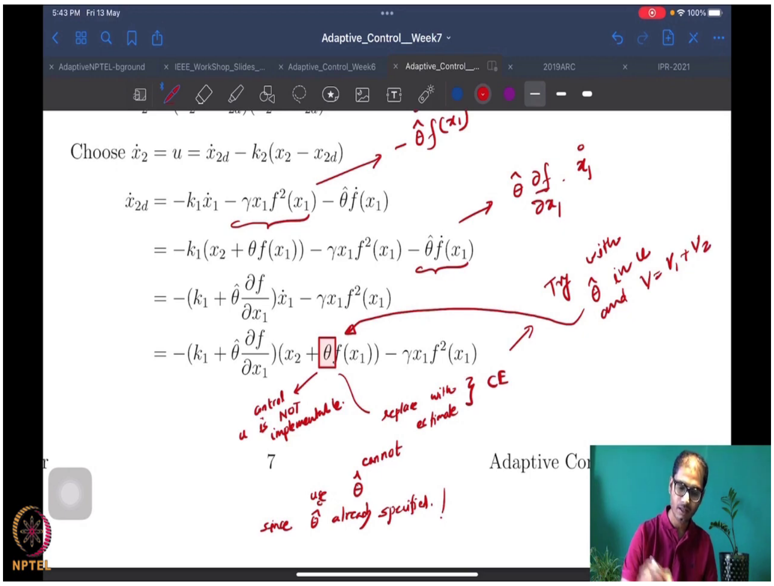

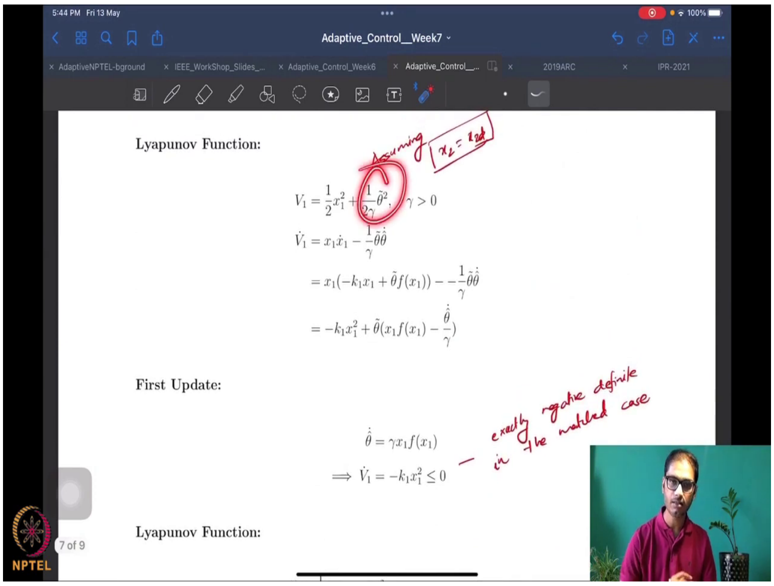

So, this is again implementable, but the x1 dot, which is this guy contains the unknown theta, So, now this control is still not implemented. So, what we choose as control u, which is the actual control is not implementable. So, of course, we want to replace its CE estimate, now, we cannot replace it with a theta hat, I encourage you to do this, like try this erroneous step of trying to replace this with theta hat itself. And then carrying out Lyapunov analysis.

How would you carry out a Lyapunov analysis? You would simply I would say, So, what I want to say is try with theta hat in u here, and V equals V1 plus V2. So, and because, of course, you remember that V already contains, a theta V1 already contains a theta tilde term.

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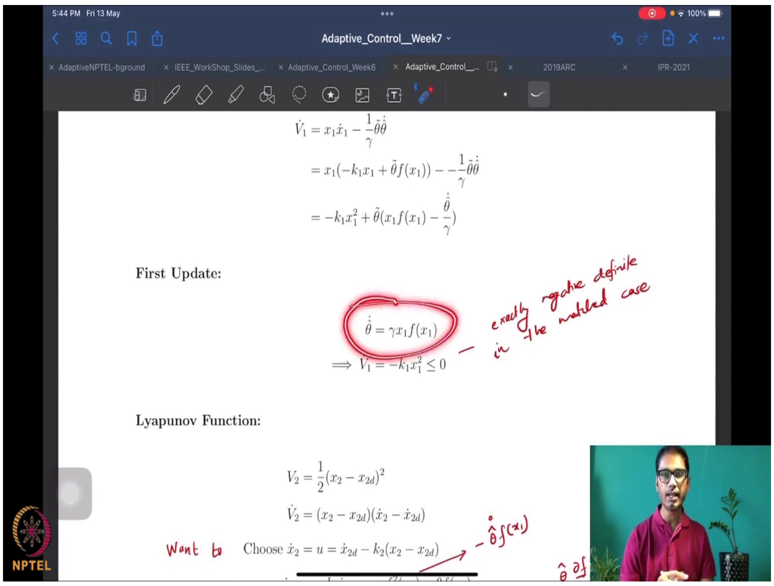

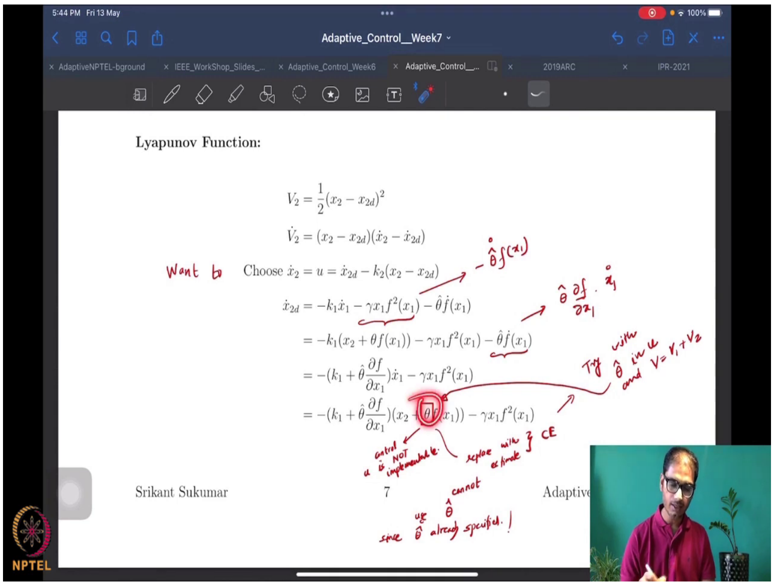

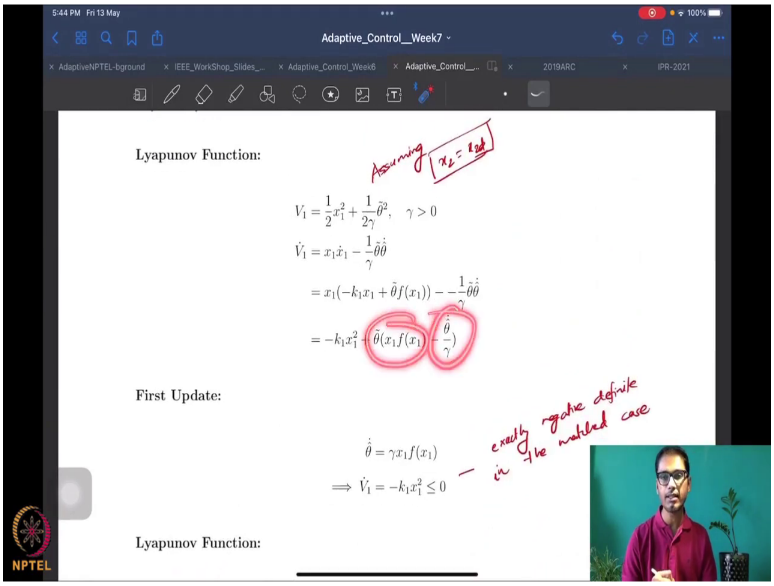

So, what is the problem, the problem that occurs is that you already chosen a theta hat dot. And because of that, you will see that this term will create a theta tilde term of course, because you instead of giving a putting a theta here, we will put a theta hat and this creates a theta tilde term, which you cannot cancel anymore, because the theta tilde term you got already cancelled by the theta hat dot earlier. So, there is no more terms remaining to be used to cancel this error.

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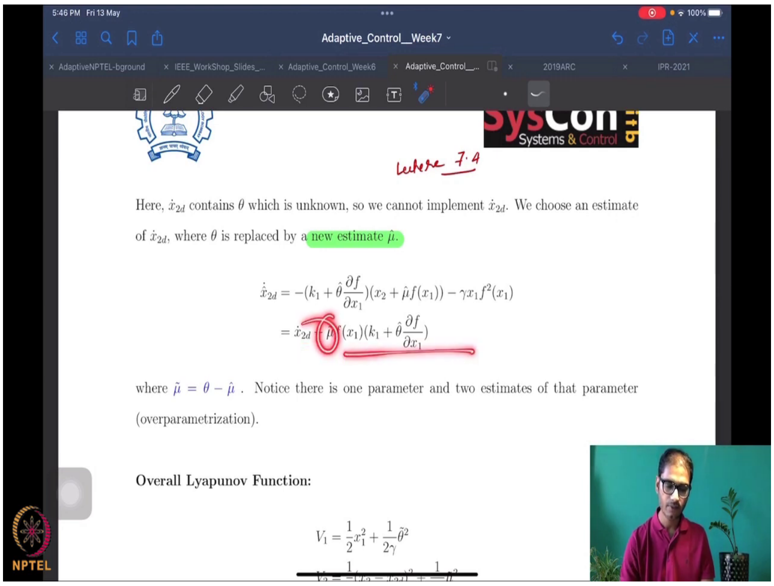

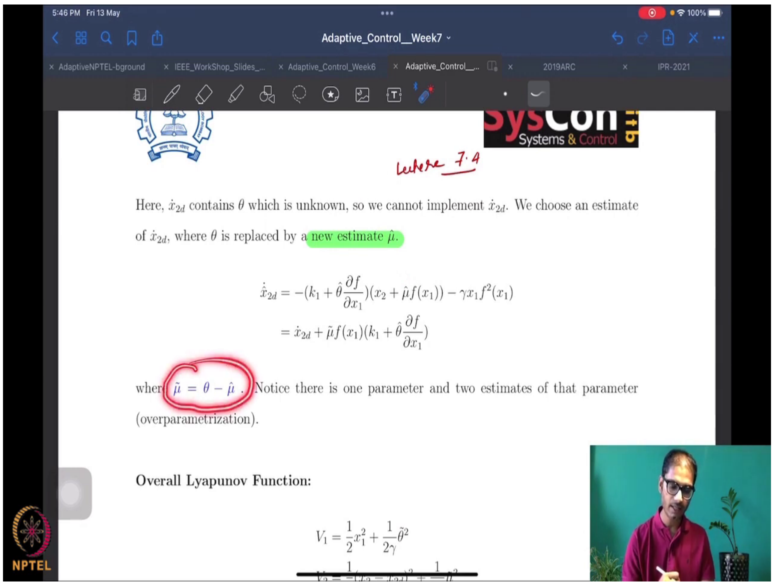

So, how do we resolve this issue? That is where we come to in lecture. In today's lecture, which is lecture 7.4. I know this was a rather longish introduction into the previous lecture, but since this is slightly more involved, I wanted to repeat what we did. So the steps are simple, they may be confusing for you in the beginning, but if you revise it, it is actually simple steps. Just a little bit more bookkeeping, all we are doing a little bit more careful bookkeeping here.

So, let us look at this. So, x2 desired dot contains a, theta of course, we cannot implement this. So, we replace theta with a new estimate. Because like I said the old estimate and please feel free to try in fact, I encourage you to try the old estimate, you will always be left with one term which in theta tilde, which you cannot cancel anymore, because you are left with no more terms from the V1 dot that we can be used to cancel because you already chose a theta hat.

So, we use a new estimate mu is, so mu hat. So, the new estimate mu hat replaces the theta. And of course, when you basically try to write it in terms of the earlier x2 desired, so, we call it x2 desired hat, x2 desired hat dot, if you replace in terms of earlier x2 desired, this becomes x2 desired dot plus this mu tilde times this term. So, it is just because there was a theta here. So, there was a theta here, instead of the theta we put in the mu hat and mu tilde is defined as theta minus mu hat. So, this is simply that term. So this term multiplied by this term mu tilde. So, this was a real x2 desired dot.

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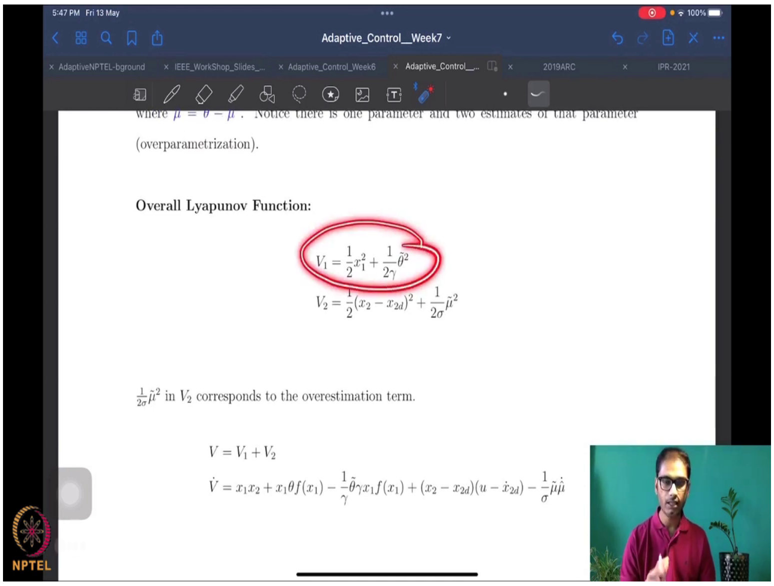
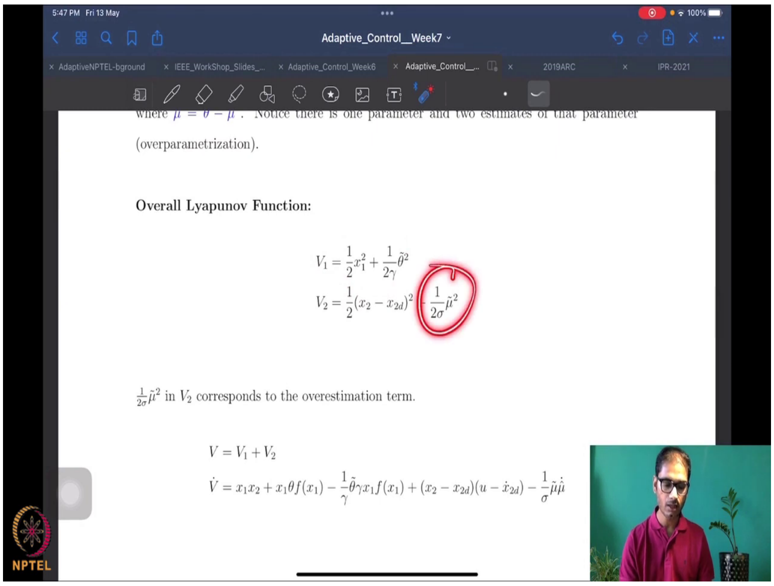
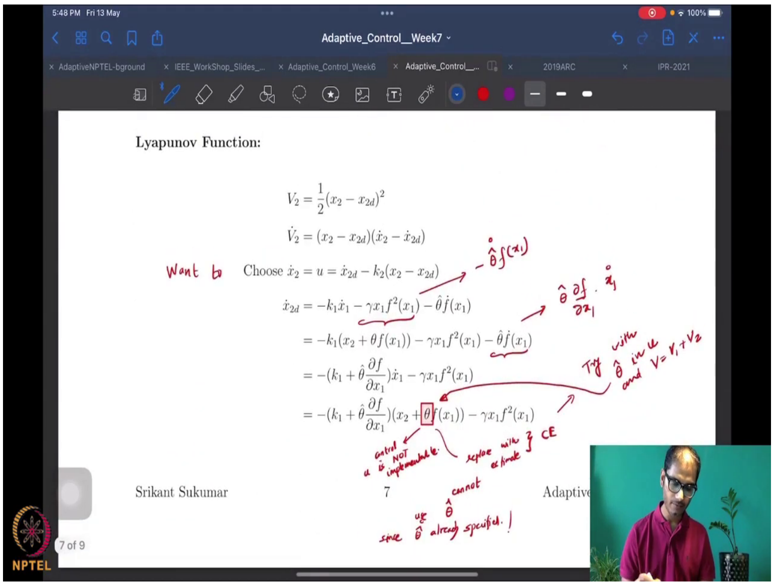
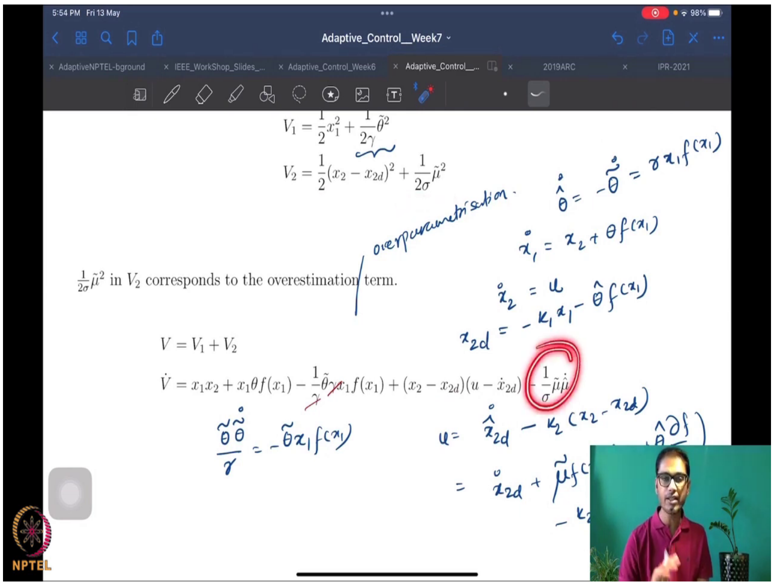
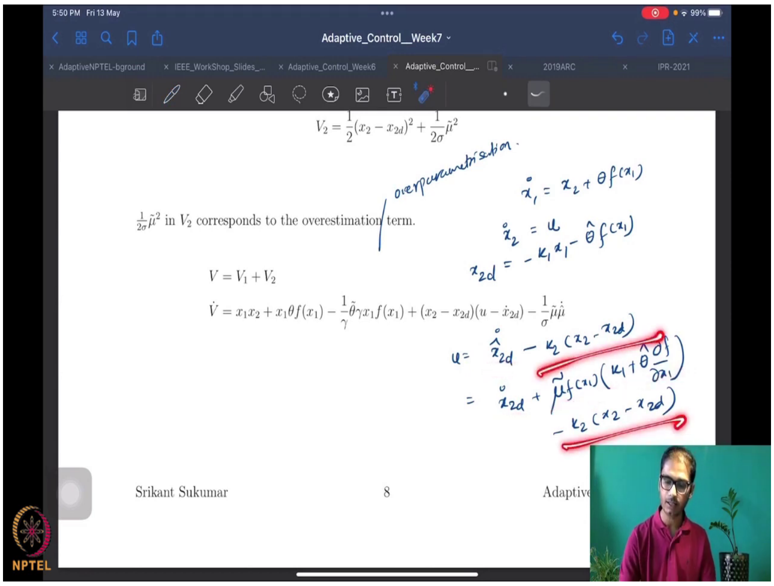
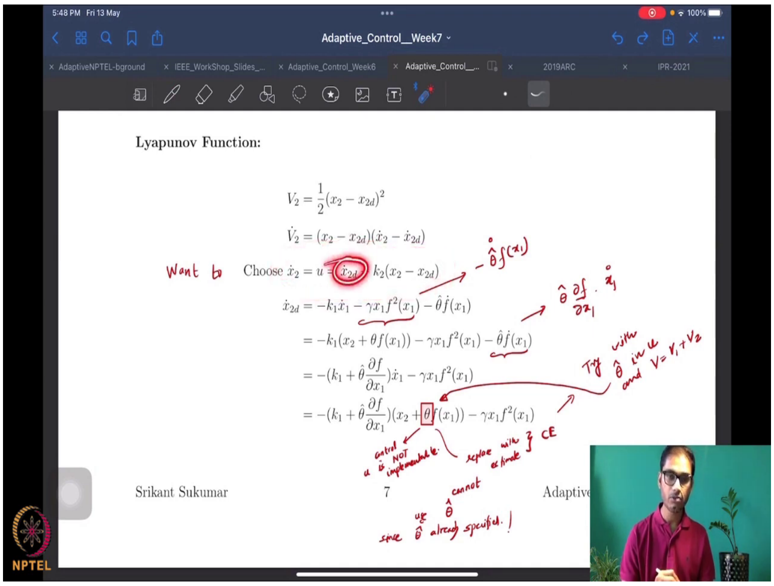





Adaptive_Control__Week7
Lyapunov Function:
$V_2 = \frac{1}{2}(x_2 - x_{2d})^2$
$\dot{V}_2 = (x_2 - x_{2d})(\dot{x}_2 - \dot{x}_{2d})$
Want to
Choose $\dot{x}_2 = u = \dot{x}_{2d} - k_2(x_2 - x_{2d})$
$\dot{x}_{2d} = -k_1\dot{x}_1 - \gamma x_1 f^2(x_1) - \hat{\theta}\dot{f}(x_1)$
$= -k_1(x_2 + \theta f(x_1)) - \gamma x_1 f^2(x_1) - \hat{\theta}\dot{f}(x_1)$
$= -(k_1 + \hat{\theta}\frac{\partial f}{\partial x_1})\dot{x}_1 - \gamma x_1 f^2(x_1)$
$= -(k_1 + \hat{\theta}\frac{\partial f}{\partial x_1})(x_2 + \theta f(x_1)) - \gamma x_1 f^2(x_1)$
Srikant Sukumar
7
Adaptive
7 of 9
NPTEL


Adaptive_Control__Week7
$V_1 = \frac{1}{2}x_1^2 + \frac{1}{2\gamma}\tilde{\theta}^2$
$V_2 = \frac{1}{2}(x_2 - x_{2d})^2 + \frac{1}{2\sigma}\tilde{\mu}^2$
$\frac{1}{2\sigma}\tilde{\mu}^2$ in V_2 corresponds to the overestimation term.
overparametrization
$V = V_1 + V_2$
$\dot{V} = x_1x_2 + x_1\theta f(x_1) - \frac{1}{\gamma}\tilde{\theta}\gamma x_1 f(x_1) + (x_2 - x_{2d})(u - \dot{x}_{2d}) - \frac{1}{\sigma}\tilde{\mu}\dot{\hat{\mu}}$
NPTEL


Adaptive_Control__Week7
$V_2 = \frac{1}{2}(x_2 - x_{2d})^2 + \frac{1}{2\sigma}\tilde{\mu}^2$
$\frac{1}{2\sigma}\tilde{\mu}^2$ in V_2 corresponds to the overestimation term.
overparametrization
$V = V_1 + V_2$
$\dot{V} = x_1x_2 + x_1\theta f(x_1) - \frac{1}{\gamma}\tilde{\theta}\gamma x_1 f(x_1) + (x_2 - x_{2d})(u - \dot{x}_{2d}) - \frac{1}{\sigma}\tilde{\mu}\dot{\hat{\mu}}$
Srikant Sukumar
8
Adaptive
NPTEL



So now, we choose a different V2. And instead of choosing it as just this term, we add a term corresponding to the parameter error. So for the same parameter, what do we want to do? We are creating another Lyapunov candidate function. So, for this special unknown case situation, we are creating a different Lyapunov function, create different Lyapunov function.

Now, this was already there, nothing new here, the new term is here. Why? Because we introduce a new estimate for the same parameter theta. So, this is what is mentioned here, 1 over 2 sigma mu delta square in V2 corresponds to the over estimation term. So, this is known as overestimation term, so or over parameterization. So, we also use the term over parameterization. What does it mean? It means that I use the multiple estimates for the same parameter. So, this is called over parameterization. So, now we carefully do this analysis, let us carefully do analysis.

Let us look at V as V1 plus V2 and compute the V dot. Now, remember earlier V1 dot was computed assuming that x2 is in fact equal to x2 desired. Now, we cannot do that. So, I am going to write what is x1 dot? x1 dot is x2 plus theta f x1 and x2 dot is u and what did we choose as u, u was, let us see, u is this, u is not exactly this guy, u is not this guy. u is in fact, x2 hat desired dot.

So, basically let me carefully write all the terms here. So, that we have everything at our disposal, everything at our disposal. So, we have x2 desired as equal to minus k1 x1 minus theta hat f x1 and we have u is equal to x2 desired hat dot minus k2 x2 minus x2 desired minus k2 x2 minus x2 desired and this x2 hat desired dot is nothing but x2 desired dot plus mu tilde.

I am simply copying from here f x1 k1 plus theta cap del f, del x1. So, all these things. We have several ingredients here. So, we have several ingredients here I have copied all of them. So, u is in terms of the x2 hat desired dot and x2 hat desired dot is in terms of x2 desired dot. So, this is minus k2 x2 minus x2 desired. So, this x2 hat desired, this guy and minus k2 x2 minus x2 desired is this guy, x2 desired itself is this.

So, now, I take the derivatives diligently complete the analysis. So, V1 dot has x1 x1 dot which is x1 x2 and this term which is, I forgot to write 1 which is I believe the theta hat dot which is gamma x1 f x1. So, theta hat dot, theta hat dot equals minus theta tilde dot equals gamma x1 f x1.

So, gamma x1, f x1, so, this is what we have, So, V1 dot is x1 x1 dot which is x1 x2 and this term gives me theta tilde, theta tilde dot by gamma which is theta tilde times x1, f x1. So, this is actually theta tilde times f1, x1 f x1 this is what I get from V1 dot. Let us see I am trying to wonder, wait a second this is theta, so, I believe this term gives me let me write this out this term gives me in V dot theta tilde, theta tilde dot divided by gamma and I think we are fine here, since just theta theta tilde dot is minus theta x1, f x1 minus theta tilde so this should be equal to minus theta tilde x1, f x1.

Let us see what is there in the next step x1, x1 dot actually gives me x1 x2 plus x1 theta f x1 this is correct. And this term gives me. So, this is where there is a fault, this should be, not, this should not be there minus theta tilde. This actually cancels out. So, these 2 terms actually cancel out, this is fine. This is absolutely. This is written in a more complicated way than I expected.

So, the first 2 terms are just x1 x1 dot and this term is theta tilde, theta tilde dot this term is of course, from the second term x2 minus x2 desired u minus x2 dot from here and the last term is this. So, this is the only piece that we have not selected yet. So this is absolutely okay.

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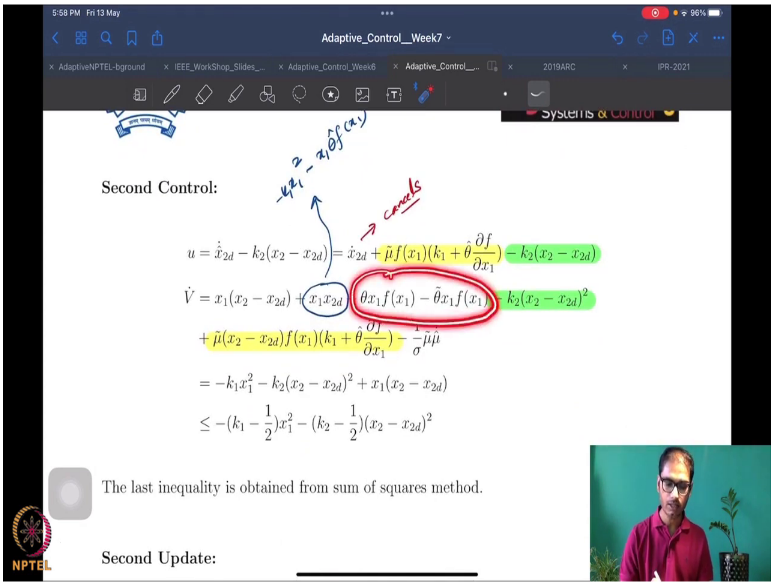
Adaptive_Control__Week7
Second Control:
$u = \dot{x}_{2d} - k_2(x_2 - x_{2d}) = \dot{x}_{2d} + \tilde{\mu} f(x_1)(k_1 + \hat{\theta}\frac{\partial f}{\partial x_1}) - k_2(x_2 - x_{2d})$
$\dot{V} = x_1(x_2 - x_{2d}) + x_1 x_{2d} + \theta x_1 f(x_1) - \tilde{\theta} x_1 f(x_1) - k_2(x_2 - x_{2d})^2$
$+ \tilde{\mu}(x_2 - x_{2d}) f(x_1)(k_1 + \hat{\theta}\frac{\partial f}{\partial x_1}) - \frac{1}{\sigma}\tilde{\mu}\dot{\hat{\mu}}$
$= -k_1 x_1^2 - k_2(x_2 - x_{2d})^2 + x_1(x_2 - x_{2d})$
$\leq -(k_1 - \frac{1}{2})x_1^2 - (k_2 - \frac{1}{2})(x_2 - x_{2d})^2$
The last inequality is obtained from sum of squares method.
Second Update:
cancels
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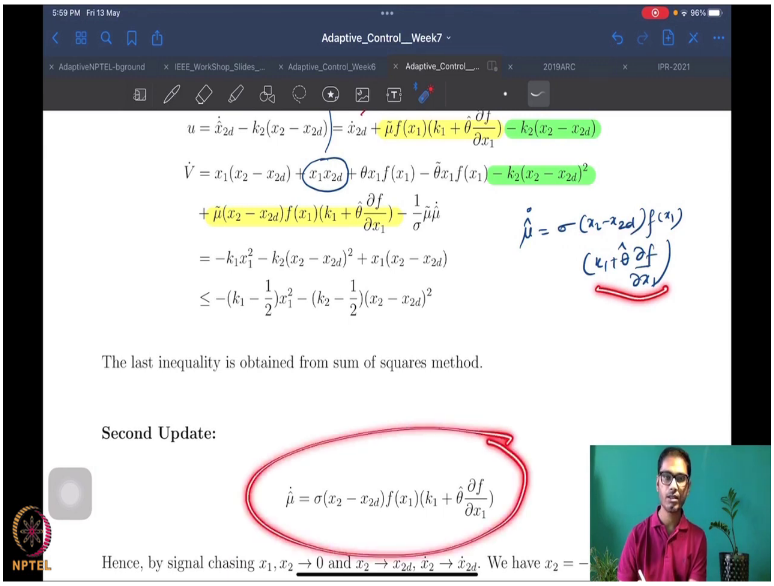
Adaptive_Control__Week7
$u = \dot{x}_{2d} - k_2(x_2 - x_{2d}) = \dot{x}_{2d} + \tilde{\mu} f(x_1)(k_1 + \hat{\theta}\frac{\partial f}{\partial x_1}) - k_2(x_2 - x_{2d})$
$\dot{V} = x_1(x_2 - x_{2d}) + x_1 x_{2d} + \theta x_1 f(x_1) - \tilde{\theta} x_1 f(x_1) - k_2(x_2 - x_{2d})^2$
$+ \tilde{\mu}(x_2 - x_{2d}) f(x_1)(k_1 + \hat{\theta}\frac{\partial f}{\partial x_1}) - \frac{1}{\sigma}\tilde{\mu}\dot{\hat{\mu}}$
$= -k_1 x_1^2 - k_2(x_2 - x_{2d})^2 + x_1(x_2 - x_{2d})$
$\leq -(k_1 - \frac{1}{2})x_1^2 - (k_2 - \frac{1}{2})(x_2 - x_{2d})^2$
The last inequality is obtained from sum of squares method.
Second Update:
$\dot{\hat{\mu}} = \sigma(x_2 - x_{2d}) f(x_1)(k_1 + \hat{\theta}\frac{\partial f}{\partial x_1})$
Hence, by signal chasing $x_1, x_2 \to 0$ and $x_2 \to x_{2d}$, $\dot{x}_2 \to \dot{x}_{2d}$. We have $x_2 = -$
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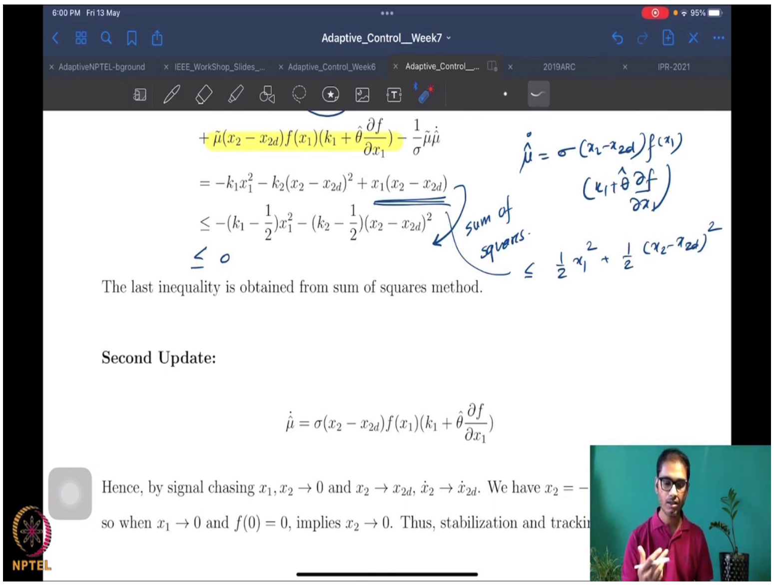
Adaptive_Control__Week7
$+ \tilde{\mu}(x_2 - x_{2d}) f(x_1)(k_1 + \hat{\theta}\frac{\partial f}{\partial x_1}) - \frac{1}{\sigma}\tilde{\mu}\dot{\hat{\mu}}$
$= -k_1 x_1^2 - k_2(x_2 - x_{2d})^2 + x_1(x_2 - x_{2d})$
$\leq -(k_1 - \frac{1}{2})x_1^2 - (k_2 - \frac{1}{2})(x_2 - x_{2d})^2$
≤ 0
sum of squares
$\leq \frac{1}{2}x_1^2 + \frac{1}{2}(x_2 - x_{2d})^2$
The last inequality is obtained from sum of squares method.
Second Update:
$\dot{\hat{\mu}} = \sigma(x_2 - x_{2d}) f(x_1)(k_1 + \hat{\theta}\frac{\partial f}{\partial x_1})$
Hence, by signal chasing $x_1, x_2 \to 0$ and $x_2 \to x_{2d}$, $\dot{x}_2 \to \dot{x}_{2d}$. We have $x_2 = -$
so when $x_1 \to 0$ and $f(0) = 0$, implies $x_2 \to 0$. Thus, stabilization and trackin
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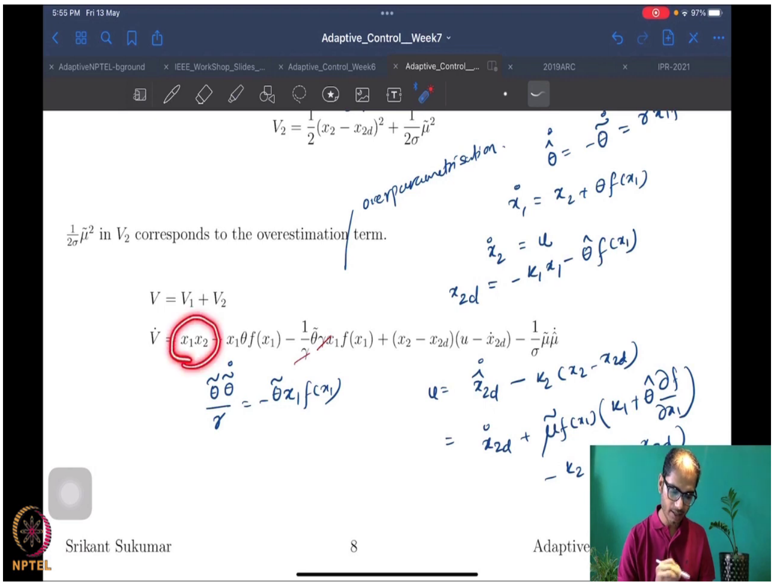

So, now, if I go to the next step and of course, I already wrote the expression for u, which is in terms of x2 hat dot, x2 d hat dot and that is this entire expression. So, what you get is first what I do is I write x2 in terms of the x2 desired. So if I write x2 in terms of the x2 desired. What do I get? Please sorry, not x2 in terms of the x2 desired, but in terms of the error.

So, basically what I will do is I will write this guy x1 x2 as x1 x2 minus x2 desired plus x1 x2 desired. So, this is just x1 x2 and this is just from x1 theta of x1 just the dynamics and this is again from the update law. And this term now, minus k2 x2 minus x2 desired is because of this term. So, now everything else is this term, this term. So, all of this so, this term gives me this term all right then let me see if I can change colors then this term gives me this term.

And this term will cancel out, because there is an x2 d dot. So, that term the first term in fact cancels out. So, this is cancels, so this term in fact cancels out. So, that is it and then I am left with this guy which is yet to be chosen which is yet to be chosen. So, this is just this is coming from this with just x2 minus x2 desired multiplied this is coming from this with just x2 minus x2 desired multiplied because of the Lyapunov function derivatives.

Now, if I look here, if I look at this x1 times x2 desired. So, what is x1 times x2 desired? Actually x2 desired is this. So, x1 times x2 desired is, if I expand this term. It is minus k1 x1 squared plus x1 theta hat f x1, x1 theta hat f x1. So I get a from x1 x2 desired, I get a minus k1 x1 squared. And this x1 theta hat f x1 cancels this guy.

Because theta minus theta tilde is minus theta hat. So I think this is fine. I hope the signs are okay. Sorry, this this, sorry that is what I am wondering. This should be a negative. Now, now this negative will cancel theta minus theta tilde x1 f x1, which is actually equal to theta hat f x1. So this is actually equal to theta hat f x1, so this cancels with this. So you get this minus k1 x1 squared left from here. So this just leaves this much. And that is it. And then this is a nice mixed term x1 and x2 minus x2 desired and then there is a nice negative quadratic term in this if, if I choose my mu cap dot, as sigma times x2 minus x2 desired f x1 k1 plus theta, del f, del x1.

So, if I make this choice, this term and this term cancel out so all I am left with is this nice negative term here, nice negative term here from this guy, and then this mixed term here, because everything else just cancels out.

And then of, then what? So, that is what is sort of written here, you see this is the update law sigma x2 minus x2 desired f x1, k1 plus theta cap del f del x1. This is exactly what is left here as you would expect, and from here to here, of course, this is sum of squares. Again, we are already used to this I hope. So this term is simply less than equal to half x1 squared plus half x2 minus x2 desired squared. This is just using a b is less than equal to a squared plus b squared. So there is the standard sum of squares.

We do this again and again. So I hope you are used to it. So once you get to this, that if k1 and k2 are greater than half, you have this. You know what a nice negative semi definiteness, only negative semi definite as of course, because now theta tilde and mu tilde also states are 2 more states. So obviously, this is not negative definite, it is only negative semi definite.

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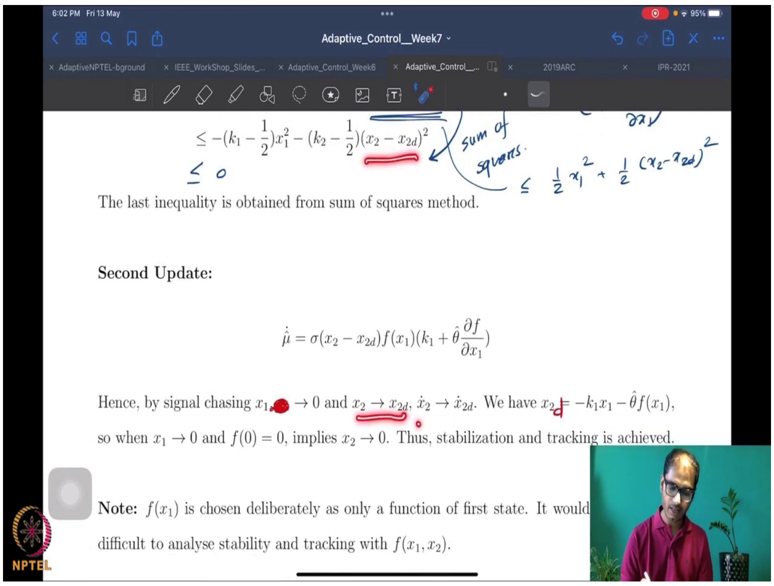

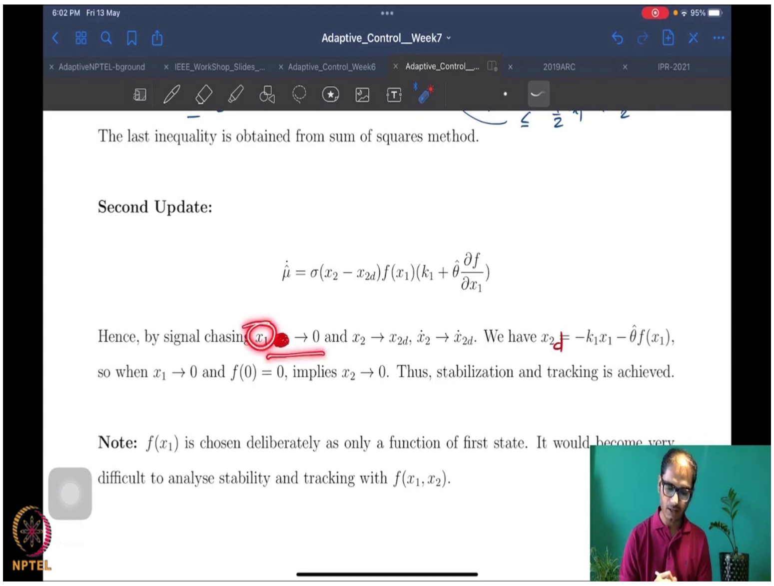

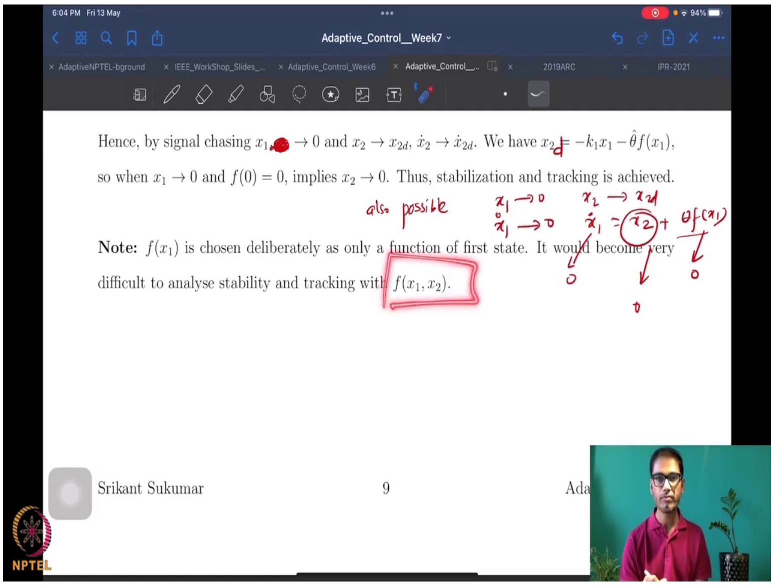

But, you know very well that I can use signal chasing and Barbalat's lemma to prove that x1 and x2, in fact, go to 0, sorry, we can prove that x1 goes to 0 and x2 goes to x2 desired. So let me let me be very careful here, let us see. Wait a second, we can prove x1 goes to 0. So, not this yet. And we can prove x2 goes to x2 desired. And, we can also prove x2 desired x2 dot goes to x2 desired dot, so, what we have is that x2 is this guy. So if x1 goes to 0, and f0 is going to 0, so this is actually x2 desired. So if x1 is going to 0 as we already proved. So I apologize, give me a second, there is some misbehavior from the pen.

So we already have x1 going to 0 directly from here by using signal chasing and Barbalat's lemma, and we have x2 going to x2 desired, and the derivative is also convergent, this is standard. So you have x1 going to 0. And we also have x2 going to x2 desired. Now we look at x2 desired more carefully. So, what happens to x2 desired and I know that as t goes to infinity x1 goes to 0. And further if I assume that f 0, 0, like before, a reasonable assumption, then I also have this term going to 0. So I have the whole thing going to 0. I have the whole thing going to 0. So this is fine.

I believe you can also show also possible via another route. So you can show x1 goes to 0, x2 goes to x2 desired and I believe you can also show x1 dot goes to 0. And from here, x1 dot is equal to x2 plus theta f x1 dot. And if x1 is going to 0, and this is going to 0, because x1 is going to 0, then this also has to go to 0. So, this is another way of going to 0, both identical, does not matter.

So in this case, one of the caveats is that the function f is deliberately chosen to be a function of the first state only if this does not happen. Doing stability analysis will be very difficult. And you can also see that you are saying that f x1, f of 0 is 0 and things like that. So if it was a function of both the states then you have to talk about x2 also being 0 and things like that. So this is a rather complicated situation, if you have a function of both states is again something that is more of a well, I mean, there will be a much more tougher exercise and research exercise and may or may not work out for all cases.

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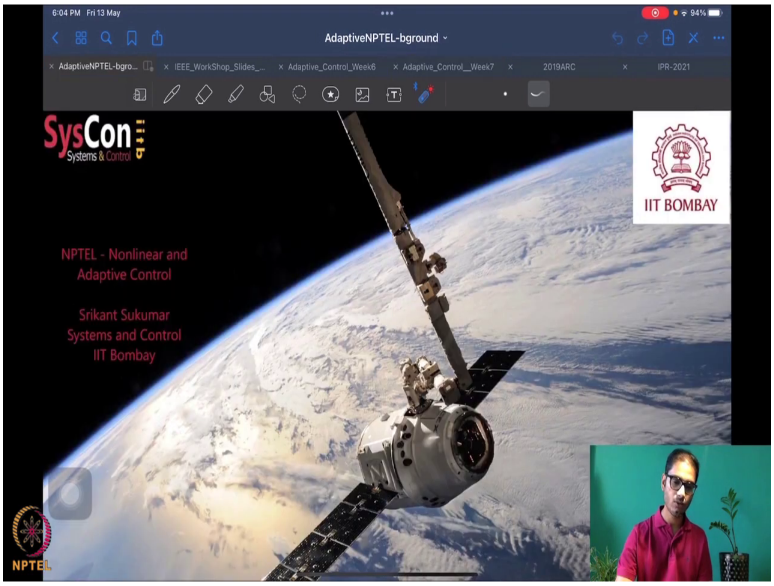

So, what did we see today? We have essentially seen how to complete the stability proofs and in fact, adaptive control design for this unmatched parameter case. This of course, turned out to be way more involved. We had to do over parameterization we had to introduce a second estimate, and in the Lyapunov function also you notice that we have 2 more 2 additional terms one due theta tilde, another due to mu tilde. So this was an over parameterization. And the analysis itself was a little bit more involved than before.

But since the problem is also more involved, this is of course reasonable. And you can see and also start to think that the Ortega type construction that worked for the matched case you cannot actually use in the unmatched case. So, that you understand that the Lyapunov function now is significantly more complicated and more involved. So, backstepping is the way that you will have to go for the more complicated cases. So, this is where we stop today. And I see you again in the next session. Thanks.